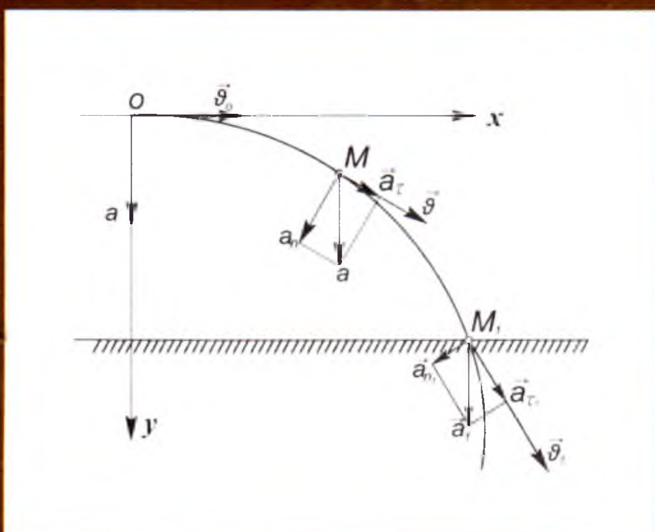
K.KENJAYEV

NAZARIY MEXANIKA MISOL VA MASALALARDA

2-qism

KINEMATIKA



Toshkent - 2020

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

TOSHKENT ARXITEKTURA QURILISH INSTITUTI

K. KENJAYEV

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II-qism

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"Shafoat Nur Fayz"
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O‘quv qo‘llanmada “Nazariy mexanika” fanining “Kinematika” bo‘limini nuqta kinematikasi, qattiq jismning ilgarilanma va qo‘zg‘almas o‘q atrofidagi aylanma harakati, nuqtaning murakkab harakati, qattiq jismning tekislikka parallel harakati boblari bo‘yicha qisqacha nazariy ma’lumotlar, masalalar yechish tartibi, masalalar yechish namunalari hamda mustaqil o‘rganish uchun ko‘p variantli masalalar taqdim etilgan.

O‘quv qo‘llanmani tuzishda: Engineering mechanics statics. J.L. Meriam, L.G. Kraige 2007, Statics and Dynamics. R.C. Hibbeler 2013, Theoretical mechanics. Vasile Szolga 2010, Engineering mechanics. R.S. Khurmi 2011, xorijiy adabiyotlardan foydalanildi.

O‘quv qo‘llanma TAQI o‘quv ishlari bo‘yicha prorektori tomonidan 2015 yil 26 iyulda tasdiqlangan “Nazariy mexanika” fani bo‘yicha ishchi o‘quv dasturi asosida tuzilgan bo‘lib, “Qurilish” yo‘nalishida ta’lim oluvchi talabalar uchun “Nazariy mexanika” fanidan yordamchi o‘quv qo‘llanma sifatida tavsiya etiladi.

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MUNDARIJA

SO'Z BOSHI

Nazariy mexanikaning kinematika bo'limida moddiy nuqtaning harakati harakatni yuzaga keltiruvchi sabablarga bog'lanmagan holda o'rganiladi. Shuning uchun odatda kinematika "harakat geometriyasi" ham deyiladi.

Kinematika bo'limi Statika va Dinamika bo'limlaridan keyinroq, XIX asrda, nazariy mexanikaning alohida bo'limi sifatida shakllangan. Kinematikaning rivojlanishida mashinasozlikning rivojlanishi, turli xil mashina va mexanizmlarning yaratilishi va ishlatilishi asosiy sabablardan hisoblanadi.

Qurilish yo'nalishlarida ta'lim oluvchi talabalar uchun turli xil mashinalarni tashkil etuvchi mexanizmlarning kinematik va dinamik xususiyatlarni o'rganish, harakat qonunlarini keltirib chiqarish muhim vazifa hisoblanadi. Buning uchun talabalar kursning nazariy asoslarini chuqur o'rganishi va amaliy masalalarni yechish malakalariga ega bo'lishlari lozim.

O'quv qo'llanmani tuzishda: Engineering mechanics statics. J.L. Meriam, L.G. Kraige 2007, Statics and Dynamics. R.C.Hibbeler 2013, Theoretical mechanics. Vasile Szolga 2010, Engineering mechanics. R.S. Khurmi 2011, xorijiy adabiyotlardan foydalanildi.

Taqdim etilayotgan o'quv qo'llanmasida nuqta kinematikasi, qattiq jismning ilgarilanma va qo'zg'almas o'q atrofidagi aylanma harakati, moddiy nuqtaning murakkab harakati, qattiq jismning tekislikka parallel harakati mavzulari bo'yicha qisqacha nazariy ma'lumotlar, masalalar yechish tartibi, masalalar yechish namunalari va talabalarga mustaqil yechish uchun ko'p variantli masalalar keltirilgan.

I –BOB.
1-§. Kinematikaning asosiy tushunchalari

Nazariy mexanikaning kinematika bo'limida moddiy nuqta va absolyut qattiq jismning harakati shu harakatni vujudga keltirgan sabablarga bog'lanmagan holda faqat geometrik nuqtai nazardan o'r-ganiladi.

Harakat tushunchasi harakatlanuvchi moddiy nuqta (yoki absolyut qattiq jism), vaqt va fazo tushunchalari bilan chambarchas bog'liqdir.

Ko'chish va harakat tushunchalari nazariy mexanikaning asosiy tushunchalari hisoblanadi. *Moddiy nuqtaning ma'lum vaqt ichida fazoda biror sanoq sistemasiga nisbatan bir holatdan boshqa holatga ixtiyoriy ravishda o'tishi ko'chish deyiladi.*

Nuqtaning boshlang'ich holatdan oxirgi holatga aniq bir usulda vaqtga bog'liq holda o'tishi esa harakat deyiladi.

Klassik mexanikada fazo uch o'lchovli, absolyut qo'zg'almas Evklid fazosi deb qaraladi va undagi barcha o'lchamlar Evklid geometriyasi asosida olib boriladi.

Vaqt ob'yektiv borliqda ro'y beruvchi hodisalarning qancha davom etishini ifodalaydi va u absolyut deb qaraladi. Vaqt barcha sanoq sistemalarida bir xil o'tadi va bir sistemaning ikkinchi sistemaga nisbatan harakatiga bog'liq bo'lmaydi. SI sistemasida sekund vaqt birligi hisoblanadi.

Harakatlanayotgan moddiy nuqtaning fazoda biror sanoq sistemasiga nisbatan holati bilan vaqt orasidagi bog'lanishni ifodalovchi tenglama nuqtaning harakat qonunini ifodalaydi. Agar moddiy nuqtaning biror sanoq sistemasiga nisbatan harakat qonuni berilgan bo'lsa, uning traektoriyasi, tezligi va tezlanishini aniqlash mumkin bo'ladi. *Traektoriya deb – moddiy nuqta yoki absolyut qattiq jismning harakatlanishi tufayli tekislik yoki fazoda qoldirgan iziga aytiladi.*

Kinematikaning asosiy vazifasi moddiy nuqta va absolyut qattiq jismning harakat qonunlarini o'rganishdan iborat.

