O‘ZBEKISTON MILLIY UNIVERSITETI HUZURIDAGI PEDAGOG KADRLARNI QAYTA TAYYORLASH VA ULARNING MALAKASINI OSHIRISH TARMOQ (MINTAQAVIY) MARKAZI

AMALIY MASALALARNI SONLI YECHISH

MODULI BO‘YICHA

O‘QUV-USLUBIY MAJMUA

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O‘ZBEKISTON RESPUBLIKASI
OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

OLIY TA’LIM TIZIMI PEDAGOG VA RAHBAR KADRLARINI QAYTA TAYYORLASH VA ULARNING MALAKASINI OSHIRISHNI TASHKIL ETISH BOSH ILMIY - METODIK MARKAZI

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«AMALIY MASALALARNI SONLI YECHISH»

modulining

O‘ Q U V – U S L U B I Y M A J M U A S I

Toshkent – 2024

Mazkur o‘quv-uslubiy majmua Oliy ta’lim, fan va innovatsiyalar vazirligining 2023-yil 25-avgustdagi 391-sonli buyrug’i bilan tasdiqlangan o‘quv reja va dastur asosida tayyorlandi.

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O‘quv-uslubiy majmua O‘zbekiston Milliy universiteti Kengashining qarori bilan nashrga tavsiya qilingan (2024-yil 20-yanvardagi № 4/2 -sonli bayonnomasi)

MUNDARIJA

I. ISHCHI DASTUR

Kirish

Dastur O‘zbekiston Respublikasining 2020-yil 23-sentabrda tasdiqlangan “Ta’lim to‘g‘risida”gi Qonuni, O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi PF–4947-son, 2019 yil 27 avgustdagi “Oliy ta’lim muassasalari rahbar va pedagog kadrlarining uzluksiz malakasini oshirish tizimini joriy etish to‘g‘risida”gi PF–5789-son, 2019 yil 8 oktabrdagi “O‘zbekiston Respublikasi oliy ta’lim tizimini 2030 yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g‘risida”gi PF–5847-sonli, 2019 yil 23 sentabrdagi “Oliy ta’lim muassasalari rahbar va pedagog kadrlarining malakasini oshirish tizimini yanada takomillashtirish bo‘yicha qo‘shimcha chora-tadbirlar to‘g‘risida”gi 797-sonli qarorlarida belgilangan ustuvor vazifalar mazmunidan kelib chiqqan holda tuzilgan bo‘lib, u oliy ta’lim muassasalari pedagog kadrlarining kasb mahorati hamda innovatsion kompetentligini rivojlantirish, sohaga oid ilg‘or xorijiy tajribalar, yangi bilim va malakalarni o‘zlashtirish, shuningdek amaliyotga joriy etish ko‘nikmalarini takomillashtirishni maqsad qiladi.

Ma’lumki mamlakatimiz mustaqilligi milliy ta’lim sohasida tub islohotlarni amalga oshirish uchun zamin yaratdi. Zamonaviy talablar inobatga olingan holda, oliy o‘quv yurtlarining pedagog kadrlarini qayta tayyorlash yo‘nalishlari bo‘yicha qayta tayyorlash va malaka oshirishning o‘quv dasturlarini muntazam takomillashtirib borish ishlarini tashkil etish bugungi kunning dolzarb vazifalaridan biri xisoblanadi.

Bu kursda tinglovchilar barcha fanlardan olgan bilimlarini qo‘llab fizik jarayonlar uchun matematik modellar yaratish, amaliy masalalar qo‘yish, yaratilgan matematik modellarning adekvatligini tekshirish, qo‘yilgash masala uchun yechish usullarini tanlash, chekli ayirmali sxemalar yaratish, sxemalarning turg‘unligini ta’minlash, hosil qilingan sonli tenglamalarni yechish usullarini tanlash, algoritmlar yaratish va dasturlar tuzish, dasturni sozlash, hisoblash eksperimentlarini o‘tkazish, olingan natijalarni tahlil qilish va natijalarni jadval, grafik yoki animasion ko‘rinishlarda (vizual) taqdim etish kabi ko‘nikmalarni oladi.

Bu ko‘nikmalarni olish davomida tinglovchilar barcha matematik fanlarining bir birini to‘ldirishi, hayotiy masalalarni yechishda ularning qanchalik zarurligini to‘laroq tushunib yetadilar, bu masalalarni yechishda informasion texnologiyalarning rolini va yangi texnologiyalardan foydalanish ilmiy-texnika rivojiga salmoqli ta’sir ko‘rsatishiga amin bo‘ladilar.

Modulning maqsadi va vazifalari

Oliy ta’lim muasasalari pedagog kadrlarini qayta tayyorlash va ularning malakasini oshirish kursining **maqsadi** pedagog kadrlarning innovatsion yondoshuvlar asosida o‘quv-tarbiyaviy jarayonlarni yuksak ilmiy-metodik darajada loyihalashtirish, sohadagi ilg‘or tajribalar, zamonaviy bilim va malakalarni o‘zlashtirish va amaliyotga joriy etishlari uchun zarur bo‘ladigan kasbiy bilim, ko‘nikma va malakalarini takomillashtirish, shuningdek ularning ijodiy faolligini rivojlantirishdan iborat

Kursning **vazifalariga** quyidagilar kiradi:

“**Amaliy matematika**” yo‘nalishida pedagog kadrlarning kasbiy bilim, ko‘nikma, malakalarini takomillashtirish va rivojlantirish;

- pedagoglarning ijodiy-innovatsion faollik darajasini oshirish;

-pedagog kadrlar tomonidan zamonaviy axborot-kommunikatsiya texnologiyalari, zamonaviy ta’lim va innovatsion texnologiyalar sohasidagi ilg‘or xorijiy tajribalarning o‘zlashtirilishini ta’minlash;

-o‘quv jarayonini tashkil etish va uning sifatini ta’minlash borasidagi ilg‘or xorijiy tajribalar, zamonaviy yondashuvlarni o‘zlashtirish;

“**Amaliy matematika**” yo‘nalishida qayta tayyorlash va malaka oshirish jarayonlarini fan va ishlab chiqarishdagi innovatsiyalar bilan o‘zaro integratsiyasini ta’minlash.

Modul bo‘yicha tinglovchilarning bilimi, ko‘nikmasi, malakasi va kompetensiyalariga qo‘yiladigan talablar

“**Amaliy matematika**” o‘quv fanini o‘zlashtirish jarayonida amalga oshiriladigan masalalar doirasida:

Tinglovchi:

- oshkormas sxemalarning turg‘unligini;
- absolyut va shartli turg‘un ayirmali sxemalarni;
- Simmetrik t-giperbolik sistemalar uchun Laks, Godunov, Rusanov ayirmali sxemalari va ularning turg‘unligini bilishi kerak.
- - iyerarxiya prinsipidai foydalanib, matematik modellar qurish;
- maltus va Ferxyulst-Pirl modellarini o‘zlashtirish;
- «Yirtqich-o‘lja» sistemasining o‘zaro munosabat modelini qo‘llash;
- ayirmali sxema turg‘unligini isbotlashning energetik usullaridan ulardan ***foydalana olishi kerak***;
- giperbolik sistemalar uchun chegaraviy shartlarga rioya etish;
- simmetrik t-giperbolik sistemalar uchun aralash masalarni qo‘llash;
- to‘r tenglamalar sistemasini yechishda iterasion usullardan foydalanish;
- giperbolik tenglamalarni to‘r usuli bilan yechish ***ko‘nikmalariga ega bo‘lishi kerak***.

Modulni tashkil etish va o‘tkazish bo‘yicha tavsiyalar

“Amaliy matematika” kursi ma’ruza va amaliy mashg‘ulotlar shaklida olib boriladi.

Kursni o‘qitish jarayonida ta’limning zamonaviy metodlari, axborot-kommunikasiya texnologiyalari qo‘llanilishi nazarda tutilgan:

1. ma’ruza darslarida zamonaviy kompyuter texnologiyalari yordamida prezentasion va elektron-didaktik texnologiyalardan;

2. o‘tkaziladigan amaliy mashg‘ulotlarda texnik vositalardan, ekspress-so‘rovlar, test so‘rovlari, aqliy hujum, guruhli fikrlash, kichik guruhlar bilan ishlash, kollokvium o‘tkazish, va boshqa interaktiv ta’lim usullarini qo‘llash nazarda tutiladi.

Modulning o‘quv rejadagi boshqa modullar bilan bog‘liqligi va uzviyligi

“Amaliy masalalarni sonli yechish” moduli mazmuni o‘quv rejadagi “Tabiiy jarayonini matematik modellashtirish” moduli hamda mutaxassislik o‘quv modullari bilan uzviy bog‘langan holda pedagoglarning ta’lim jarayonida kasbiy pedagogik tayyorgarlik darajasini oshirishga xizmat qiladi.

Modulning oliy ta’limdagi o‘rni

Modulni o‘zlashtirish orqali tinglovchilar amaliy masalalarning matematik modellari, ularni yechish usullari va dasturiy ta’minotlar yaratish, hisoblash tajribalarini o‘tkazish, olingan natijalarni tahlil etish, amalda qo‘llash va baholashgadoir kasbiy kompetentlikka ega bo‘ladilar.

Modul bo‘yicha soatlar taqsimoti

№	Modul mavzulari	Auditoriya uquv yuklamasi		
		Jami	jumladan	
			Nazariy	Amaiy mashg’ulot
1.	Simmetrik t- giperbolik sistemalarning asosiy tushunchalari.	6	2	4
2.	Ayirmali sxema turg‘unligini isbotlashning energetik usullari.	4	2	2
3.	Differensial tenglamalarni ayirmali tenglamalar bilan approksimasiya qilish.	4	2	2
4.	Oshkor va oshkor emas ayirmali sxemalar.	4	2	2
	Jami:	**18**	**8**	**10**

NAZARIY MASHG‘ULOTLAR MAZMUNI

1-mavzu. Simmetrik t- giperbolik sistemalarning asosiy tushunchalari.

1.1. Giperbolik sistemalarning kanonik shakli.

1.2. Giperbolik sistemalar uchun chegaraviy shartlar.

1.3. Simmetrik t-giperbolik sistemalar uchun aralash masala.

2-mavzu. Ayirmali sxema turg‘unligini isbotlashning energetik usullari.

2.1. Aprior baho. Nostatsionar gaz dinamikasi tenglamalari.

2.2. Statsionar Eyler tenglamalari. Saint-Venant tenglamalari.

2.3. Xususiy differensial tenglamalar uchun chegaraviy masalani yechishning sonli usullari.

3-mavzu. Differensial tenglamalarni ayirmali tenglamalar bilan approksimatsiya qilish.

3.1. To‘r tenglamalar sistemasini yechish.

3.2. Chegaraviy shartlarni approksimatsiya etish.

3.3. To‘r tenglamalar sistemasini yechish.

4-mavzu. Oshkor va oshkormas ayirmali sxemalar.

4.1. Oshkormas sxemalarning turg‘unligi.

4.2. Absolyut va shartli turg‘un ayirmali sxemalar.

4.3. Simmetrik t-giperbolik sistemalar uchun Laks, Godunov, Rusanov ayirmali sxemalari va ularning turg‘unligi.

AMALIY MASHG‘ULOTLAR MAZMUNI

1-amaliy mashg‘ulot. Giperbolik sistemalarning kanonik shakli. Giperbolik sistemalar uchun chegaraviy shartlar. (4 soat).

2-amaliy mashg‘ulot. Ayirmali sxema turg‘unligini isbotlashning energetik usullari. Aprior baho. Nostatsionar gaz dinamikasi tenglamalari. Statsionar Eyler tenglamalari. Saint-Venant tenglamalari. Xususiy differensial tenglamalar uchun chegaraviy masalani yechishning sonli usullari. (2 соат).

3-amaliy mashg‘ulot. Differensial tenglamalarni ayirmali tenglamalar bilan approksimatsiya qilish. To‘r tenglamalar sistemasini yechish. To‘r tenglamalar sistemasini yechishda iteratsion usullar. Giperbolik tenglamalarni to‘r usuli bilan yechish. (2 soat).

4-amaliy mashg‘ulot. Oshkor va oshkormas ayirmali sxemalar. Oshkormas sxemalarning turg‘unligi. Absolyut va shartli turg‘un ayirmali sxemalar. Simmetrik t-giperbolik sistemalar uchun Laks, Godunov, Rusanov ayirmali sxemalari va ularning turg‘unligi. (2 soat).

Amaliy mashg‘ulotlarni tashkil etish bo‘yicha ko‘rsatma va tavsiyalar

Amaliy mashg‘ulotlarda tinglovchilar o‘quv modullari doirasidagi ijodiy topshiriqlar, keyslar, o‘quv loyihalari, texnologik jarayonlar bilan bog‘liq vaziyatli masalalar asosida amaliy ishlarni bajaradilar.

Amaliy mashg‘ulotlar zamonaviy ta’lim uslublari va innovatsion texnologiyalarga asoslangan holda o‘tkaziladi. Bundan tashqari, mustaqil holda o‘quv va ilmiy adabiyotlardan, elektron resurslardan, tarqatma materiallardan foydalanish tavsiya etiladi.

Mustaqil malaka oshirishni tashkil etish bo‘yicha ko‘rsatma va tavsiyalar

Mustaqil malaka oshirish quyidagi shakllarni o‘z ichiga oladi: ochiq o‘quv mashg‘ulotlari va mahorat darslarini tashkil etish; iqtidorli va iste’dodli talabalar bilan ishlash; ilmiy konferensiyalarda ma’ruza bilan qatnashish; ilmiy jurnallarda maqolalar chop etish; ko‘rgazma va tanlovlarda ishtirok etish; ilmiy loyihalarda ishtirok etish; xalqaro (impakt-faktorli) nashrlarda maqolalar e’lon qilish; ixtiro (patent), ratsionalizatorlik takliflari, innovatsion ishlanmalarga mualliflik qilish; monografiya, mualliflik ijodiy ishlar katalogini tayyorlash va nashrdan chiqarish;

o‘quv adabiyotlari (darslik, o‘quv qo‘llanma, metodik qo‘llanma)ni tayyorlash va nashrdan chiqarish; falsafa doktori (PhD) darajasini olish uchun himoya qilingan dissertatsiyaga ilmiy rahbarlik qilish.

Pedagog kadrlarning mustaqil malaka oshirish natijalari elektron portfolio tizimida o‘z aksini topadi.

Mustaqil malaka oshirish davrida pedagoglar asosiy ish joyi bo‘yicha pedagogik amaliyotdan o‘tadilar. Pedagogik amaliyot davrida pedagog asosiy ish joyi bo‘yicha kafedraning yetakchi professor-o‘qituvchilarini 2 ta darsini kuzatadilar va tahlil qiladilar hamda kafedra a’zolari ishtirokida talabalar guruhi uchun 1 ta ochiq dars o‘tkazadilar. Ochiq dars tahlili hamda pedagog tomonidan kuzatilgan darslar xulosalari kafedraning yig‘ilishida muhokama etiladi va tegishli kafedraning bayonnomasi bilan rasmiylashtiriladi.

Shuningdek, mustaqil malaka oshirish jarayonida tinglovchi qo‘yidagi bilim va ko‘nikmalarini rivojlantirishi lozim:

− ta’lim, fan va ishlab chiqarishni integratsiyalashni tashkil etish, kadrlar buyurtmachilari va mehnat bozori ehtiyojlarini hisobga olgan holda o‘quv rejalari va fanlar dasturlarini shakllantirish;

− o‘quv mashg‘ulotlarining har xil turlari (ma’ruzalar, amaliy mashg‘ulotlar, laboratoriya mashg‘ulotlari, kurs ishlari loyihalari, malaka bo‘yicha amaliy mashg‘ulotlar)ni tashkillashtirish;

− talabalar o‘rtasida milliy mustaqillik g‘oyalari asosida ma’naviy-axloqiy va tarbiyaviy ishlarni olib borish, ta’lim jarayoni qatnashchilari bilan o‘zaro munosabatlarda etika normalari va nutq madaniyati, talabalarning bilim va ko‘nikmalarini nazorat qilishni tashkil etish va ilmiy-metodik ta’minlash, iqtidorli talabalarni qidirib topish, tanlash va ular bilan ishlash metodlarini bilish va amalda qo‘llash;

− oliy ta’limda menejment va marketing asoslarini bilish va amaliy faoliyatga tatbiq etish.

mustaqil ta’lim olish yo‘li bilan o‘z bilimlarini takomillashtirish.

O‘QITISH SHAKLLARI

– Mazkur modul bo‘yicha quyidagi o‘qitish shakllaridan foydalaniladi:

– ma’ruzalar, amaliy mashg‘ulotlar (ma’lumotlar va texnologiyalarni anglab olish, aqliy qiziqishni rivojlantirish, nazariy bilimlarni mustahkamlash);

– davra suhbatlari (ko‘rilayotgan loyiha yechimlari bo‘yicha taklif berish qobiliyatini oshirish, eshitish, idrok qilish va mantiqiy xulosalar chiqarish);

– bahs va munozaralar (loyihalar yechimi bo‘yicha dalillar va asosli argumentlarni taqdim qilish, eshitish va muammolar yechimini topish qobiliyatini rivojlantirish).

II. MODULNI O‘QITISHDA FOYDALANILADIGAN INTREFAOL TA’LIM METODLARI

“KWHL” metodi

Metodning maqsadi: Mazkur metod tinglovchilarni yangi axborotlar tizimini qabul qilishi va bilimlarni tizimlashtirishi uchun qo‘llaniladi, shuningdek, bu metod tinglovchilar uchun mavzu bo‘yicha qo‘yidagi jadvalda berilgan savollarga javob topish mashqi vazifasini belgilaydi.

Izoh. *KWHL:*
Know – nimalarni bilaman?
Want – nimani bilishni xohlayman?
How - qanday bilib olsam bo‘ladi?
Learn - nimani o‘rganib oldim?

“KWHL” metodi	
1. Nimalarni bilaman: -	***2. Nimalarni bilishni xohlayman, nimalarni bilishim kerak:*** -
3. Qanday qilib bilib va topib olaman: -	***4. Nimalarni bilib oldim:*** -

“W1H” metodi

Metodning maqsadi: Mazkur metod tinglovchilarni yangi axborotlar tizimini qabul qilishi va bilimlarni tizimlashtirishi uchun qo‘llaniladi, shuningdek, bu metod tinglovchilar uchun mavzu bo‘yicha qo‘yidagi jadvalda berilgan oltita savollarga javob topish mashqi vazifasini belgilaydi.

What?	Nima? (ta’rifi, mazmuni, nima uchun ishlatiladi)	
Where?	Qaerda (joylashgan, qaerdan olish mukin)?	
What kind?	Qanday? (parametrlari, turlari mavjud)	
When?	Qachon? (ishlatiladi)	
Why?	Nima uchun? (ishlatiladi)	
How?	Qanday qilib? (yaratiladi, saqlanadi, to‘ldiriladi, tahrirlash mumkin)	

"SWOT-tahlil" metodi

Metodning maqsadi: mavjud nazariy bilimlar va amaliy tajribalarni tahlil qilish, taqqoslash orqali muammoni hal etish yo'llarni topishga, bilimlarni mustahkamlash, takrorlash, baholashga, mustaqil, tanqidiy fikrlashni, nostandart tafakkurni shakllantirishga xizmat qiladi.

S – (strength)	• kuchli tomonlari
W – (weakness)	• zaif,kuchsiz tomonlari
O – (opportunity)	• imkoniyatlari
T – (threat)	• xavflar

2.1-rasm.

"VEER" metodi

Metodning maqsadi: Bu metod murakkab, ko'ptarmoqli, mumkin qadar, muammoli xarakteridagi mavzularni o'rganishga qaratilgan. Metodning mohiyati shundan iboratki, bunda mavzuning turli tarmoqlari bo'yicha bir xil axborot beriladi va ayni paytda, ularning har biri alohida aspektlarda muhokama etiladi. Masalan, muammo ijobiy va salbiy tomonlari, afzallik, fazilat va kamchiliklari, foyda va zararlari bo'yicha o'rganiladi. Bu interfaol metod tanqidiy, tahliliy, aniq mantiqiy fikrlashni muvaffaqiyatli rivojlantirishga hamda o'quvchilarning mustaqil g'oyalari, fikrlarini yozma va og'zaki shaklda tizimli bayon etish, himoya qilishga imkoniyat yaratadi. "Veer" metodidan ma'ruza mashg'ulotlarida individual va juftliklardagi ish shaklida, amaliy va seminar mashg'ulotlarida kichik guruhlardagi ish shaklida mavzu yuzasidan bilimlarni mustahkamlash, tahlili qilish va taqqoslash maqsadida foydalanish mumkin.

Metodni amalga oshirish tartibi:

trener-o‘qituvchi ishtirokchilarni 5-6 kishidan iborat kichik guruhlarga ajratadi;

trening maqsadi, shartlari va tartibi bilan ishtirokchilarni tanishtirgach, har bir guruhga umumiy muammoni tahlil qilinishi zarur bo‘lgan qismlari tushirilgan tarqatma materiallarni tarqatadi;

har bir guruh o‘ziga berilgan muammoni atroflicha tahlil qilib, o‘z mulohazalarini tavsiya etilayotgan sxema bo‘yicha tarqatmaga yozma bayon qiladi;

navbatdagi bosqichda barcha guruhlar o‘z taqdimotlarini o‘tkazadilar. Shundan so‘ng, trener tomonidan tahlillar umumlashtiriladi, zaruriy axborotlr bilan to‘ldiriladi va mavzu yakunlanadi.

Muammoli savol					
1-usul		**2-usul**		**3-usul**	
afzalligi	kamchiligi	afzalligi	kamchiligi	afzalligi	kamchiligi
Xulosa:					

“Keys-stadi” metodi

«Keys-stadi» - inglizcha so‘z bo‘lib, («case» – aniq vaziyat, hodisa, «stadi» – o‘rganmoq, tahlil qilmoq) aniq vaziyatlarni o‘rganish, tahlil qilish asosida o‘qitishni amalga oshirishga qaratilgan metod hisoblanadi. Mazkur metod dastlab 1921 yil Garvard universitetida amaliy vaziyatlardan iqtisodiy boshqaruv fanlarini o‘rganishda foydalanish tartibida qo‘llanilgan. Keysda ochiq axborotlardan yoki aniq voqea-hodisadan vaziyat sifatida tahlil uchun foydalanish mumkin.

“Keys metodi” ni amalga oshirish bosqichlari

Ish bosqichlari	Faoliyat shakli va mazmuni
1-bosqich: Keys va uning axborot ta’minoti bilan tanishtirish	✓ yakka tartibdagi audio-vizual ish; ✓ keys bilan tanishish (matnli, audio yoki media shaklda); ✓ axborotni umumlashtirish; ✓ axborot tahlili; ✓ muammolarni aniqlash
2-bosqich: Keysni aniqlashtirish va o‘quv topshirig‘ni belgilash	✓ individual va guruhda ishlash; ✓ muammolarni dolzarblik ierarxiyasini aniqlash; ✓ asosiy muammoli vaziyatni belgilash
3-bosqich: Keysdagi asosiy muammoni tahlil etish orqali o‘quv topshirig‘ining yechimini izlash, hal etish yo‘llarini ishlab chiqish	✓ individual va guruhda ishlash; ✓ muqobil yechim yo‘llarini ishlab chiqish; ✓ har bir yechimning imkoniyatlari va to‘siqlarni tahlil qilish; ✓ muqobil yechimlarni tanlash
4-bosqich: Keys yechimini shakllantirish va asoslash, taqdimot.	✓ yakka va guruhda ishlash; ✓ muqobil variantlarni amalda qo‘llash imkoniyatlarini asoslash; ✓ ijodiy-loyiha taqdimotini tayyorlash; ✓ yakuniy xulosa va vaziyat yechimining amaliy aspektlarini yoritish

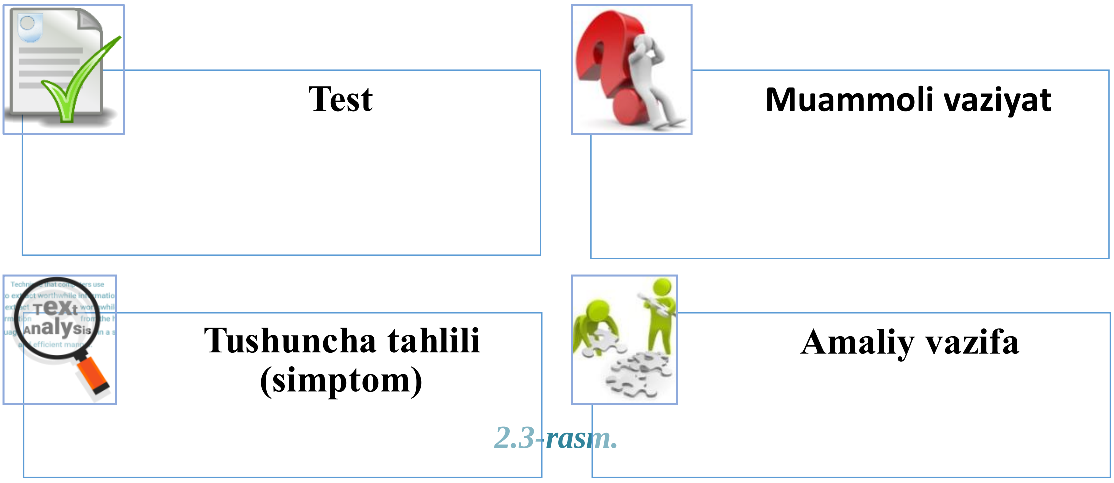
“Assesment” metodi

Metodning maqsadi: mazkur metod ta’lim oluvchilarning bilim darajasini baholash, nazorat qilish, o‘zlashtirish ko‘rsatkichi va amaliy ko‘nikmalarini tekshirishga yo‘naltirilgan. Mazkur texnika orqali ta’lim oluvchilarning bilish faoliyati turli yo‘nalishlar (test, amaliy ko‘nikmalar, muammoli vaziyatlar mashqi, qiyosiy tahlil, simptomlarni aniqlash) bo‘yicha tashhis qilinadi va baholanadi.

Metodni amalga oshirish tartibi:

“Assesment” lardan ma’ruza mashg‘ulotlarida talabalarning yoki qatnashchilarning mavjud bilim darajasini o‘rganishda, yangi ma’lumotlarni bayon qilishda, seminar, amaliy mashg‘ulotlarda esa mavzu yoki ma’lumotlarni o‘zlashtirish darajasini baholash, shuningdek, o‘z-o‘zini baholash maqsadida individual shaklda foydalanish tavsiya etiladi. Shuningdek, o‘qituvchining ijodiy yondashuvi hamda o‘quv maqsadlaridan kelib chiqib, assesmentga qo‘shimcha topshiriqlarni kiritish mumkin.

Har bir katakdagi to‘g‘ri javob 5 ball yoki 1-5 balgacha baholanishi mumkin.



2.3-rasm.

"Insert" metodi

Metodni amalga oshirish tartibi:

- o'qituvchi mashg'ulotga qadar mavzuning asosiy tushunchalari mazmuni yoritilgan matnni tarqatma yoki taqdimot ko'rinishida tayyorlaydi;
- yangi mavzu mohiyatini yorituvchi matn ta'lim oluvchilarga tarqatiladi yoki taqdimot ko'rinishida namoyish etiladi;
- ta'lim oluvchilar individual tarzda matn bilan tanishib chiqib, o'z shaxsiy qarashlarini maxsus belgilar orqali ifodalaydilar. Matn bilan ishlashda talabalar yoki qatnashchilarga quyidagi maxsus belgilardan foydalanish tavsiya etiladi:

Belgilar	Matn
"V" – tanish ma'lumot.	
"?" – mazkur ma'lumotni tushunmadim, izoh kerak.	
"+" bu ma'lumot men uchun yangilik.	
"– " bu fikr yoki mazkur ma'lumotga qarshiman?	

Belgilangan vaqt yakunlangach, ta'lim oluvchilar uchun notanish va tushunarsiz bo'lgan ma'lumotlar o'qituvchi tomonidan tahlil qilinib, izohlanadi, ularning mohiyati to'liq yoritiladi. Savollarga javob beriladi va mashg'ulot yakunlanadi.

III. NAZARIY MATERIALLAR

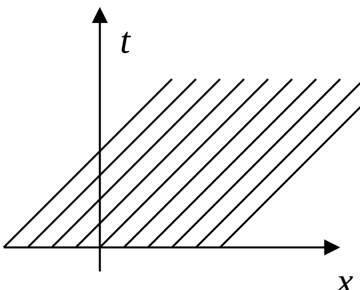
1mavzu: Simmetrik t- giperbolik sistemalarning asosiy tushunchalari

Reja

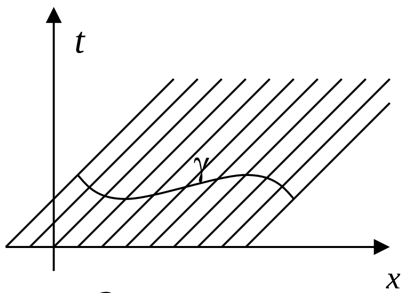
1.1. Giperbolik sistemalarning kanonik shakli.
1.2. Giperbolik sistemalar uchun chegaraviy shartlar.
1.3. Simmetrik t-giperbolik sistemalar uchun aralash masala.
1.4. Giperbolik sistemalar uchun chegaraviy shartlar
1.5. Simmetrik t-giperbolik sistemalar uchun aralash masala.

Matematik fizika kursidan xususiy hosilali differensial tenglamalar uchun qo‘yiladigan masalalarning xos namunalari bilan tanishmiz. Dastlab, eng sodda xususiy hosilali $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$ tenglamaga to‘xtalamiz. Uning umumiy yechimini olish uchun oddiy differensial tenglamalar kursidan ma’lum bo‘lgan yechimni qurish usulini amalga oshiramiz. (x, t) tekisligida shunday to‘g‘ri chiziqlarni quramizki, ular bo‘ylab $\frac{dx}{dt}=1$

tenglik o‘rinli bo‘lsin (1- rasm). Bunday to‘g‘ri chiziqlarning (har birining) tenglamasi $x - t = const$ ko‘rinishida ifodalanishi mumkin. Ushbu to‘g‘ri chiziqlarning har biri uchun *const* qiymati o‘zgarmas bo‘ladi. O‘zgarmaslarning qiymatlari bu to‘g‘ri chiziqlarni nomerlab chiqadi desak bo‘ladi. $x–t=c$ tenglamadagi o‘zgarmas c soni, ushbu tenglama orqali beriladigan to‘g‘ri chiziqlar oilasiga mansub bo‘lgan to‘g‘ri chiziqning tartib raqami deb aytamiz.



1-rasm.



2 -rasm.

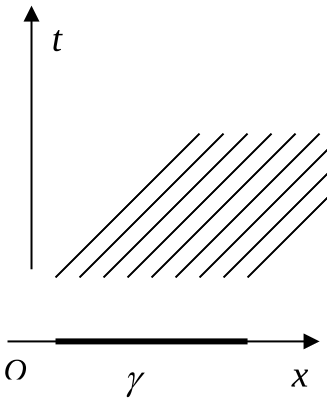
Biror $u(x, t)$ funksiyani qaraymiz va uning $\frac{du}{dt}$ hosilasini x-t=c to‘g‘ri chiziq bo‘ylab hisoblaymiz. Bunda $u(x, t)$ funksiyani differensiallanuvchi deb faraz qilishimiz zarurligi ayondir. «Differensiallanuvchi» so‘zining o‘rniga, «silliq» so‘zini ishlatamiz. Yana ham aniqroq bo‘lishi uchun «silliq» so‘zi qaralayotgan funksiya, biz o‘tkazmoqchi bo‘lgan tavtishlar yoki keltirib chiqarishlarning o‘rinli bo‘lishi uchun zarur bo‘ladigan tartibgacha hosilaga (hatto uzluksiz hosilaga) ega ekanligini anglatadi. Ushbu atamadan keyinchalik tez-tez foydalanamiz. Shunday qilib, $\frac{du}{dt}$ hosilani $\frac{dx}{dt}=1$ to‘g‘ri chiziq bo‘ylab hisoblaymiz: $\frac{du}{dt}=\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}\frac{dx}{dt}=\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}$

Ushbu hosila uchun keltirib chiqarilgan formulasidan ko‘rinib turibdiki, $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$ tenglama $\frac{dx}{dt}=1$ to‘g‘ri chiziqlarning har biri bo‘ylab $u(x,\ t)$ funksiyaning o‘zgarmasligini anglatadi. Albatta, bu o‘zgarmas har xil to‘g‘ri chiziqlarda har xil bo‘lishi mumkin. Shunday qilib, $u(x,\ t)$ funksiyaning $(x,\ t)$ nuqtadagi qiymati faqatgina nuqta yotuvchi to‘g‘ri chiziq tartib raqamiga bog‘liq bo‘ladi, ya’ni $u(x,t)=f(x-t)$ bo‘ladi (x-t ifodaning qiymati to‘g‘ri chiziqning tartib raqami deb ataladi).

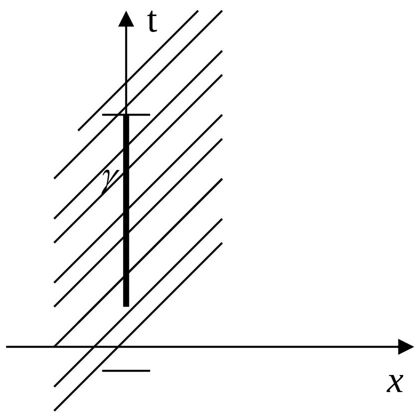
u(x,t) funksiyaning $\frac{\partial u}{\partial t},\ \frac{\partial u}{\partial x}$ hosilalari mavjud bo‘lishi uchun $f(\xi)$ funksiya differensiallanuvchi bo‘lishi talab etiladi. Bunda $\frac{\partial u}{\partial t}=-f'(x-t),\ \frac{\partial u}{\partial x}=f'(x-t)$ bo‘ladi.

Bu yerdan esa ixtiyoriy silliq f funksiyasi $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$ tenglamaning yechimini berishi yaqqol ko‘rinib turibdi. $u=f(x-t)$ formula ushbu tenglamaning umumiy yechimini beradi deb aytamiz.

Endi bu tenglama uchun qo‘yilishi mumkin bo‘lgan masalalarni muhokama qilishga o‘tamiz. Bunda masala deganda, yechimlar majmuasidan yagona yechimni ajratish uchun zarur bo‘ladigan qo‘shimcha shartlar majmuasini tushunamiz. (x,t) tekisligida x-t=const to‘g‘ri chiziqlar yo‘lakchasini qaraymiz. 2- rasmda x-t= *const* to‘g‘ri chiziqlarning har biri bilan faqat bir nuqtada kesishuvchi biror γ egri chiziqni tasvirladik. γ egri chiziq $x=\xi(s),\quad t=\tau(s)$ parametrik ko‘rinishda berilgan bo‘lsin va bu egri chiziq bo‘ylab $\varphi=\varphi(s)$ funksiyasi berilgan bo‘lsin. To‘g‘ri chiziqlarda $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$ tenglamani qanoatlantiruvchi u=u(x,t) funksiyani shunday aniqlashimiz mumkinki, y γ egri chiziq nuqtalarida $\varphi=\varphi(s)$: $u\big|_{\gamma}=\varphi$ berilgan qiymatlarni qabul qilsin. Haqiqatdan ham, yechim *u=f(x-t)* ko‘rinishda bo‘lishi kerak. f funksiyaning ko‘rinishini quyidagi tarzda aniqlashimiz mumkin. Har bir x-t ifodaning qiymati uchun tenglamadan s kattalikni topamiz. Bu s nuqtasi x-t=*const* to‘g‘ri chiziq bilan egri chiziqning kesishish nuqtasiga mos keladi va shartimiz bo‘yicha yagona bo‘ladi. Bundan keyin $f(x-t)=\varphi(s)$ deb olamiz. Agar $\xi(s),\ \ \tau(s),\ \ \varphi(s)$ silliq funksiyalar bo‘lsa ($\xi'(s)-\tau'(s)\neq 0$), u holda biz ko‘rgan *f(x-t)* funksiya ham silliq bo‘lishini va demak u o‘rganilayotgan tenglamaning yechimi bo‘lishini isbotlash mumkin.



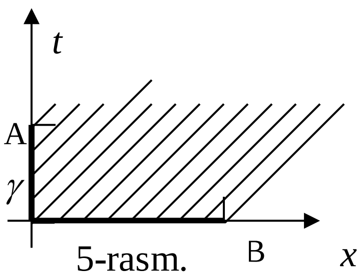
3- rasm.



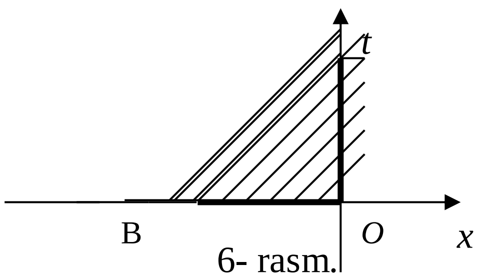
4 - rasm.

Hozir keltiriladigan ko‘rgazmali tasavvurlar, matematik fizika tenglamalarining muhim bir sinfining nazariyasidagi asosiy dalillarni tezroq va soddaroq bayon etish uchun zarur.

Dastlabki masalaga qaytamiz. γ egri chiziq sifatida 3, 4 rasmda ko‘rsatilganidek, x o‘qida yotuvchi kesmani yoki t o‘qida yotuvchi biror kesmani tanlashimiz mumkin. Hattoki γ egri chiziq sifatida bir-biri bilan tutashgan x o‘qi va t o‘qida yotuvchi kesmalarni (5- rasm) ham olish mumkin. Bunda shuni ta’kidlash lozimki, AO va OB kesmada berilgan $\varphi(s)$ funksiya qismlari, x=t to‘g‘ri chiziqda differensiallanuvchi va O nuqtadan o‘tuvchi f(x-t) funksiyani aniqlab berishini ta’minlashimiz zarur.



5-rasm.



6- rasm.

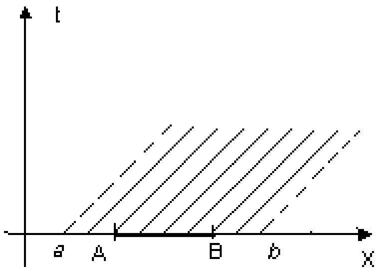
Savol. Buni ta’minlash uchun, funksiyasining elementlari AO va OV chiziqlar ustida qanday shartlarni bajarishi lozim?

6 rasmda tasvirlangan *x,t* o‘qlarining kesmalarini masalaning qo‘yilishidagi γ egri chiziq sifatida ishlatish mumkin emas, chunki AO kesma bilan kesishuvchi *x-t=const* to‘g‘ri chiziqlar, BO kesma bilan ham kesishadi. Har bir *x-t=const* to‘g‘ri chizig‘i buylab *u*(*x*,*t*) funksiyaning qiymati o‘zgarmaydi, natijada φ(s) funksiyaning qiymatini OA kesmada ixtiyoriy berish mumkin emas. φ(s) funksiyaning OA kesmadagi qiymatlari, uning qiymatlarini VO kesmada berilgandan so‘ng, bir

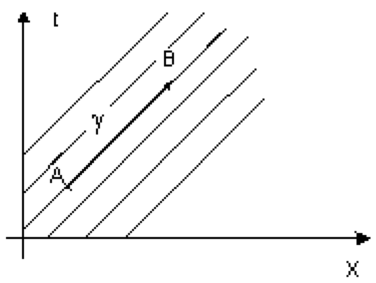
qiymatli aniqlanadi.

Endi yechimning yagonaligi haqidagi savolga to‘xtalamiz. Faraz qilamiz $\varphi(s)$ funksiyaning qiymati x o‘qining AB kesmasida berilgan bo‘lsin. Bu holatda yechim AV kesma bilan kesuvchi *x-t= const* to‘g‘ri chiziqlar hosil qilgan yo‘lakcha ichida bir qiymatli aniqlanadi. Agar $\varphi(s)$ funksiyani katta *ab* kesmaga silliq ravishda davom ettirsak
(7 rasm), u holda biz chegaralari shtrixlar bilan belgilangan kengroq yo‘lakchada yechimni qurib bilamiz. $\varphi(s)$ funksiyani bunday davom ettirish usullari ko‘p bo‘lganligi sababli, yechim ham AB kesmada, $\varphi(s)$ funksiyani berish bilan kengroq yo‘lakchada bir qiymatli aniqlanmaydi.

AB bilan kesishuvchi *x-t=const* to‘g‘ri chiziqlar yo‘lakchasi yagonalik sohasi deyiladi. Yana bir holatni o‘rganib chiqamiz. γ egri chiziq sifatida *x-t=const* to‘g‘ri chiziqlarning birida yotuvchi AB kesmani olamiz. Masalan AB kesmasi *x-t=0* to‘g‘ri chizig‘i ustida yotsin (8 rasm).



7-rasm.



8-rasm.

Bu holda $u|_\gamma = \varphi(s)$ shartdagi $\varphi(s)$ funksiyani ixtiyoriy berish mumkin emas, chunki kesma bo‘ylab hosila $\frac{du}{dt} = 0$, bu yerdan esa $\varphi(s)$ funksiyaning o‘zgarmasligi kelib chiqadi. Aks xolda

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0, \\ u|_\gamma = \varphi(s) \end{cases}$$

masala hech qanday yechimga ega bo‘lmaydi. Agar biz $\varphi(s) = \varphi_0 = const$ deb tanlasak, u holda masala $u = f(x-t)$ yechimga ega bo‘ladi, bu yerda $f(\xi)$ funksiya faqat $f(0)=\varphi_0$ shartga bo‘ysunadi, qolgan to‘g‘ri chiziqlar ustida ixtiyoriy bo‘ladi. Bu holda yagonalik sohasi bitta *x-t*=0 to‘g‘ri chiziqdan iborat bo‘ladi.

Biz, qo‘shimcha shartlarni berish mumkin bo‘lgan egri chiziqlarni ixtiyoriy tanlash mumkin emasligini ko‘rdik. *x-t=const* to‘g‘ri chiziqlarga nisbatan egri chiziqning joylashishini e’tiborga olishimiz zarur. Bu to‘g‘ri chiziqlar $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ tenglamaning *xarakteristikalari* deb ataladi. Hozircha biz xarakteristika muhim tushunchasiga biror umumiy ta’rif bermaymiz. Bunday ta’rif keyingi paragrafda beriladi.

Keyinchalik, odatda γ egri chiziq sifatida x o‘qi kesmasini tanlab va faqat $t \geq 0$ vaqt uchun bu kesmaga tayanadigan xarakteristik yo‘lakchada $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$ tenglama yechimi izlanadi. Bu holda $u|_{\gamma}=\varphi(s)$ qo‘shimcha shartni boshlang‘ich shart yoki boshlang‘ich berilganlar (qiymatlar) deb aytiladi. $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$ tenglama uchun barcha aytilganlarni $\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0$ tenglama uchun ham deyarli so‘zma-so‘z takrorlanishi mumkinligini qayd qilamiz. Ushbu holatda *x-t=const* to‘g‘ri chiziqlar rolini $\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0$ tenglamaning xarakteristikalari deb ataladigan *x-at= const* to‘g‘ri chiziqlar o‘ynagani sababli uning umumiy yechimi $u=f(x-at)$ ko‘rinishda yoziladi.

Endi ikkita bog‘liqmas (erkin) tenglamalardan iborat

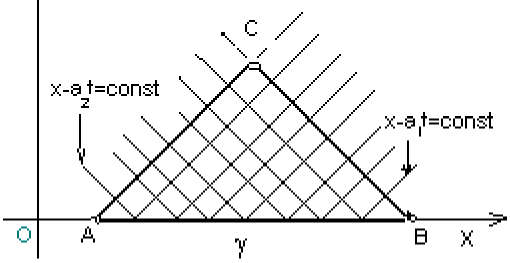
$$\begin{cases} \frac{\partial u_1}{\partial t}+a_1\frac{\partial u_1}{\partial x}=0 \\ \frac{\partial u_2}{\partial t}+a_2\frac{\partial u_2}{\partial x}=0 \end{cases}$$

sistemaning murakkabroq misolini tavtish qilamiz.

Sistema birinchi tenglamasining yechimi $u_1=f(x-a_1t)$, ikkinchisining yechimi $u_2=g(x-a_2t)$ ko‘rinishda bo‘ladi. Sistemamiz uchun x o‘qkishning AB kesmasida (ya’ni t=0 da) boshlang‘ich qiymatlarni beramiz. AB kesmani avvalgidek γ bilan belgilaymiz. $u_1|_{\gamma}=\varphi(x),\ \ u_2|_{\gamma}=\psi(x)$.

9- rasmda $u_1(x,t)$ va $u_2(x,t)$ qiymatlarini aniqlab bo‘ladigan (x,t) tekislikning $(t \geq 0)$ yarim tekisligidagi yarim yo‘lakchalari tasvirlangan. Ko‘rgazmali bo‘lishi uchun biz a_1, a_2 koeffitsiyentlarni turli $(a_1>0,\ a_2<0)$ иshoralar bilan oldik.

Sistema yechimlari haqida faqat AB kesmaga tayanuvchi ikkala xarakteristik yo‘lakchaning kesishidan (to‘plam nazariyasi ma’nosida) iborat ABC uchburchak ichida gapirish ma’noga ega, chunki faqat ushbu uchburchak ichida yechim bir qiymatli aniqlanadi.



9-rasm.

Tabiiyki sistemaning $x-a_1t=const$, $x\text{-}a_2t=const$ xarakteristikalari bilan chegaralangan ABC uchburchak xarakteristik uchburchak deb ataladi. Hozir biz ko‘rgan sistema misoli g‘ayri tabiiy ko‘rinishi mumkin. Shuning uchun birinchi qarashda murakkab ko‘ringan, yuqorida o‘rganilgan holga keltirilishi mumkin bo‘lgan sistemani namoyish qilamiz:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0} \dfrac{\partial p}{\partial x} = 0 \\ \dfrac{\partial p}{\partial t} + \rho_0 c_0^2 \dfrac{\partial u}{\partial x} = 0 \end{cases} \qquad (1)$$

Bu sistema tinchlangan muhitda yassi tovush to‘lqinlarini (kichik qo‘zg‘alishlarning) tarqalishini tavsiflaydi. Bu yerda u – qo‘zg‘atilgan muhit tezligi, p – ushbu muhit bosimi. O‘zgarmas ρ_0, c_0 tinchlangan muhit xossalari bilan bog‘liq, ρ_0- uning zichligi, c_0 esa siqilishni xarakterlovchi o‘zgarmas. (1) tenglamalar sistemasi akustika tenglamalar sistemasi deb ham ataladi. Bu tenglamalarning keltirib chiqarilishini fizika kursida yoki tutash muhitlar mexanikasidan topish mumkin. Biz hozir tovush to‘lqinlarini tavsiflovchi sistemani sodda almashtirish va o‘zgaruvchilarni almashtirish bilan yuqorida keltirilgan oddiy ko‘rinishdagi sistemaga keltirish mumkinligini ko‘rsatamiz. Shu maqsadda, sistemaning ikkinchi tenglamasini $\dfrac{1}{\rho_0 c_0^2}$ ga ko‘paytiramiz. Hosil qilingan $\dfrac{\partial \frac{p}{\rho_0 c_0}}{\partial t} + c_0 \dfrac{\partial u}{\partial x} = 0$ tenglamani, birinchi $\dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0} \dfrac{\partial p}{\partial x} = 0$ tenglamaga qo‘shamiz. Natijada $\dfrac{\partial (u + \frac{p}{\rho_0 c_0})}{\partial t} + c_0 \dfrac{\partial (u + \frac{p}{\rho_0 c_0})}{\partial x} = 0$ tenglamani hosil qilamiz. Agar yuqoridagi amallarda qo‘shishni ayirish bilan almashtirsak, u holda shunga o‘xshash boshqa tenglama hosil bo‘ladi:

$$\frac{\partial (u - \frac{p}{\rho_0 c_0})}{\partial t} - c_0 \frac{\partial (u - \frac{p}{\rho_0 c_0})}{\partial x} = 0$$

Endi

$$\begin{cases} \dfrac{\partial u_1}{\partial t} + c_0 \dfrac{\partial u_1}{\partial x} = 0 \\ \dfrac{\partial u_2}{\partial t} - c_0 \dfrac{\partial u_2}{\partial x} = 0 \end{cases}$$

sistema ko‘rinishiga kelish uchun,

$$u + \frac{p}{\rho_0 c_0} = u_1, \; u - \frac{p}{\rho_0 c_0} = u_2$$

belgilashlarni kiritish kifoya.

Ma’lumki, bunday sistemaning umumiy yechimi quyidagi ko‘rinishga ega:

$$u_1 = f(x - c_0 t), \quad u_2 = g(x + c_0 t).$$

u_1, u_2 ni u, p orqali ifodalab

$$u+\frac{p}{\rho_0 c_0}=f(x-c_0t),\quad u-\frac{p}{\rho_0 c_0}=g(x+c_0t)$$

yoki

$$u=\frac{f(x-c_0t)+g(x+c_0t)}{2},\quad p=\rho_0 c_0\frac{f(x-c_0t)-g(x+c_0t)}{2}$$

ifodalarni olamiz. Bu formulalar tovush tarqalishi tenglamalarining umumiy yechimini namoyish qiladi.

Har biri faqatgina bitta erkli funksiyaga bog‘liq bo‘lgan bir nechta birinchi tartibli erkli tenglamalar sistemasiga keltirish mumkin bulgan sistemalar giperbolik tenglamalar deb ataluvchi sinfga tegishli bo‘ladi. Kelgusida bunday sistemalar mukammalroq o‘rganiladi. Faraz qilamiz $t=0$ boshlang‘ich vaqtda va biror $x_1<x<x_2$ intervalda p bosimning taqsimoti va u tezlik ma’lum bo‘lsin. Yuqorida ko‘rganimizdek, ushbu boshlang‘ich berilganlar yechimni o‘zining asosi bilan (x_1, x_2) intervalga tayanuvchi xarakteristik uchburchakda bir qiymatli tarzda aniqlaydi. Bu uchburchak $t>0$, $x\text{-}c_0t>x_1$, $x_0+c_0t<x_2$ tengsizliklar bilan aniqlanadi.

$u\pm\frac{p}{\rho_0 c_0}$ kattaliklar nemis matematigi Riman nomi bilan riman invariantlari deb ataladi. $u+\frac{p}{\rho_0 c_0}=f(x-c_0t)$ formula ushbu riman invarianti shaklini o‘zgartirmasdan c_0 tezlik bilan o‘ng tomonga ko‘chirishini ko‘rsatadi. Ushbu c_0 kattalikni qo‘zg‘atilgan tovush to‘lg‘inlari tarqalishining tezligi yoki qisqacha tovush tezligi deb atalishiga asos bo‘ladi. Shunga o‘xshash $u-\frac{p}{\rho_0 c_0}=g(x+c_0t)$ formula boshqa riman invariantini shaklini o‘zgartirmasdan c_0 tezlik bilan chap tomonga qarab ko‘chirishni ko‘rsatadi.

f, g funksiyalarni shunday tanlaymizki, yuqordagi formulalar yordamida olingan yechimlar $x_1\le x\le x_2$ kesmada berilgan $u(x,0)=u|_{t=0}=\varphi(x)$, $p(x,0)=p|_{t=0}=\psi(x)$ boshlang‘ich qiymatlarni qanoatlantirsin. Shu maqsadda

$$\varphi(x)=\frac{f(x)+g(x)}{2},\quad \frac{\psi(x)}{\rho_0 c_0}=\frac{f(x)-g(x)}{2}$$

deb olish yetarlidir. Bu yerdan

$$f(z)=\varphi(z)+\frac{\psi(z)}{\rho_0 c_0},\quad g(z)=\varphi(z)-\frac{\psi(z)}{\rho_0 c_0},$$

$$\begin{cases} u+\dfrac{p}{\rho_0 c_0}=\varphi(x-c_0t)+\dfrac{\psi(x-c_0t)}{\rho_0 c_0} \\ u-\dfrac{p}{\rho_0 c_0}=\varphi(x+c_0t)-\dfrac{\psi(x+c_0t)}{\rho_0 c_0} \end{cases}$$

Endi akustika tenglamasi uchun Koshi masalasi deb ataluvchi masalaning yechimini beruvchi formulani olish qiyin emas:

$$u=\frac{\varphi(x-c_0t)+\varphi(x+c_0t)}{2}+\frac{\psi(x-c_0t)-\psi(x+c_0t)}{2\rho_0c_0}$$

$$p=\frac{\psi(x-c_0t)+\psi(x+c_0t)}{2}+\rho_0c_0\frac{\varphi(x-c_0t)-\varphi(x+c_0t)}{2}$$

(1) sistema uchun Koshi masalasi quyidagicha qo‘yiladi: (1) sistemaning berilgan $u|_{t=0}=\varphi(x),\quad p|_{t=0}=\psi(x)$ boshlang‘ich shartlarini qanoatlantiruvchi yechimini topish talab qilinadi. Yuqorida keltirilgan mulohazalar Koshi masalasi yechimining xarakteristik uchburchak ichida mavjudligi, hamda yagonaligini isbotlashga imkon beradi. Ko‘p hollarda birinchi tartibli

$$\begin{cases}\dfrac{\partial u}{\partial t}+\dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x}=0\\[2mm]\dfrac{\partial p}{\partial t}+\rho_0c_0^2\dfrac{\partial u}{\partial x}=0\end{cases}$$

sistema o‘rniga *p* bosim uchun sistemaning birinchi tenglamasini *x* bo‘yicha, ikkinchisini *t* bo‘yicha differensiallab, so‘ngra ulardan $\frac{\partial^2 u}{\partial x\partial t}$ **aralash hosilani yo‘qotish bilan hosil bo‘ladigan ikkinchi tartibli** $\frac{\partial^2 p}{\partial t^2}-c_0^2\frac{\partial^2 p}{\partial x^2}=0$ **tenglama qaraladi.**

Bu ikkinchi tartibli tenglama odatda torning kichik tebranishlari tenglamasi deb ataladi. Tor, tortilgan ip tebranishini tavsiflovchi tenglama ham shunday ko‘rinishda bo‘ladi. Aynan tor tebranishini tadqiq qilish jarayonida u dastlab matematik tadqiqotlarda paydo bo‘ldi.

Bu tenglama quyidagi ko‘rinishda yozilishi mumkin.

$$\left(\frac{\partial}{\partial t}-c_0\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}+c_0\frac{\partial}{\partial x}\right)p=0$$

$\frac{\partial p}{\partial t}+c_0\frac{\partial p}{\partial x}$ ifodani q orkali ifodalab, bu tenglamaga ekvivalent birinchi tartibli sistemaga kelamiz.

$$\begin{cases}\dfrac{\partial p}{\partial t}+c_0\dfrac{\partial p}{\partial x}=q\\[2mm]\dfrac{\partial q}{\partial t}-c_0\dfrac{\partial q}{\partial x}=0\end{cases}$$

Sistema tenglamalaridan ikkinchisi umumiy yechimga ega. (Keyinchalik ixtiyoriy silliq va G funksiyaning biror boshqa g funksiyaning hosilasi orqali belgilangan ikkinchi ko‘rinishi bizga qulay bo‘ladi) p uchun $\frac{\partial p}{\partial t}+c_0\frac{\partial p}{\partial x}=2c_0g'(x+c_0t)$ tenglamasini uning umumiy yechimini

$p-g(x+c_0t)=f(x-c_0t),$ $\qquad p(x,t)=f(x\text{-}c_0t)+g(x+c_0t)$ ko‘rinishda yozish imkonini beruvchi

$$\frac{\partial[p-g(x+c_0t)]}{\partial t}+c_0\frac{\partial[p-g(x+c_0t)]}{\partial x}=0$$

ko‘rinishda yozish mumkin. Umumiy yechim uchun yozilgan formulani birinchi bo‘lib (1747yil) Dalamber topgan. 1748 yilda Eyler *f, g* funksiyalarni $p(x,t)$, $p_t(x,t)$ funksiyalarning $t=0$ dagi

$$p(x,0)=\varphi(x),\ \mathrm{p_t}(x,0)=\psi(x) \qquad (2)$$

qiymatlari orqali ifodaladi. Bu uni Dalamber formulasi deb ham ataladigan (Dalamber uni qonuniy emas deb hisoblashiga qaramasdan)

$$p(x,t)=\frac{\varphi(x+c_0t)+\varphi(x-c_0t)}{2}+\frac{1}{2c_0}\int_{x-c_0t}^{x+c_0t}\psi(\xi)d\xi$$

formulaga olib keldi.

Biroq atama qabul qilindi va biz uni qo‘llab quvvatladik. Bu formula

$$\frac{\partial^2 p}{\partial t^2}-c^2{}_0\frac{\partial^2 p}{\partial x^2}=0$$

tenglama uchun Koshi masalasining yechimini beradi. Ushbu ikkinchi tartibli tenglama uchun Koshi masalasi quyidagicha qo‘yiladi: (2) boshlang‘ich qiymatlarni qanoatlantiruvchi yechimlarni topish talab etiladi. Koshi masalasi yechimi formulasini hosil qilish uchun, umumiy yechimning $p(x,t)=f(x-c_0t)+g(x+c_0t)$ ko‘rinishida *f, g* funksiyalarni

$$f(x)+g(x)=p(x,0)=\varphi(x)$$
$$-cf'(x)+c_0g'(x)=p_t(x,0)=\psi(x)$$

shartlardan aniqlashimiz kerak. Birinchi tenglikni differensiallab, ikkinchisi bilan birgalikda yechib funksiyalarning qiymatlarini topamiz:

$$f'(x)=\frac{1}{2}\varphi^1(x)-\frac{1}{2c_0}\psi(x),\ \ g'(x)=\frac{1}{2}\varphi^1(x)+\frac{1}{2c_0}\psi(x).$$

Bu tengliklarni integrallaymiz:

$$f(x)=\frac{1}{2}\varphi(x)-\frac{1}{2c_0}\int_{x_0}^{x}\psi(\xi)d\xi+a,\ \ g(x)=\frac{1}{2}\varphi(x)+\frac{1}{2c_0}\int_{x_0}^{x}\psi(\xi)d\xi+b$$

Bu yerda x_0 – boshlang‘ich qiymatlar beriladigan sohadagi ixtiyoriy nuqta, *a* ва *b* o‘zaro bog‘liq bo‘lmagan o‘zgarmaslar. $f(x)+g(x)=\varphi(x)$ tenglikdan *b=-a* tenglikni hosil qilamiz. Demak:

$$p(x,t)=f(x-c_0t)+g(x+c_0t)=\left[\frac{\varphi(x-c_0t)}{2}-\frac{1}{2c_0}\int_{x_0}^{x-c_0t}\varphi(\xi)d\xi+a\right]+$$

$$+\left[\frac{\varphi(x+c_0t}{2}+\frac{1}{2c_0}\int_{x_0}^{x+c_0t}\varphi(\xi)d\xi-a\right]=\frac{\varphi(x+c_0t)+\varphi(x-c_0t)}{2}+\frac{1}{2c_0}\int_{x-c_0t}^{x+c_0t}\Psi(\xi)d\xi$$

Shunday qilib

Koshi masalasi yechimi formulasi isbotlandi. (1) sistema uchun Koshi masalasi yechimining mavjudligi va yagonaligini yechimning oshkor formulalarini keltirib chiqarish bilan ko‘rsatgan edik. Biroq odatda yagonalik teoremasini isbotlash uchun energiyaning saqlanish qonuni bilan aloqador tushunchalardan foydalaniladi. Ushbu usul yordamida yagonalik teoremasi isbotini keltiramiz.

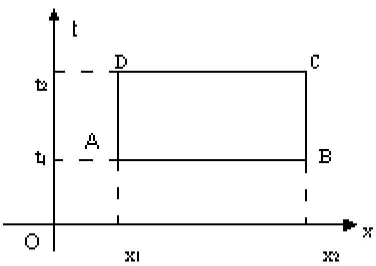
$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} = 0 \\ \dfrac{\partial p}{\partial t} + \rho_0 c_0^2 \dfrac{\partial u}{\partial x} = 0 \end{cases}$$

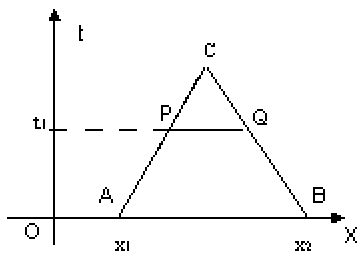
tenglamalar sistemasidan birinchisini $\rho_0 u$ ko‘paytuvchiga, ikkinchisini $\dfrac{p}{\rho_0 c_0^2}$ ko‘paytuvchiga ko‘paytiramiz. Natijalarni qo‘shib

$$\frac{\partial\left[\rho_0\left(\dfrac{u^2}{2} + \dfrac{p^2}{2\rho_0^2 c_0^2}\right)\right]}{\partial t} + \frac{\partial pu}{\partial x} = 0$$

ayniyatga kelamiz. Bu ayniyatdan ixtiyoriy bo‘lakli silliq yopiq kontur bo‘yicha

$$\oint -\left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2}\right)dx + pudt = 0$$





ekanligi kelib chikadi.

10-rasm. 11-rasm.

Bu integral tenglik tovush tarqalishi tenglamalarining silliq yechimlari uchun energiyaning saqlanish qonuni deyiladi. «Energiyaning saqlanish qonuni» atamasini tushuntirish uchun 10- rasmda tasvirlangan AVSDA to‘g‘ri burchakli kontur integral ayniyatni qo‘llaymiz va quyidagi tenglikni hosil qilamiz:

$$\int_D^C \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2}\right)dx = \int_A^B \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2}\right)dx + \int_A^D pudt - \int_B^C pudt.$$

Bu tenglikdagi $\int_A^B \rho_0 \frac{u^2}{2} dx$ t_0 - boshlang‘ich vaqtda $x_1<x<x_2$ intervalda joylashgan gazning kinetik energiyasini tasvirlaydi; $\int_A^B \frac{\rho^2}{2\rho_0 c_0^2} dx$ - ushbu gazning siqilish potensial energiyasi.

$\int_C^D \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx$ integral t_1 vaqtdagi to‘liq energiya.

$\int_A^D pudt - \int_B^C pudt$ ayirma (t_0, t_1) vaqt intervalida gaz ustida bajargan ishni namoyish qiladi. Bu mulohazalardan keyin integral ayniyatni energiyaning saqlanish qonuni (yoki energiya integrali) deb atalishi tushunarli bo‘lishi kerak. Endi energiyaning saqlanish qonunidan yagonalik teoremasini isbotlashda qanday foydalanish mumkinligini ko‘rsatamiz. $t = 0$ da x o‘qining AB $(x_1 \le x \le x_2)$ kesmasida $u|_{t=0} = \varphi(x), \quad p|_{t=0} = \psi(x)$ boshlang‘ich qiymatlarni beramiz. Yechimning yagonaligini chap va o‘ng tomondan $AC \quad (x - c_0 t = x_1)$ va $BC \quad (x + c_0 t = x_2)$ xarakteristikalar bilan chegaralangan, AVS xarakteristik uchburchak ichida (11-rasm) isbotlaymiz.

Bu uchburchakni $t=t_1$ to‘g‘ri chiziqning PQ kesmasi bilan kesamiz, so‘ngra ABQP konturga energiya saqlanish qonunining integral shaklini qo‘llaymiz:

$$\int_P^Q \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx = \int_A^B \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx -$$

$$- \int_A^P \left[(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \quad) dx - pudt \right] + \int_B^Q \left[\left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx - pudt \right].$$

$\int_A^p \left[\left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx - pudt \right]$ integralni batafsilroq qaraymiz.