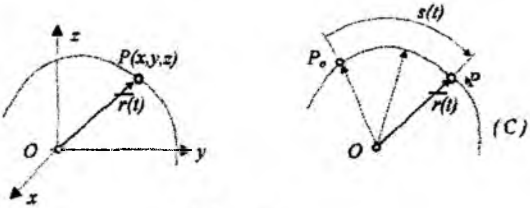
2-§. Moddiy nuqta harakatining berilish usullari

Kinematikada nuqtaning harakati vektor, koordinatalar va tabiiy usulda beriladi.

1. Vektor usuli.

Harakatdagi *M* nuqtaning Oxyz sanoq sistemasiga nisbatan holati *O* markazdan o'tkazilgan $\vec{r}$ radius – vektor bilan aniqlanadi (15.1-rasm). M nuqta harakatlanganda vaqt o'tishi bilan uning radius – vektori $\vec{r}$ miqdor va yo'nalish jihatdan o'zgaradi, ya'ni skalyar argument t ning vektorli funksiyasidan iborat bo'ladi:

$$\vec{r} = \vec{r}(t)\,. \qquad (1.1)$$

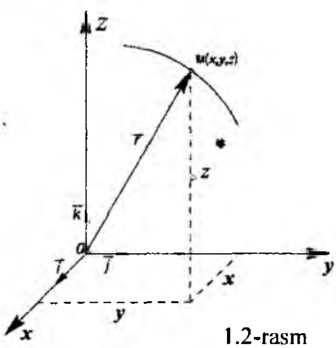


1.1-rasm

Agar $\vec{r}(t)$ funksiyasi ma'lum bo'lsa, nuqtaning fazodagi holati vaqtning har bir payti uchun aniq bo'ladi. Shu sababli (1.1) tenglama nuqta harakatining vektor ko'rinishdagi kinematik tenglamasi deyiladi. Ko'riladigan masalalarda $\vec{r}(t)$ funksiya bir qiymatli, uzluksiz va kamida ikkinchi tartibli hosilaga ega bo'lishi lozim.

2. Koordinatalar usuli.

M nuqta Oxyz sanoq sistemasiga nisbatan harakatlanayotgan bo'lsin. Nuqtaning holatini uning uchta x,y,z Dekart koordinatalari orqali aniqlash mumkin (1.2-rasm).



1.2-rasm

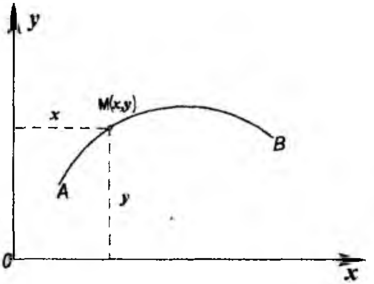
Nuqta harakatlanganda uning koordinatalari vaqt o'tishi bilan o'zgaradi, ya'ni ular t vaqtning funksiyasidan iborat bo'ladi:

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t). \end{cases} \quad (1.2)$$

Agar nuqta koordinatalari bilan vaqt orasidagi munosabatlar berilgan bo'lsa, nuqtaning istalgan paytdagi holatini aniqlash mumkin bo'ladi. Shu sababli (1.2) tenglamalar nuqta harakatining Dekart koordinatalaridagi kinematik tenglamalarini ifodalaydi.

(1.2) tenglamalar nuqta traektoriyasining parametrik tenglamalarini ham ifodalaydi. Bunda parametr sifatida t vaqt olingan.

(1.2) tenglamalardan t vaqtni yo'qotib, nuqtaning kordinatalar formasidagi traektoriya tenglamasi aniqlanadi.



1.3-rasm

M nuqtaning O koordinatalar boshiga nisbatan radius-vektorini

$\vec{r}$, koordinata o'qlarining birlik yo'naltiruvchi vektorlarini $\vec{i}, \vec{j}, \vec{k}$ bilan belgilasak (1.2- rasm), harakatning vektor va Dekart koordinatalari orqali aniqlash usullari orasidagi bog'lanishni ifodalovchi quyidagi tenglama o'rinli bo'ladi:

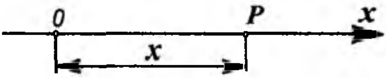
$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} \quad (1.3)$$

Agar nuqta xy tekisligida harakatlansa (1.3 - rasm), nuqtaning tekislikdagi harakat tenglamalari quyidagi ko'rinishda bo'ladi:

$$\begin{cases} x = x(t), \\ y = y(t). \end{cases} \quad (1.4)$$

Nuqta to'g'ri chiziqli harakatda bo'lsa (1.4-rasm), harakat traektoriyasi bo'ylab x o'qini yo'naltiramiz. Bu holda nuqtaning to'g'ri chiziqli harakat tenglamasi quyidagi ko'rinishda yoziladi

$$X=x(t) \quad (1.5)$$



1.4-rasm.

3. Tabiiy usul.

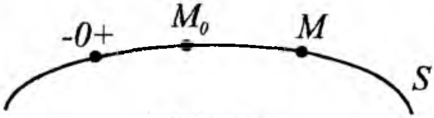
Harakatlanayotgan nuqtaning trayektoriyasi oldindan ma'lum bo'lsa, nuqta harakatini tabiiy usulda aniqlash qulay. Nuqtaning trayrktoriyasi to'g'ri chiziqdan yoki egri chiziqdan iborat bo'ladi. Trayektoriyada qo'zg'almas O nuqtani olib, bu nuqtaga nisbatan yoy koordinatasini o'tkazamiz (1.5-rasm). Harakatlanayotgan M nuqtaning trayektoriyadagi holatini O nuqtadan trayektoriya bo'yicha OM=S yoy koordinatasi bilan aniqlaymiz. O nuqtadan bir tomonga qo'yilgan masofani musbat, ikkinchi tomonga qo'yilgan masofani manfiy deb hisoblaymiz. Vaqtning o'tishi bilan harakatlanayotgan nuqtadan qo'zg'almas O nuqtagacha bo'lgan OM masofa o'zgaradi, ya'ni koordinatasi vaqtning funksiyasidan iborat:

$$S=f(t) \tag{1.6}$$

Bu munosabatga **nuqtaning tabiiy usuldagi harakat tenglamasi** yoki **harakat qonuni** deyiladi.

Agar f(t) funksiya ma'lum bo'lsa, u holda t vaqtning har bir payti uchun OM ni aniqlab, O nuqtadan trayektoriya bo'yicha qo'yamiz. Natijada M nuqtaning berilgan t paytdagi holati aniqlanadi. Shunday qilib, uqtaning harakatini tabiiy uslda aniqlash uchun uning trayektoriyasida O qo'zg'almas nuqta (hisoblash boshi) va yoy koordinatasining hisoblash yo'nalishi hamda S=f(t) harakat tenglamasi bo'lishi kerak. Nuqtaning S yoy koordinatasi bilan trayektoriya ustidan o'tgan *OM* yo'li doimo bir xil bo'lavermaydi.

Agar M nuqtaningharakati O qo'zg'almas nuqtadan boshlanib $\Delta t = t - t_0$ vaqt oralig'ida doimo musbat yo'nalishi bo'yicha bo'lsa, t vaqtda nuqtaning yoy koordinatasi bilan Δt vaqt oralig'ida o'tilgan yo'l o'zaro teng.