Bu integral $x - c_0 t = const$ xarakteristika bo‘ylab olingani uchun $dx = c_0 dt,$ natijada uni yana quyidagicha yozish mumkin:

$$\int_A^P \left[\left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) c_0 dt - \rho udt \right] = \frac{\rho_0 c_0}{2} \int_A^p \left(u - \frac{p}{\rho_0 c_0} \right)^2 dt \ge 0$$

Biz xarakteristikaning AP kesmasi bo‘yicha integralning nomanfiyligini ko‘rsatdik. Shunga o‘xshash BQ bo‘ylab $dx=-c_0dt$ tenglikdan foydalanib,

$$\int_B^Q \left[\left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx - pudt \right] = -\frac{\rho_0 c_0}{2} \int_B^Q \left(u + \frac{p}{\rho_0 c_0} \right)^2 dt \le 0$$

ekanligiga ishonch hosil qilish mumkin. Isbotlangan tengsizliklardan va energiya saqlanish qonunining integral ayniyatidan

$$\int_P^Q \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx \le \int_A^B \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx$$

kelib chiqadi. Agar t=0 da, r=0, u=0 ga ega bo‘lsak, u holda

$$\int_P^Q \left(\rho_0 \frac{u^2}{2} + \frac{p^2}{2\rho_0 c_0^2} \right) dx \le 0$$

bo‘ladi. Natijada PQ da u, p miqdorlar ham nolga teng bo‘lishi kerak. Bu yerdan xarakteristik uchburchak asosida berilgan nol qiymatli boshlang‘ich qiymatlar, zaruriy tarzda uchburchakning ichidagi hamma nuqtalarda $u = 0, \quad p = 0$ ekanligi kelib chiqadi.

Endi yagonalik teoremasini isbotlash oson bo‘ladi. Agar u_1, p_1 va u_2, p_2 lar AV kesmada bir xil boshlang‘ich qiymatlarni qanoatlantiruvchi chiziqli sistema yechimi bo‘lsa, u holda ularning ayirmasi u_1-u_2, p_1- p_2 ham shu sistemaning nolga teng bo‘lgan boshlang‘ich qiymatlarini qanoatlantiruvchi yechimi bo‘ladi. Yuqorida isbotlanganligiga asosan butun xarakteristik uchburchak ichida u_1-u_2≡0, p_1- p_2≡0 bo‘ladi. Yagonalik teoremasi isbotlandi.

Ba’zida akustika tenglamalari uchun Koshi masalasini yechish emas, balki bu yechim sohaning x=0, x=l chegaralarida biror qo‘shimcha shartlarni qanoatlantirishini talab qiluvchi $u(x,0) = \varphi(x), \quad p(x,0) = \psi(x), \quad 0 \le x \le l$ boshlang‘ich qiymatlar bo‘yicha $0 \le x \le l, \quad t \ge 0$ sohada yechimni izlashdan iborat aralash masala deb ataluvchi masalani yechishga to‘g‘ri keladi. Masalan bu qo‘shimcha shartlar $u(0,t)=0$, $u(l,t)=0$ tengliklardan iboratligini talab qilish mumkin. Bunday qo‘yilgan masala $x=0$, $x=l$ mahkamlangan devorlar o‘rtasida gaz tebranishini o‘rganish bilan bog‘liq. Bu yerda qo‘yilgan aralash masalaning yechimi quyidagi ko‘rinishda bo‘lishi kerak:

$$u = \frac{f(x - c_0 t) + g(x + c_0 t)}{2}, \quad p = \rho_0 c_0 \frac{f(x - c_0 t) - g(x + c_0 t)}{2}$$

bu yerda f, g funksiyalar

$$f(z) = \varphi(z) + \frac{\psi(z)}{\rho_0 c_0}, \quad g(z) = \varphi(z) - \frac{\psi(z)}{\rho_0 c_0}$$

formulalar bilan $u\big|_{t=0} = \varphi(x); \quad p\big|_{t=0} = \psi(x)$ boshlang‘ich qiymatlar orqali aniqlanadi. Bu formulalar $f(z)$, $g(z)$ funksiyalarni faqat $0 \le z \le l$ da aniqlaydi, ya’ni $x - c_0 t \ge 0, \quad x + c_0 t \le l$ xarakteristik uchburchak ichida yechimni qurish imkonini beradi.

Yechimni butun $0 \le x \le l$ yo‘lakchada qurish uchun va $x=0$, l da $u=0$ chegaraviy shartlarining bajarilishi uchun, boshlang‘ich qiymatlarni butun x o‘qiga sun’iy usulda davom ettirishdan foydalanamiz. *f(z), g(z)* funksiyalarni $0 \le z \le l$ da aniqlab, ularni barcha z qiymatlarida shunday davom ettiramizki $f(z) = -g(-z),\quad f(l\text{-}z) = \text{-}g(l+z)$ bo‘lsin. Ishonch hosil qilish mumkinki, agar $f(0)+g(0)=0,\ f(l)+g(l)=0$ bo‘lsa, u holda bunday davom ettirish o‘rinli va yagona usulda amalga oshirish mumkin. Haqiqatdan ham, agar biz $0 \le z \le l$ da *f(z), g(z)* funksiyalarni bilsak, u holda *f(-z)=-g(z), -f(z)=g(-z)* formulalar bu funksiyalarni $-l \le z \le 0$ da aniqlashga imkon beradi. *f(0)+g(0)=0* tenglik funksiyalarni davom ettirishgacha va davom ettirishdan keyin ham *z*=0 dagi qiymatlarining ustma-ust tushushini ta’minlaydi. Endi *f(z), g(z)* ni butun to‘g‘ri chiziqda *2l* davrli davriy funksiyalar bilan davom ettiramiz:

$$f(z+2l) = f(z), \qquad g(z+2l) = g(z)$$

Bunaqa uzluksiz davom ettirish imkoniyati $f(l)+g(l)=0$ shart va $f(-l) = -g(l),\ g(\text{-}l) = \text{-}f(l)$ davom ettirish shartlaridan kelib chikadigan $f(l) = f(-l),$ $g(l) = g(-l)$ tengliklar bilan ta’minlanadi. Butun *z* o‘qiga davom ettirilgan *f(z), g(z)* funksiyalar uzluksiz va birinchi hamda ikkinchi tartibli uzluksiz hosilalarga ega deb faraz qilamiz. Qurilgan yechim *x* va *t* bo‘yicha birinchi va ikkinchi tartibli uzluksiz xususiy hosilalarga ega ekanligi ko‘rinib turibti.

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} = 0, \\ \dfrac{\partial p}{\partial t} + \rho_0 c_0^2 \dfrac{\partial u}{\partial x} = 0; \end{cases} \qquad u(0,\ t) = u(l,t) = 0$$

tenglamani va chegaraviy shartlarni t vaqt bo‘yicha differensiallasak, u_t, p_t hosilalar ham shunga o‘xshash

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial p_t}{\partial x} = 0, \\ \dfrac{\partial p_t}{\partial t} + \rho_0 c_0^2 \dfrac{\partial u_t}{\partial x} = 0 \end{cases}$$

sistema va $u_t(0,t) = u_t(l,t) = 0$ chegaraviy shartlarni qanoatlantiradi. Dastlabki tenglamalardan foydalanib u_t, p_t ifodalarni u_x, p_x orqali ifodalash mumkin va bu ifodalar quyidagi tenglama va chegaraviy shartlarni qanoatlantirishiga ishonch hosil qilish mumkin:

$$\begin{cases} \dfrac{\partial p_x}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial(\rho_0^2 c_0^2 u_x)}{\partial x} = 0, \\ \dfrac{\partial(p_0^2 c_0^2 u_x)}{\partial t} + \rho_0 c_0^2 \dfrac{\partial p_x}{\partial x} = 0; \end{cases} \qquad p_x(0,\ t) = p_x(l,\ t) = 0$$

Dastlabki tenglamalar va ulardan olingan u_t, p_t, u_x, p_x hosilalar ishtirok etgan tenglamalardan iborat sistema kengaytirilgan tenglamalar sistemasi deb ataladi. Bunday sistemalar yechim xossalarini o‘rganishda keng qo‘llaniladi va muhim rol o‘ynaydi. $u(0,t) = u(l,t) = 0$ chegaraviy shartlar bilan berilgan aralash masala yechimining yagonaligini isbotlash uchun, yana energiya integralidan foydalanish mumkin.

$$\int_0^l \left[\rho_0 \frac{u^2(x,t)}{2} + \frac{p^2(x,t)}{2\rho_0 c_0^2} \right] dx = \int_0^l \left[\rho_0 \frac{u^2(x,0)}{2} + \frac{p^2(x,0)}{2\rho_0 c_0^2} \right] dx + \int_0^t p(0,t)\, u(0,t) dx -$$

$$- \int_0^t p(l,t)\, u(l,t) dt = \int_0^l \left[\rho_0 \frac{\varphi^2(x)}{2} + \frac{\psi^2(x)}{2\rho_0 c_0^2} \right] dx$$

tenglikdan

$\varphi(x) = 0, \quad \psi(x) = 0$ nolga teng bo‘lgan boshlang‘ich qiymatlarga nol qiymatli yechim mos kelishi kelib chiqadi.

$$\begin{cases} u_t(0,t) = -\dfrac{1}{\rho_0} p_x(0,t) = -\dfrac{1}{\rho_0} \psi'(x), \\ p_t(0,t) = -\rho_0 c_0^2 u_x(0,t) = -\rho_0 c_0^2 \varphi'(x), \end{cases}$$

$$\begin{cases} p_x(0,t) = \psi'(x) \\ \rho_0^2 c_0^2 u_x(0,t) = \rho_0^2 c_0^2 \varphi'(x) \end{cases}$$

boshlang‘ich shartlarni qanoatlantiruvchi kengaytirilgan sistemaning u_t, p_t, p_x, $\rho_0^2 c_0^2 u_x$ yechimlari uchun integral ayniyatni qo‘llab, dastlabki sistema uchun energiya integrali bilan birga

$$\int_0^l u^2(x,t) dx, \int_0^l u_t^2(x,t) dx, \int_0^l u_x^2(x,t) dx,$$

$$\int_0^l p^2(x,t) dx, \int_0^l p_t^2(x,t) dx \quad \int_0^l p_x^2(x,t) dx$$

integrallarni boshlang‘ich berilgan funksiyalarning

$$\int_0^l \varphi^2(x) dx, \quad \int_0^l \psi^2(x) dx, \quad \int_0^l (\varphi')^2 dx, \quad \int_0^l (\psi')^2 dx$$

integrallari orqali baholashga imkon beruvchi

$$\int_0^l \left[\rho_0 \frac{u_t^2(x,t)}{2} + \frac{p_t^2(x,t)}{2\rho_0 c_0^2} \right] dx = \int_0^l \left\{ \frac{[\psi'(x)]^2}{2\rho_0} + \frac{\rho_0 c_0^2}{2} [\varphi'(x)]^2 \right\} dx,$$

$$\int_0^l \left[\rho_0 \frac{p_x^2(x,t)}{2} + \frac{\rho_0^2 c_0^2 u_x^2(x,t)}{2} \right] dx = \int_0^l \left\{ \rho_0 \frac{[\psi'(x)]^2}{2} + \frac{\rho_0^2 c_0^2}{2} [\varphi'(x)]^2 \right\} dx$$

tengliklarga kelamiz.

Aralash masala yechimini vaqtning $0 < t < T$ intervalida qarab, energiya integrali ayniyatini t bo‘yicha integrallab quyidagi tengliklarni olish qiyin emas:

$$\int_0^T \int_0^l \left[\rho_0 \frac{u^2(x,t)}{2} + \frac{1}{2\rho_0 c_0^2} p^2(x,t) \right] dx dt = T \int_0^l \left\{ \rho_0 \frac{\varphi^2(x)}{2} + \frac{\psi^2(x)}{2\rho_0 c_0^2} \right\} dx,$$

$$\int_0^T \int_0^l \left[\rho_0 \frac{u_t^2(x,t)}{2} + \frac{1}{2\rho_0 c_0^2} p_t^2(x,t) \right] dx dt = T l_0^2 \int_0^l \left\{ \rho_0 \frac{[\varphi'(x)]^2}{2} + \frac{[\psi'(x)]^2}{2\rho_0 c_0^2} \right\} dx,$$

$$\int_0^T\int_0^l\left[\rho_0\frac{u_x^2(x,t)}{2}+\frac{1}{2\rho_0c_0^2}p_x^2(x,t)\right]dxdt=T\int_0^l\left\{\rho_0\frac{[\varphi'(x)]^2}{2}+\frac{[\psi'(x)]^2}{2\rho_0c_0^2}\right\}dx,$$

Bular yordamida

$$\int_0^T\int_0^l u^2(x,t)dxdt,\ \int_0^T\int_0^l u_t^2(x,t)dxdt,\ \int_0^T\int_0^l u_x^2(x,t)dxdt,$$

$$\int_0^T\int_0^l p^2(x,t)dxdt,\ \int_0^T\int_0^l p_t^2(x,t)dxdt,\ \int_0^T\int_0^l p_x^2(x,t)dxdt$$

integrallar baholanadi. Bunday turdagi baholar keyinchalik giperbolik sistema yechimlari xossalarini o‘rganishda asos qilib olinadi. Albatta hosilalar uchun integral ayniyat faqat o‘xshashlik uchun energiya integrali deb nomlanishi mumkin. Lekin ular uchun shunday nomlash qabul qilingan va biz undan foydalanamiz. Kelgusi tadqiqotlar uchun masalaga operator nuqtai nazaridan yondoshish ancha qulaydir.

u(0,t)=u(l,t) chegaraviy shartlarni qanoatlantiruvchi

$$\begin{cases}\dfrac{\partial u}{\partial t}+\dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x}=0\\ \dfrac{\partial p}{\partial t}+\rho_0c_0^2\dfrac{\partial u}{\partial x}=0\end{cases}$$

tenglamalar sistemasini qaraymiz.

$\varphi(x),\psi(x)\quad(0\le x\le l)$ boshlang‘ich qiymatlarni yuqoridagi masala $u(x,t),\ p(x,t)\ (0\le x\le l,\ 0\le t\le T)$ echimlariga akslantiruvchi: $\begin{pmatrix}u\\p\end{pmatrix}=R\begin{pmatrix}\varphi\\\psi\end{pmatrix}$ R operatorini aniqlaydi. $\varphi(x),\psi(x)$ boshlang‘ich qiymatlarni

$$\left\|\begin{matrix}\varphi\\\psi\end{matrix}\right\|_\Phi=\sqrt{\int_0^l\left\{\varphi^2(x)+\psi^2(x)+[\varphi'(x)]^2+[\psi'(x)]^2\right\}dx}$$

norma bilan aniqlangan F funksional fazo vektori deb qaraymiz, $u(x,t)$, $p(x,t)$ yechimlarni esa

$$\left\|\begin{matrix}u\\p\end{matrix}\right\|_U=\sqrt{\frac{1}{T}\int_0^T\int_0^l\left\{u^2(x,t)+p^2(x,t)+u_t^2(x,t)+u_x^2(x,t)+p_t^2(x,t)+p_x^2(x,t)\right\}dxdt}+$$

$$+\max_{0\le t\le T}\sqrt{\int_0^l\left[u^2(x,t)+p^2(x,t)+u_x^2(x,t)+p_x^2(x,t)\right]dx}$$

norma bilan aniqlangan U fazoning vektori deb qaraymiz.

Energiya interali yordamida olingan baholar, quyidagi ρ_0, c_0, T orqali ifodalanuvchi o‘zgarmas M bilan baholovchi tengsizlik ko‘rinishida yozilishi mumkin:

$$\left\|\begin{matrix}u\\p\end{matrix}\right\|_U\le M\left\|\begin{matrix}\varphi\\\psi\end{matrix}\right\|_\Phi .$$

Ushbu tengsizlikni esa $\Phi\xrightarrow{R}U$ akslantirishni amalga oshiruvchi R operator normasi uchun $\|R\|\le M$ baho deb tushunish mumkin. Biz bu akslantirishni

$$\varphi(0)=0,\ \psi'(0)=0,\ \varphi''(0)=0,$$

$$\varphi(l)=0,\ \psi'(l)=0,\ \varphi''(l)=0$$

muvofiqlashtirish shartlarini qanoatlantiruvchi F fazoning yetarlicha silliq ikki marta uzluksiz differensiallanuvchi $\{\varphi(x), \psi(\text{x})\}$ elementlari uchun aniqlaymiz. Funksional analizda fazoni to‘liq va to‘liqmasga farqlash qabul qilingan. To‘liq fazoda har qanday fundamental ketma-ketlik uchun bu ketma-ketlik yaqinlashadigan shu fazo elementini ko‘rsatish mumkin. Agar F fazo tarkibiga faqat ikki marta uzluksiz differensiallanuvchi $\varphi(x), \psi(\text{x})$ funksiyalarni kiritsak, ushbu fazo to‘liq deb atalmaydi. Chunki $\| \|_{\Phi}$ bo‘yicha yaqinlashishidan φ'', ψ'' ikkinchi tartibli hosilalarning yaqinlashishi haqida hech qanday xulosa chiqarish mumkin emas (φ'', ψ'' - lar normaga kirmaydi).

$\| \|_{\Phi}$ buyicha $\left\{\varphi(x) = \left|\frac{l}{2} - x\right|, \ \psi(\text{x}) = 0\right\}$ juftlikka yaqinlashuvchi ikki marta differensiallanuvchi $\{\varphi_n(x), \ \psi_n(x)\}$ juftlikdan iborat, shunday fundamental ketma-ketlik qurish mumkinki, undagi $\varphi(x)$ funksiya x=0 da hatto birinchi tartibli hosilaga ega bo‘lmaydi.

Bunday ketma-ketlikni qurish matematik analiz kursining standart mashqidir, shu sababli biz bunga to‘xtalib o‘tirmaymiz. Ikki marta uzluksiz differensiallanuvchi $\varphi(x), \psi(x)$ funksiyalardan iborat boshlang‘ich qiymatlar F fazosini va mos silliq yechimlarning U fazosining to‘liqmasligi nazariyani qurishni murakkablashtiradi. Ya’ni F fazodan olingan boshlang‘ich shartlarni qanoatlantiruvchi taqribiy yoki aniq yechimlarning fundamental ketma-ketligini qurib, bunday ketma-ketlik limiti F fazoda yotuvchi boshlang‘ich shartlarni qanoatlantiradi va U fazoda yotadi deb tasdiqlashga imkon bermaydi. Biroq bu muammoni yo‘qotish mumkin, agar F, U fazolarga ulardagi barcha silliq elementlarning mumkin bo‘lgan fundamental ketma-ketliklar limitlarini kiritsak, metrik fazoda bunday ideal elementni (fundamental ketma-ketliklar limitlarini) kiritish, fazoni to‘ldirish deb ataladi. Ratsional sonlar to‘plamini barcha haqiqiy sonlar to‘plamiga kengaytirish fazoni to‘ldirishga yaxshi misol bo‘ladi.

Metrik fazoni to‘ldirish nazariyasini mukammalroq bayoni bilan [3], [4] darsliklar bo‘yicha tanishish mumkin. Yuqorida o‘rganilgan aralash masala yechimini beruvchi chegaralangan R operatorni F fazodagi to‘ldirilgan boshlang‘ich qiymatlar to‘plamigacha kengaytirish mumkin. Shundan so‘ng R operator to‘ldirilgan boshlang‘ich qiymatlar fazosi elementlarini to‘ldirilgan silliq yechimlar U fazosi elementlariga akslantiradi. To‘ldirilgan U fazo elementlariga umumlashgan yechim nomi beriladi.

Shu bilan, boshlang‘ich berilganlar fazosida yotuvchi elementlarni, yechimlar fazosida yotuvchi funksiyalarga akslantiruvchi operatorlarga asoslangan nazariyaga hozircha yakun yasaymiz.

2-mavzu. Ayirmali sxema turg‘unligini isbotlashning energetik usullari.

2.1.Aprior baho. Nostatsionar gaz dinamikasi tenglamalari.
2.2.Statsionar Eyler tenglamalari. Saint-Venant tenglamalari.
2.3.Xususiy differensial tenglamalar uchun chegaraviy masalani

Ta'rif: $$A\frac{\partial u}{\partial t}+B\frac{\partial u}{\partial x}+C\frac{\partial u}{\partial y}=f$$

tenglamalar sistemasi (Fridrixs buyicha) simmetrik t - giperbolik sistema deyiladi, agar A, V, S matritsalar simmetrik, shu bilan birga A musbat aniqlangan bo'lsa. (*A, V, S* matritsaning barcha elementlari va o'ng tomon *f* vektor-funksiyaning yasovchilari odatdagidek *x, y, t* ning yetarlicha silliq funksiyalari deb faraz qilinadi).

<u>Misol.</u> Biz ko'rgan tovush to'lqinlari tarqalish tenglamasini quyidagicha yozish mumkin:

$$\begin{cases}\dfrac{1}{\rho_0 c_0^2}\dfrac{\partial p}{\partial t}+\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0\\ \rho_0\dfrac{\partial u}{\partial t}+\dfrac{\partial p}{\partial x}=0\\ \rho_0\dfrac{\partial v}{\partial t}+\dfrac{\partial p}{\partial y}=0\end{cases}$$

Oldingi misol bilan solishtirib, biz birinchi tenglamani $\rho_0 c_0^2$ ga bo'ldik, keyingi ikkitasini ρ_0 ko'paytirdik.

Qaralayotgan sistema matritsaviy ko'rinishida quyidagacha yoziladi:

$$\begin{pmatrix}\dfrac{1}{\rho_0 c_0^2} & 0 & 0\\ 0 & \rho_0 & 0\\ 0 & 0 & \rho_0\end{pmatrix}\frac{\partial}{\partial t}\begin{pmatrix}p\\ u\\ v\end{pmatrix}+\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\frac{\partial}{\partial x}\begin{pmatrix}p\\ u\\ v\end{pmatrix}+\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}\frac{\partial}{\partial y}\begin{pmatrix}p\\ u\\ v\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

Ushbu sistemadagi

$$A=\begin{pmatrix}\dfrac{1}{\rho_0 c_0^2} & 0 & 0\\ 0 & \rho_0 & 0\\ 0 & 0 & \rho_0\end{pmatrix},\quad \mathrm{B}=\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ \mathrm{C}=\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}$$

matritsalar yuqorida keltirilgan ta'rifning barcha shartlarini qanoatlantiradi va shuning uchun tovush tarqalish tenglamasi simmetrik t – giperbolik sistemani tashkil qiladi.

Simmetrik t-giperbolik sistemalarning yechimlari qanoatlantiradigan ba'zi muhim tengliklarni keyingi bobda batafsil o'rganamiz. Ushbu tengliklar akustika tenglamalari yoki Maksvell tenglamalarining yechimlari uchun energiyaning saqlanish qonunini umumlashtiruvchi energiya integrallari nomi bilan ataladi. Hozircha mavjud bo'lgan simmetrik giperbolik sistemalar nazariyasi bu ayniyatlarga asoslanadi.

$A(x,y,t,u)\frac{\partial u}{\partial t}+B(x,y,t,u)\frac{\partial u}{\partial x}+C(x,y,t,u)\frac{\partial u}{\partial y}=f(x,y,t,u)$ (bu yerda A, V, S- matritsalar, u – n o‘lchovli vektor-funksiya) sistema uchun xarakteristikalarni shunday S sirtlar kabi aniqladikki, S sirtga o‘tkazilgan normal (τ,ξ,η) vektori

$$\det\|\tau A+\xi B+\eta C\|=0$$

tenglikni qanoatlantiradi.

Ko‘rish mumkinki, bu ta’rif izlanayotgan funksiyalar to‘plamini ixtiyoriy chiziqli xosmas almashtirishda va dastlabki tenglamani ularning ixtiyoriy chiziqli kombinatsiyasi bilan almashtirganda o‘zgarmaydigan sirtni ajratadi.

Aynan $u=Tv$ ni qo‘yamiz (T=T(x,y,t) – xosmas matritsa). U holda v funksiya

$$AT\frac{\partial v}{\partial t}+BT\frac{\partial v}{dx}+CT\frac{\partial v}{dy}=f-\left(A\frac{\partial T}{\partial t}+B\frac{\partial T}{dx}+C\frac{\partial T}{dy}\right)v$$

sistemani qanoatlantiradi.

Sistema tenglamalarini ularni chiziqli kombinatsiyalari bilan almashtirish, sistemani chapdan xosmas Q matritsaga ko‘paytirishga ekvivalent. Bunda tenglamalar sistemasi

$$QAT\frac{\partial v}{\partial t}+QBT\frac{\partial v}{\partial x}+QCT\frac{\partial v}{\partial y}=Qf-Q\left(A\frac{\partial T}{\partial t}+B\frac{\partial T}{\partial x}+C\frac{\partial T}{\partial y}\right)v$$

ko‘rinishni oladi. Agar Q xos matritsa bo‘lsa, bu tenglamalar sistemasi dastlabki tenglamalar sistemasiga ekvivalent bo‘lmas edi.

Bunday almashtirilgan sistemalar uchun xarakteristika tenglamasini yozamiz:

$$\det\|\tau QAT+\xi QBT+\eta QCT\|=0$$

Matritsalar ko‘paytmasining determinanti haqidagi teoremaga asosan

$$\det\|\tau QAT+\xi QBT+\eta QCT\|=\det\|Q(\tau A+\xi B+\eta C)T\|=\det\|Q\|\det\|\tau A+\xi B+\eta C\|\det\|\|\|T\|$$

$$\det\|Q\|\neq 0,\ \det\|\mathrm{T}\|\neq 0$$

tengsizlikka asosan

$$\det\|\tau A+\xi B+\eta C\|=0 \qquad (3)$$

tenglama

$$\det\|\tau QAT+\xi QBT+\eta QCT\|=0$$

tenglamaga ekvivalent.

Xarakteristika tushunchasining izlanayotgan funksiyalar to‘plamini xosmas chiziqli almashtirishiga nisbatan va tenglamalar sistemasini ixtiyoriy unga teng kuchli bo‘lgan tenglamalar chiziqli kombinatsiyasidan tashkil topgan sistemaga nisbatan invariantligi isbotlandi.

(3) tenglikni qanoatlantiruvchi (τ,ξ,η) vektorlar to‘plami konus bo‘lib, har bir (τ,ξ,η) vektor bilan unga kolleniar $(k\tau,k\xi,k\eta)$ ko‘rinishidagi vektorlar ham bu tenglikni qanoatlantiradi. Bunday tenglama bilan aniqlanadigan konus xarakteristika uchun normallar konusi yoki qisqacha xarakteristik normallar konusi deb ataladi. Agar *A*, *V*,

S koeffitsientlar matritsasi x,y,t koordinatalarga bog‘liq bo‘lsa, u holda xarakteristik normallar konusi

$$\det\left\| \tau A(x,y,t) + \xi B(x,y,t) + \eta C(x,y,t \right\| = 0$$

x,y,t fazoning har bir nuqtasida o‘zgacha bo‘ladi.

Xarakteristika ta’rifini ikkinchi tartibli bitta tenglama holida ham beramiz:

$$A\frac{\partial^2 u}{\partial t^2} + 2B\frac{\partial^2 u}{\partial t \partial x} + C\frac{\partial^2 u}{\partial x^2} = f(x,t,u,u_x,u_t)$$

erkli o‘zgaruvchilar soni ikkita bo‘lgan hol bilan chegaralanamiz. Ko‘p sondagi erkli o‘zgaruvchilar holida xam, xarakteristikalar aynan shunday aniqlanadi.

$$\varphi = \varphi(x,t), \quad \alpha = \alpha(\mathrm{x,\ t}), \quad \frac{\mathrm{D}(\varphi,\alpha)}{\mathrm{D(x,\ t)}} \neq 0$$

yangi koordinatalar sistemasiga o‘tamiz. Bu koordinatalar sistemasida tenglama quyidagicha yoziladi:

$$\left[A\left(\frac{\partial\varphi}{\partial t}\right)^2 + 2B\left(\frac{\partial\varphi}{\partial t}\right)\left(\frac{\partial\varphi}{\partial x}\right) + C\left(\frac{\partial\varphi}{\partial x}\right)^2 \right]\frac{\partial^2 u}{\partial\varphi^2} + 2\left[A\frac{\partial\varphi}{\partial t}\frac{\partial\alpha}{\partial t} + B\frac{\partial\varphi}{\partial t}\frac{\partial\alpha}{\partial x} + B\frac{\partial\varphi}{\partial x}\frac{\partial\alpha}{\partial t} + C\frac{\partial\varphi}{\partial x}\frac{\partial\alpha}{\partial x} \right]\times$$

$$\times\frac{\partial^2 u}{\partial\varphi\partial\alpha} + \left[A\left(\frac{\partial\alpha}{\partial t}\right)^2 + 2B\left(\frac{\partial\alpha}{\partial t}\right)\left(\frac{\partial\alpha}{\partial x}\right) + C\left(\frac{\partial\alpha}{\partial x}\right)^2 \right]\frac{\partial^2 u}{\partial\alpha^2} + \left[A\frac{\partial^2\varphi}{\partial t^2} + 2B\frac{\partial^2\varphi}{\partial t\partial x} + C\frac{\partial^2\varphi}{\partial x^2} \right]\frac{\partial u}{\partial\varphi} +$$

$$+\left[A\frac{\partial^2\alpha}{\partial t^2} + 2B\frac{\partial^2\alpha}{\partial t\partial x} + C\frac{\partial^2\alpha}{\partial x^2} \right]\frac{\partial u}{\partial x} = f\left(x,t,u,u_\varphi\varphi_x + u_\varphi\alpha_x, u_\varphi\varphi_t + u_\alpha\alpha_t\right)$$

Endi faraz qilamiz, biror $\varphi = \varphi_0 = const$ egri chiziqda u funksiya (α o‘zgaruvchining funksiyasi sifatida) va uning barcha birinchi tartibli hosilalari berilgan bo‘lsin. u funksiya va uning $\frac{\partial u}{\partial\varphi}$ hosilasi ma’lumligi biz uchun juda muhim. Ularni α bo‘yicha differensiallab (ya’ni egri chiziq buylab), bu egri chiziqda

$$\frac{\partial u}{\partial\alpha}, \frac{\partial^2 u}{\partial\alpha^2}, \frac{\partial^2 u}{\partial\varphi\partial\alpha}$$

ifodalarni topamiz.

Agar

$$A\left(\frac{\partial\varphi}{\partial t}\right)^2 + 2B\left(\frac{\partial\varphi}{\partial t}\right)\left(\frac{\partial\varphi}{\partial x}\right) + C\left(\frac{\partial\varphi}{\partial x}\right)^2 \neq 0$$

bo‘lsa, tenglama yordamida $\frac{\partial^2 u}{\partial\varphi^2}$ ni topishimiz mumkin.

$$A\left(\frac{\partial\varphi}{\partial t}\right)^2 + 2B\left(\frac{\partial\varphi}{\partial t}\right)\left(\frac{\partial\varphi}{\partial x}\right) + C\left(\frac{\partial\alpha}{\partial x}\right)^2 = 0 \qquad (4)$$

tenglikni qanoatlatiruvchi $\varphi(x,t) = const$ $(\mathrm{grad}\ \ \varphi \neq 0)$ egri chiziqlar

$$A\frac{\partial^2 u}{\partial t^2} + 2B\frac{\partial^2 u}{\partial t\partial x} + C\frac{\partial^2 u}{\partial x^2} = f$$

tenglamaning xarakteristikalari deb ataladi.

Xarakteristikalar sistema uchun qanday rol o‘ynagan bo‘lsa, tenglama uchun ham xuddi shunday muhim rol o‘ynaydi. Ikkinchi tartibli tenglamalar uchun xarakeristikani ta’riflashda, yuqorida qaralgan sistemadagi kabi, (4) tenglikni unga ekvivalent bo‘lgan

$$A\tau^2+2B\tau\xi+C\xi^2=0$$

tenglikka almashtirish mumkin. Bu yerda (τ,ξ) - tadqiq qilinadigan egri chiziqqa o‘tkazilgan normal vektori. Agar $\varphi(x,t)=\varphi_0$ egri chiziq xarakteristika bo‘lsa, u xolda *u ye*chim yshbu xarakteristika bo‘ylab quyidagi tenglikni qanoatlantiradi:

$$2\left[A\frac{\partial\varphi}{\partial t}\frac{\partial\alpha}{\partial t}+B\frac{\partial\varphi}{\partial t}\frac{\partial\alpha}{\partial x}+B\frac{\partial\varphi}{\partial x}\frac{\partial\alpha}{\partial t}+C\frac{\partial\varphi}{\partial x}\frac{\partial\alpha}{\partial x}\right]\frac{\partial}{\partial\alpha}\left(\frac{\partial u}{\partial\varphi}\right)+\left[A\left(\frac{\partial\alpha}{\partial t}\right)^2+2B\frac{\partial\alpha}{\partial t}\frac{\partial\alpha}{\partial x}+C\left(\frac{\partial\alpha}{\partial x}\right)^2\right]\times$$

$$\times\frac{\partial^2u}{\partial\alpha^2}+\left[A\frac{\partial^2u}{\partial t^2}+2B\frac{\partial^2\varphi}{\partial t\partial x}+C\frac{\partial^2\varphi}{\partial x^2}\right]\frac{\partial u}{\partial\varphi}+\left[A\frac{\partial^2\alpha}{\partial t^2}+2B\frac{\partial^2\alpha}{\partial t\partial x}+C\frac{\partial^2\alpha}{\partial x^2}\right]\frac{\partial u}{\partial\alpha}=$$

$$=f\left(x,t,u,u_\varphi\varphi_x+u_\alpha\alpha_x;u_\varphi\varphi_t+u_\alpha\alpha_t\right)$$

Bu tenglikni xarakteristika bo‘ylab $u,\frac{\partial u}{\partial\varphi}$ orasidagi munosabat deb qarash mumkin.

Ikkinchi tartibli tenglama uchun Koshi masalasi quyidagicha quyiladi. Biror $\varphi=const$ chiziqlar ustida $u,\frac{\partial u}{\partial\varphi}$ qiymatlarni berib, bu chiziq atrofida yechimni aniqlashga xarakat qilishimiz kerak. Agar $\varphi=const$ egri chiziq xarakteristika bo‘lsa, unda ushbu chiziqda Koshi masalasini quyib bo‘lmaydi. Egri chizikda *u* funksiya qiymatini berib, xarakteristika uchun yozilgan tengliklardan u_φ qiymatni aniqlashimiz mumkin. Aytish joizki ba’zan, yechimni aniqlash uchun xarakteristika ustida *u* funksiya qiymatini berish yetarli, lekin bunaqa masalani quyish, Koshi masalasi deb atash notabiiy bo‘ladi.

Misol 1. $\frac{\partial^2u}{\partial t^2}-\frac{\partial^2u}{\partial x^2}=0$ tenglama uchun xarakteristikalar.

$\left(\frac{\partial\varphi}{\partial t}\right)^2-\left(\frac{\partial\varphi}{\partial x}\right)^2=\left(\frac{\partial\varphi}{\partial t}-\frac{\partial\varphi}{dx}\right)\left(\frac{\partial\varphi}{\partial t}+\frac{\partial\varphi}{\partial x}\right)=0$ tenglik bilan aniqlanadi.

$\frac{\partial\varphi}{\partial t}+\frac{\partial\varphi}{\partial x}=0$ tenglamaning umumiy yechimi $\varphi=\varphi(x-t)$ ko‘rinishga ega, $\frac{\partial\varphi}{\partial t}-\frac{\partial\varphi}{\partial x}$ =0 tenglamaning umumiy yechimi $\varphi=\varphi(x+t)$ ko‘rinishda bo‘ladi. $\varphi(x-t)=const$ yoki $\varphi(x+t)=const$ tengliklar, qaralayotgan tenglamaning xarakteristikalari bo‘ladigan ikkita $x\pm t=const$ to‘g‘ri chiziqlar oilasini aniqlaydi.

2. $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ Laplas tenglamasi uchun $\left(\frac{\partial\varphi}{\partial t}\right)^2+\left(\frac{\partial\varphi}{\partial x}\right)^2=0$ xarakteristikalar tenglamasi haqiqiy yechimga ega emas.

3. $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ issiqlik o‘tkazuvchanlik tenglamasi $\left(\frac{\partial \varphi}{\partial x}\right)^2=0$ xarakteristik tenglamaga olib keladi. Bu tenglamaning umumiy yechimi $\varphi=\varphi(t)$ ko‘rinishda bo‘ladi. $\varphi(\mathrm{t})=\mathrm{const}\quad(\mathrm{t}=const)$ xarakteristikalar esa x o‘qiga parallel chiziqlardan iborat. $u(x,0)$ boshlang‘ich qiymatlar bo‘yicha, $t>0$ uchun temperaturani aniqlash masalasi xarakteristika ustida berilgan masaladan iborat. Aynan shu sababli issiqlik o‘tkazuvchanlik tenglamasi ikkinchi tartibli bo‘lsa ham, boshlang‘ich shart sifatida faqat bitta shart beriladi. Issiqlik o‘tkazuvchanlik tenglamasi uchun boshlang‘ich masala aslida Koshi masalasi emas, ammo adabiyotlarda uni Koshi masalasi deb atalishi tez-tez uchraydi.

Xususiy hosilali tenglamalarni xarakteristik tenglama xossalari bo‘yicha sinflarga bo‘linishi tabiiy ekanligi ko‘rinib turibti. Yuqorida aniqlangan giperbolik sistemalar tushunchasi shu asosda kiritilgan edi. Endi elliptik sistema yoki tenglama ta’rifini beramiz.

$$A\frac{\partial u}{\partial x}+B\frac{\partial u}{\partial y}+C\frac{\partial u}{\partial z}=f$$

sistema elliptik deyiladi, agar uning xarakteristik tenglamasi

$$det\|\xi A+\eta B+\zeta C\|=0,\quad \xi^2+\eta^2+\zeta^2>0$$

shartlarni qanoatlantiruvchi (ξ,η,ζ) haqiqiy yechimga ega bo‘lmasa. Bitta

$$A\frac{\partial^2 u}{\partial x^2}+2B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}=f$$

tenglamaga elliptiklik ta’rifini berish uchun uning

$$A\xi^2+2B\xi\,\eta+C\eta^2=0$$

xarakteristik tenglamasini qarash kerak va u $(\xi,\eta),\quad \xi^2+\eta^2>0$ bo‘lgan haqiqiy yechimlarga ega bo‘lmasligi talab qilinadi.

Elliptik sistemasiga

$$\begin{cases}\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=0,\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=0\end{cases}$$

Koshi-Riman tenglamalar sistemasi misol bo‘ladi. Elliptik tenglamaga

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Laplas tenglamasi misol bo‘ladi.

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}=0$$

issiqlik o‘tkazuvchanlik tenglamasi, xarakteristik tenglama sifatida ikkita ustma-ust tushuvchi tenglamalarga ajraluvchi $\xi^2 = 0$ tenglamalarga ega. U elliptik va giperbolik orasidagi oraliq parabolik sinfga tegishli.

Parabolik tenglamalarning ta’rifini bermasdan lekin unda nafaqat yuqori hosilalar oldidagi koeffitsientlar, balki boshqa koeffitsientlarni ham inobatga olinishini ta’kidlashimiz mumkin. Paragraf yakunida quyidagi holga e’tiboringizni jalb qilamiz. Yangi noma’lum funksiyalarga o‘tish va tenglamalarni ularni chiziqli kombinatsiyasiga almashtirish, xarakteristikalarni o‘zgartirishsiz qoldirishga qaramasdan, shunday holatlar bo‘ladiki, bir xil hodisalarni tavsiflash uchun har xil xarakteristikalarga ega tenglama va sistemalar qo‘llanishi mumkin. Misol keltiramiz.

$$\begin{cases} \dfrac{1}{c_0^2}\dfrac{\partial p}{\partial t} + \rho_0 \dfrac{\partial u}{\partial x} + \rho_0 \dfrac{\partial v}{\partial y} = 0 \\ \rho_0 \dfrac{\partial u}{\partial t} + \dfrac{\partial p}{\partial x} = 0 \\ \rho_0 \dfrac{\partial v}{\partial t} + \dfrac{\partial p}{\partial y} = 0 \end{cases}$$

akustika tenglamalarida, agar ulardan birinchisini t bo‘yicha differensiallab va natijadan ikkinchi va uchinchilarini mos ravishda x bo‘yicha va u bo‘yicha differensiallab ayirsak, ikkinchi tartibli bitta tenglamaga kelamiz:

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial y^2} = 0$$

Uning $\varphi_t^2 - c_0^2\left(\varphi_x^2 + \varphi_y^2\right) = 0$ xarakteristik tenglamasi dastlabki sistemaning $\varphi_t\left[\varphi_t^2 - c_0^2\left(\varphi_x^2 + \varphi_y^2\right)\right] = 0$ xarakteristika tenglamasidan farq qiladi. Bu almashtirishlarda xarakteristikalarni invariant qoldiruvchi almashtirishlar ro‘yxatiga kirmagan differensiallash amalidan foydalanildi.

3-mavzu. Differensial tenglamalarni ayirmali tenglamalar bilan approksimatsiya qilish.

3.1. To‘r tenglamalar sistemasini yechish.
3.2. Chegaraviy shartlarni approksimatsiya etish.
3.3. To‘r tenglamalar sistemasini yechish.

To`r tenglamalarini yechish usullari yoritilgan ushbu bobda dastlab Puasson tenglamasi uchun Dirixle masalasiga mos differensial va ayirmali masala qo`yiladi, so`ngra ayirmali masalani yechishga imkon beradigan universal metodlardan biri bo`lgan dekompositsiya metodi, o`zgaruvchilarni ajratish metodi, to`r qurish algoritmlarini realizasiya qilish, oddiy iterasiya usuli, Puasson tenglamasi uchun ayirmali Dirixle masalasini optimal iterasiya parametrlarini tanlash orqali yechish algoritmi keltirilgan.

Elliptik tenglamalar uchun chegaraviy masalalarni ayirmali approksimasiyalaganda chiziqli algebraik tenglamalar sistemasiga kelindi (ayirmali yoki to`r tenglamalari). Bunday sistemaning matritsasi $A$ juda katta tartibli bo`lib, uning tartibi N to`rning tugunlari soniga teng bo`ladi. Masalan, agar har bir o`zgaruvchi $x_1, x_2, ..., x_p$ bo`yicha h qadamli to`r olsak $(h_1 = h_2 = ... = h_p = h)$ tugunlar soni $N = O\left(\frac{1}{h^p}\right)$ ga teng bo`ladi, bu yerda p–o`lchamlar soni. Ikki va uch o`lchamli masalalar uchun tenglamalar soni $N \approx 10^4 - 10^6$ dan katta bo`ladi (masalan, $h = 1/100$ bo`lganda). Bundan tashqari, algebraik sistemaning matritsasida juda ko`p nol elementlar mavjud bo`lib, u maxsus (lentasimon) strukturaga ega va nihoyat, yomon shartlangan matritsadan iborat bo`ladi, ya’ni, matritsaning eng katta xos qiymatining eng kichik xos qiymatiga nisbati juda ham katta bo`ladi $(\sim 10^3 \div 10^4)$ va u $O(h^{-2})$ tartibli miqdor bo`ladi.

Elliptik to`r tenglamalarining ushbu jihatlari ularni sonli yechish uchun maxsus tejamli algoritmlarni ishlab chiqishni taqozo etadi.

Samarali bevosita yechish metodlari, odatda, to`r tenglamalarining o`ta tor, ammo, muhim sinfini yechishda qo`llaniladi. Bundan tashqari, bevosita yechish metodlari iterasiya metodlarida yuqori qatlamdagi operatorning teskarisini topishda mos ravishda tanlab olinadi.

Hozirda Puasson tenglamasi uchun ayirmali chegaraviy masalalarni dekart, qutb, silindrik va sferik koordinatalar sistemasida samarali yechishga imkon beradigan ikkita bevosita yechish metodi mavjud. Ulardan biri – dekompozisiya metodi yoki faktorizasiyaga ega bo`lgan juft-toq yo`qotish usuli, ushbu metod Gauss yo`qotish usulining takomillashtirilgan formasidan iborat. Ikkinchi metod – o`zgaruvchilarni ajratish metodi bo`lib, u Furyening tez almashtirish algoritmiga asoslangan.

Har ikkala metod uchun ikki o`lchamli masalani yechishga sarflanadigan arifmetik amallar soni uchun quyidagi baho o`rinli $\sim Q = O(N^2 \log_2 N)$, bu yerda N– bitta yo`nalish bo`yicha to`r tugunlari soni.

Ketma-ket yaqinlashuvchi iterasiya metodlari ixtiyoriy sohadagi yanada umumiy masalalarga, o`zgaruvchan koeffisientlarga ega bo`lgan umumiy ko`rinishdagi tenglamalarni yechishga qo`llanilishi mumkin.

To`r tenglamalarini yechish usullari

g chegaraga ega bo`lgan to`g`ri to`rtburchakli $\overline{G} = \{0 \le x_\alpha \le l_\alpha,\ \alpha = 1,2\}$ sohada Puasson tenglamasi uchun ayirmali Dirixle masalasini qaraylik:

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = -f(x),\ \ x = (x_1, x_2) \in G, u|_g = \mu(x).. \qquad (1.1)$$

$\overline{G}$ sohada h_1 va h_2 qadamlar bilan ayirmali to`r kiritamiz:

$$\omega_h = \{i_1 h_1, i_2 h_2 \in \overline{G}, i_\alpha = 0,1,\dots,N_\alpha, h_\alpha N_\alpha = l_\alpha, \alpha = 1,2\}.$$

$\gamma_h = \{x_{i_1 i_2} \in g\}$ esa to`rning chegaraviy tugunlari bo`lsin. Differensial masala (15.1) ga mos keluvchi ayirmali masala quyidagi ko`rinishga ega

$$\begin{aligned} &\Lambda y = -f(x), x \in \omega_h,\ y|_{\gamma_h} = \mu(x), \\ &\Lambda = \Lambda_1 + \Lambda_2, \Lambda_\alpha y = y_{\bar{x}_\alpha x_\alpha}, \alpha = 1,2, \\ &y_{i_1 i_2} = y(i_1 h_1, i_2 h_2). \end{aligned} \qquad (1.2)$$

Dekompozisiya metodi. Ayirmali masala (1.2) ni vektorli tenglamalar sistemasi ko`rinishiga keltiramiz

$$\begin{aligned} &-Y_{j-1} + CY_j - Y_{j+1} = F_j,\ j = 1,2,\dots,N_2 - 1, \\ &Y_0 = F_0, Y_{N_2} = F_{N_2}, \end{aligned} \qquad (1.3)$$

bu yerda Y_j va F_j–vektorlar bo`lib, ularning komponentalari sifatida $y_{i_j} = y(ih_1, jh_2)$ yechimning va $f_{i_j} = f(ih_1, jh_2)$ o`ng tomonning ω_h to`rning

j–ustunidagi qiymatlari qaraladi, C–esa kvadrat matritsa bo`lib, uni quyida aniqlaymiz.

Haqiqatan ham, agar (1.2) tenglamaning o`ng tomonini chegaraga yaqin tugunlarda o`zgartirsak, u holda chegaraviy tugunlar $i=0, i=N_1$ da $y_{i_j}=0$ deb hisoblash mumkin.

Tenglama (1.2) ni quyidagi ko`rinishda yozib olamiz:

$$\begin{aligned} & -y_{i,j-1}+(2y-h_2^2 y_{\bar{x}_1 x_1})_{ij}-y_{i,j+1}=h_2^2\varphi_{ij}, \\ & 1\le i\le N_1-1, 1\le j\le N_2-1, \\ & y_{0j}=y_{N_1 j}=0,\ 0<j<N_2, \\ & y_{i0}=\mu_{i0},\ y_{iN_2}=\mu_{iN_2},\ 0<i<N_1, \end{aligned} \qquad (1.4)$$

bu yerda

$$\begin{aligned} & \varphi_{ij}=f_{ij}\ agar 1<i<N_1-1, 1\le j\le N_2-1, \\ & \varphi_{1j}=f_{1j}+\frac{1}{h_1^2}\mu_{0j},\ \varphi_{N_1-1,1}=f_{N_1-1,j}+\frac{1}{h_1^2}\mu_{N_1 j}. \end{aligned} \qquad (1.5)$$

Vektorlar Y_j va F_j ni kiritamiz:

$$\begin{aligned} & Y_j=(y_{1j}, y_{2j},\dots,y_{N_1-1,j}),\ j=0,1,\dots,N_2, \\ & F_j=(h_2^2 f_{1j}+\frac{h_2^2}{h_1^2}\mu_{0j}, h_2^2 f_{2j},\dots,h_2^2 f_{N_1-2,j}, h_2^2 f_{N_1-1,j}+\frac{h_2^2}{h_1^2}\mu_{N_1 j}), \\ & j=1,2,\dots,N_2-1, \\ & F_j=(\mu_{1j},\mu_{2j},\dots,\mu_{N_1-1,j}),\ agar\ j=0, N_2. \end{aligned}$$

Ayirmali operator C ni quyidagicha aniqlaymiz:

$$\begin{aligned} & (CY_j)_i=(2y-h_2^2 y_{\bar{x}_1 x_1})_{ij},\ 0<i<N_1, \\ & y_{0j}=y_{N_1 j}=0. \end{aligned}$$

Bundan va (1.4) dan kelib chiqadiki, ayirmali masala (1.2) vektorli tenglamalar sistemasi (1.3) ga ekvivalent.

Dekompozisiya yoki toq-juft yo`qotish usulini bayon qilishga o`tamiz, bunda $N_2=2^n$ deb hisoblaymiz. Bu metodning asosiy g`oyasida tenglamalar (1.3) dan dastlab toq nomerli $Y_j$ vektorlar, so`ngra indekslari 2, 4, 8 va hokazolarga karrali bo`lgan juft nomerli $Y_j$ vektorlar yo`qotiladi.

Indekslarning

$$j = 2,4,6,\ldots,N_2 - 2, \quad \text{bu yerda } N_2 = 2^n$$

qiymatlari uchun quyidagi uchta tenglamani yozamiz

$$\begin{aligned} -Y_{j-2} + CY_{j-1} - Y_j &= F_{j-1}, \\ -Y_{j-1} + CY_j - Y_{j+1} &= F_j, \\ -Y_j + CY_{j+1} - Y_{j+2} &= F_{j+1}. \end{aligned}$$

Ikkinchi tenglamaga C operator bilan ta'sir etamiz va har uchala tenglamani qo`shib, quyidagi "qisqartirilgan" sistemani hosil qilamiz:

$$\begin{aligned} &-Y_{j-2} + C^{(1)}Y_j - Y_{j+2} = F_j^{(1)}, \\ &j = 2,4,6,\ldots,N_2 - 2, \\ &Y_0 = F_0,\ Y_{N_2} = F_{N_2}, \end{aligned} \quad (1.6)$$

u faqat juft indeksli noma'lumlarga bog`liq bo`ladi. bu yerda quyidagi belgilashlar kiritilgan

$$\begin{aligned} C^{(1)} &= [C^{(0)}]^2 - 2E, \\ F_j^{(1)} &= F_{j-1}^{(0)} + C^{(0)}F_j^{(0)} + F_{j+1}^{(0)}, \end{aligned}$$

bu yerda

$$C^{(0)} = C,\ F_j^{(0)} = F_j.$$

Agar tenglama (1.6) dan juft nomerli Y_j lar topilgan bo`lsa, u holda toq nomerli noma'lumlarni ushbu tenglamalardan aniqlash mumkin

$$\begin{aligned} &C^{(0)}Y_j = F_j^{(0)} + Y_{j+1} + Y_{j-1}, \\ &j = 1,3,5,\ldots,N_2 - 1. \end{aligned}$$

Tenglama (1.3) dan juft nomerli vektorlarni yo`qotganimizdek, sistema (1.6) dan nomeri $j$ 2 ga karrali bo`lgan, ammo 4 ga karrali bo`lmagan va hokazo noma'lumlarni yo`qotamiz. Natijada barcha noma'lumlarni ketma-ket topish uchun tenglamalar sistemasiga ega bo`lamiz, ular ushbu tenglamalarni yechish yo`li bilan topiladi

$$\begin{aligned} &C^{(k-1)}Y_j = F_j^{(k-1)} + Y_{j-2^{k-1}} + Y_{j+2^{k-1}}, \\ &j = 2^{k-1}, 3\cdot 2^{k-1}, 5\cdot 2^{k-1}, \ldots, (N_2 - 1)\cdot 2^{k-1}, \\ &Y_0 = F_0, Y_{N_2} = F_{N_2}, \end{aligned} \quad (1.7)$$

bu yerda $C^{(k)}$ va $F_j^{(k)}$ quyidagi rekurrent formulalar bo`yicha aniqlanadi:

$$
\begin{aligned}
&C^{(k)} = \left[C^{(k-1)}\right]^2 - 2E,\ k = 1,2,\dots,n-1,\\
&C^{(0)} = C,\\
&F_j^{(k)} = F_{j-2^{k-1}}^{(k-1)} + C^{(k-1)}F_j^{(k-1)} + F_{j+2^{k-1}}^{(k-1)},\\
&j = 2^k, 2\cdot 2^k, 3\cdot 2^k, \dots, N_2 - 2^k,\\
&k = 1,2,\dots,n-1.
\end{aligned}
\qquad (1.8)
$$

Dekompozisiya algoritmi, $C^{(k)}$ operatorni faktorizasiyalash (sodda operatorlarga ajratish) g`oyasiga asoslangan:

$$
\begin{aligned}
&C^{(k)} = \prod_{l=1}^{2^k}(C - \mu_l E),\\
&\mu_l = 2\cos\frac{(2l-1)\pi}{2^{k+1}},
\end{aligned}
\qquad (1.9)
$$

bu o`z navbatida $C^{(k)}$ operatorning teskarisini topish masalasini uch nuqtali ayirmali operatorning progonka metodi bilan ketma-ket teskarisini topishga keltiriladi.

Haqiqatan ham ushbu tenglamani yechish talab qilingan bo`lsin

$$C^{(k)}\vartheta = \varphi .$$

Agar $C^{(k)}$ operatorning (1.9) ko`rinishidagi ifodalanishini hisobga olsak, u holda ϑ ni topish masalasi quyidagi tenglamalarni ketma-ket yechish masalasiga keltiriladi,

$$
\begin{aligned}
&(C - \mu_1 E)\vartheta^{(1)} = \varphi,\\
&(C - \mu_l E)\vartheta^{(l)} = \vartheta^{(l-1)},\ l = 2,3,\dots,2^k,
\end{aligned}
$$

bunda izlanayotgan yechim $\vartheta = \vartheta^{(2k)}$ dan iborat. Yuqorida yozilgan tenglamalardan har biri ushbu uch nuqtali ayirmali tenglamani tashkil etadi

$$
\begin{aligned}
&2\vartheta^{(l)} - h_2^2\vartheta_{\bar{x}_1x_1}^{(l)} - \mu_l\vartheta^{(l)} = \vartheta^{(l-1)},\ h_1 \le x_1 \le l_1 - h_1,\\
&\vartheta^{(l)}(0) = \vartheta^{(l)}(l_1) = 0,
\end{aligned}
$$

u progonka metodi bilan yechiladi.

$F_j^{(k)}$ ni hisoblashda quyidagi algoritmdan foydalaniladi. Vektorlar $p_j^{(k)}$ va $q_j^{(k)}$ kiritiladi, ular orqali $F_j^{(k)}$ ushbu formula orqali ifodalanadi

$$F_j^{(k)} = C^{(k)}p_j^{(k)} + q_j^{(k)}. \qquad (1.10)$$

Vektorlar $p_j^{(k)}$ va $q_j^{(k)}$ larni hisoblash algoritmini hosil qilish uchun (1.10) ni, tenglama (1.8) ga qo`yamiz:

$$\begin{aligned} C^{(k)} p_j^{(k)} + q_j^{(k)} = C^{(k-1)}[p_{j-2^{k-1}}^{(k-1)} + p_{j+2^{k-1}}^{(k-1)} + q_j^{(k-1)}] + \\ + (C^{(k-1)})^2 p_j^{(k-1)} + p_{j-2^{k-1}}^{(k-1)} + q_{j+2^{k-1}}^{(k-1)}, \end{aligned} \quad (1.11)$$

so`ngra

$$q_j^{(k)} = 2 p_j^{(k)} + q_{j-2^{k-1}}^{(k-1)} + q_{j+2^{k-1}}^{(k-1)} \quad (1.12)$$

deb olamiz va tenglama (1.11) ni, munosabat $C^{(k)} = [C^{(k-1)}]^2 - 2E$ ni inobatga olgan holda yozib olamiz

$$(C^{(k-1)})^2 (p_j^{(k)} - p_j^{(k-1)}) = C^{(k-1)} (p_{j-2^{k-1}}^{(k-1)} + p_{j+2^{k-1}}^{(k-1)} + q_j^{(k-1)}).$$

Oxirgi tenglamani $C^{(k-1)}$ ga qisqartirib, ushbuga ega bo`lamiz

$$\begin{aligned} & C^{(k-1)} S_j^{(k-1)} = p_{j-2^{k-1}}^{(k-1)} + p_{j+2^{k-1}}^{(k-1)} + q_j^{(k-1)}, \\ & p_j^{(k)} = p_j^{(k-1)} + S_j^{(k-1)}, \end{aligned}$$

Bunda $q_j^{(k)}$ formula (1.12) ga asosan aniqlanadi.

Munosabat (1.10) ni (1.7) ga qo`yish quyidagini beradi

$$C^{(k-1)}[Y_j - p_j^{(k-1)}] = q_j^{(k-1)} + Y_{j-2^{k-1}} + Y_{j+2^{k-1}}.$$

Shunday qilib, $F_j^{(k)}$ o`rniga $p_j^{(k)}$ va $q_j^{(k)}$ vektorlarni kiritish orqali masala (1.2) ning yechimini topish masalasini faqat $C^{(k-1)}$ operatorning teskarisini topish va tenglamaning o`ng tomonini hisoblashda vektorlarni qo`shishga olib keladi.

Hisoblashlar quyidagi ketma-ketlikda olib boriladi:

1. Boshlang`ich qiymatlar $p_0^{(k)}$, $q_0^{(k)}$ beriladi:

$$q_j^{(0)} = F_j, p_j^{(0)} = 0, j = 1,2,...,N_2 - 1.$$

2. Barcha $k = 1,2,...,n-1$ lar uchun ushbu tenglama yechiladi

$$C^{(k-1)} S_j^{(k-1)} = q_j^{(k-1)} + p_{j-2^{k-1}}^{(k-1)} + p_{j+2^{k-1}}^{(k-1)}$$

va $p_j^{(k)}$ va $q_j^{(k)}$ vektorlar barcha

$$j = 2^k, 2 \cdot 2^k, 3 \cdot 2^k, ..., N_2 - 2^k$$

lar uchun quyidagi formulalar bo`yicha hisoblanadi

$$p_j^{(k)} = p_j^{(k-1)} + S_j^{(k-1)},$$
$$q_j^{(k)} = 2p_j^{(k)} + q_{j-2^{k-1}}^{(k-1)} + q_{j+2^{k-1}}^{(k-1)}.$$

3. Ushbu tenglamalar yechiladi

$$C^{(k-1)}S_j^{(k-1)} = q_j^{(k-1)} + Y_{j-2^{k-1}} + Y_{j+2^{k-1}},$$
$$Y_0 = F_0, Y_{N_2} = F_{N_2}$$

va barcha $j = 2^{k-1}, 3\cdot 2^{k-1}, \ldots, N_2 - 2^{k-1}, k = n, n-1, \ldots, 1$ lar uchun izlanayotgan yechim

$$Y_j = p_j^{(k-1)} + S_j^{(k-1)}$$

hisoblanadi.

Dekompozisiya metodi $Q = O(N_1 N_2 \log_2 N_2)$ arifmetik amal va noma'lumlar soniga nisbatan 1.5 marta ko`p kompyuter xotirasi talab qiladi.

Endi o`zgaruvchilarni ajratish metodini qaraymiz, bu metod ham to`r tenglamalarini bevosita yechishga imkon beradi. Ayirmali masala (1.2) ni birjinsli chegaraviy shartlari bilan qaraylik

$$\Lambda y = -\varphi, y\big|_{\gamma_h} = 0, \tag{1.13}$$

bu yerda φ masala (1.2) ning o`ng tomoni *f* dan faqatgina chegaraga yaqin nuqtalarda farq qiladi, $i_1 = 1, i_1 = N_1 - 1$ da $\frac{1}{h_1^2}\mu$ ga, $i_2 = 1, i_2 = N_2 - 1$ da esa $\frac{1}{h_2^2}\mu$ ga farqlanadi.

$\mu_k(jh_2)$ va λ_k–mos ravishda ushbu masalaning *k* nomerli xos funksiyalari va xos qiymatlari bo`lsin

$$\begin{aligned} &\Lambda_2\mu_k + \lambda_k\mu_k = 0, h_2 \le x_2 \le l_2 - h_2, \\ &\mu_k(0) = \mu_k(l_2) = 0. \end{aligned} \tag{1.14}$$

Ma'lumki, ular uchun quyidagi formulalar o`rinli

$$\mu_k(jh_2) = \sqrt{\frac{2}{l_2}}\sin\frac{k\pi j}{N_2}, \lambda_k = \frac{4}{h_2^2}\sin^2\frac{k\pi h_2}{2l_2}$$
$$k = 1, 2, \ldots, N_2 - 1.$$

Ayirmali masala (1.13) ning yechimini ushbu yig`indi ko`rinishida izlaymiz

$$\begin{aligned} &y_{ij} = \sum_{k=1}^{N_2-1} c_k(ih_1)\mu_k(jh_2) \\ &i = 1, 2, \ldots, N_1 - 1,\ j = 1, 2, \ldots, N_2 - 1, \end{aligned} \tag{1.15}$$

bu yerda c_k Furye koeffisienti bo`lib, u $x_1 = ih_1$ ga bog`liq bo`ladi.

Ifoda (1.15) ni tenglama (1.13) ga qo`yib

$$\begin{aligned} \Lambda y = \Lambda_1 y + \Lambda_2 y = \sum_{k=1}^{N_2-1} [\mu_k(jh_2)\Lambda_1 c_k(ih_1) + c_k(ih_1)\Lambda_2 \mu_k(jh_2)] = \\ = \sum_{k=1}^{N_2-1} \varphi_k(ih_1)\mu_k(jh_2), \end{aligned} \tag{1.16}$$

tenglamaga ega bo`lamiz, bu yerda $\varphi_k(ih_1) - \varphi(x)$ funksiyaning Furye koeffisienti:

$$\varphi_k(ih_1) = \sum_{j=1}^{N_2-1} \varphi(ih_1, jh_2)\mu_k(jh_2)h_2.$$

Formula (1.14) ni, xuddi shuningdek μ_k funksiyalarning ortoganalligini e'tiborga olsak, tenglama (1.16) dan koeffisientlar c_k ni aniqlash uchun quyidagi masalaga kelamiz

$$\begin{aligned} &\Lambda_1 c_k - \lambda_k c_k = -\varphi_k,\ h_1 \le x_1 \le l_1 - h_1, \\ &c_k(0) = c_k(l_1) = 0, \end{aligned} \tag{1.17}$$

bu yerda $k = 1,2,...,N_2 - 1$.

Bundan ko`rinadiki, $c_k(ih_1)$ koeffisient $x_1 = ih_1$ ning funksiyasi sifatida, har bir $k$ uchun progonka metodi bilan topiladi. Hammasi bo`lib $N_2 - 1$ marta progonka metodi algoritmi qo`llaniladi.

So`ngra $c_k(ih_1)$ koeffisientlarni bilgan holda formula (1.15) bo`yicha masala (1.13) ning yechimi topiladi. Furye koeffisientlari φ_k ni hisoblash va yechimlar y_{ij} ni topish aynan bir xil formulalar bo`yicha olib boriladi. Bu yerda yig`indini hisoblash umumiy bo`lib hisoblanadi

$$\vartheta_j = \sum_{k=1}^{N-1} z_k \sin\frac{k\pi j}{N},\ j = 1,2,...,N-1.$$

Buning uchun Furyening tez almashtirish maxsus algoritmidan foydalaniladi, u ko`rsatilgan yig`indilarni $q = 5N\log_2 N$ arifmetik amal sarflab hisoblash imkonini beradi $(N = 2^n)$, yig`indini odatdagi qo`shishda $O(N^2)$ arifmetik amal sarflanadi. Ayirmali Dirixle masalasi (1.2) ni yechishga $O(N_1 N_2 \log_2 N_2$ arifmetik amallar sarflanadi. Dekompozisiya metodini o`zgaruvchilarni ajratish metodi bilan birgalikda qo`llash mumkin.