1.5-rasm

Agar t_0 boshlang'ich vaqtda nuqta M_0 holatda bo'lip, Δt vaqtdan keyin M holatni egallasa, u holda Δt oralig'ida nuqtaning bir tomonga harakatlanishi natijasida o'tilgan yo'l

$$S = \int_{t_0}^{t} f'(t)dt$$

formula bilan aniqlanadi.

Takrorlash uchun sovollar

1. Kinematika fani nimani o'rganadi?
2. Kinematika asosiy tushunchalarini ta'riflab bering.
3. Nuqtaning harakati qanday usullarda beriladi?
4. Nuqtaning vektor ko'rinishidagi harakat tenglamasini yozing.
5. Nuqtaning harakati koordinatalar usulida berilganda harahat tenglamalari qanday ko'rinishda yoziladi?
6. Nuqtaning harakati tabiiy usulda berilganda harakat tenglamasi qanday ko'rinishda yoziladi?
7. Trayektoriya nima?
8. Harakat deb nimaga aytiladi?
9. Ko'chish deb nimaga aytiladi?
10. Nuqtaning harakat qonunini tariflang.

3-§ Nuqtaning tezligi

Tezlik deb berilgan sanoq sistemasida har qanday vaqt onida moddiy nuqta harakatining qanchalik ildamligi va uning yo'nalishini ifodalaydigan vektor kattalikika aytiladi.

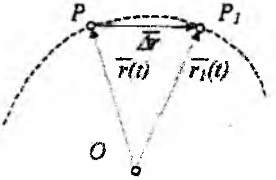
1. Harakat qonuni vektor usulida berilgan nuqtaning tezligi.

Agar nuqtaning harakati vektor usulda $\vec{r} = \vec{r}(t)$ tenglama bilan berilgan bo'lsa, nuqtaning berilgan ondagi tezlik vektori uning radius vektoridan vaqt bo'yicha olingan birinchi tartibli hosilaga teng bo'ladi:

$$\vec{\vartheta} = \frac{d\vec{r}}{dt} \qquad (1.7)$$

Tezlik vektori nuqta traektoriyasiga harakat yo‘nalishi bo‘yicha o‘tkazilgan urinma bo‘ylab yo‘naladi (1.6-rasm).

$$\overline{v}_m = \frac{\Delta r(t)}{\Delta t}$$



1.6-rasm

2. Harakati koordinatalar usulida berilgan nuqtaning tezligi.

Agar nuqtaning harakati koordinatalar usulida

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t), \end{cases} \quad (1.8)$$

tenglamalar bilan berilgan bo‘lsa, nuqta tezligining biror qo‘zg‘almas Dekart koordinata o‘qidagi proeksiyasi mos koordinatasidan vaqt bo‘yicha olingan birinchi tartibli hosilaga teng bo‘ladi.

Shuning uchun:

$$\vartheta_x = \frac{dx}{at}, \quad \vartheta_y = \frac{dy}{at}, \quad \vartheta_z = \frac{dz}{at}. \quad (1.9)$$

Agar tezlikning koordinataga o‘qlaridagi proeksiyalari ma’lum bo‘lsa, uning moduli

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2 + \vartheta_z^2} \quad (1.10)$$

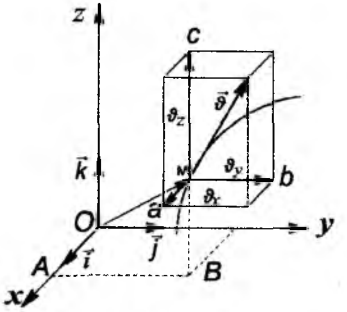
formula bilan, yo‘nalishi esa

$$\cos\left(\vec{\vartheta}\,\widehat{}\,\vec{i}\right) = \frac{\vartheta_x}{\vartheta}, \cos\left(\vec{\vartheta}\,\widehat{}\,\vec{j}\right) = \frac{\vartheta_y}{\vartheta}, \cos(\vec{\vartheta}\,\widehat{}\,\vec{k}) = \frac{\vartheta_z}{\vartheta}. \quad (1.11)$$

formulalar yordamida aniqlanadi. Bunda $\vec{i}, \vec{j}, \vec{k}$ lar Dekart koordinata o‘qlarining birlik vektorlari (1.7-rasm).

Agar nuqta tekislikda harakatlansa, uning harakati

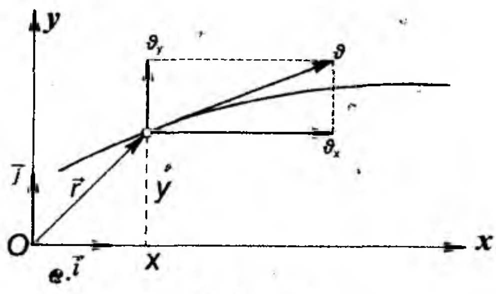
$$\begin{cases} x = x(t), \\ y = y(t), \end{cases} \qquad (1.12)$$



1.7-rasm.

tenglamalar bilan beriladi. Bunday holda tezlik moduli va yo‘nalishi quyidagicha aniqlanadi (1.8-rasm):

$$\cos\left(\vec{\vartheta}\,\hat{}\,\vec{i}\right) = \frac{\vartheta_x}{\vartheta}, \quad \cos\left(\vec{\vartheta}\,\hat{}\,\vec{j}\right) = \frac{\vartheta_y}{\vartheta}. \quad \vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} \qquad (1.13)$$



1.8-rasm.

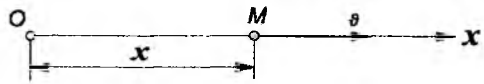
Nuqtaning *ox* o‘qi bo‘ylab to‘g‘ri chiziqli harakati

$$y = x(t) \qquad (1.14)$$

tenglama bilan beriladi.

Bunday holda nuqta tezligining moduli tezlik vektorining koordinata o'qidagi proeksiyasining absolyut qiymatiga teng bo'ladi (1.9-rasm)

$$\vartheta = |\vartheta_x| = \left|\frac{dx}{dt}\right| \qquad (1.15)$$



1.9-rasm

3. Harakati tabiiy usulda ifodalangan nuqtaning tezligi.

Agar nuqta berilgan traektoriya bo'ylab s=s(t) qonun asosida harakatlansa, tezlik vektori quyidagi formula orqali ifodalanadi:

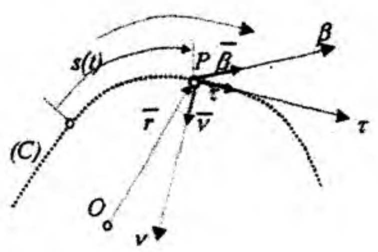
$$\vec{\vartheta} = \frac{ds}{dt}\vec{\tau}^0 \qquad (1.16)$$

(1.16) da $\frac{ds}{dt}$ hosila $\vec{\vartheta}$ tezlikning urinmadagi proeksiyasi ϑ ni ifodalaydi va tezlikning algebraik qiymati deyiladi.