4-mavzu. Oshkor va oshkormas ayirmali sxemalar.

Reja

4.1. Oshkormas sxemalarning turg‘unligi.
4.2. Absolyut va shartli turg‘un ayirmali sxemalar.
4.3. Simmetrik t-giperbolik sistemalar uchun Laks, Godunov, Rusanov ayirmali sxemalari va ularning turg‘unligi.

Ayirmali approksimatsiya usulida differensial tenglama va qo’shimcha shartlarga kiruvchi har bir hosila faqatgina shablonni tashkil qiluvchi tugun nuqtalarda ifodalangan ayirmali ifodalar bilan almashtiriladi. Ushbu usul juda sodda bo’lganligi bois qo’shimcha izohlarga hojat yo’q.

To’g’ri to’rtburchakli to’rda uzluksiz (va yetarlicha silliq) koeffitsiyentli differensial tenglamalar uchun ayirmali approksimatsiya usuli birinchi va ikkinchi tartibli approksimatsiyaga ega bo’lgan ayirmali sxemalarni oson tuzish imkonini beradi. Ammo, ushbu usulni murakkabroq bo’lgan hollar uchun qo’llash ancha mushkul yoki qo’llashni imkoni bo’lmaydi. Masalan, uzilishli koeffisiyentga ega bo’lgan differensial tenglamalar uchun, hisoblash sohasi to’g’ri to’rtburchak bo’lmasa, yuqori tartibli differensial tenglamalar uchun notekis to’rda va boshqa hollarda.

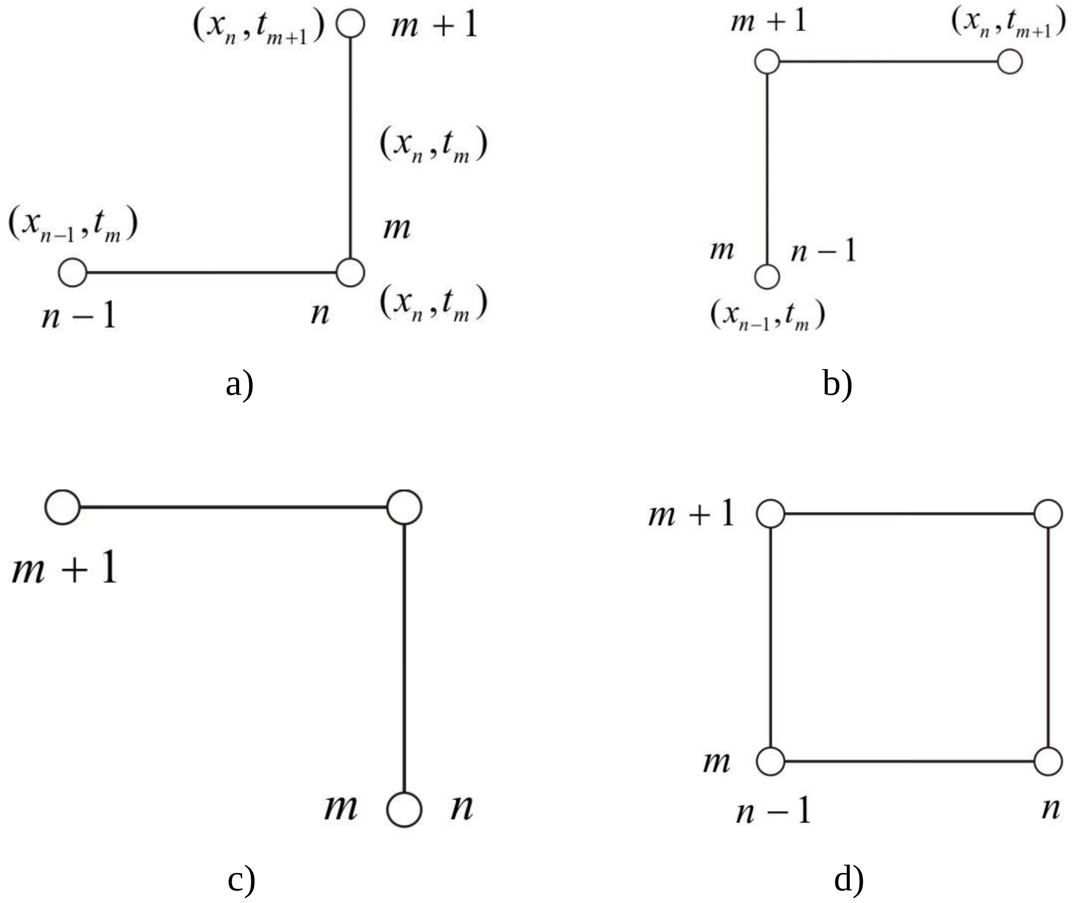
Misol. Quyidagi differensial masala uchun ayirmali sxema tuzish talab etilsin:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = f(x,t), \quad 0 < x \le 1, \qquad (2.1)$$

$$u(x,0) = \mu_1(x), \quad 0 < x \le 1, \qquad (2.2)$$

$$u(0,t) = \mu_2(t), \quad 0 \le t \le T. \qquad (2.3)$$

Ayirmali approksimatsiya usuli bo’yicha ayirmali sxema tuzish uchun shablon tanlaymiz. Buning uchun 4-(a, b, c, d) rasmlarda keltirilgan shablonlardan foydalanamiz. Ushbu shablonlardan (2.1) differensial tenglamani quyidagicha approksimatsiya qilish mumkin:



6-rasm.

a) shablon uchun:

$$\frac{y_n^{m+1} - y_n^m}{\tau} + c\frac{y_n^m - y_{n-1}^m}{n} = f(x_n, t_m)$$

b) shablon uchun:

$$\frac{y_{n-1}^{m+1} - y_{n-1}^m}{\tau} + c\frac{y_n^{m+1} - y_{n-1}^{m+1}}{n} = f(x_n - 0,5h,\ t_m + 0,5\tau)$$

c) shablon uchun:

$$\frac{y_n^{m+1} - y_n^m}{\tau} + c\frac{y_n^{m+1} - y_{n-1}^{m+1}}{h} = f(x_n - 0,5h,\ t_m + 0,5\tau)$$

d) shablon uchun:

$$\frac{y_n^{m+1}+y_{n-1}^{m+1}-y_n^m+y_{n-1}^m}{2\tau}+c\frac{y_n^{m+1}+y_n^m-y_{n-1}^{m+1}-y_{n-1}^m}{2h}=f(x_n-0{,}5h,\,t_m+0{,}5\tau)$$

Qo'shimcha shartlar barcha hollar uchun quyidagicha approksimatsiya qilinadi:

$$y_n^0=\mu_1(nh),\quad n=\overline{0,N}\,,\,h=\frac{a}{N}.$$

$$y_0^m=\mu_2(\tau m),\quad m=\overline{0,M}\,,\ \tau=\frac{T}{M}.$$

Integro-interpolyatsion usul.

Ushbu usulni balans usuli ham deb nomlashadi. $D=\{0\le x\le 1, 0\le t\le T\}$ to'g'ri to'rtburchakda

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+f(x,t), \qquad (2.4)$$

differensial tenglamani va

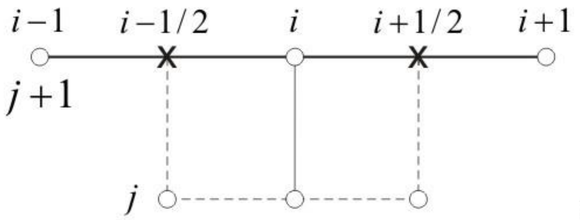
$$u(x,0)=u_0(x),\ 0\le x\le 1, \qquad (2.5)$$

$$u(0,t)=u_1(t),\ u(1,t)=u_2(t),\ 0\le t\le T, \qquad (2.6)$$

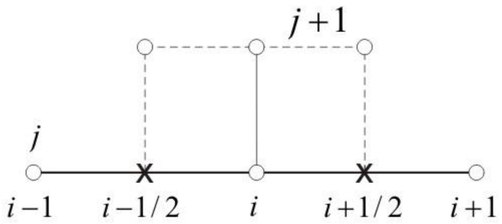
qo'shimcha shatrlarni qanoatlantiruvchi $u=u(x,t)$ funksiyani aniqlash talab etilgan bo'lsin. Ushbu masalani sonli yechish uchun $D=\{0\le x\le 1, 0\le t\le T\}$ sohada tekis to'r quramiz.

$\overline{\omega_h}=\{x_i=ih,\ i=\overline{0,N},\ h=1/N\}$ $0\le x\le 1$ kesmada h qadamli tekis to'r bo'lsin va $\overline{\omega_\tau}=\{t_j=j\tau,\ \ j=\overline{0,M},\ \tau=T/M\}$ $0\le t\le T$ kesmada τ qadamli to'r bo'lsin. U holda $\overline{\omega}_{h\tau}=\overline{\omega}_h\cdot\omega_\tau=\{(x_i t_j);\ \ x_i\ni\overline{\omega}_h,\ \ t_j\ni\omega_\tau\}$ - $D=\{0\le x\le 1, 0\le t\le T\}$ to'g'ri to'rtburchakda h va τ qadamlar bilan qurilgan to'rni anglatadi.

Integro-interpolyatsion usul yordamida (2.4) differensial tenglamani ayirmali sxema bilan approksimatsiya qilish uchun (2.4) tenglamani $x_{i-0,5}\le x\le x_{i+0,5}$, $t_{i-0,5}\le t\le t_{i+0,5}$ to'g'ri to'rtburchakda integrallaymiz:



7-rasm



8-rasm

$$\frac{1}{h\tau}\int_{x_{i-0,5}}^{x_{i+0,5}}\left[u(x,t_{j+1})+u(x,t)\right]dx=\frac{1}{h\tau}\int_{t_j}^{t_{j+1}}\left[\frac{\partial u}{\partial x}(x_{i+\frac{1}{2}},t)-\frac{\partial u}{\partial x}(x_{i-\frac{1}{2}},t)\right]dt+$$

$$+\frac{1}{h\tau}\int_{t_j}^{t_{j+1}}dt\int_{x_{i-0,5}}^{x_{i+0,5}}f(x,t)dx \qquad (2.7)$$

(2.7) tenglikka kiruvchi integrallarni quyidagicha approksimatsiyalaymiz:

$$\int_{x_{i-0,5}}^{x_{i+0,5}}u(x,t)dx\approx hu(x_i,t)$$

$$\int_{t_j}^{t_{j+1}}\frac{\partial u(x_{i+0,5},t)}{\partial x}dx\approx\tau\, u_{\bar{x},j+1}^{j+1}$$

$$\int_{t_j}^{t_{j+1}}dt\int_{x_{i-0,5}}^{x_{i+0.5}}\frac{\partial u(x_{i+0,5},t)}{\partial x}dx\approx\tau\, u_{\bar{x},i+1}^{j+1}$$

$$\int_{t_j}^{t_{j+1}}dt\int_{x_{i-0,5}}^{x_{i+0.5}}f(x,t)dx\approx h\,\tau\,0{,}5\big(f(x_i,t_j)+f(x_i,t_{j+1})\big)$$

U holda (2.7) tenglikdan quyidagi oshkormas ayirmali sxemaga ega bo’lish mumkin:

$$\frac{u_i^{j+1}-u_i^j}{\tau}=\frac{1}{h}\left[u_{\bar{x},i+1}^{j+1}-u_{\bar{x},i}^{j+1}\right]+\frac{1}{2}\left[f_i^{\,j}+f_i^{\,j+1}\right]$$

yoki

$$\frac{u_i^{j+1}-u_i^j}{\tau}=\frac{u_{i+1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1}}{h^2}+\frac{1}{2}\left[f_i^{\,j}+f_i^{\,j+1}\right].$$

Endi (2.4) tenglamani 8–rasmda ko’rsatilgan yacheyka bo’yicha integrallaymiz:

$$\frac{1}{h\tau}\int_{x_{i-0,5}}^{x_{i+0,5}}\left[u(x,t_{j+1})+u(x,t_j)\right]dx=\frac{1}{h\tau}\int_{t_j}^{t_{j+1}}\left[\frac{\partial u(x_{i+\frac{1}{2}},t)}{\partial x}-\frac{\partial u(x_{i-\frac{1}{2}},t)}{\partial x}\right]dt+$$

$$+\frac{1}{h\tau}\int_{t_j}^{t_{j+1}}dt\int_{x_{i-0,5}}^{x_{i+0,5}}f(x,t)dx \qquad (2.8)$$

(2.8) tenglamaga kiruvchi integrallardan $\int_{t_j}^{t_{j+1}}\frac{\partial u(x_{i+0,5},t)}{\partial x}\approx\tau u_{x,i+1}^{j}$ va $\frac{1}{h\tau}\int_{t_j}^{t_{j+1}}dt\int_{x_{i-0,5}}^{x_{i+0,5}}f(x,t)dx\approx\frac{h\tau}{3}(f(x_{i-1},t_j)+f(x_i,t_j)+f(x_{i+1},t_j))$ approksimatsiya qilsak oshkor ayirmali sxemaga ega bo'lish mumkin:

$$\frac{u_i^{j+1}-u_i^j}{\tau}=\frac{u_{i+1}^j-2u_i^j+u_{i-1}^j}{h^2}+\frac{1}{3}(f_{i-1}^j+f_i^j+f_{i+1}^j)$$

qo'shimcha shartlar ikkala holda ham

$$u_i^0=u_0(x_i)\quad,\quad y_0^j=\mu_1\left(t_j\right)\quad,\quad u_N^j=\mu_2\left(t_j\right)$$

ko'rinishda approksimatsiya qilinadi.

Nomalum koeffitsiyentlar usuli.

Nomalum koeffitsiyentlar usulida ayirmali sxema sifatida nomalum to'r funksiyaning shablonni tashkil qiluvchi tugun nuqtalardagi qiymatlarining chiziqli kombinasiyasi olinadi. Ushbu chiziqli kombinasiyaning koeffitsiyentlari ayirmali sxema berilgan differensial tenglamani to'r qatlamlari bo'yicha iloji boricha yuqori tartibda approksimatsiya qilish shartidan topiladi.

Masalan,

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

tenglama uchun

$$Ш(x,t)=\left\{\left(x_{i-1},t_j\right);\left(x_i,t_j\right);\ \left(x_{i+1},t_j\right);\left(x_i,t_{j+1}\right);\right\}$$

shablonda ayirmali sxema qurish talab etilgan bo'lsin. Demak ayirmali sxema

$$\alpha u_{i-1}^{j}+\beta u_i^j+\gamma u_{i+1}^j+\mu u_i^{j+1}=0. \qquad (2.9)$$

ko'rinishda ekan. u_{i-1}^{j} , u_{i+1}^{j} va u_i^{j+1} to'r funksiyalarni (x_i,t_j) nuqta atrofida Teylor qatoriga yoyib, (2.9) tenglamaga qo'yamiz.

$$\alpha u_{i-1}^{j}+\beta u_i^j+\gamma u_{i+1}^j+\mu u_i^{j+1}=$$

$$\alpha\left(u_i^j-h\frac{\partial u(x_i,t_j)}{\partial x}+\frac{h^2}{2}\frac{\partial^2 u(x_i,t_j)}{\partial x^2}+\frac{h^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial x^3}+\ldots\right)+$$

$$+\beta\left(u_i^j+\gamma(u_i^j+h\frac{\partial u(x_i,t_j)}{\partial x}+\frac{h^2}{2}\frac{\partial^2 u(x_i,t_j)}{\partial x^2}+\frac{h^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial x^3}+\ldots\right)+$$

$$+\mu\left(u_i^j+\tau\frac{\partial u(x_i,t_j)}{\partial t}+\frac{\tau^2}{2}\frac{\partial^2 u(x_i,t_j)}{\partial t^2}+\frac{h^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial t^3}+\ldots\right)=$$

$$(\alpha+\beta+\gamma+\mu)u_i^j+(\gamma-\alpha)h\frac{\partial u\left(x_i,t_j\right)}{\partial x}+\frac{h^2}{2}(\alpha+\gamma)\frac{\partial^2 u\left(x_i,t_j\right)}{\partial x^2}$$

$$+\mu\tau\frac{\partial u\left(x_i,t_j\right)}{\partial t}+O\left(h^3+\tau^2\right). \qquad (2.10)$$

(2.10) tenglikdan $\alpha u_{i-1}^{j}+\beta u_i^j+\gamma u_{i+1}^j+\mu u_i^{j+1}=\left(\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}\right)_{(x_i,t_j)}+O(h^3+\tau^2)$ bo'lishi uchun α,β,γ va μ koeffitsiyentlar quyidagi tenglamalar sistemasini qanoatlantirishi kerak:

$$\begin{cases}\alpha+\beta+\gamma+\mu=0\\ \gamma-\alpha=0\\ \alpha+\gamma=-\dfrac{2}{h^2}\\ \mu\tau=1\end{cases}. \qquad (2.11)$$

(2.11) tenglamalar sistemasini yechib, $\alpha = \gamma = -\frac{1}{h^2}$, $\mu = \frac{1}{\tau}$ va $\beta = \frac{2}{h^2} = -\frac{1}{\tau}$ ekanligini aniqlaymiz. Koeffitsiyentlarni bu qiymatlarini (2.10) ga qo'yamiz:

$$-\frac{u_{i-1}^j}{h^2} + \frac{2u_i^j}{h^2} - \frac{u_i^j}{\tau} - \frac{u_{i+1}^j}{h^2} + \frac{u_i^{j+1}}{\tau} = 0 \quad \text{yoki}$$

$$\frac{u_i^{j+1} - u_i^j}{\tau} - \frac{u_{i-1}^j - 2u_i^j + u_{i+1}^j}{h^2} = 0 \qquad (2.12)$$

Demak,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

differensial tenglamani $Ш(x_i, t_i) = \{(x_{i-1}, t_j); (x_i, t_j); (x_{i+1}, t_j); (x_i, t_{j+1});\}$ shablonda approksimatsiya qiluvchi ayirmali sxema quyidagi ko'rinishga ega ekan:

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \frac{u_{i-1}^j - 2u_i^j + u_{i+1}^j}{h^2}.$$

Endi $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ differensial tenglamani

$$Ш(x_i, t_j) = \{(x_i, t_j) \quad (x_{i-1}, t_{j+1}) \quad (x_i, t_{j+1}) \quad (x_{i+1}, t_{j+1})\}$$

shablonda approksimatsiya qiluvchi ayirmali tenglamani topamiz. Bu holda ayirmali sxema

$$\alpha u_i^j + \beta u_{i-1}^{j+1} + \gamma u_i^{j+1} + \mu u_{i+1}^{j+1} = 0. \qquad (2.13)$$

ko'rinishda bo'ladi. (2.13) da u_{i-1}^{j+1}, u_i^{j+1}, u_{i+1}^{j+1} to'r funksiyalarni (x_i, t_j) nuqta atrofida Teylor qatoriga yoyamiz:

$$u_i^{j+1} = u_i^j + \tau \frac{\partial u(x_i, t_j)}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} + \frac{\tau^3}{6}\frac{\partial^3 u(x_i, t_j)}{\partial t^3} + \frac{\tau^4}{24}\frac{\partial^4 u(x_i, t_j)}{\partial t^4} + \ldots$$

$$u_{i-1}^{j+1} = u_i^j - h\frac{\partial u(x_i, t_j)}{\partial x} + \tau \frac{\partial u(x_i, t_j)}{\partial t} + \frac{h^2}{2}\frac{\partial^2 u(x_i, t_j)}{\partial x^2} + \frac{\tau^2}{2}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} -$$

$$-h\tau\frac{\partial^2 u(x_i,t_j)}{\partial x\partial t}-\frac{h^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial x^3}+\frac{\tau^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial t^3}+\frac{h^2\tau}{2}\frac{\partial^3 u(x_i,t_j)}{\partial x^2\partial t}-\frac{h\tau^2}{2}\frac{\partial^3 u(x_i,t_j)}{\partial x\partial t^2}+\dots$$

$$u_{i+1}^{j+1}=u_i^j+h\frac{\partial u(x_i,t_j)}{\partial x}+\tau\frac{\partial u(x_i,t_j)}{\partial t}+\frac{h^2}{2}\frac{\partial^2 u(x_i,t_j)}{\partial x^2}+\frac{\tau^2}{2}\frac{\partial^2 u(x_i,t_j)}{\partial t^2}+$$

$$+h\tau\frac{\partial^2 u(x_i,t_j)}{\partial x\partial t}+\frac{h^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial x^3}+\frac{\tau^3}{6}\frac{\partial^3 u(x_i,t_j)}{\partial t^3}+\frac{h^2\tau}{2}\frac{\partial^3 u(x_i,t_j)}{\partial x^2\partial t}+\frac{h\tau^2}{2}\frac{\partial^3 u(x_i,t_j)}{\partial x\partial t^2}+\dots$$

U holda

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=\alpha u(x_i,t_j)+\beta\Big[u(x_i,t_j)-hu'_x(x_i,t_j)+u'_t(x_i,t_j)+$$

$$+0{,}5h^2u''_{xx}(x_i,t_j)+0{,}5\tau^2u''_{tt}(x_i,t_j)-h\tau u''_{xt}(x_i,t_j)-\frac{h^3}{6}u'''_{xxx}(x_i,t_j)+\frac{\tau^3}{6}u'''_{ttt}(x_i,t_j)+$$

$$+\frac{h^2\tau}{2}u'''_{xxt}(x_i,t_j)-\frac{h\tau^2}{2}u'''_{xtt}(x_i,t_j)+\frac{h^4}{24}u''''_{xxxx}(x_i,t_j)-\frac{h^3\tau}{6}u''''_{xxxt}(x_i,t_j)+$$

$$+\frac{h^2\tau^2}{4}u''''_{xxtt}(x_i,t_j)-\frac{h\tau^3}{6}u''''_{xttt}(x_i,t_j)+\frac{\tau^4}{24}u''''_{tttt}(x_i,t_j)+\dots\Big]+$$

$$+\gamma\Big[u(x_i,t_j)+\tau u'_t(x_i,t_j)+0{,}5\tau^2u''_{tt}(x_i,t_j)+\frac{\tau^3}{6}u'''_{ttt}(x_i,t_j)+\frac{\tau^4}{24}u''''_{ttt}(x_i,t_j)+\dots\Big]+$$

$$+\mu\Big[u(x_i,t_j)+hu'_x(x_i,t_j)+u'_t(x_i,t_j)+$$

$$+0{,}5h^2u''_{xx}(x_i,t_j)+0{,}5\tau^2u''_{tt}(x_i,t_j)+h\tau u''_{xt}(x_i,t_j)+\frac{h^3}{6}u'''_{xxx}(x_i,t_j)+\frac{\tau^3}{6}u'''_{ttt}(x_i,t_j)+$$

$$+\frac{h^2\tau}{2}u'''_{xxt}(x_i,t_j)+\frac{h\tau^2}{2}u'''_{xtt}(x_i,t_j)+\frac{h^4}{24}u''''_{xxxx}(x_i,t_j)+\frac{h^3\tau}{6}u''''_{xxxt}(x_i,t_j)+$$

$$+\frac{h^2\tau^2}{4}u''''_{xxtt}(x_i,t_j)+\frac{h\tau^3}{6}u''''_{xttt}(x_i,t_j)+\frac{\tau^4}{24}u''''_{tttt}(x_i,t_j)+\dots\Big]$$

O’xshash hadlarni ixchamlab, quyidagiga ega bo’lamiz:

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=[\alpha+\beta+\gamma+\mu]u(x_i,t_j)+h(\mu-\beta)u'_x(x_i,t_j)+$$

$$+\tau(\beta+\gamma+\mu)u'_t(x_i,t_j)+0{,}5h^2(\beta+\mu)u''_{xx}(x_i,t_j)+0{,}5\tau^2(\beta+\gamma+\mu)u''_{tt}(x_i,t_j)+$$

$$+h\tau(\mu-\beta)u''_{xt}(x_i,t_j)+\frac{h^3}{6}(\mu-\beta)u'''_{xxx}(x_i,t_j)+\frac{\tau^3}{6}(\beta+\gamma+\mu)u'''_{ttt}(x_i,t_j)+$$

$$+0{,}5h^2\tau(\beta+\mu)u'''_{xxt}(x_i,t_j)+0{,}5h^2\tau(\mu-\beta)u'''_{xtt}(x_i,t_j)+\frac{h^4}{24}(\beta+\mu)u''''_{xxxx}+$$

$$+\frac{h^3\tau}{6}(\mu-\beta)u''''_{xxxt}(x_i,t_j)+\frac{h^2\tau^2}{4}(\beta+\mu)u''''_{xxxt}(x_i,t_j)+\frac{h\tau^3}{6}(\mu-\beta)u''''_{xttt}(x_i,t_j)+$$

$$+\frac{h\tau^3}{6}(\mu-\beta)u''''_{xttt}(x_i,t_j)+\frac{\tau^4}{24}(\beta+\mu)u''''_{tttt}(x_i,t_j)+\ldots.$$

Oxirgi tenglikda quyidagilarni bajarilishini talab qilamiz:

$$\begin{cases}\alpha+\beta+\gamma+\mu=0\\ \beta+\gamma+\mu=\dfrac{1}{\tau}\\ \beta+\mu=-\dfrac{2}{h^2}\\ \mu-\beta=0\end{cases}.$$

Bu tenglamalar sistemasini yechib, noma'lum α,β,γ,μ parametrlar qiymatini aniqlaymiz:

$$\alpha=-\frac{1}{\tau},\quad \beta=\mu=-\frac{1}{h^2},\quad \gamma=\frac{1}{\tau}+\frac{1}{h^2}.$$

Parametrlarning ushbu qiymatlarida

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=\left(\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}\right)_{(x_i,t_j)}-0{,}5\tau u''_{tt}(x_i,t_j)+\frac{\tau^2}{6}u'''_{ttt}(x_i,t_j)-$$

$$-\frac{h^2}{12}u''''_{xxxx}(x_i,t_j)-\frac{\tau^2}{2}u''''_{xxxx}(x_i,t_j)+-\frac{h^2\tau^4}{12}u''''_{tttt}(x_i,t_j)+\ldots.. \qquad (2.13')$$

α,β,γ,μ parametrlarning topilgan qiymatlarini (2.13) tenglamaga qo'ysak, biz qurgan ayirmali sxemaning ko'rinishi kelib chiqadi:

$$\frac{u_i^{j+1}-u_i^j}{\tau}-\frac{u_{i-1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1}}{h^2}=\frac{\partial u(x_i,t_j)}{\partial t}-\frac{\partial^2 u(x_i,t_j)}{\partial t^2}+O(\tau,h^2).$$

Demak, $Ш(x_i,t_j)=\{(x_i,t_j)\ \ (x_{i-1},t_{j+1})\ \ (x_i,t_{j+1})\ \ (x_{i+1},t_{j+1})\}$ shablonda qurilgan ayirmali sxema oshkormas bo'lib, berilgan differensial masalani τ bo'yicha birinchi tartib bilan va h bo'yicha ikkinchi tartib bilan approksimatsiya qilar ekan.

Chegaraviy shartlarni ayirmali approksimatsiya qilish.

Faraz qilaylik, $D=\{0\le x\le 1\,,\,0\le t\le T\}$ sohada quyidagi differensial masala berilgan bo'lsin:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ 0\le x\le 1\,,\ 0\le t\le T\,, \tag{2.14}$$

$$u(x,0)=\mu(x)\,,\ u_x(0,t)=\mu_1(t)\,,\ u_x(1,t)=\mu_2(t)$$

Bu yerda $u_x(0,t)=\mu_1(t)$ chegaraviy shartni

$$\frac{y_1^{j+1}-y_0^{j+1}}{h}=\mu_1(t_{j+1})$$

ayirmali tenglama bilan approksimatsiya qilish mumkin. Malumki, ushbu ayirmali hosila berilgan chegaraviy shartni $O(h)$ bilan approksimatsiya qiladi. Bu esa masalani yechishdagi umumiy aniqlikni pasayishiga olib keladi. Bunday holatdan chiqish uchun chegaraviy shartlarni ayirmali approksimatsiya qilish usullari bilan tanishamiz.

Fiktiv nuqtalar usuli. $0\le x\le 1$ kesma tashqarisida $x_{-1}=x_0-h$ tugun nuqta kiritamiz va ushbu x_{-1} nuqtada ham berilgan (2.14) tenglama o'rinli deb hisoblaymiz. $i=0$ nuqtada (2.14) tenglamani approksimatsiya qiluvchi ayirmali tenglamani yozamiz.

$$\frac{y_0^{j+1}-y_0^j}{\tau}=\frac{y_{-1}^j-2y_0^j+y_1^j}{h^2} \tag{2.15}$$

Chap chegaraviy shartni markaziy ayirmali hosila bilan approksimatsiya qilamiz.

$$\frac{y_1^j-y_{-1}^j}{2h}=\mu_1(t_j)\,. \tag{2.16}$$

Endi (2.16) dan $y_{-1}^j=y_1^j-2h\mu_1$ ni aniqlab, uni (2.15) tenglikga qo'ysak va soddalashtirsak,

$$\frac{y_1^j - y_0^j}{h} = \mu_1\left(t_j\right) + \frac{h}{2\tau}\left(y_0^{j+1} - y_0^j\right) \qquad (2.17)$$

ayirmali tenglamaga ega bo'lish mumkin. Bu tenglamadan y_0^{j+1} ni oshkor holda aniqlash mumkin.

Approksimatsiya xatoligini kamaytirish usuli.

$u(x_1,t)$ funksiyani (x_0,t) nuqta atrofida Teylor qatoriga yoyamiz:

$$u(x_1,t) = u(x_0,t) + hu_x(x_0,t_j) + \frac{h^2}{2}u_{xx} + ...$$

Chegaraviy shartga ko'ra $u_x(0,t) = \mu_1(t)$ va $u_{xx} = u_t$ ekanligini hisobga olib, ularni Teylor qatoriga qo'yamiz:

$$u(x_1,t) = u(x_0,t) + h\mu_1(t) + \frac{h^2}{2}u_t(x_0,t) + ...$$

Bu yerda $u_t \approx y_0^{j+1} - y_0^j / \tau$ ekanligini hisobga olsak, yana (2.17) chegaraviy shartga ega bo'lish mumkin. O'ng chegaraviy shartga nisbatan ham bayon etilgan amallarni qo'llash mumkin.

Misol. (3.1.1)-(3.1.2) ko'chish tenglamasini approksimatsiya qiluvchi (qarang §3.5) (3.1.3) Laks sxemasini tadqiq qilishdan tushunarliki, vaqt bo'yicha τ qadamni $\tau = O(h)$ kabi tanlansa mazkur sxema turg'un bo'ladi. Vaqt bo'yicha τ qadamni $\tau = O(h^2)$ kabi tanlansa ham sxema turg'un bo'ladi. Agar $h \to 0$ da τ ni

$$\tau = h^2 / \mu, \quad \mu = const > 0 \qquad (2.1.1)$$

ko'rinishda olinsa (3.1.3) Laks sxemasi (3.1.1)-(3.1.2) ko'chirish tenglamasini approksimatsiya qiladimi?

Yechish. Dastlab Laks sxemasi approksimatsiya xatoligining bosh hadini topamiz. Buning uchun (3.1.3) tenglikka quyidagi Teylor qatoriga $(jh, n\tau)$ nuqta atrofida yoyilgan quyidagi ifodalarni qo'yamiz:

$$u_j^{n+1} = u_j^n + \tau u_t + \frac{\tau^2}{2} u_{tt} + O(\tau^3);\ u_{j\pm1}^n = u_j^n \pm \tau u_x + \frac{h^2}{2} u_{xx} + O(h^3).$$

Natijada Laks sxemasining birinchi differensial yaqinlashishini topamiz:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = -\frac{\tau}{2}\frac{\partial^2 u}{\partial t^2} + \frac{h^2}{2\tau}\frac{\partial^2 u}{\partial x^2}. \qquad (2.1.2)$$

Ushbu tenglikka τ uchun yozilgan (2.1.1) ifodani qo'yamiz va quyidagi tenglikni hosil qilamiz:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = -\frac{h^2}{2\mu}\frac{\partial^2 u}{\partial t^2} + \frac{\mu}{2}\frac{\partial^2 u}{\partial x^2}. \qquad (2.1.3)$$

(2.1.3.) tenglikdan ko'rinadiki, $h \to 0$ da Laks sxemasi (3.1.1) tenglamani emas, balki

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \frac{\mu}{2}\frac{\partial^2 u}{\partial x^2}$$

parabolik tenglamani approksimatsiya qiladi. Laks sxemasi shartli approksimatsiya qiluvchi sxemaga misol bo'ladi.

Approksimatsiya xatoligi.

L, L_h- aniqlanish sohalari mos holda Φ va Φ_h, qiymatlar sohalari mos holda F va F_h bo'lgan operatorlar bo'lsin. Bundan keyin L, L_h operatorlarni mos holda differenial va ayirmali operatorlar deb ataymiz.

Ayirmali L_h operator differensial L operatorni $n-$tartib bilan approksimatsiya qiladi deyiladi, agarda shunday musbat $\tilde{h}$ va C doimiylari mavjud bo'lsaki, barcha $h < \tilde{h}$ lar uchun quyidagi tengsizlik o'rinli bo'lsa

$$\left\| L_h(u)_h - (Lu)_h \right\|_{F_h} \le Ch^n.$$

L_h operator L operatorni x_i nuqtada n chi tartib bilan approksimatsiya qiladi deyiladi, agarda shunday $\tilde{h}$ va C doimiylari mavjud bo'lib, barcha $h \le \tilde{h}$ lar uchun

$$\left| (L_h(u)_h - (Lu)_h)_{x=x_i} \right| \le Ch^n$$

tengsizlik o'rinli bo'lsa.

Quyidagi

$$Lu = f\,,\ u \in \Phi\,,\ f \in F\,,$$

$$lu = g\,,\ g \in G\,, \qquad (3.1)$$

differensial masala berilgan bo'lsin.

$$L_h\varphi^h = f^h\,,\ \varphi^h \in \Phi_h\,,\ f^h \in F_h\,,$$

$$l_h\varphi^h = g^h\,,\ g^h \in G_h \qquad (3.2)$$

ayirmali sxemalar oilasini qaraylik. Bu ayirmali masalalar to'plamini kelgusida ayirmali sxemalar, ayirmali masalalar yechimlari to'plamini ayirmali sxemalar yechimi deb ataymiz.

(3.2) ayirmali sxema berilgan (3.1) differensial masalani n chi tartib bilan approksimatsiya qiladi deyiladi, agarda shunday $\tilde{h}$, C_1 va C_2 musbat doimiylari mavjud bo'lsaki, barcha $h < \tilde{h}$ lar uchun

$$\left\| L_h(u)_h - f^h \right\|_{F_h} \le C_1 h^{n_1}\,,$$

$$\left\| l_h(u)_h - g^h \right\|_{G_h} \le C_2 h^{n_2}\,,$$

$$n = \min(n_1, n_2)$$

tengsizliklar o'rinli bo'lsa,

$\psi_h = L_h(u)_h - (Lu)_h$ to'r funksiya ayirmali approksimatsiya xatoligi deyiladi. Berilgan (3.1) differensial masala va (3.2) ayirmali masalalar yechimlari ayirmasi $Z^h = \varphi^h - u$ (3.2) ayirmali sxemaning xatoligi deyiladi.

1-misol. $Lu_{\bar{x}x} = \dfrac{u_{x,i} - u_{\bar{x},i}}{h} = \dfrac{u_{i-1} - 2u_i + u_{i+1}}{h^2}$

ayirmali operator $Lu = \dfrac{d^2u}{dx^2}$ operatorni $x = x_i$ nuqtada h bo'yicha ikkinchi tartib bilan approksimatsiya qilishini ko'rsatish mumkin. Buning uchun $u_{\bar{x}x,i}$ ikkinchi tartibli ayirmali hosiladagi u_{i-1} va u_{i+1} larni Teylor qatoriga yoysak

$$u_{\bar{x}x,i} - u''(x_i) = \frac{h^2}{12} u^{IV}(x_i) + O(h^4)$$

ekanligi tasdiqlanadi.

2-misol. $Lu = u^{IV}(x)$ differensial operatorni $L_h u = u_{\bar{x}x\bar{x}x}$ ayirmali operator bilan $(x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2})$ shablonda approksimatsiya qilish mumkin.

$$L_h u = \frac{1}{h^2}\left[u_{\bar{x}x}(x_{i+1}) - 2u_{\bar{x}x}(x_i) + u_{\bar{x}x}(x_{i-1})\right] =$$

$$= \frac{1}{h^4}\left(u_{i+2} - 4u_{i+1} + 6u_i - 4u_{i-1} + u_{i-2}\right) \qquad (*)$$

(*) dagi u_{i-2}, u_{i-1}, u_i, u_{i-2} larni $x = x_i$ nuqtada Teylor qatoriga yoysak $u_{\bar{x}x\bar{x}x,i} - u^{IV}(x_i) = \frac{h^2}{6} u^{IV}(x_i) + O(h^4)$, yani ayirmali operator L_h L operatorni ikkinchi tartib bilan approksimatsiya qiladi.

Diskretlashtirish. Kelishilganlik.

Diskretlashtirish. Xususiy hosilali differensial tenglama (tenglamalar sistemasi) ni algebraik tenglamalar sistemasiga keltirish uchun bir necha variantlardan birini tanlash mumkin. Eng ko’p qo’llanadigan usullar chekli ayirmali usullar, chekli elementlar usuli va spektral usul bo’lib hisoblanadi.

Diskretlashtirishda bu usullardan birini tanlash berilgan differensial tenglamada (tenglamalar sistemasida) vaqt bo’yicha hosila qatnashishi yoki qatnashmasligiga bog’liq.

Vaqt bo’yicha hosila qatnashgan hollarda chekli ayirmali usuldan foydalanadi. Faqatgina fazoviy koordinatalar bo’yicha diskretlashtirishda chekli ayirmali usuldan tashqari chekli elementlar usuli, spektral usul yoki chekli hajmlar usulini qo’llash mumkin.

Kelishilganlik. Diskretlashitirish natijasida hosil bo’lgan algebraik tenglamalar sistemasi berilgan xususiy hosilali differensial tenglama (tenglamalar sistemasi) bilan kelishilgan deyiladi, agarda to’r yacheykalari o’lchamlari nolga intilganda algebraik tenglamalar sistemasi to’rning har bir tugun nuqtasida berilgan xususiy hosilali differensial tenglamaga ekvivalent bo'lsa.

Ayirmali masalaning yechimi differensial masala yechimiga yaqinlashish uchun kelishilganlik sharti bajarilishi zarur. Ammo, bu yetarli emas, chunki to'r yacheykalari o'lchamlari nolga intilganda algebraik tenglamalar sistemasi berilgan differensial tenglamaga ekvivalent bo'lsada, algebraik tenglamalar sistemasi yechimi berilgan differensial tenglama yechimiga intilishi kelib chiqmasligi mumkin. Misol sifatida shartli turg'un ayirmali sxemalarni keltirish mumkin. Agar turg'unlik sharti buzilsa, algebraik tenglamalar sistemasi berilgan differensial tenglamaga ekvivalent bo'lsada, taqribiy yechim uzoqlashuvchi bo'ladi.

Misol. Quyidagi chegaraviy masala berilgan bo'lsin.

$$\frac{\partial \overline{T}}{\partial t} = \alpha \frac{\partial^2 \overline{T}}{\partial x^2},\ 0 < x < 1,\ 0 < t \le t_{max} \tag{3.3}$$

$$\overline{T}(0,t) = b, \overline{T}(1,t) = d, \tag{3.4}$$

$$\overline{T}(x,0) = T_0(x), 0 \le x \le 1. \tag{3.5}$$

Bu yerda T $-$berilgan differensial masalaning aniq yechimini bildiradi.

(3.3) tenglamani diskretlashtirish uchun hosilalarni ularga ekvivalent bo'lgan chekli ayirmali ifodalar bilan almashtirish mumkin.

$$\frac{\overline{T}_i^{n+1} - \overline{T}_i^n}{\Delta t} = \frac{\alpha(\overline{T}_{i-1}^n - 2\overline{T}_i^n + \overline{T}_{i+1}^n)}{\Delta x^2} \tag{3.6}$$

(3.6) da Δt va Δx lar mos holda vaqt bo'yicha va fazoviy koordinata x bo'yicha to'r qadamlaridir. $T_i^n - T$ ning (i,n) tugun nuqtadagi qiymatiga mos keladi.

(3.6) ni quyidagicha yozish mumkin:

$$T_i^{n+1} = T_i^n + \frac{\alpha \Delta t}{\Delta x^2}(T_{i-1}^n - 2T_i^n + T_{i+1}^n) \tag{3.7}$$

agar $\frac{\partial^2 \widetilde{T}}{\partial x^2}$ hosila $n+1$ vaqt qatlamida diskretlashtirilsa, u holda oshkormas ayirmali sxemaga ega bo'lish mumkin:

$$sT_{i-1}^{n+1} - (1+2s)T_i^{n+1} + sT_{i+1}^{n+1} = -T_i^n, \tag{3.8}$$

bu yerda $s = \alpha \Delta t / \Delta x^2$.

Shunday qilib, (3.3) differensial tenglamani diskretlashtirishda quyidagi oshkor va oshkormas

$$T_i^{n+1} = sT_{i-1}^n + (1-2s)T_i^n + sT_{i+1}^n \qquad (3.9)$$

$$sT_{i-1}^{n+1} - (1+2s)T_i^{n+1} + sT_{i+1}^{n+1} = -T_i^n \qquad (3.10)$$

ayirmali sxemalarga ega bo'ldik.

I. (3.9) oshkor ayirmali sxema uchun kelishilganlik shartini tekshirish uchun bu tenglamaga berilgan differensial tenglamani (i,n) tugun nuqtadagi aniq yechimini anglatuvchi $\overline{T}_i^n$ ni qo'yamiz.

$$\overline{T}_i^{n+1} = s\overline{T}_{i-1}^n + (1-2s)\overline{T}_i^n + s\overline{T}_{i+1}^n \qquad (3.11)$$

Endi (3.11) tenglamani berilgan differensial tenlama (3.3) ga mosligini (x_i, t_n) tugun nuqtada qanchalik yaqinligini aniqlashimiz zarur. (3.11) tenglamadagi ayrim hadlarni (x_i, t_n) nuqta atrofida Teylor qatoriga yoyib, soddalashtirsak quyidagi munosabatga ega bo'lish mumkin:

$$\left[\frac{\partial \overline{T}}{\partial t}\right]_i^n - \alpha \left[\frac{\partial^2 \overline{T}}{\partial t^2}\right]_i^n + \mathrm{E}_i^n = 0, \qquad (3.12)$$

bu yerda

$$\mathrm{E}_i^n = 0{,}5\Delta t \left[\frac{\partial^2 \overline{T}}{\partial t^2}\right]_i^n - \alpha\left(\frac{\Delta x^2}{12}\right)\left[\frac{\partial^4 \overline{T}}{\partial t^4}\right]_i^n + O(\Delta t^2, \Delta t^4)$$

(3.13)

ko'rinib turibdiki, (3.13) differensial tenglama (3.3) differensial tenglamadan approksimatsiya xatoligi deb ataluvchi E_i^n qo'shimcha had bilan farq qilib turibdi. Ushbu qo'shimcha hadning paydo bo'lishi esa $\frac{\partial^2 \overline{T}}{\partial t^2}$ va $\frac{\partial^2 \overline{T}}{\partial x^2}$ hosilalarni diskretlashtirish natijasi bilan bog'liq. (3.12) da to'r yacheykalari o'lchamlari

$(\Delta x^2, \Delta t)$ kichik qilib tanlansa, approksimatsiya xatoligi E_i^n fiksirlangan qandaydir (x_i, t_n) nuqtada nolga intiladi. $\Delta x \to 0$, $\Delta t \to 0$ dagi limitda (3.9) tenglama (3.3) differensial tenglamaga ekvivalent bo'lib qoladi. Bu xossa esa kelishilganlik deyiladi. (3.3) differensial tenglamaga asosan quyidagi munosabatlar o'rinli:

$$\frac{\partial^2 \overline{T}}{\partial t^2} = \alpha \frac{\partial}{\partial t} \frac{\partial^2 \overline{T}}{\partial x^2} = \alpha \frac{\partial^2}{\partial x^2} \frac{\partial \overline{T}}{\partial x} = \alpha^2 \frac{\partial^4 \overline{T}}{\partial x^4} \tag{3.14}$$

Shu sababli approksimatsiya xatoligi E_i^n ifodasini quyidagicha qayta yozish mumkin:

$$\mathrm{E}_i^n = 0{,}5\Delta x^2 (s - \frac{1}{6}) \left[\frac{\partial^4 \overline{T}}{\partial t^4} \right]_i^n + O(\Delta t^2, \Delta t^4) \tag{3.15}$$

$s = \frac{1}{6}$ bo'lsa, (3.15) dagi birinchi had nolga teng bo'ladi va approksimatsiya xatoligi $O(\Delta t^2, \Delta t^4)$ bo'ladi.

II. Endi (3.10) oshkormas ayirmali sxemani berilgan (3.3) differensial tenglama bilan kelishilganligini tekshiramiz.

(3.10) tenglamaga (3.3) differensial tenglamani (x_i, t_n) tugun nuqtadagi aniq yechimini anglatuvchi T_i^n ni qo'yamiz:

$$\frac{\overline{T}_i^{n+1} - \overline{T}_i^n}{\Delta t} - \alpha \frac{\overline{T}_{i-1}^{n+1} - 2\overline{T}_i^{n+1} + \overline{T}_{i+1}^{n+1}}{\Delta x^2} = 0. \tag{3.16}$$

(3.16) dagi $\overline{T}_{i-1}^{n+1}$ va $\overline{T}_{i+1}^{n+1}$ larni $(i, n+1)$ tugun nuqta atrofida Teylor qatoriga yoyamiz:

$$\frac{\overline{T}_i^{n+1} - \overline{T}_i^n}{\Delta t} - \alpha \left\{ \left[\overline{T}_{xx} \right]_i^n + (\frac{\Delta x^2}{12}) \left[\overline{T}_{x^2} \right]_i^n + (\frac{\Delta x^4}{360}) \left[\overline{T}_{x^4} \right]_i^{n+1} + \ldots \right\} = 0.$$

Endi oxirgi munosabatdagi $\overline{T}_i^{n+1}$, $\left[\overline{T}_{xx} \right]_i^{n+1}$ va hokozolarni (x_i, t_n) nuqta atrofida Teylor qatoriga yoysak, quyidagiga ega bo'lamiz:

$$\left[\overline{T}_t \right]_i^n - 0{,}5\Delta t \left[\overline{T}_{tt} \right]_i^n + \frac{\Delta t^2}{6} \left[\overline{T}_{t^3} \right]_i^n + \ldots - \alpha \left[\overline{T}_{xx} \right]_i^n +$$

$$\Delta t \left[\overline{T}_{xxt} \right]_i^n + 0{,}5\Delta t^2 \left[\overline{T}_{xxtt} \right]_i^n + \ldots +$$

$$+\frac{\Delta x^2}{12}([\bar{T}_{x^4}]_i^n+\Delta t[\bar{T}_{x^4t}]_i^n+...)+\frac{\Delta x^4}{360}([\bar{T}_{x^4}]_i^n+...)...=0 \qquad (3.17)$$

Agar $\bar{T}_t=\alpha\bar{T}_{xx}$, $\bar{T}_{tt}=\alpha^2\bar{T}_{x^4}$, $\bar{T}_{ttt}=\alpha^3\bar{T}_{x^6}$, $s=\frac{\alpha\Delta t}{\Delta x^2}$, $\Delta x^2=\alpha\Delta t/s$

tengliklardan foydalansak, (3.17) tenglama quyidagi ko’rinishga keladi:

$$[\bar{T}_t-\alpha\bar{T}_{xx}]_i^n+E_i^n=0. \qquad (3.18)$$

Bu yerda approksimatsiya xatoligi

$$\mathrm{E}_i^n=-0,5\Delta t(1+\frac{1}{6s})[T_{tt}]_i^n+\frac{\Delta t^2}{3}(1+\frac{1}{4s}+\frac{1}{120s^2})[\bar{T}_{tttt}]_i^n+... \qquad (3.19)$$

Ko’rinib turibdiki $\Delta t\to 0$ da $E_i^n\to 0$, (3.18) tenglama berilgan (3.3) differensial tenglama bilan ustma-ust tushadi. Bu esa (3.10) oshkormas ayirmali sxema (3.3) differensial tenglama bilan kelishilganligini bildiradi. (3.19) ni (3.13) bilan solishtirib, shuni aytish mumkinki oshkormas ayirmali sxemada $O(\Delta x^4)$ tartib bilan taminlovchi s ni (3.19) dan topish mumkin emas.

Turg’unlik.

Xususiy hosilali differensial tenglamalarni dikretlashtirishda hosil bo’ladigan algebraik tenglamalar sistemasini yechishda $(x_i,t_n),(i=\overline{1,N_1}),n=\overline{1,N_2}$ tugun nuqtadagi xatolikni ξ_i^n bilan belgilaymiz.

$$\xi_i^n=T_i^n-{}^*T_i^n \qquad (3.20)$$

(3.20) da T_i^n , $*T_i^n$ lar mos xolda algebraik tenglamalar sistemasining aniq va taqribiy yechimlaridir.

Diskretlashtirishda hosil bo’ladigan chiziqli algebraik tenglamalar sistemasining xatoliklari ξ_i^n ham xuddi shu chiziqli algebraik tenglamalar sistemasini qanoatlantiradi. Masalan, (3.9) oshkor ayirmali sxemadan foydalansak, yuqoridagi fikrimiz $*T_i^{n+1}$

$$*T_i^{n+1}=s*T_i^{n+1}+(1-2s)*T_i^n+s*T_{i+1}^n \qquad (3.21)$$

tenglamani qanoatlantirishini anglatadi.

Agar algebraik tenglamalar sistemasini aniq yechimi T_i^n ham (3.9) tenglamani qanoatlantirishini eslasak va (3.21), tenglamaga (3.20) ni hisobga olsak ξ_i^n xatolikka nisbatan quyidagi bir jinsli algebraik tenglamaga ega bo'lamiz:

$$\xi_i^{n+1} = s\xi_{i-1}^n + (1-2s)\xi_i^n + s\xi_{i+1}^n \qquad (3.22)$$

Agar boshlang'ich va chegaraviy shartlar berilgan deb faraz qilsak, u holda barcha boshlang'ich xatoliklar ξ_i^0 $(i=\overline{1,N_1-1})$, shuningdek chegaraviy xatoliklar ξ_0^n va $\xi_{N_1}^n (n=0,...N_2)$ lar (3.20) tenglikka asosan nolga teng bo'ladi.

Ayirmali sxemalar turg'unligini tahlil qiluvchi matrisali usul va Neyman usullari eng ko'p qo'llaniladigan usullardir. Ushbu usullar asosida hisoblash algoritmining haqiqiy yechimi va taqribiy yechimi o'rtasidagi farq yoki xatolikni o'sishi yoki kamayishini bashorat qilish yotadi.

Misol. $\dfrac{\partial u}{\partial t} - \dfrac{\partial u}{\partial x} = \varphi(x,t), -\infty < x < +\infty, 0 \le t \le 1,$ tenglamaga qo'yilgan $u(x,0) = \psi(x), -\infty < x < +\infty$ Koshi masalasini aproksimasiya qiluvchi

$$L_h u^{(h)} = \begin{cases} \dfrac{u_m^{n+1} - u_m^n}{\tau} - \dfrac{u_m^n - u_{m-1}^n}{h}, \\ u_m^0, \\ m = 0, \pm 1, \pm 2, ..., n = 0, 1, 2, ..., \end{cases} \qquad f^{(h)} = \begin{cases} \varphi(x_m, t_n), \\ \psi(x_m), m = 0, \pm 1, \pm 2, ..., \\ n = 0, 1, 2, ..., \end{cases}$$

ayirmali sxemaning turg'unligini tekshiring.

Yechish. Dastlab sxemani $u_m^{n+1} = r\, u_{m+1}^n + (1+r)\, u_m^n, \quad r = \tau / h$ ko'rinishda yozib olamiz va quyidagi normalarni aniqlaymiz:

$\left\| u^{(h)} \right\|_{U_h} = \max\limits_{m,n} \left| u_m^n \right|$, $\left\| f^{(h)} \right\|_{F_h} = \max\limits_{m} \left| \psi(x_m) \right| + \max\limits_{m,n} \left| \varphi(x_m, t_n) \right|$. Agar $r \le 1$ bo'lsa

$$\left| u_m^{n+1} \right| \le \left| r\, u_{m+1}^n + (1-r)\, u_m^n + \tau\, \varphi(x_m, t_n) \right| \le r \left| u_{m+1}^n \right| + (1-r) \left| u_m^n \right| + \tau \left| \varphi(x_m, t_n) \right|$$

bo'ladi. Demak

$$\left|u_m^{n+1}\right| \le \max_m \left|u_m^n\right| + \tau \max_{m,n} \left|\varphi(x_m, t_n)\right|.$$

Hosil bo'lgan

$$\max_m \left|u_m^0\right| = \max_m \left|\psi(x_m)\right|,$$

$$\left|u_m^1\right| \le \max_m \left|u_m^0\right| + \tau \max_{m,n} \left|\varphi(x_m, t_n)\right|,$$

$$\left|u_m^2\right| \le \max_m \left|u_m^2\right| + \tau \max_{m,n} \left|\varphi(x_m, t_n)\right|,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots,$$

$$\left|u_m^n\right| \le \max_m \left|u_m^{n-1}\right| + \tau \max_{m,n} \left|\varphi(x_m, t_n)\right|$$

tengsizliklarni qo'shib, $\max_m \left|u_m^n\right| \le \max_m \left|\psi(x_m)\right| + \tau\, n \max_{m,n} \left|\varphi(x_m, t_n)\right|$ tengsizlikni hosil qilamiz. Natijada

$$\max_{m,n} \left|u_m^n\right| \le \max_m \left|\psi(x_m)\right| + \tau\, N \max_{m,n} \left|\varphi(x_m, t_n)\right| \le K\left(\max_m \left|\psi(x_m)\right| + \max_{m,n} \left|\varphi(x_m, t_n)\right|\right)$$

ekanligi tushunarli, bu yerda $K = \max(1, T),\ T = \tau\, N.$ Shunday qilib $r \le 1$ bo'lganda ayirmali sxema turg'un va differensial masalani approksimatsiya qiladi. Demak, Laks teoremasiga asosan ayirmali masalaning yechimi differensial masala yechimiga yaqinlashadi.

Oshkor ayirmali sxema turg'unligini tekshirish uchun matrisali usulni qo'llash.

Matrisali usulni mohiyati xatoliklarni aniqlovchi tenglamalar sistemasini matrisa ko'rinishiga keltiriladi. Undan so'ng mos matrisaning xos qiymatlarini aniqlash orqali turg'unlik tahlil qilinadi.

Ushbu usulni (3.9) oshkor ayirmali sxemaga nisbatan qo'llashni ko'rib o'tamiz.

(3.22) da $i = \overline{1, N_1 - 1}$ larni qo'yib, chegaraviy xatoliklar barcha n lar uchun $\xi_0^n = \xi_{N_1}^n = 0$ ekanligini hisobga olib quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\xi_1^{n+1} = (1 - 2s)\xi_1^n + s\xi_2^n$$

$$\xi_2^{n+1} = s\xi_1^n + (1-2s)\xi_2^n + s\xi_3^n$$

$$\dots\dots\dots\dots\dots\dots\dots\dots \qquad (3.23)$$

$$\xi_i^{n+1} = s\xi_{i-1}^n + (1-2s)\xi_i^n + s\xi_{i+1}^n$$

$$\xi_{N_{11}-1}^{n+1} = s\xi_{N_1-2}^n + (1-2s)\xi_{N_1}^n + s\xi_{i+1}^n$$

bu tenglamalar sistemasini esa quyidagi matrisali ko'rinishda yozish mumkin:

$$\xi^{n+1} = A\xi^n, \quad n = 0,1,..., \qquad (3.24)$$

bu yerda A - $N_1 - 1$ tartibli kvadrat matrisa , ξ^n esa $N_1 - 1$ ta elementdan iborat ustun vektor.

Ularning ko'rinishi qo'yidagicha:

$$A = \begin{bmatrix} (1-2s) & s & & & \\ s & (1-2s) & s & & \\ & s & & s & \\ \dots & \dots & \dots & \dots & \dots \\ & & & s & (1-2s) \end{bmatrix}, \quad \xi^n = \begin{bmatrix} \xi_1^n \\ . \\ . \\ . \\ \xi_i^n \\ . \\ . \\ . \\ \xi_{N_1-1} \end{bmatrix}.$$

Agar A matrisaning xos qiymatlari λ_m lar har xil va absolyut qiymatlari birdan kichik yoki teng bo'lsa, yani

$$|\lambda_m| \le 1 \qquad (3.25)$$

barcha m lar uchun bajarilsa n ning ortishi bilan ξ^n xatoliklar chegaralanganligini ko'rsatish mumkin.

Ilmiy manbalardan ma'lumki r diagonalli A matrisaning xos qiymatlari quyidagicha aniqlanadi:

$$\lambda_m = 1 - 4s\sin^2\left(\frac{m\pi}{2(N_1-1)}\right), \quad m = \overline{1, N_1 - 1} \qquad (3.26)$$

(3.25) turg'unlik sharti bo'lib, faqatgina quyidagi tengsizlikini qanoatlantiruvchi s ni qiymatidan foydalanishni taqozo etadi!

$$-1 \le 1 - 4s\sin^2\left(\frac{m\pi}{2(N_1-1)}\right) \le 1 \qquad (3.27)$$

Ushbu tengsizlikning o'ng qismi barcha m va s larda bajariladi. Ammo, tengsizlikning chap qismi bajarilishi uchun

$$s \cdot \sin^2\left(\frac{m\pi}{2(N_1-1)}\right) \le \frac{1}{2},$$

bo'lishi zarur. Bu esa $s \le \frac{1}{2}$ bo'lganda barcha m lar uchun bajariladi.

Yuqoridagi fikrdan (3.9) oshkor ayirmali sxema $s = \frac{\alpha \Delta t}{\Delta x^2} \le \frac{1}{2}$ da turg'un ekan.

IV.AMALIY MASHG'ULOT MATERIALLARI

1-amaliy mashg'ulot :Chiziqli giperbolik tenglamalarni uzilishga ega bo'lgan boshlang'ich shartlar bilan yechish

Differensial masalaning qo'yilishi

Bir o'lchovli ko'chish tenglamasi uchun Koshi masalasini qaraymiz

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0, \qquad a = const,\ a > 0,\ \ x \in (-\infty, +\infty), t \in (0, T]$$

бошлангич функцияси финит булган $u(x,0)$: (1)

$u(x,0) = 0$ агар $x \in (-\infty, l_1) \cup (l_2, +\infty)$ булса $u(x,0) = u_0(x)$ агар $x \in [l_1, l_2]$,

$$u_0(x)=\begin{cases}\frac{1}{l_2-l_1}(x-l_1), & \text{"чап учбурчак" - 1 шакл,}\\ 1, & \text{"тугрибурчакли учбурчак" - 2 шакл,}\\ 0.5\left(1-\cos\left(\frac{2\pi}{l_2-l_1}(x-l_1)\right)\right), & \text{"косинус" - 3 шакл ,}\\ \begin{cases}-\frac{2}{3(l_{11}-l_1)}(x-l_1)+1, & x\in[l_1,l_{11})\\ \frac{1}{3}, & x\in[l_{11},l_{22}]\\ \frac{2}{3(l_2-l_{22})}(x-l_2)+1, & x\in(l_{22},l_2]\end{cases}, & \text{"тишсимон" - 4 шакл,}\\ \begin{cases}-\frac{2}{3(l_{12}-l_1)}(x-l_1)+1, & x\in[l_1,l_{12})\\ \frac{2}{3(l_2-l_{12})}(x-l_2)+1, & x\in[l_{12},l_2]\end{cases}, & \text{"M" - 5 шакл,}\\ \frac{1}{l_2-l_1}(l_2-x), & \text{"унг учбурчак" - 6 шакл.}\end{cases}$$

(1) tenglama uchun Koshi masalasining aniq yechimi [4] ga asosan quyidagicha bo‘ladi:

$$u(x,t)=\begin{cases}0, & x-at<l_1\\ u_0(x-at), & l_1\le x-at\le l_2.\\ 0, & x-at>l_2\end{cases} \qquad (2)$$

Ushbu yechimdan kelgusida ayirmali sxemalarning aniqligini tekshirish uchun foydalaniladi. Bundan tashqari taqribiy yechimning C, L_1, L_2, W_2^1 fazodagi $\Omega=(-\infty,+\infty)*[0,T]$:

$$\|f\|_C=\max_{\Omega}|f|,\quad \|f\|_{L_1}=\int_0^T\int_{-\infty}^{+\infty}|f|dxdt,\quad \|f\|_{L_2}=\left(\int_0^T\int_{-\infty}^{+\infty}f^2dxdt\right)^{1/2},\quad \|f\|_{W_2^1}=\left(\int_0^T\int_{-\infty}^{+\infty}(f_x')^2dxdt\right)^{1/2};$$

$R=(-\infty,+\infty)$, $t=T$ dagi normalarning ayirmali analoglarida xatolik baholanadi.

Masalani sonli yechish uchun (x,t) sohada $\omega_{h\tau}=\omega_h\times\omega_\tau$, $\omega_h=\{x_i=ih,\ i=0,1,2,...\}$, $\omega_\tau=\{t_j=j\tau,\ j=0,1,2,...\}$ tekis to‘r quramiz va $\gamma=\frac{a\tau}{h}$ Kurant sonini belgilaymiz.

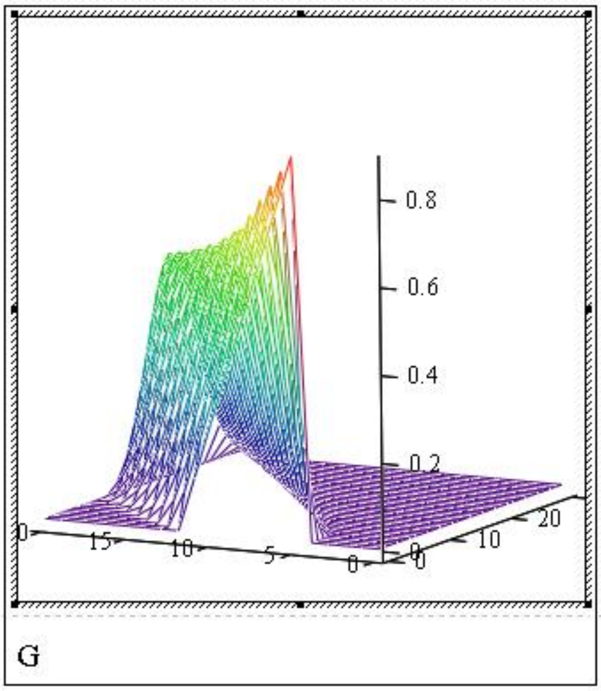
1-misol. Chap ayirmali sxema.

$$y_i^{n+1}=(1-\gamma)y_i^n+\gamma y_{i-1}^n \qquad (3)$$

Sxema vaqt va fazoviy o‘zgaruvchi x bo‘yicha birinchi tartibli approksimatsiyaga ega. $\gamma\le 1$ da ayirmali sxema turg‘un va $\gamma=1$ da sxema aniq yechimni beradi. (1) tenglama va boshlang‘ich shartlar bilan berilgan masalani (3) ayirmali sxema yordamida sonli yechish uchun Mathcad tizimida dastur tuzamiz:

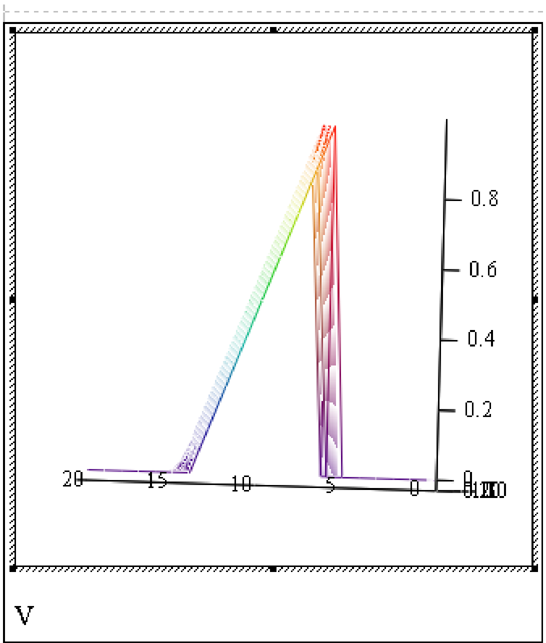
```
M := 30      N := 20

n := 0..M     j := 0..N     h := 1/N          τ := 1/M

x_j := j·h     t_n := τ·n
                    l1 := 0.2        l2 := 0.6

G := | for j ∈ 0..N
     |    u_{0,j} ← | 1/(l2 − l1)·(l2 − x_j)   if l1 < x_j < l2
     |              | 0  otherwise
     | for n ∈ 0..M − 1
     |   for j ∈ 1..N − 1
     |     | for j ∈ 1..N − 1
     |     |
     |     |   u_{n+1,j} ← u_{n,j} − (τ/h)(u_{n,j} − u_{n,j−1})
     |     |
     |     | u
     | u
```

1 – rasm. Mathcad tizimidagi dastur.