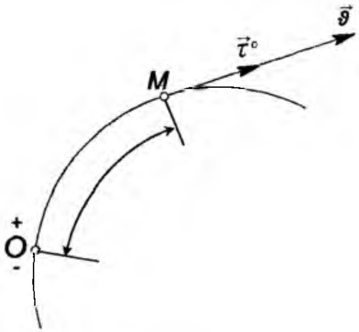
ϑ_τ ning absolyut qiymati tezlikning moduliga teng bo'ladi:

$$\vartheta = |\vartheta_\tau| = \left|\frac{ds}{dt}\right| \qquad (1.17)$$

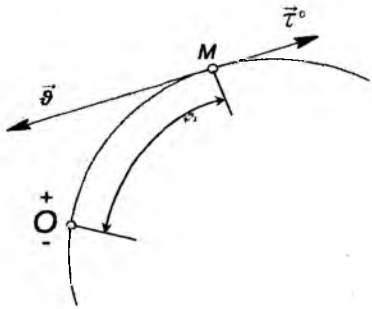
Bunda $\frac{ds}{dt} > 0$ bo'lsa, yoy koordinatasi s orta boradi va nuqta tezligi $\vec{\vartheta}$ ning yo'nalishi $\vec{\tau}^0$ bilan ustma – ust tushadi. Agar $\frac{ds}{dt} < 0$ bo'lsa, yoy koordinatasi s kamaya boradi va $\vec{\vartheta}$ tezlik vektori $\vec{\tau}^0$ga qarama-qarshi yo'naladi (1.10a,b,c-rasmlar).



1.10a-rasm



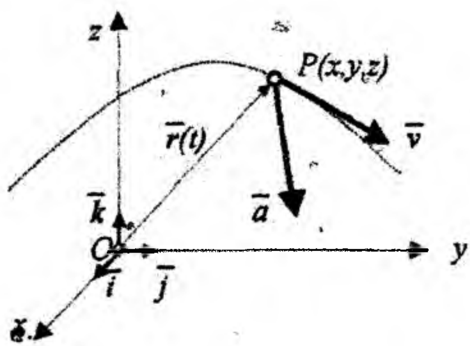
1.10b-rasm



1.10c-rasm

4-§ Nuqtaning tezlanishi

Harakatdagi nuqta tezligining vaqt o'tishi bilan miqdor va yo'nalish jihatidan o'zgarishini ifodalovchi vektor kattalik tezlanish deyiladi.



1.11a-rasm

Harakati vektor usulida berilgan nuqtaning tezlanishi.

Nuqtaning harakati vektor usulida

$$\vec{r} = \vec{r}(t) \qquad (1.18)$$

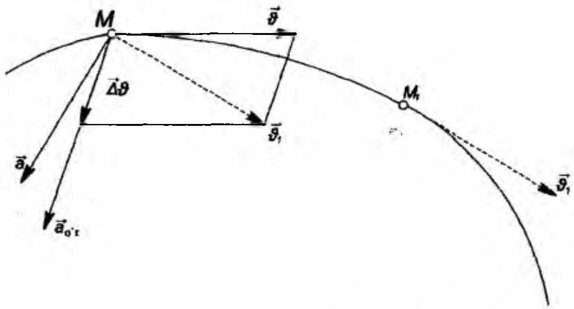
tenglama bilan berilganda, uning tezligi

$$\vec{\vartheta} = \frac{d\vec{r}}{at} \qquad 1.19)$$

bo‘lishini e’tiborga olsak, nuqtaning tezlanish vektori uning tezlik vektoridan vaqt bo‘yicha olingan birinchi tartibli hosilaga yoki radius vektoridan vaqt bo‘yicha olingan ikkinchi tartibli hosilaga teng bo‘ladi:

$$\vec{a} = \frac{d\vec{\vartheta}}{at} = \frac{d^2\vec{r}}{at^2} \qquad (1.20)$$

Nuqta bir tekislikda yotuvchi traektoriya bo‘ylab harakatlansa, tezlanish vektori, o‘rtacha tezlanish $\vec{a}_{ur}$ kabi, traektoriya tekisligida yotadi hamda traektoriyaning botiq tomoniga yo‘naladi.



1.11 b-rasm

Agar nuqtaning traektoriyasi bir tekislikda yotmaydigan egri chiziqdan iborat bo‘lsa, tezlanish vektori egrilik tekisligida yotadi va traektoriyaning botiq tomoniga yo‘naladi (1.11a,b-rasm).

2. Harakati koordinatalar usulida berilgan nuqtaning tezlanishi.

Nuqtaning harakati koordinatalar usulida berilganda nuqta tezligining koordinata o‘qlaridagi proeksiyalari

$$\vartheta_x = \frac{dx}{dt}, \quad \vartheta_y = \frac{dy}{dt}, \quad \vartheta_z = \frac{dz}{dt} \qquad (1.21)$$

formulalar yordamida aniqlangan edi.

Nuqta tezlanishining biror o‘qdagi proeksiyasi nuqta tezligining mazkur o‘qdagi proeksiyasidan vaqt bo‘yicha olingan birinchi tartibli hosilaga yoki radius vektoridan vaqt bo‘yicha olingan ikkinchi tartibli hosilaga teng bo‘ladi.

Shuning uchun:

$$a_x = \frac{d\vartheta_x}{dt} = \frac{d^2x}{dt^2}, \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}, \quad a_z = \frac{d\vartheta_z}{dt} = \frac{d^2z}{dt^2}. \qquad (1.22)$$

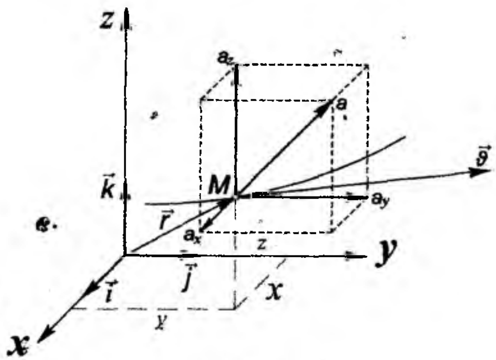
Tezlanishning koordinata o‘qlaridagi proeksiyalari ma’lum bo‘lsa, uning moduli

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} \qquad (1.23)$$

formula bilan, yo‘nalishi esa,

$$\cos(\vec{a}\,\hat{}\,\vec{i}) = \frac{a_x}{a}, \quad \cos(\vec{a}\,\hat{}\,\vec{j}) = \frac{a_y}{a}, \quad \cos(\vec{a}\,\hat{}\,\vec{k}) = \frac{a_z}{a} \qquad (1.24)$$

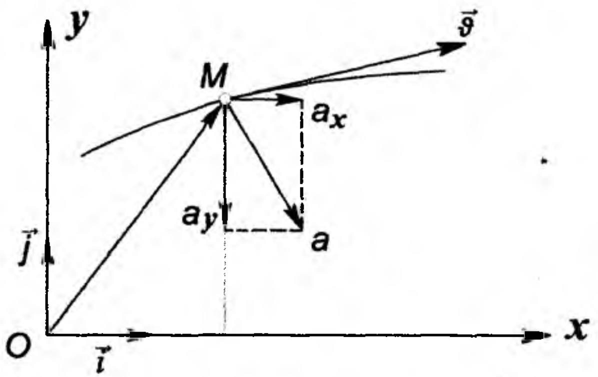
formulalar yordamida aniqlanadi. Bunda $\vec{i}, \vec{j}, \vec{k}$ lar koordinata o‘qlarining birlik vektorlari (1.12-rasm).