2 – rasm. Taqribiy yechim.

1-rasmda Mathcad tizimida chap ayirmali sxema uchun tuzilgan. 2-rasmada esa sonli yechimning grafigi tasvirlangan.



3-rasm.Aniq yechim.

G – V =

	14	15	16	17	18	19
12	$-151.923\cdot10^{-9}$	$29.620\cdot10^{-3}$	$6.057\cdot10^{-3}$	$886.504\cdot10^{-6}$	$95.146\cdot10^{-6}$	7.581·10
13	$-372.050\cdot10^{-9}$	$34.312\cdot10^{-3}$	$7.628\cdot10^{-3}$	$1.231\cdot10^{-3}$	$147.904\cdot10^{-6}$	13.419·10
14	$-817.732\cdot10^{-9}$	$39.247\cdot10^{-3}$	$9.407\cdot10^{-3}$	$1.658\cdot10^{-3}$	$220.124\cdot10^{-6}$	22.384·10
15	$-1.651\cdot10^{-6}$	$44.408\cdot10^{-3}$	$11.396\cdot10^{-3}$	$2.174\cdot10^{-3}$	$315.959\cdot10^{-6}$	35.567·10
16	$-3.110\cdot10^{-6}$	$41.447\cdot10^{-3}$	$13.597\cdot10^{-3}$	$2.789\cdot10^{-3}$	$439.845\cdot10^{-6}$	54.260·10
17	$-5.535\cdot10^{-6}$	$38.684\cdot10^{-3}$	$16.009\cdot10^{-3}$	$3.510\cdot10^{-3}$	$596.459\cdot10^{-6}$	79.965·10
18	$-9.387\cdot10^{-6}$	$36.105\cdot10^{-3}$	$18.632\cdot10^{-3}$	$4.343\cdot10^{-3}$	$790.667\cdot10^{-6}$	114.398·10
19	$-15.278\cdot10^{-6}$	$33.697\cdot10^{-3}$	$21.463\cdot10^{-3}$	$5.295\cdot10^{-3}$	$1.027\cdot10^{-3}$	159.483·10
20	$-23.994\cdot10^{-6}$	$31.450\cdot10^{-3}$	$24.501\cdot10^{-3}$	$6.373\cdot10^{-3}$	$1.312\cdot10^{-3}$	217.349·10
21	$-36.528\cdot10^{-6}$	$29.351\cdot10^{-3}$	$27.742\cdot10^{-3}$	$7.582\cdot10^{-3}$	$1.649\cdot10^{-3}$	290.327·10
22	$-54.098\cdot10^{-6}$	$27.392\cdot10^{-3}$	$31.183\cdot10^{-3}$	$8.926\cdot10^{-3}$	$2.045\cdot10^{-3}$	380.934·10
23	$-78.182\cdot10^{-6}$	$25.562\cdot10^{-3}$	$34.819\cdot10^{-3}$	$10.410\cdot10^{-3}$	$2.504\cdot10^{-3}$	491.868·10
24	$-110.541\cdot10^{-6}$	$23.853\cdot10^{-3}$	$38.646\cdot10^{-3}$	$12.037\cdot10^{-3}$	$3.031\cdot10^{-3}$	625.988·10
25	$-153.240\cdot10^{-6}$	$22.255\cdot10^{-3}$	$42.660\cdot10^{-3}$	$13.811\cdot10^{-3}$	$3.631\cdot10^{-3}$	786.304·10
26	$-208.669\cdot10^{-6}$	$20.762\cdot10^{-3}$	$46.855\cdot10^{-3}$	$15.734\cdot10^{-3}$	$4.310\cdot10^{-3}$	975.960·10
27	$-279.568\cdot10^{-6}$	$19.364\cdot10^{-3}$	$51.227\cdot10^{-3}$	$17.809\cdot10^{-3}$	$5.071\cdot10^{-3}$	1.198·10
28	$-369.033\cdot10^{-6}$	$18.054\cdot10^{-3}$	$55.769\cdot10^{-3}$	$20.037\cdot10^{-3}$	$5.921\cdot10^{-3}$	1.456·10
29	$-480.536\cdot10^{-6}$	$16.826\cdot10^{-3}$	$60.477\cdot10^{-3}$	$22.419\cdot10^{-3}$	$6.862\cdot10^{-3}$	1.754·10
30	$-617.930\cdot10^{-6}$	$15.672\cdot10^{-3}$	$65.345\cdot10^{-3}$	$24.956\cdot10^{-3}$	$7.899\cdot10^{-3}$	2.095·10

4-rasm. Xatolik.

Mustaqil yechish uchun misollar.

1. Quyidagi ayirmali sxemalar yordamida (1) differensial tenglamaning $u_0(x)$ boshlang‘ich shartlarni qanoatlantiruvchi taqribiy yechimlarini toping. Ayirmali sxema approksimatsiya tartibini va taqribiy yechim xatoligini aniqlang.

1.1. Lax-Wendroff sxemasi

$$y_i^{n+1} = y_i^n - \gamma\left(F_p - F_l\right), \text{ бунда}$$

$$F_p = 0.5\left(y_{i+1}^n + y_i^n\right) - 0.5\gamma\left(y_{i+1}^n - y_i^n\right), F_l = 0.5\left(y_i^n + y_{i-1}^n\right) - 0.5\gamma\left(y_i^n - y_{i-1}^n\right)$$

1.2. Markaziy ayirmali sxema

$$y_i^{n+1} = y_i^n + 0.5\gamma(y_{i+1}^n - y_{i-1}^n) .$$

1.3. Lax ayirmali sxemasi

$$y_i^{n+1} = 0.5\left(y_{i+1}^n + y_{i-1}^n\right) - 0.5\gamma\left(y_{i+1}^n - y_{i-1}^n\right).$$

1.4. SHASTA oqim korreksiyali sxema

$$y_i^{n+1} = \tilde{y}_i^n - \tilde{F}_p + \tilde{F}_m, \text{ , бунда}$$

$$\tilde{y}_i^n = y_i^n - \frac{1}{2}\left[\gamma\left(y_{i+1}^n + y_i^n\right) - \gamma(y_i^n + y_{i-1}^n)\right] + \left[\nu\left(y_{i+1}^n - y_i^n\right) - \nu(y_i^n - y_{i-1}^n)\right],$$

$$F_p = \frac{1}{8}(\tilde{y}_{i+1} - \tilde{y}_i), \tilde{F}_p = S \cdot \max\left\{0, \min\left[S \cdot (\tilde{y}_{i+2} - \tilde{y}_{i+1}), \left|F_p\right|, S \cdot (\tilde{y}_i - \tilde{y}_{i-1})\right]\right\},$$

$$S = sign\left(\tilde{y}_{i+1} - \tilde{y}_i\right), \nu = \gamma^2/2 + 1/8$$

1.5. Cheklagichli sxema

$$y_i^{n+1} = y_i^n - \gamma\left(y_i^n - y_{i-1}^n\right) - 0.5\gamma\left(\alpha_{i+1/2}\left(y_{i+1}^n - y_i^n\right) - \alpha_{i-1/2}\left(y_i^n - y_{i-1}^n\right)\right), \textit{бунда}$$

$$\alpha_{i+\frac{1}{2}} = \alpha\left(R_{i+\frac{1}{2}}\right), R_{i+1/2} = \frac{y_i^n - y_{i-1}^n}{y_{i+1}^n - y_i^n},$$

$$\alpha(R) = \begin{cases} 0, & R \le 0, \\ \dfrac{(a+b(1-\delta))R}{(a+b)(1-\delta)}, & 0 < R < 1-\delta, \\ \dfrac{a+bR}{a+b}, & |R-1| \le \delta, \\ \dfrac{(a+b(1-\delta))R - 2a\delta}{(a+b)(1-\delta)}, & 1+\delta < R < 2, \\ 2, & R \ge 2 \end{cases}$$

$0 < \delta < 1, a = const, b = const, a + b \ne 0$ sxem ikkinchi va $a = 1/3, b = 2/3$- bo‘lganda uchinchi tartibli approksimatsiyaga ega.

1.5. V.V.Rusanov sxemasi

$$y_i^{n+1} = y_i^n - \tilde{a}y_{\overset{0}{x}} - \tilde{\mu}y_{\overset{0}{x}\overset{0}{x}} - \tilde{\nu}y_{\bar{x}\bar{x}xx} + \tilde{\beta}y_{\bar{x}\overset{0}{x}x}, \text{ бунда}$$

$$\tilde{a} = \left(\beta_{30} + \beta_{32}\right)a, \quad \tilde{\mu} = \beta_{21}\beta_{32}\tilde{a}^2\tau, \quad \tilde{\nu} = \omega_{32}\frac{h^4}{\tau}, \quad \tilde{\beta} = \beta_{32}\beta_{21}\tilde{a}^3\tau^2 + \beta_{30}\theta_{31}\tilde{a}h^2,$$

$$\beta_{10} = 1/3, \beta_{21} = 2/3, \beta_{30} = 1/4, \beta_{32} = 3/4, \theta_{31} = -2/3$$

Sxemaning turg‘unligi ω_{32} parametrning tanlanishiga bog‘liq. $\omega_{32} = -0.125$ sxema aniq yechimni beradi.

1.6. Oshkor ayirmali sxema

$$y_i^{n+1} = \left(1 - \frac{3}{2}\gamma\right) y_i^n + \gamma\left(2y_{i-1}^n - \frac{1}{2}y_{i-2}^n\right).$$

1.7. Crank-Nicolson ayirmali sxemasi

$$y_i^{n+1} = y_i^n - 0.25\gamma\left(y_{i+1}^n - y_{i-1}^n + y_{i+1}^{n+1} - y_{i-1}^{n+1}\right).$$

1.8. Oshkormas chap ayirmali sxema

$$y_i^{n+1} = \frac{\gamma}{1+\gamma} y_{i-1}^{n+1} + \frac{1}{1+\gamma} y_i^n \ .$$

1.9, 10 Oshkormas Lax-Wendroff sxemasi(σ = 1; σ = 0.5).

$$y^{n+1} = y^n - 0.5\sigma\gamma y_{\overset{0}{x}}^{n+1} - 0.5(1-\sigma)\gamma\, y_{\overset{0}{x}}^n + 0.5\sigma\gamma^2 y_{\bar{x}x}^{n+1} + 0.5(1-\sigma)\gamma^2 y_{\bar{x}x}^n \quad \text{бунда}$$

$$y_{\overset{0}{x}} = y_{i+1} - y_{i-1}, \ y_{\bar{x}x} = y_{i-1} - 2y_i + y_{i+1}$$

1.11. «Kabare» ayirmali sxemasi

$$y_i^{n+1} = (1-2\gamma)(y_i^n - y_{i-1}^n) + y_{i-1}^{n-1}.$$

Birinchi qatlamda yechimni chap ayirmali sxema bilan topiladi.

1.12. Monotonlashtiruvchilarli «Kabare» ayirmali sxemasi

$$\tilde{y}_i = (1-\gamma)y_i^n + \gamma y_{i-1}^n - \tau Q_{i-1} + \alpha G_i^n + (1-\alpha)G_{i-1}^n,$$

$$y_i^{(\max)} = \max\left(y_i^n,\ y_{i-1}^n\right),\ y_i^{(\min)} = \min\left(y_i^n,\ y_{i-1}^n\right),$$

$$y_i^{n+1} = \begin{cases} y_i^{(\max)}, \text{если } \tilde{y}_i > y_i^{(\max)} \\ \tilde{y}_i \\ y_i^{(\min)}, \text{если } \tilde{y}_i < y_i^{(\min)} \end{cases},$$

$$G_i^1 = 0,\ G_i^{n+1} = \tilde{y}_i - y_i^{n+1}, Q_{i-1} = \frac{y_{i-1}^n - y_{i-1}^{n-1}}{\tau} + a\frac{y_i^n - y_{i-1}^n}{h}.$$

1.13. Oshkormas ayirmali sxema

$$y_{i+1}^{n+1} = \frac{2-\gamma}{\gamma} y_i^n + y_{i-1}^n - \frac{2-\gamma}{\gamma} y_i^{n+1}.$$

1.14. Oshkormas ayirmali sxema

$$y_i^{n+1} = y_i^n + \frac{\gamma}{2+\gamma} y_{i-1}^{n+1} - \frac{\gamma}{2+\gamma} y_{i+1}^n.$$

1.15, 16. Cheklagichli ayirmali sxemalar

$$y_i^{n+1} = y_i^n - 0.5\gamma(1+\gamma)(y_i^n - y_{i-1}^n) - 0.5\gamma(1+\gamma)(y_{i+1}^n - y_i^n) + 0.5\gamma\left(\alpha_{i+1/2}(y_{i+1}^n - y_i^n) - \alpha_{i-1/2}(y_i^n - y_{i-1}^n)\right),$$

$$\alpha_{i+\frac{1}{2}} = \alpha_{*0}\left(R_{i+\frac{1}{2}}\right), R_{i+1/2} = \frac{y_i^n - y_{i-1}^n}{y_{i+1}^n - y_i^n},\ * = 2, 3,$$

$$\alpha_{20}(R) = \begin{cases} 1-\gamma, & R \le 0, \\ 1-\gamma - \frac{2R}{\gamma}(1-\gamma), & 0 < R \le \frac{\gamma}{2-\gamma}, \\ R-\gamma, & \frac{\gamma}{2-\gamma} < R \le \gamma, \\ 0, & \gamma < R \le R^*, \\ -\frac{2}{\gamma}(1-\gamma)(R-R^*), & R^* < R \le R^* + \frac{\gamma(1+\gamma)}{2(1-\gamma)}, \\ -1-\gamma, & R > R^* + \frac{\gamma(1+\gamma)}{2(1-\gamma)}. \end{cases}$$

Fazoviy o‘zgaruvchi bo‘yicha approksimatsiyani ko‘tarish uchun cheklagichni boshqa ko‘rinishda yozish mumkin. Sun’iy diffuziya koeffitsientini quyidagicha olish mumkin:

$$\alpha_{30}(R) = \begin{cases} 1-\gamma, & R \le 0, \\ 1-\gamma - \frac{2R}{\gamma}(1-\gamma), & 0 < R \le \frac{\gamma}{2-\gamma}, \\ R-\gamma, & \frac{\gamma}{2-\gamma} < R \le \frac{1+3\gamma}{4}, \\ \frac{1-R}{3}, & \frac{1+3\gamma}{4} < R \le R^*, \\ -\frac{2}{\gamma}(1-\gamma)(R-R^*) + \frac{1-R^*}{3}, & R^* < R \le R^{**}, \text{где } R^{**} = \frac{(6-7\gamma)R^* + \gamma(3\gamma+4)}{6(1-\gamma)} \\ -1-\gamma, & R > R^{**}. \end{cases}$$

Ushbu sxema Lax-Wendroff sxemasiga cheklagichli monotonlashtiruvchi kiritish yo‘li bilan hosil qilingan.

1.17. R. P. Fedorenko ayirmali sxemasi

$$y_i^{n+1} = y_i^n - \gamma(y_i^n - y_{i-1}^n) - 0.5\sigma\gamma(\gamma - \gamma^2)(y_{i+1}^n - 2y_i^n + y_{i-1}^n), \text{ бунда}$$

$$\sigma = \begin{cases} 1, \text{ агар } & |y_{i+1}^n - 2y_i^n + y_{i-1}^n| < \lambda|y_i^n - y_{i-1}^n| \\ 0 \text{ - акс холда.} \end{cases}$$

λ=0 da birinchi tartibli chap ayirmali sxemasiga aylanadi. $\lambda = \infty$da ikkinchi tartibli Laksa-Vendroff sxemasiga aylanadi.

1.18. Silliqlashtiruvchili sxema

$$y_i^{n+1}=\gamma y_{i-1}^n+(1-\gamma)y_i^n+\Delta_2 f(\xi),\quad \text{бунда}$$

$$f(\xi)=\begin{cases}1, & \text{агар } \xi\le 1\\ 1/\xi, & \text{агар } \xi<1\end{cases},\quad \xi=\left|\Delta_2/\Delta_1\right|$$

$$\Delta_1=\left(y_i^n-y_{i-1}^n\right)\min(\gamma,1-\gamma),\ \Delta_2=(1-\gamma)y_{i-1}^n-\frac{1-\gamma}{1+\gamma}y_{i-1}^{n+1}-\gamma\frac{(1-\gamma)}{1+\gamma}y_i^n$$

1.19. K. I. Babenko («kvadrat») sxemasi

$$y_{i+1}^{n+1}=y_i^n-\frac{1-\gamma}{1+\gamma}\left(y_i^{n+1}-y_{i+1}^n\right).$$

1.19. K. I. Babenkoning cheklagich shaklidagi korreksiya sxemasi

K. I. Babenko s korreksiey tipa «limitera» [11]

Regulyarizovannaya sxema K. I. Babenko budet viglyadet tak (dlya a=1):

$$y_i^{n+1}=y_i^n+\left(-2\gamma\tilde{y}_{\bar{x}}+(1-\gamma)(\mu_{-1}-1)\tilde{y}_{-1,t}\right)/\left(1+\gamma+(1-\gamma)\mu\right),\ \text{где}$$

$$\tilde{y}_{\bar{x}}=y_i^n-y_{i-1}^n,\ \tilde{y}_{-1,t}=y_{i-1}^{n+1}-y_{i-1}^n$$

$$R_{-1}=\frac{y_{\bar{x}}}{y_{-1,t}}=\gamma\frac{y_i^n-y_{i-1}^n}{y_{i-1}^{n+1}-y_{i-1}^n},\ R=\gamma\frac{y_{i+1}^n-y_i^n}{y_i^{n+1}-y_i^n},\ \mu_{-1}=\mu(R_{-1}),\ \mu=\mu(R),$$

$$\mu=\mu(R)=\begin{cases}-1, & R\le -R^*-\dfrac{1-\gamma}{2}\\[2mm] \dfrac{2(R+R^*)}{1-\gamma}, & -R^*-\dfrac{1-\gamma}{2}<R\le -R^*\\[2mm] 0, & -R^*<R\le -\dfrac{1-\gamma}{2}\\[2mm] 1+\dfrac{2R}{1-\gamma}, & -\dfrac{1-\gamma}{2}<R\le 0\\[2mm] 1, & R>0\end{cases}$$

2-amaliy mashg‘ulot: Kvazichiziqli giperbolik tenglamani sonli yechish
Masalaning qo‘yilishi: Qvazichiziqli giperbolik tenglamalar uchun Riman masalasini qaraylik

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}(\mathbf{U})}{\partial x}=\mathbf{0},\quad A=\frac{\partial \mathbf{F}}{\partial \mathbf{U}}=\Omega_R\Lambda\Omega_L \qquad (1)$$

Bu yerda

$$\mathbf{U}=\mathbf{U}(t,x)=[U_1,\ldots,U_n]^T,\ \mathbf{F}(\mathbf{U})=[F_1,\ldots,F_n]^T,\ t\ge 0,\ -\infty<x<\infty.$$

Ω_R va Ω_L - matritsalar mos ravishda A matritsaning o‘ng va chap xos vektorlaridan iborat matritsa. Boshlang‘ich sharti

$$U_0(x)=f(x)$$

Riman masalasi uchun oshkor Godunov sxemasi

$$\frac{\mathbf{U}_i^{k+1}-\mathbf{U}_i^{k}}{\Delta t}+\frac{\mathbf{F}_{i+1/2}-\mathbf{F}_{i-1/2}}{\Delta x}=\mathbf{0}, \qquad \mathbf{F}_{i\pm 1/2}=\mathbf{F}(\mathbf{U}_{i\pm 1/2}). \tag{2}$$

Bu sxemanida $F(U)=\frac{U^2}{2}$ qaraylik.

(1) tenglama va boshlang‘ich shartlar bilan berilgan masalani (2) ayirmali sxema yordamida sonli yechish uchun Mathcad tizimida dastur tuzamiz:

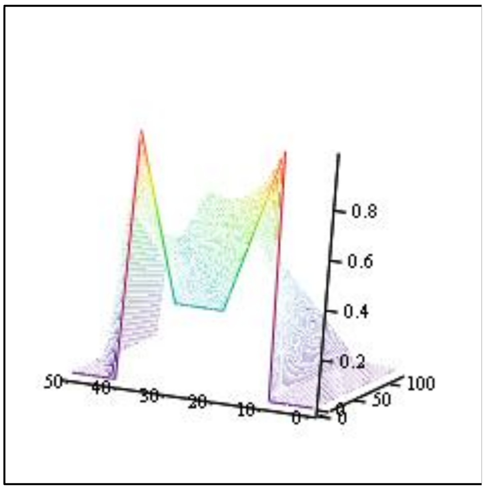
$M := 120 \qquad N := 50$

$h := \frac{1}{N} \qquad \tau := \frac{1}{M}$

$n := 0..M \qquad j := 0..N \qquad l1 := 0.2 \qquad l11 := 0.4 \qquad f(v) := \frac{v^2}{2}$

$x_j := j\cdot h \qquad t_n := \tau\cdot n \qquad l2 := 0.8 \qquad l22 := 0.6$

```
G := | for j ∈ 0..N
     |   u[0,j] ← | −2/(3·(l11 − l1))·(x[j] − l1) + 1  if l1 ≤ x[j] < l11
     |            | 1/3  if l11 ≤ x[j] ≤ l22
     |            | 2/(3·(l2 − l22))·(x[j] − l2) + 1  if l22 < x[j] ≤ l2
     |            | 0  otherwise
     | for n ∈ 0..M − 1
     |   | u[n+1,0] ← ((u[n,1] + u[n,0])/2)
     |   | for j ∈ 1..N − 1
     |   |   for j ∈ 1..N − 1
     |   |     u[n+1,j] ← u[n,j] − τ/h·(f(u[n,j]) − f(u[n,j−1]))
     | u
```



G

1 – rasm. Mathcad tizimidagi dastur.

2 – rasm. Taqribiy yechim.

1-rasmda Mathcad tizimida Riman masalasi uchun Godunov sxemasining dasturi va 2-rasmda esa sonli yechimning grafigi tasvirlangan.

3-amaliy mashg‘ulot: Xususiy differensial tenglamalar uchun chegaraviy masalani yechishda sonli usullardan foydalanish. Elliptik turdagi differensial tenglamalarni ayirmali tenglamalar bilan approksimatsiya qilish.

Ishdan maqsad: Maple, Mathematica, Mathcad, MatLab tizimlarda Xususiy differensial tenglamalar uchun chegaraviy masalani yechishda sonli usullardan foydalanish. Elliptik turdagi differensial tenglamalarni ayirmali tenglamalar bilan approksimatsiya qilish.

Masalaning qo‘yilishi: Xususiy differensial tenglamalar uchun chegaraviy masalani yechishda sonli usullardan foydalanish. Elliptik turdagi differensial tenglamalarni ayirmali tenglamalar bilan approksimatsiya qilish.

Chekli ayirmali sxemalar. Ayirmali approksimatsiya.

Misol(Laplas operatorining ayirmali approksimatsiyasi)

Faraz kilamiz D:= {(x,y) : $a \le x \le b \quad c \le y \le d$ } soxada u=f(x,y) funksiyasi berilgan bulsin. Laplas operatorining ayirmali approksi-

matsiyasini kurish va uning xatoligini xisoblash talab kilingan bulsin. Shu maksadda misol sifatida $a := 0 \; b := 1 \; c := 0 \; d := 1$ deb olib D soxani x - yunalishi buyicha $n := 10$ bulakga $hx := \frac{b-a}{n}$ kadam bilan va y - yunalishi buyicha $m := 10$ bulakga $hy := \frac{b-a}{m}$ kadam bilan bulib

$x := a, a+hx..b \quad y := c, c+hy..d$ tugun nuktalarni xosil kilamiz. Funksiya misoli sifatida $u(x,y) := \sin\left[\left(x^2+y^2\right)\cdot 10\right]$ funksiyani olamiz. Laplas operatoridagi ikkinchi tartibli xususiy xosilalarni ayirmali munosabatlar bilan almashtirib approksimatsiya xatoligi funksiyalari kiymatlarini barcha tugun nuktalarda xisoblaymiz.

$$Rx(x,y) := \frac{u(x+hx,y) - 2\cdot u(x,y) + u(x-hx,y)}{hx^2} - \frac{d^2}{dx^2}u(x,y)$$

$$Ry(x,y) := \frac{u(x,y+hy) - 2\cdot u(x,y) + u(x,y-hy)}{hx^2} - \frac{d^2}{dy^2}u(x,y)$$

$$R(x,y) := Rx(x,y) + Ry(x,y)$$

$$i := 1,2..n-1 \quad j := 1,2..m-1 \quad S_{i,j} := R(i\cdot hx,\, j\cdot hy)$$

$$\max(S) = 136.593$$

Mustaqil bajarish uchun topshiriqlar

ABCD kvadratda $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Laplas tenglamasiga qo‘yilgan Dirhle masalsini yeching. A(0;0), B(0;1),C(1;1),D(1;0) va h=0,25;

Variant nomeri	$u\|_{AB}$	$u\|_{BC}$	$u\|_{CD}$	$u\|_{AD}$
1	$30y$	$30(1-x^2)$	0	0
2	$30y$	$30\cos\frac{\pi x}{2}$	$30\cos\frac{\pi y}{2}$	$30x^2$
3	$50y(1-y^2)$	0	0	$50\sin\pi x$
4	$20y$	20	$20y^2$	$50x(1-x)$
5	0	$50x(1-x)$	$50y(1-y^2)$	$50x(1-x)$
6	$30\sin\pi y$	$30x$	$30y$	$30x(1-x)$
7	$30(1-y)$	$30\sqrt{x}$	$30y$	$30(1-x)$
8	$50\sin\pi y$	$50\sqrt{x}$	$50y^2$	$50\sin\pi x$
9	$40y^2$	40	40	$40\sin\frac{\pi x}{2}$
10	$50y^2$	$50(1-x)$	0	$60x(1-x^2)$
11	$20y^2$	20	$20y$	$10x(1-x)$
12	$40\sqrt{y}$	$40(1-x)$	$20y(1-y)$	0
13	$20\cos\frac{\pi y}{2}$	$30x(1-x)$	$30y(1-y^2)$	$20(1-x^2)$
14	$30y^2(1-y)$	$50\sin\pi x$	0	$10x^2(1-x)$
15	$20y$	$20(1-x^2)$	$30\sqrt{y}(1-y)$	0
16	$30(1-y^2)$	$30x$	30	30
17	$30\cos\frac{\pi y}{2}$	$30x^2$	$30y$	$30\cos\frac{\pi x}{2}$
18	0	$50\sin\pi x$	$50y(1-y^2)$	0
19	$20\sqrt{y}$	20	$20y^2$	$40x(1-x)$
20	$50y(1-y)$	$20x^2(1-x^2)$	0	$40x(1-x^2)$
21	$20\sin\pi y$	$30x$	$30y^2$	$20x(1-x)$
22	$40(1-y)$	$30\sqrt{x}$	$30y$	$40(1-x)$
23	$20\sin\pi y$	$50\sqrt{x}$	$50y^2$	$20\sin\pi x$
24	40	40	$40y^2$	$40\sin\frac{\pi}{2}(1-x)$
25	$30y^2$	$30(1-x)$	0	$40x^2(1-x)$
26	$25y^2$	25	$25y$	$20x(1-x)$
27	$15\sqrt{y}$	$15(1-x)$	$30y(1-y)$	0
28	$30\cos\frac{\pi y}{2}$	$20x(1-x)$	$20y(1-y^2)$	$30(1-x^2)$
29	$10y^2(1-y)$	$30\sin\pi x$	0	$15x(1-x^2)$
30	$15y$	$25(1-x^2)$	$30\sqrt{y}(1-y)$	0

4-amaliy mashg‘ulot: Oshkor va oshkor emas ayirmali sxemalar.

Ishdan maqsad: Maple, Mathematica, Mathcad, MatLab tizimlarda Giperbolik va parabolik turdagi tenglamalarni to‘r usuli bilan yechish. Oshkor va oshkor emas ayirmali sxemalar.

Masalaning qo‘yilishi: Giperbolik va parabolik turdagi tenglamalarni to‘r usuli bilan yechish. Oshkor va oshkor emas ayirmali sxemalar.

{0<x<l , 0<t<T} soxada

$$\frac{d}{dx}u = \frac{d^2}{dx^2}u + f(x,t)$$

Issiklik utkazish tenglamasini kanoatlantiruvchi va t=0 da

$u(x,o) = u0(x)$ $\qquad 0 \le x \le 1$

xamda x=0 va x=l da

$u(0,t) = \mu1(t)$ $\qquad u(l,t) = \mu2(t)$

chegaraviy shartlarni kanoatlantiruvchi u(x,t) funksiya topilsin. Aniklik uchun $l := 1$ $\quad T := 1$ $\quad f(x,t) := -1$ $\quad u0(x) := x^2$ $\quad \mu1(t) := t$ $\quad \mu2(t) := t + 1^2$ deb olamiz. Oshkor sxemani kurish maksadida soxani x- yun alishi buyicha $N := 5$ bulakga bulib tugun nuktalarni $h := \frac{1}{N}$ $i := 0..N$ $\; x_i := i \cdot h$ xisoblaymiz. Oshkor sxema shartli turgun bulganligi sababli $r := 0.5$ $\quad (r < 0.5)$ turgunlik koeffitsientini kiritamiz. Unda t- yunalishi buyicha bulinish kadami $\tau := r \cdot h^2$ formula yerdamida xisoblanadi va bulakchalar soni $K := \mathrm{ceil}\left(\frac{T}{\tau}\right)$ orkali, tugun nuktalar $n := 0..K$ $\quad t_n := n \cdot \tau$ formulalar yerdamida xisoblanadi.Ushbu tugun nuktalarda anik yechim $u_{i,n} := t_n + (x_i)^2$ yerdamida xisoblanadi. Oshkor sxemada takribiy yechim katlam buyicha topiladi.

```
y := | for i ∈ 0..N
     |   y[i,0] ← u0(x[i])
     | for n ∈ 0..K
     |   | y[0,n] ← μ1(t[n])
     |   | y[N,n] ← μ2(t[n])
     | for n ∈ 0..K − 1
     |   | a ← r
     |   | c ← 1 + 2·r
     |   | b ← r
     |   | α[1] ← 0
     |   | β[1] ← μ1(t[n+1])
     |   | for j ∈ 1..N − 1
     |   |   | α[j+1] ← b / (c − α[j]·a)
     |   |   | β[j+1] ← (a·β[j] + y[j,n] + τ·f(x[j], t[n+1])) / (c − α[j]·a)
     |   | y[N,n+1] ← μ2(t[n+1])
     |   | for j ∈ N − 1..1
     |   |   y[j,n+1] ← α[j+1]·y[j+1,n+1] + β[j+1]
     | y
```

Mustaqil bajarish uchun topshiriqlar

$G=\{0\le x\le 1;\ 0<t<0,5\}$ sohada

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\quad u(x,0)=f(x),\ \left.\frac{\partial u}{\partial t}\right|_{t=0}=\Phi(x),\ u(0,t)=\varphi(t),\ u(1,t)=\psi(t)$$

Differensial masalani to‘r metodi bilan / bo‘lganda oshkor sxemadan foydalanib yeching.

№ 1. $f(x)=x(x+1),\ \Phi(x)=\cos x,\ \varphi(t)=0,\ \psi(t)=2(t+1)$.

№ 2. $f(x)=x\cos\pi x,\ \Phi(x)=x(2-x),\ \varphi(t)=2t,\ \psi(t)=-1$.

№ 3. $f(x)=\cos\frac{\pi x}{2},\ \Phi(x)=x^2,\ \varphi(t)=1+2t,\ \psi(t)=0$.

№ 4. $f(x)=(x+0,5)(x-1)$, $\Phi(x)=\sin(x+0,2)$, $\varphi(t)=t-0,5$, $\psi(t)=3t$.

№ 5. $f(x)=2x(x+1)+0,3$, $\Phi(x)=2\sin x$, $\varphi(t)=0,3$, $\psi(t)=4,3+t$.

№ 6. $f(x)=(x+0,2)\sin\frac{\pi x}{2}$, $\Phi(x)=1+x^2$, $\varphi(t)=0$, $\psi(t)=1,2(t+1)$.

№ 7. $f(x)=x\sin\pi x$, $\Phi(x)=(x+1)^2$, $\varphi(t)=2t$, $\psi(t)=0$.

№ 8. $f(x)=3x(1-x)$, $\Phi(x)=\cos(x+0,5)$, $\varphi(t)=2t$, $\psi(t)=0$.

№ 9. $f(x)=x(2x-0,5)$, $\Phi(x)=\cos 2x$, $\varphi(t)=t^2$, $\psi(t)=1,5$.

№ 10. $f(x)=(x+1)\sin\pi x$, $\Phi(x)=x^2+x$, $\varphi(t)=0$, $\psi(t)=0,5t$.

№ 11. $f(x)=(1-x)\cos\frac{\pi x}{2}$, $\Phi(x)=2x+1$, $\varphi(t)=2t+1$, $\psi(t)=0$.

№ 12. $f(x)=0,5x(x+1)$, $\Phi(x)=x\cos x$, $\varphi(t)=2t^2$, $\psi(t)=1$.

№ 13. $f(x)=0,5(x^2+1)$, $\Phi(x)=x\sin 2x$, $\varphi(t)=0,5+3t$, $\psi(t)=1$.

№ 14. $f(x)=(x+1)\sin\frac{\pi x}{2}$, $\Phi(x)=1-x^2$, $\varphi(t)=0,5t$, $\psi(t)=2$.

№ 15. $f(x)=x^2\cos\pi x$, $\Phi(x)=x^2(x+1)$, $\varphi(t)=0,5t$, $\psi(t)=t-1$.

№ 16. $f(x)=(1-x^2)\cos\pi x$, $\Phi(x)=2x+0,6$, $\varphi(t)=1+0,4t$, $\psi(t)\ \ 0$.

№ 17. $f(x)=(x+0,5)^2$, $\Phi(x)=(x+1)\sin x$, $\varphi(t)=0,5(0,5+t)$, $\psi(t)=2,25$.

№ 18. $f(x)=1,2x-x^2$, $\Phi(x)=(x+0,6)\sin x$, $\varphi(t)=0$, $\psi(t)=0,2+0,5t$.

№ 19. $f(x)=(x+0,5)(x+1)$, $\Phi(x)=\cos(x+0,3)$, $\varphi(t)=0,5$, $\psi(t)=3-2t$.

№ 20. $f(x)=0,5(x+1)^2$, $\Phi(x)=(x+0,5)\cos\pi x$, $\varphi(t)=0,5$, $\psi(t)=2-3t$.

№ 21. $f(x)=(x+0,4)\sin\pi x$, $\Phi(x)=(x+1)^2$, $\varphi(t)=0,5t$, $\psi(t)=0$.

№ 22. $f(x)=(2-x)\sin\pi x$, $\Phi(x)=(x+0,6)^2$, $\varphi(t)=0,5t$, $\psi(t)=0$.

№ 23. $f(x)=x\cos\frac{\pi x}{2}$, $\Phi(x)=2x^2$, $\varphi(t)=0$, $\psi(t)=t^2$.

№ 24. $f(x)=(x+0,4)\cos\frac{\pi x}{2}$, $\Phi(x)=0,3(x^2+1)$, $\varphi(t)=0,4$, $\psi(t)=1,2t$.

№ 25. $f(x)=(1-x^2)+x$, $\Phi(x)=2\sin(x+0,4)$, $\varphi(t)=1$, $\psi(t)=(t+1)^2$.

№ 26. $f(x)=0,4(x+0,5)^2$, $\Phi(x)=x\sin(x+0,6)$, $\varphi(t)=0,1+0,5t$, $\psi(t)=0,9$.

№ 27. $f(x)=(x+0,5)^2\cos\pi x$, $\Phi(x)=(x+0,7)^2$, $\varphi(t)=0,5$, $\psi(t)=2t-1,5$.

№ 28. $f(x)=(x+2)(0,5x+1)$, $\Phi(x)=2\cos\left(x+\frac{\pi}{6}\right)$,

$\varphi(t)=2$, $\psi(t)=4,5-3t$.

№ 29. $f(x)=(x^2+1)(1-x)$, $\Phi(x)=1-\sin x$, $\varphi(t)=1$, $\psi(t)=0,5t$.

№ 30. $f(x)=(x+0,2)\sin\frac{\pi x}{2}$, $\Phi(x)=1+x^2$, $\varphi(t)=0,6t$, $\psi(t)=1,2$.

V. GLOSSARIY

Termin	O‘zbek tilidagi sharhi	Ingliz tilidagi sharhi
TVD (total variation dimension) to‘la variatsiyaning o‘smasligi	–funksiya to‘la variatsiyasi o‘smasligini ta’minlovchi sxema	
ENO(essentially nonoscillatory)-	o‘ta ossilyatsiyalanmaydigan	
WENO (weighted essentially nonoscillatory)-	yuqori tartibli osilyatsiyalanmaydigan sxema	
MUSCL monotone upstream schems for conservation laws -	saqlanish qonun uchun monoton chap ayirmali sxema	
Umumlashgan yechim	integral tenglama shakldа yozilgan tenglamani qanoatlantiruvchi yechim.	
To‘r	tartiblashgan nuqtalar to‘plami;	
To‘r funksiya	tartiblashgan nuqtalar to‘plamida aniqlangan funksiyaning qiymatlari;	
Iteratsion metod	yechimga ketma–ket yaqinlashish usuli;	
To‘r tenglama	to‘rning nuqtalarida noma’lumlarni qiymati qatnashadigan tenglamalar;	
Turg‘unlik	yechimning boshlang‘ich berilganlarga uzluksiz bog‘liqligi;	
Spektr radius	matritsa xos sonlarining eng kattasi;	
O‘tish matritsasi	iteratsion ketma–ketlikda bir qadamdan ikkinchi qadamga o‘tishda kupaytiriladigan matritsa	
Diskret masala	tugun nuqtalarda yechimni topish haqidagi masala	
Approksimatsiya	yaqinlashtirish.	
Gibrid sxema	masala qaralayotgan sohaning turli qismida turlicha o‘zgaradigan sxema	
Konservativ sxema	saqlanish qonunini approksimatsiya qiliuvchi sxema.	
Shock-capturing methods (skvoznogo scheta)	butun soha bo‘yicha sonli yechimni bitta sxema bo‘yicha hisoblash	
Normativ-huquqiy hujjatlar —	umummajburiy davlat ko‘rsatmalari sifatida huquqiy normalarni belgilashga, o‘zgartirishga yoki bekor qilishga qaratilgan rasmiy hujjatdir.	normative-legal documents - are official documents aimed at establishing, changing or abolishing legal norms as universal state instructions.

Qonun —	O‘zbekiston Respublikasida eng muhim va barqaror ijtimoiy munosabatlarni tartibga soladigan masalalar bo‘yicha, O‘zbekiston Respublikasi Oliy Majlisi tomonidan yoki referendum o‘tkazish yo‘li bilan qabul qilinadigan oliy yuridik kuchga ega bo‘lgan normativ hujjat.	Law — Normative document of the highest legal force, adopted by the Oliy Majlis of the Republic of Uzbekistan or by way of a referendum on the most important and stable issues of regulating social relations in the Republic of Uzbekistan
Normativ-huquqiy hujjatlar qabul qilish huquqiga ega bo‘lgan organlar yoki mansabdor shaxslar —	O‘zbekiston Respublikasi Oliy Majlisining palatalari, O‘zbekiston Respublikasining Prezidenti, O‘zbekiston Respublikasi Vazirlar Mahkamasi, vazirliklar, davlat qo‘mitalari va idoralar, mahalliy davlat hokimiyati organlari normativ-huquqiy hujjatlar qabul qilish huquqiga ega bo‘lgan organlar yoki mansabdor shaxslar hisoblanadi.	organizations or officials with the right to receive normative-legal documents — Chambers of the Oliy Majlis of the Republic of Uzbekistan, the President of the Republic of Uzbekistan, the Cabinet of Ministers of the Republic of Uzbekistan, ministries, state committees and departments, local state authorities are the persons or officials authorized to adopt normative-legal acts
qonun osti hujjatlari —	O‘zbekiston Respublikasi Prezidentining farmonlari va qarorlari, O‘zbekiston Respublikasi Vazirlar Mahkamasining qarorlari, vazirliklar, davlat qo‘mitalari va idoralarning buyruqlari hamda qarorlari, mahalliy davlat hokimiyati organlarining qarorlari	Decrees and resolutions of the President of the Republic of Uzbekistan, resolutions of the Cabinet of Ministers of the Republic of Uzbekistan, orders and resolutions of ministries, state committees and agencies, resolutions of local state authorities.
bakalavriat —	o‘rta maxsus, kasb-hunar ta’limi negizida oliy ta’lim yo‘nalishlaridan biri bo‘yicha fundamental bilimlar beradigan, o‘qish muddati to‘rt yildan kam bo‘lmagan tayanch oliy ta’lim	bachelor's degree — Basic higher education with a period of study of not less than four years, providing fundamental knowledge in one of the directions of higher education on the basis of secondary special, vocational education
magistratura —	bakalavriat negizida o‘qish muddati kamida ikki yil bo‘lgan aniq mutaxassislik bo‘yicha oliy ta’lim	master's degree — higher education in a specific specialty with a duration of study at least two years on the basis of a bachelor's degree
bakalavr, magistr —	oliy ta’limning tegishli bosqichiga muvofiq dasturlarni muvaffaqiyatli o‘zlashtirgan shaxslarga beriladigan akademik darajalar	Bachelor, Master — academic degrees awarded to persons who have successfully mastered the programs in accordance with the relevant stage of higher education

oliy ma'lumot darajasi —	shaxs tomonidan oliy ta'limning muayyan o'quv rejalari va fanlar dasturini mazkur ma'lumot haqida tegishli davlat hujjati berilgan holda, o'zlashtirishi natijasi	level of higher education — the result of a person mastering certain curricula and science programs of higher education with the issuance of the relevant state document on this information
oliy ma'lumot haqida davlat hujjati (diplom) —	akkreditatsiyadan o'tgan oliy ta'lim muassasalari bitiruvchilariga beriladigan va ularning oliy ta'limning o'quv rejalari va fanlar dasturini bajarganliklarini tasdiqlovchi davlat namunasidagi hujjat. Hujjat uzluksiz ta'limning keyingi bosqichlarida o'qishni davom ettirish yoki olingan akademik darajaga muvofiq ishlash huquqini beradi	state document on higher education (diploma) — a state-recognized document issued to graduates of accredited higher education institutions and confirming their completion of the curriculum and subject program of higher education. The document entitles the holder to continue one`s studies at the later stages of continuing education or work in accordance with the academic degree received.
oliy ta'lim yo'nalishlari va mutaxassisliklari klassifikatori —	oliy ma'lumotli kadrlar tayyorlash uchun bakalavriat ta'limi yo'nalishlari va magistratura mutaxassisliklarining tizimlashtirilgan ro'yxati	classifier of directions and specialties of higher education— a systematized list of directions of Bachelor's education and master's specialties for training of personnel with higher education.
oliy ta'limning davlat ta'lim standarti —	muayyan ta'lim sohasiga (soha tarkibiga) qo'yiladigan malaka talablari, ta'lim mazmuni, bitiruvchilar umumiy tayyorgarligining zaruriy va yetarli darajasini, kadrlar tayyorlash sifatini baholash darajalarini belgilaydigan etalon darajasi	state educational standard of higher education — qualification requirements for a particular field of education (structure of the field), the content of education, the standard level that determines the necessary and sufficient level of general training of graduates, the level of assessment of the quality of training
malaka talablari —	uzluksiz ta'lim tegishli bosqichi bitiruvchisining umumiy bilim va kasb tayyorgarligi darajasiga qo'yiladigan talablar	qualification requirements — requirements for the level of general knowledge and professional training of the graduate of the relevant stage of continuing education
o'qitishning me'yoriy muddati —	ta'lim oluvchilar tomonidan o'quv rejalari va fanlar dasturi o'zlashtirilishi uchun belgilangan muddat	normative duration of teaching — the period set by students for mastering the curriculum and science program

o‘quv fanlari bloki —	o‘quv rejalari va fanlar dasturlarining kadrlar tayyorlash jarayonida aniq maqsad va vazifalarga erishish uchun muayyan bilim sohasi yoki faoliyatning o‘zlashtirilishini ta’minlaydigan o‘quv fanlarini birlashtiruvchi tarkibiy qismi	educational block — curricula and science programs are an integral part of the curriculum, ensuring the mastery of a particular field of knowledge or activity to achieve specific goals and objectives in the process of training
o‘quv rejasi —	oliy ta’limning muayyan bakalavriat ta’lim yo‘nalishi yoki magistratura mutaxassisligi bo‘yicha o‘quv faoliyati turlari, o‘quv fanlari va kurslarining tarkibi, ularni o‘rganishning izchilligi va soatlardagi hajmini belgilaydigan hujjat	academic plan (curriculum) — a document defining the types of educational activities, the composition of academic disciplines and courses, the sequence of their study and the number of hours in a particular bachelor's or master's degree in higher education
o‘quv fani —	ta’lim muassasasida o‘rganish uchun fan, texnika, san’at, ishlab chiqarish faoliyatining muayyan sohasidan saralab olingan bilimlar, o‘quv va ko‘nikmalar tizimi	educational science — system of knowledge, training and skills selected for study in an educational institution from a specific field of science, technology, art, production activities
o‘quv semestri —	oliy ta’lim muassasasida o‘quv yilining yarmini tashkil etuvchi o‘zaro bog‘langan fanlarning ma’lum majmuini o‘zlashtirishga mo‘ljallangan va ular bo‘yicha yakuniy nazorat bilan tugallanadigan qismi	academic semester — part of a higher education institution intended for mastering a certain set of interconnected disciplines that make up half of the academic year and ending with the final control over them
o‘quv fani dasturi —	ta’lim mazmuni, uning talabalar tomonidan o‘zlashtirilishining eng maqbul usullari, axborot manbalari ko‘rsatilgan normativ hujjat	educational program — normative document indicating the content of education, the most optimal methods of its mastering by students, sources of information
malaka amaliyoti —	o‘quv jarayonining nazariy bilimlarni mustahkamlash, amaliy ko‘nikma va o‘quv hosil qilish, o‘quv rejalari va fanlar dasturlarning ma’lum (yakuniy) qismidagi mavzu bo‘yicha materiallar to‘plash uchun o‘tkaziladigan bir qismi	qualification practice — part of the educational process to consolidate theoretical knowledge, develop practical skills and curriculum, to collect materials on the topic in a particular (final) part of the curriculum and science programs

yakuniy davlat attestatsiyasi —	bakalavr yoki magistr darajasiga qo‘yiladigan malaka talablariga muvofiq holda, ma’lum talab va tartibotlar vositasida (fanlar bo‘yicha davlat attestatsiyasi, bitiruv malakaviy ishi yoki magistrlik dissertatsiyasi himoyasi) bitiruvchi tomonidan oliy ta’lim o‘quv reja va dasturlarining bajarilishi sifatini baholash	final state attestation — assessment of the quality of implementation of higher education curricula and programs by the graduate in accordance with the qualification requirements for the bachelor's or master's degree, through certain requirements and procedures (state certification in disciplines, defense of graduate work or master's dissertation)
o‘qitish sifatini nazorat qilish —	talabaning bilim saviyasini tekshirish va uning o‘quv dasturini o‘zlashtirish darajasini aniqlash	control of the quality of teaching — check the level of knowledge of the student and determine the level of mastery of his curriculum
ta’lim sifatini nazorat qilish —	o‘qitish mazmuni va natijalarining davlat ta’lim standartlari talablariga muvofiqligini tekshirish	control of the quality of education — checking that the content and results of training meet the requirements of state educational standards
oliy ta’lim muassasasi attestatsiyasi —	oliy ta’lim muassasasida kadrlar tayyorlash mazmuni, darajasi va sifatining OT DTS talablariga muvofiqligini aniqlovchi tadbir	attestation of higher education institution — an event that determines the content, level and quality of training in higher education institutions in accordance with the requirements of SES
oliy ta’lim —	uzluksiz ta’limning yuqori malakali mutaxassislar tayyorlovchi mustaqil turi. Oliy ta’lim muassasalarida amalga oshiriladi. Oliy ta’lim ikki bosqichdan iborat: bakalavriat va magistratura	higher education — an independent type of continuing education that trains highly qualified professionals. It is carried out in higher education institutions. Higher education consists of two stages: bachelor's and master's
Korrupsiya —	shaxsning o‘z mansab yoki xizmat mavqeidan shaxsiy manfaatlarini yoxud o‘zga shaxslarning manfaatlarini ko‘zlab moddiy yoki nomoddiy naf olish maqsadida qonunga xilof ravishda foydalanishi, xuddi shuningdek bunday nafni qonunga xilof ravishda taqdim etish	corruption — unlawful use of one's position or position for personal gain or material or intangible benefits for the benefit of others, as well as illegal provision of such benefits
korrupsiyaga oid huquqbuzarlik —	korrupsiya alomatlariga ega bo‘lgan, sodir etilganligi uchun qonun hujjatlarida javobgarlik nazarda tutilgan qilmish	corruption offense — an act with signs of corruption, for which the legislation provides for liability
Klassifikator —	oliy ma’lumotli kadrlar tayyorlash yo‘nalishlari va mutaxassisliklarining tizimlashtirilgan ro‘yxati.	Classifier — a systematized list of areas and specialties of higher education

Yo‘nalish —	5-bosqichning o‘quv rejalar va fanlar dasturi bo‘yicha oliy ta’lim muassasasi bitiruvchisi tomonidan egallangan va beriladigan «bakalavr» akademik darajasi doirasida kasb faoliyatining muayyan turini bajarishni ta’minlovchi bazaviy va fundamental bilimlar, uquvlar va ko‘nikmalar kompleksi.	Direction — A set of basic and fundamental knowledge, skills and abilities acquired by a graduate of a higher education institution in accordance with the curriculum and science program of the 5th stage and providing a certain type of professional activity within the "bachelor's" academic degree
Mutaxassislik —	5A-bosqichning o‘quv rejalar va fanlar dasturi bo‘yicha oliy ta’lim muassasasi bitiruvchisi tomonidan egallangan va beriladigan «magistr» akademik darajasi doirasida kasb faoliyatining muayyan turini bajarishni ta’minlovchi muayyan mutaxassislik bo‘yicha bilimlar, uquvlar va ko‘nikmalar kompleksi.	Specialty — A set of knowledge, training and skills in a specific specialty, provided by a graduate of the higher education institution in the curriculum and science program of the 5A stage, to perform a certain type of professional activity within the academic degree of "master"
Kredit —	biror fanni o‘zlashtirish uchun sarflanadigan talabaning ish hajmining o‘lchovi	Credit — a measure of a student's workload required to master a subject
kredit, kredit-soat	o‘quv ishlari hajmini o‘lchashning yagonalashtirilgan birligi	credit, credit-hour — a unified unit of measuring the volume of educational work
Registrator ofisi	fanlarga talabalarni qayd qilish va ularning o‘quv davridagi barcha o‘zlashtirish ko‘rsatkichlarini qayd qilishning markazlashtirilgan xizmat turi	registrar's office — a centralized type of service for recording students in subjects and recording all their mastery indicators during the study period
edvayzer	talabaga mutaxassislik bo‘yicha o‘quv traektoriyasini tanlash va o‘quv davridagi fanlarni o‘zlashtirish bo‘yicha yordam beruvchi mutaxassislik kafedrasi o‘qituvchisi	Adviser — Teacher of the specialty department, which helps the student to choose the educational trajectory of the specialty and master the disciplines of the study period
namunaviy o‘quv reja	o‘quv fanlarini o‘rganish ketma-ketligi va hajmini belgilovchi o‘quv rejasi	Curriculum — a curriculum that defines the sequence and scope of the study of academic subjects
ishchi o‘quv rejasi	talabalarning shaxsiy o‘quv rejalari asosida shakllantirilgan o‘quv rejasi, o‘qituvchilarning o‘quv ishlari yuklamalarini hisoblash uchun asos bo‘ladigan hujjat	the curriculum, formed on the basis of individual curricula of students, is a document that serves as a basis for calculating the workload of teachers

kredit ta'lim tizimi	o'quv jarayonini tashkil etish shakli bo'lib, talabalarga o'z o'quv traektoriyalarini muayyan chegarada belgilash imkonini beradi, mustaqil va ijodiy bilim olishni rag'batlantirishga yo'naltiriladi, o'zlashtirilgan bilimlar hajmi kreditlarda o'lchanadi	credit education system — is a form of organization of the educational process, which allows students to set their own educational trajectories within certain limits, is aimed at encouraging independent and creative learning, the amount of acquired knowledge is measured in credits
mustaqil ish	mavzular bo'yicha mustaqil ta'limga ajratilgan ish bo'lib, o'quv-uslubiy adabiyotlar va tavsiyalar bilan ta'minlanadi, testlar, nazorat ishlari, kollokviumlar, referatlar, bayon va hisobotlar shaklida nazorat qilinadi	self-study activity — is a work devoted to independent study on the subject, is provided with educational and methodical literature and recommendations, is supervised in the form of tests, control works, colloquiums, abstracts, statements and reports
tyutor	fan bo'yicha amaliy mashg'ulotlarni olib boruvchi va maslahat beruvchi, talabalarning mustaqil ishlarini tashkil qiluvchi va bajarilishini ta'minlovchi	Tutor — conducting and advising on practical training in science, organizing and ensuring the independent work of students
Forum	fan mavzulari bo'yicha telegram kanallari yoki masofaviy ta'lim platformalarida fikr almashish	Forum — exchange of views on science topics on telegram channels or distance learning platforms
kollokvium	o'quv modulining nazariy qismining o'zlashtirilishini tekshirish maqsadida suhbat uyushtirish	Colloquium — conduct a conversation to check the mastery of the theoretical part of the training module
keys-stadi	ishlab chiqarishdagi muammoli vaziyatlar bo'yicha belgilangan shakldagi topshiriqlar bo'yicha yechim izlash	case study — search for solutions to tasks in the prescribed form on problematic situations in production
kurs ishi	fan yoki fanlar majmuasi (korxonalar iqtisodiyoti, menejment asoslari, ekologiya va atrof-muhit muhofazasi, fuqaro himoyasi va h.k.) muammolari bo'yicha belgilangan uslubiy qo'llanmalar asosida bajariladigan belgilangan uslubiy qo'llanmalar asosida yoziladigan yozma va hisob ishlari	course work — written and accounting work on the basis of established methodological guidelines on the problems of science or a set of disciplines (economics of enterprises, basics of management, ecology and environmental protection, civil protection, etc.)
Edvayzer	o'qish davri bo'yicha shaxsiy o'quv traektoriyasini tanlash va ta'lim dasturini o'zlashtirishga yordam beruvchi o'qituvchi	Advisor — a teacher who helps to select an individual learning trajectory for the study period and to master the curriculum

Fasilitator	guruhlardagi faoliyat natijasini samarali baholash, muammoning ilmiy yechimini topishga yo‘naltirish, guruhdagi komunikatsiyani rivojlantirish kabi vazifalarni bajaradi	Facilitator — effectively evaluates the results of group work, focuses on finding a scientific solution to the problem, develops group communication
Moderator	qabul qilingan qoidalarga amal qilish tekshiradi, tinglovchilarning mustaqil fikrlash va ishlash qobiliyatlarni rivojlantirish, bilish faolyatini faollashtirishga yordam beradi. Ma’lumotni, seminarni, treninglar va davra suhbatlarini boshqaradi, fikrlarni umumlashtiradi	Moderator — checks compliance with accepted rules, helps to develop students' independent thinking and working skills, activates cognitive activity. Manages information, seminars, trainings and roundtables, summarizes ideas
Supervayzer	quyidagi to‘rt fazifani bajaradi: o‘qituvchi sifatida o‘rgatadi, fasilitatorlik, maslahatchi, ekspert vazifani bajaradi	Supervisor — performs the following four functions: teaches as a teacher, facilitator, consultant, expert

VI. ADABIYOTLAR RO‘YXATI

I. O‘zbekiston Respublikasi Prezidentining asarlari:

1. Mirziyoev SH.M. Buyuk kelajagimizni mard va olijanob xalqimiz bilan birga quramiz. – T.: “O‘zbekiston”, 2017. – 488 b.
2. Mirziyoev SH.M. Milliy taraqqiyot yo‘limizni qat’iyat bilan davom ettirib, yangi bosqichga ko‘taramiz. 1-jild. – T.: “O‘zbekiston”, 2017. – 592 b.
3. Mirziyoev SH.M. Xalqimizning roziligi bizning faoliyatimizga berilgan eng oliy bahodir. 2-jild. T.: “O‘zbekiston”, 2018. – 507 b.
4. Mirziyoev SH.M. Niyati ulug‘ xalqning ishi ham ulug‘, hayoti yorug‘ va kelajagi farovon bo‘ladi. 3-jild.– T.: “O‘zbekiston”, 2019. – 400 b.
5. Mirziyoev SH.M. Milliy tiklanishdan – milliy yuksalish sari. 4-jild.– T.: “O‘zbekiston”, 2020. – 400 b.

II. Normativ-huquqiy hujjatlar:

6. O‘zbekiston Respublikasining Konstitutsiyasi.–T.:O‘zbekiston, 2018.
7. O‘zbekiston Respublikasining 2020 yil 23 sentabrda qabul qilingan “Ta’lim to‘g‘risida”gi O‘RQ-637-sonli Qonuni.
8. O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevral “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi 4947-sonli Farmoni.
9. O‘zbekiston Respublikasi Prezidentining 2018 yil 21 sentabr “2019-2021 yillarda O‘zbekiston Respublikasini innovatsion rivojlantirish strategiyasini tasdiqlash to‘g‘risida”gi PF-5544-sonli Farmoni.

10. Oʻzbekiston Respublikasi Prezidentining 2019 yil 27 may “Oʻzbekiston Respublikasida korrupsiyaga qarshi kurashish tizimini yanada takomillashtirish chora-tadbirlari toʻgʻrisida”gi PF-5729-son Farmoni.

11. Oʻzbekiston Respublikasi Prezidentining 2019 yil 27 avgust “Oliy ta’lim muassasalari rahbar va pedagog kadrlarining uzluksiz malakasini oshirish tizimini joriy etish toʻgʻrisida”gi PF-5789-sonli Farmoni.

12. Oʻzbekiston Respublikasi Prezidentining 2019 yil 8 oktabr “Oʻzbekiston Respublikasi oliy ta’lim tizimini 2030 yilgacha rivojlantirish konsepsiyasini tasdiqlash toʻgʻrisida”gi PF-5847-sonli Farmoni.

13. Oʻzbekiston Respublikasi Prezidenti Shavkat Mirziyoevning 2020 yil 25 yanvardagi Oliy Majlisga Murojaatnomasi.

14. Oʻzbekiston Respublikasi Vazirlar Mahkamasining 2001 yil 16 avgustdagi «Oliy ta’limning davlat ta’lim standartlarini tasdiqlash toʻgʻrisida»gi 343-sonli Qarori.

15. Oʻzbekiston Respublikasi Vazirlar Mahkamasining 2015 yil 10 yanvardagi “Oliy ta’limning Davlat ta’lim standartlarini tasdiqlash toʻgʻrisida”gi 2001 yil 16 avgustdagi “343-sonli qororiga oʻzgartirish va qoʻshimchalar kiritish haqida”gi 3-sonli qarori.

Ш. Maxsus adabiyotlar:

16. А. И. Лобанов, И. Б. Петров. Математическое моделирование нелинейных процессов : учебник для академического бакалавриата. Москва : Издательство Юрайт, 2019. — 255 с. —ISBN 978-5-9916-8897-0. — Текст : электронный // ЭБС Юрайт [сайт]. — URL: https://urait.ru/bcode/437003.

17. Введение в математическое моделирование. Под ред. В.П.Трусова. М., Логос, 2005.

18. Высшая математика на компьютере в программе Maple 14: учебное пособие по лабораторным работам / С.Т. Касюк, А.А. Логвинова. — Челябинск: Издательский центр ЮУрГУ, 2011. — 57 с.

19. Исроилов М.И. Ҳисоблаш методлари, II. -Т., 2009.

20. Jo’rayev G’.U., Xudoyberganov M.O’, Baxramov S.A. Ayirmali sxemalar nazariyasi elementlari.Toshkent. 2014y. 96b.

21. Ayirmali sxemalar nazariyasi. Darslik.Ch.B.Normurodov.-Termiz:Surxon-nashr, 2021.240 bet.

22. Г.Куликовский, Н.В.Погорелов, А.Ю.Семенов Математические вопросы численного решения гиперболических систем уравнений. М: Физматлит, 2001.

23. Р.Д.Алоев, Метод диссипативных интегралов энергии для разностных схем. Новосибирск. 1994г.

24. С.К. Годунов, Уравнения математической физики М.Наука,1979г.

25. Ишмуҳамедов Р.Ж., М.Мирсолиева. Ўқув жараёнида инновацион таълим технологиялари. – Т.: «Fan va texnologiya», 2014. 60 б.

26. Музафаров Х.А., Баклушин М.Б., Абдураимов М.Г. Математическое моделирование. Ташкент, Университет. 2002.

27. Муслимов Н.А ва бошқалар. Инновацион таълим технологиялари. Ўқув-методик қўлланма. – Т.: “Sano-standart”, 2015. – 208 б.

28. Олий таълим тизимини рақамли авлодга мослаштириш концепцияси. Европа Иттифоқи Эрасмус+ дастурининг кўмагида. https://hiedtec.ecs.uni-ruse.bg/pimages/34/3._UZBEKISTAN-CONCEPT-UZ.pdf

29. Самарский А. А., Михайлов А. П. Математическое моделирование. М., Наука, 2005.

30. Самарский А.А., Гулин А.В. Численные методы. –М., Наука, 1989.

31. Ўрунбаев Э., Муродов Ф. Компьютер алгебраси тизимларининг амалий татбиқлари. – СамДУ нашри – Самарқанд, 2003, 96 б.

32. Усмонов Б.Ш., Ҳабибуллаев Р.А. Олий ўқув юртларида ўқув жараёнини кредит-модуль тизимида ташкил қилиш. Ўқув қўлланма. Т.: “Tafakkur” нашриёти, 2020 й. 120 бет.

IV. Internet saytlar:

33. http://edu.uz – O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi.

34. http://lex.uz – O‘zbekiston Respublikasi Qonun hujjatlari ma’lumotlari milliy bazasi.

35. http://bimm.uz – Oliy ta’lim tizimi pedagog va rahbar kadrlarini qayta tayyorlash va ularning malakasini oshirishni tashkil etish bosh ilmiy-metodik markazi.

36. http://ziyonet.uz – Ta’lim portali ZiyoNET.

37. http://natlib.uz – Alisher Navoiy nomidagi O‘zbekiston Milliy kutubxonasi.