1.12-rasm

Agar nuqta Oxyz tekisligida harakatlansa (1.13-rasm), $a_z = \dot{\vartheta}_z = \ddot{z} = 0$ bo'lib, tezlanish miqdori va yo'nalishi quyidagi formulalar bilan aniqlanadi:

$$a = \sqrt{ax^2 + ay^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2};$$
$$\cos(\vec{a}\,\hat{}\,\vec{i}) = \frac{a_x}{a}, \quad \cos(\vec{a}\,\hat{}\,\vec{j}) = \frac{a_y}{a}. \qquad (1.25)$$



1.13-rasm

Agar nuqta Ox o'qi bo'ylab to'g'ri chiziqli harakat qilsa (1.14-rasm), tezlanish moduli

$$a = |a_x| = |\ddot{x}| \qquad (1.26)$$

formula bilan aniqlanadi. Agar $\ddot{x} > 0$ bo'lsa, tezlanish vektori $\vec{a}$ Ox o'qining musbat yo'nalishi bo'yicha, $\ddot{x} < 0$ bo'lsa, manfiy yo'nalishi bo'yicha yo'naladi.

1.14-rasm.

3. Harakati tabiiy usulda berilgan nuqtaning tezlanishi.

Nuqtaning harakati tabiiy usulda berilganda uning tezligi quyidagicha ifodalanar edi:

$$\vec{\vartheta} = \frac{ds}{dt}\vec{\tau}^0 = \vartheta\vec{\tau}^0 . \qquad (1.27)$$

Tezlanish vektori, tezlik vektoridan vaqt bo'yicha olingan birinchi tartibli hosilaga teng bo'ladi:

$$\vec{a} = \frac{d\vec{\vartheta}}{dt} = \frac{d\vartheta}{dt}\vec{\tau}^0 + \vartheta\frac{d\vec{\tau}^0}{dt} = \frac{d\vartheta}{dt}\vec{\tau}^0 + \vartheta\frac{d\vec{\tau}^0}{ds}\frac{ds}{dt} \qquad (1.28)$$

Analitik geometriyadan ma'lumki

$$\frac{d\vec{\tau}^0}{ds} = \frac{1}{\rho}\vec{n}^0 \qquad (1.29)$$

bunda ρ – traektoriyaning egrilik radiusi, $\vec{n}^0$ – traektoriyaga o'tkazilgan bosh normal birlik vektori.

Bularni e'tiborga olsak,

$$\vec{a} = \frac{d\vartheta}{dt}\vec{\tau}^0 + \frac{\vartheta^2}{\rho}\vec{n}^0 \qquad (1.30)$$

Bu ifodada $\frac{d\vartheta}{dt}\vec{\tau}^0$ vektor kattalik traektoriyaga M nuqtada o'tkazilgan urinma bo'ylab yo'naladi va *urinma tezlanish* deyiladi:

$$\vec{a}_\tau = \frac{d\vartheta}{dt}\vec{\tau}^0 . \qquad (1.31)$$

$\frac{\vartheta^2}{\rho}\vec{n}^0$ vektor kattalik traektoriyaga M nuqtada o'tkazilgan bosh normal bo'ylab yo'naladi va *normal tezlanish* deyiladi:

$$\vec{a}_n = \frac{\vartheta^2}{\rho}\vec{n}^0 . \qquad (1.32)$$

Urinmaning birlik vektori $\vec{\tau}^0$ va bosh normalning birlik vektori $\vec{n}^0$ traektoriyaning M nuqtasiga o'tkazilgan egrilik tekisligida yotganligi tufayli, tezlanish vektori ham mazkur egrilik tekisligida yotadi. Shu sababli tezlanishning binormaldagi tashkil etuvchisi nolga teng bo'ladi.

Tezlanishning tabiiy koordinata o'qlaridagi proeksiyalari quyidagicha aniqlanadi:

$$a_\tau = \frac{d\vartheta}{dt} = \frac{d^2s}{dt^2},$$

$$a_n = \frac{\vartheta^2}{\rho} . \qquad (1.33)$$

Tezlanish vektori urinma tezlanish $\vec{a}_\tau$ va normal tezlanish $\vec{a}_n$ larning geometrik yig'indisiga teng bo'ladi:

$$\vec{a} = \vec{a}_\tau + \vec{a}_n \qquad (1.34)$$

Bu tezlanishlar o'zaro perpendikulyar yo'nalganidan, to'la tezlanish moduli

$$a = \sqrt{a_\tau^2 + a_n^2} \qquad (1.35)$$

yoki

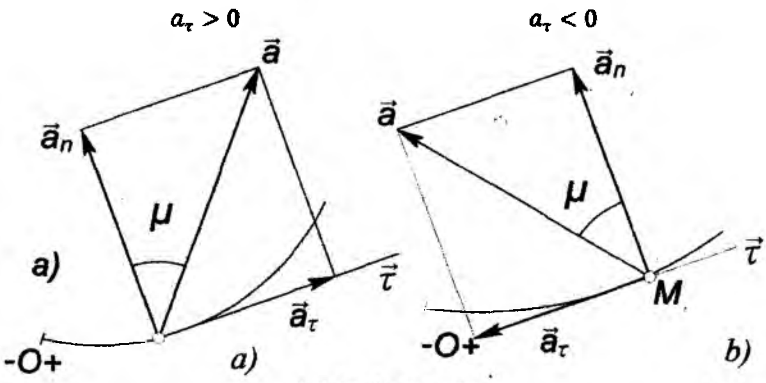
$$a = \sqrt{(\frac{d\theta}{at})^2 + (\frac{\theta^2}{\rho})^2} \qquad (1.36)$$

formula bilan, yo'nalishi esa

$$tg\mu = \frac{|a_\tau|}{a_n} \qquad (1.37)$$

formula bilan aniqlanadi (1.15a,b- rasmlar).

Bunda $\vec{a}_n$ har doim traektoriyaning botiq tomoniga yo'naladi ($a_n > 0$), $\vec{a}_\tau$ proeksiyaning ishorasiga bog'liq holda $M\tau$ o'qning musbat yoki manfiy tomoniga qarab yo'naladi (1.15a,b-rasmlar).



1.15 a,b-rasmlar

5-§ Nuqta harakatining xususiy hollari

Nuqtaning tezlanishi tabiiy koordinata o‘qlaridagi tashkil etuvchilari orqali quyidagicha yoziladi:

$$\vec{a} = \frac{d\vartheta}{at}\vec{\tau}^0 + \frac{\vartheta^2}{\rho}\vec{n}^0 \qquad (1.38)$$

Nuqtaning tezlanishiga qarab harakat turlarini aniqlash mumkin.

1. **To‘g‘ri chiziqli tekis harakat.**

Nuqtaning traektoriyasi to‘g‘ri chiziqdan iborat bo‘lsa, $\rho = \infty$ bo‘ladi.

Bunday holda

$$a_n = \frac{\vartheta^2}{\rho} = 0 \qquad (1.39)$$

bo‘lib, nuqtaning tezlanishi faqat urinma tezlanishdan iborat bo‘ladi:

$$a = a_\tau = \frac{d\vartheta_\tau}{at} \qquad (1.40)$$

Bunday holda nuqtaning tezligi faqat miqdor jihatdan o‘zgaradi $(\rho = \infty)$. *Shuning uchun ham urinma tezlanish tezlikning son qiymati jihatdan o‘zgarishini ifodalaydi.*

Nuqtaning harakati davomida doimo $\vec{a}_\tau = 0$, $\vec{a}_n = 0$ ya’ni $\vec{a} = 0$ bo‘lsa, $\frac{d\vartheta_\tau}{at} = 0$ bo‘lib, $\vartheta = |\vartheta_\tau| = const$ bo‘ladi.

$\frac{\vartheta^2}{\rho} = 0$ bo‘lganidan $\rho = \infty$ ekanligi kelib chiqadi.

Bunday holda nuqta to‘g‘ri chiziqli tekis harakatda bo‘ladi.

2. **Egri chiziqli tekis harakat.**

Nuqta egri chiziqli tekis harakatda bo‘ladi, agarda tezlikning son qiymati harakat davomida doimo o‘zgarmas holda saqlansa:

$$\vartheta = const .$$

Bunday holda

$$a_\tau = \frac{d\vartheta}{at} = 0 \qquad (1.41)$$

bo‘lib, nuqtaning tezlanishi faqat normal tezlanishdan iborat bo‘ladi:

$$a = a_n = \frac{\vartheta^2}{\rho} . \qquad (1.42)$$

Bunda nuqtaning normal tezlanishi $\vec{a}_n$ doimo egri chiziqning botiq tomoniga yo'nalgan bosh normal bo'ylab yo'naladi. $\vartheta = const$ bo'lgani uchun, bu tezlanish nuqtaning tezligi vaqt o'tishi bilan faqat yo'nalishini o'zgartirishidan hosil bo'ladi. *Shu sababli, normal tezlanish nuqta tezligining yo'nalish jihatdan o'zgarishini ifodalaydi.*

Agar $\vartheta = \frac{ds}{at}$ ekanligini e'tiborga olsak, $(\vartheta = \vartheta_0)$

$$ds = \vartheta dt \qquad (1.43)$$

Bu tenglikni mos chegaralar bo'yicha integrallasak

$$\int_{s_0}^{s} ds = \int_0^t \vartheta_0 dt$$

yoki

$s = s_0 + \vartheta_0 t$

tenglama hosil bo'ladi.

Agar $s_0 = 0$ bo'lsa, $s = \vartheta_0 t$ (1.44)

(1.44)tenglama nuqtaning egri chiziqli tekis harakati tenglamasi deyiladi.

3. Egri chiziqli tekis o'zgaruvchan harakat.

Agar nuqtaning harakati davomida doimo $a_\tau = const$ bo'lsa, bunday harakat tekis o'zgaruvchan harakat deyiladi.

Agar t=0 da $s=s_0$ va $\vartheta=\vartheta_0$ bo'lsa,

$$a_\tau = \frac{d\vartheta}{at} = \frac{d^2s}{at^2} \qquad (1.45)$$

tenglamadan

$$d\vartheta = a_\tau ds \qquad (1.46)$$

tenglik hosil bo'ladi. $a_\tau = const$ ekanligini e'tiborga olib, (1.46) tenglikni mos chegaralar bo'yicha integrallasak

$$\int_{\vartheta_0}^{\vartheta} d\vartheta = \int_0^t a_\tau ds$$

yoki

$$\vartheta = \vartheta_0 + a_\tau t \qquad (1.47)$$

(1.47) egri chiziqli tekis o‘zgaruvchan harakatdagi nuqtaning tezligini ifodalaydi.Agar

$$\vartheta = \frac{ds}{at}$$

ekanligini e’tiborga olsak, (1.47) tenglama quyidagicha yoziladi:

$$\frac{ds}{at} = \vartheta_0 + a_\tau t \qquad (1.48)$$

Bu tenglamaning har ikkala tomoni mos chegaralar bo‘yicha integrallansa, tekis o‘zgaruvchan harakat tenglamasi hosil bo‘ladi:

$$s = s_0 + \vartheta_0 t + \frac{d_\tau^2 t^2}{2} \qquad (16.43)$$

To‘g‘ri chizikli tekis o‘zgaruvchan harakat tezligi va harakat tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$\dot{x} = \vartheta_0 + a_x t, \qquad (1.49)$$

$$x = x_0 + \vartheta_0 t + \frac{a_x t^2}{2} \qquad (1.50)$$

Takrorlash uchun savollar

1. Nuqtaning tezligi deb qanday kattalikka aytiladi?
2. Nuqtaning tezligi vektor usulida qanday aniqlanadi?
3. Nuqtaning tezligi koordinatalar usulida qanday aniqlanadi?
4. Nuqtaning tezligi tabiy usulda qanday aniqlanadi?
5. Nuqtaning tezlanishi vector usulida qanday aniqlanadi?
6. Nuqtaning tezlanishi koordinatalar usulida qanday aniqlanadi?
7. Nuqtaning tezlanishi. Tabiiy usulda qanday aniqlanadi?
8. Nuqtaning urinma tezlanishi qanday harakatda yuzaga keladi?
9. Nuqtaning normal tezlanishi qanday harakatda yuzaga keladi?
10. Nuqtaning tezlanishi doimo nolga teng bo‘lsa, u qanday harakatda bo‘ladi?
11. Tekis o‘zgaruvchan harakatni ta’riflang.
12. Egri chiziqli biror M nuqtadagi egriligi qanday aniqlanadi.
13. Egrilik radiusini ta’riflang.

6-§ Nuqta harakatining tenglamalari va traektoriyasini aniqlashga doir masalalarni yechish uchun uslubiy korsatmalar

Nuqta kinematikasida nuqtaning harakat tenglamalari berilgan bo‘lib, uning traektoriyasi, tezligi, tezlanishi kabi kinematik kattaliklarni aniqlash talab etiladi.

Bunday holda nuqta kinematikasi masalalarini quyidagi tartibda yechish maqsadga muvofiq bo‘ladi:

1. Koordinatalar sistemasi tanlab olinadi;
2. Tanlangan koordinatalar sistemasida nuqtaning harakat tenglamalari tuziladi;
3. Nuqtaning harakat tenglamalarini bilgan holda, traektoriya tenglamasi tuziladi. Buning uchun harakat tenglamalaridan t vaqt yo‘qotiladi;
4. Nuqtaning harakat tenglamalarini bilgan holda, tezlikning o‘qlardagi proeksiyalarini, ular orqali esa, tezlikning miqdori va yo‘nalishi aniqlanadi;
5. Tezlikning o‘qlardagi proeksiyalarini bilgan holda, tezlanishning o‘qlardagi proeksiyalarini, ular orqali esa, tezlanishning miqdori va yo‘nalishi aniqlanadi.

Agar masalada nuqtaning traektoriyasi berilgan bo‘lsa, tezlanishni uning tabiiy o‘qlardagi proeksiyalari orqali ham aniqlash mumkin.

Bunda, masalani quyidagi tartibda yechish tavsiya etiladi:

1. Tezlanishning urinma o‘qdagi proeksiyasi aniqlanadi:

$$a_\tau = \frac{\vartheta_x a_x + \vartheta_y a_y}{\vartheta} \qquad (1.51)$$

2. Tezlanishning bosh normaldagi proeksiyasi aniqlanadi:

$$a_n = \frac{\vartheta_x a_y - \vartheta_y a_x}{\vartheta}. \qquad (1.52)$$

Ular orqali nuqtaning to‘la tezlanishi topiladi:

$$a = \sqrt{a_\tau^2 + a_n^2}. \qquad (1.53)$$

3. Traektoriyaning egrilik radiusi:

$$\rho = \frac{\vartheta^2}{a_n} \qquad (1.54)$$

formula yordamida aniqlanadi.

Harakatdagi nuqtaning fazoda qoldirgan izi uning traektoriyasi deyiladi. Nuqtaning traektoriyasi tekislikda yoki fazoda yotuvchi chiziq bo'lishi mumkin. Nuqtaning harakati uning harakat qonuni orqali ifodalanadi. Nuqtaning harakat qonuni (tenglamasi) uning tekislikda yoki fazodagi o'rni va vaqt oralig'idagi bog'lanishni ifodalaydi. Nuqtaning harakati vektor usulida berilganida ixtiyoriy vaqt onidagi o'rni kordinatalar boshidan harakatdagi nuqtaga o'tkazilgan $\vec{r}$ radius vektor orqali aniqlanadi (1.16-rasm).

$$\vec{r} = \vec{r}(t) \qquad (1.55)$$

Nuqtaning harakati koordinatalar usulida berilganda uning ixtiyoriy vaqt oralig'idagi o'rni:

a) fazoda $x = f_1(t),\ y = f_2(t),\ z = f_3(t)$

b) tekislikda $x = f_1(t),\ y = f_2(t)$ (1.56)

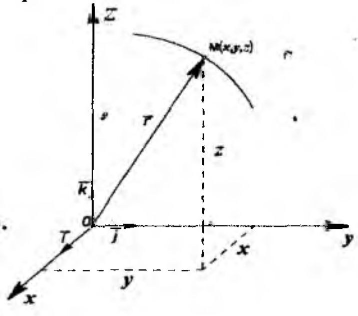
c) nuqta to'g'ri chiziqli harakatda bo'lganda $-x = f(t)$ koordinatalari orqali aniqlanadi.

Nuqtaning harakati qutb, slindrik va sferik koordinatalarda ham beriladi.

Agar nuqta harakatining traektoriyasi oldindan ma'lum bo'lsa, uning harakatini tabiiy usulda berish qulay bo'ladi. Bunday holda nuqtaning traektoriyadagi o'rni

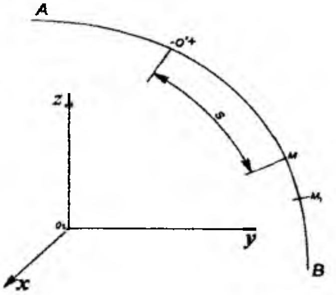
$$S=f(t) \qquad (1.57)$$

tenglama orqali aniqlanadi.



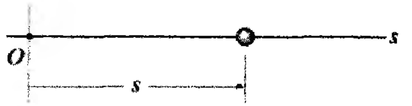
1.16-rasm

Bu ifodada S - egri chiziqli koordinata bo‘lib, traektoriya bo‘ylab tanlab olingan biror O nuqtadan hisoblanadi (1.17-rasm).



1.17-rasm

Bunda nuqtaning traektoriyasi to‘g‘ri chiziq bo‘lishi ham mumkin (1.18-rasm).



1.18-rasm

Nuqta harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar quyidagi tartibda yechiladi:

1. Qo‘zg‘almas o‘qlar sistemalari (to‘g‘ri burchakli, qutb va h. k), ularning boshi (qo‘yilish nuqtalari) tanlab olinadi;

2. Masala shartiga ko‘ra tanlab olingan koordinatalar sistemasi uchun nuqtaning harakat tenglamalari tuziladi;

3. Tuzilgan harakat tenglamalariga ko‘ra istalgan vaqt oni uchun nuqtaning o‘rni, harakatining yo‘nalishi, traektoriyasi aniqlanadi.

7-§ Nuqta harakatining tenglamalari, traektoriyasini aniqlashga doir masalalar

1. Masala

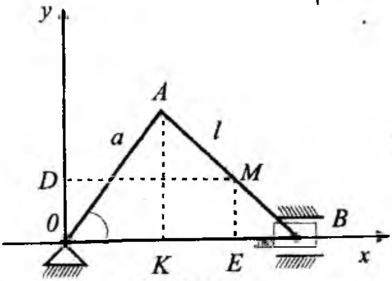
Krivoship-shatun mehanizmida OA kriviship doimiy ω burchak tezligi bilan aylanadi va OA=l , l=a . shatun o'rtasidagi M nuqtaning harakat tenglamasi va trayektoriya tenglamasini aniqlang. Shuningdek, B polzunning harakat tenglamasini toping. Harakat boshlanishida B polzun o'ngdagi eng chetki holatda bo'lsin. Koordinata o'qlari shaklda ko'rsatilgan bo'lsin.

Yechish.

M nuqtadan koordinata o'qlariga MD va ME perpendikularlar tushiramiz.

1.19 rasmdan (1.58)

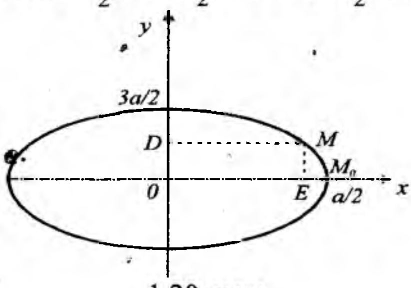
$$X=OE=OK+KE=Oa\cos\varphi+AM\cos\psi \qquad (1.58)$$



1.19-rasm

l=a bo'lgani uchun φ=ψ bo'ladi, u holda

$$x=a\cos\varphi+\frac{a}{2}\cos\varphi=\frac{3a}{2}\cos\varphi;\quad y=\frac{a}{2}\sin\varphi \qquad (1.59)$$



1.20-rasm

Bizda φ=ψ*t u holda (1.59) tenglama quyidagicha yoziladi:

$$\left.\begin{aligned} x &= \frac{3a}{2}\cos\omega t, \\ y &= \frac{a}{2}\sin\omega t. \end{aligned}\right\} \quad (1.60)$$

(1.60) tenglamalar sistemasi M nuqtaning harakat tenglamalari bo'ladi. Bu tenglamalardan vaqt *t* ni yo'qotsak, trayektoriya tenglamalarini topamiz. Sinus va kosinus funksiyalarning argumentlari bir xil bolsa, vaqt *t* ni yo'qotish uchun (1.60) tenglamalarni quyidagi ko'rinishda yozamiz:

$$\left.\begin{aligned} \cos\omega t &= \frac{2}{3}\cdot\frac{x}{a}, \\ \sin\omega t &= \frac{2}{a}\cdot y. \end{aligned}\right\} \quad (1.61)$$

(1.61) tenglamalarning ikkala tomonini kvadratga ko'taramiz:

$$\left.\begin{aligned} \cos^2\omega t &= \frac{4}{9}\cdot\frac{x^2}{a^2}, \\ \sin^2\omega t &= \frac{4}{a^2}\cdot y^2. \end{aligned}\right\} \quad (1.62)$$

O'zaro qo'ship quyidagini hosil qilamiz:

$$\frac{4x^2}{9a^2}+\frac{4y^2}{a^2}=1 \text{ yoki } \frac{x^2}{\frac{9a^2}{4}}+\frac{y^2}{\frac{a^2}{4}}=1 \quad (1.63)$$

(1.63) tenglama M nuqtaning trayektoriya tenglamasi. Trayektoriya yarim o'qlari $\frac{3a}{2}$ va $\frac{a}{2}$ ga teng bolgan va ellipsdan iborat (1.20-rasm)

Endi B polzuning harakat tenglamasini topamiz.

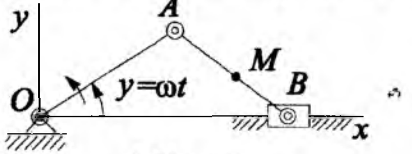
1.19-rasmdan:

$x_B=OB=acoscos\varphi+lcos\varphi+acos\varphi=2acos\varphi$ *yoki* $x_B=2acos\varphi$ (1.64)

(1.64) tenglama B polzuning harakat tenglamasini ifodalaydi.

2. *masala.*

OA krivoship φ=10 rad/s doimiy burchak tezlik bilan aniqlanadi. Uzunlik OA=AB=80 sm. Shatun o'rtasidagi M nuqtaning harakat tenglamasi va traektoriyasi, shuningdek B polzunning harakat tenglamasi topilsin; harakat boshlanganida B polzun o'ngdagi eng chetki holatda bo'lgan; koordinata o'qlari rasmda ko'rsatilgan (1.21-rasm)



1.21-rasm

Yechish. Koordinata boshi sifatida O nuqtani tanlab, Ox o'qini gorizontal, Oy o'qini vertikal holda o'tkazamiz. Mehanizmning berilgan holati uchun M nuqtaning koordinatalarini aniqlaymiz:

$$x = OA\cos\varphi + \frac{OA}{2}\cos\varphi = \frac{3}{2}OA\cos\varphi = 1.5OA\cos\omega t$$

$$y = \frac{OA}{2}\sin\varphi = \frac{OA}{2}\sin\omega t$$

Demak, berilgan mehanizm M nuqtasining harakat qonuni quyidagicha ko'rinishda bo'lar ekan:

$$\left.\begin{aligned} x_m &= 120\cos 10t \\ y_m &= 40\sin 10t \end{aligned}\right\}$$

Mehanizm "M" nuqtasining traektoriyasini aniqlash uchun traektoriya tenglamasini tuzamiz. Buning uchun harakat qonunini ifodalovchi tenglamalardan parametr t ni qisqartiramiz.

$$\left(\frac{x}{120}\right)^2 = \cos^2 10t,$$

$$\left(\frac{y}{40}\right)^2 = \sin^2 10t.$$

Yozilgan tenglamalardan

$$\frac{x^2}{120^2} + \frac{y^2}{40^2} = 1$$

Hosil bo'lgan tenglama ellips tenglamasi hisoblanadi.

"B" nuqtaning harakat tenglamasini aniqlaymiz:

$$x = 2OA\cos\omega t = 160\cos 10t$$

3. masala. Avtomabil to'g'ri chiziqli yo'lda o'zgarmas 20 m/s tezlik bilan harakatlanadi; uning R=1 m radiusli gardishida yotuvchi nuqtaning harakat tenglamasi va traektoriyasi aniqlansin. G'ildirakni sirpanmasdan g'ildiraydi deb hisoblansin; koordinata boshini Ox o'q sifatida olingan yo'lning harakat boshlanadigan nuqtasida olinsin.

Yechish. Avtomobil g'ildiragi gardishidagi nuqtaning harakat tenglamasi quyidagi ko'rinishda yoziladi.

$$x = R\varphi - d\sin\varphi$$

$$y = R - d\cos\varphi$$

Masalada R=1 φ=ωt. Shuning uchun

$$\omega = \frac{v}{r} = \frac{20}{1} 20 \frac{1}{s}$$

Natijada avtomobil g'ildiragi gardishidagi nuqtaning harakat tenglamasi quyidagi ko'rinishda yoziladi:

x=20t-sin20t y=1-cos20t

4. masala. Nuqtaning harakati

$$x = \vartheta_0 t\cos\alpha \quad (1.65)$$

$$y = \vartheta_0 t\sin\alpha - \frac{gt^2}{2} \quad (1.66)$$

tenglamalar bilan berilgan. Bundagi ϑ_0 va g – lar doimiy miqdorlar.

Nuqtaning traektoriyasi, maksimal ko'tarilish balandligi va bunday holatda gorizontal yo'nalishda s siljishi, hamda qancha uzoqqa borishi aniqlansin (1.22-rasm).

Yechimi.

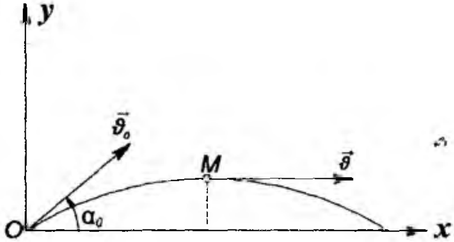
Traektoriyaning tenglamasini aniqlash uchun nuqtaning harakat tenglamalarining biridan t vaqtni topib, ikkinchi tenglamaga qo'yamiz:

$$t = \frac{x}{\vartheta_0 \cos\alpha} \quad (1.67)$$

$$y = tg\alpha \cdot x - \frac{g}{2\vartheta_0^2 \cos^2\alpha} x^2 \quad (1.68)$$

(1.68) ifoda parabola tenglamasidir.

Nuqtaning traektoriyasi mazkur parabolaning $x \geq 0$ shartni qanoatlantiruvchi qismidan iborat (1.22-rasm).



1.22-rasm

Nuqta eng yuqori holatga ko'tarilguncha o'tgan vaqt va maksimal ko'tarilish balandligini aniqlash uchun tezlikning koordinata o'qlaridagi proeksiyalarini aniqlaymiz:

$$\vartheta_x = \dot{x} = \vartheta_0 \cos\alpha\,,$$

$$\vartheta_y = \dot{y} = \vartheta_0 \sin\alpha - gt\,. \qquad (1.69)$$

Nuqta maksimal balandlikni egallaganda uning tezligi x o'qiga parallel bo'ladi. Shu sababli

$$\vartheta_y = 0$$

yoki

$$\vartheta_0 \sin\alpha - gt_1 = 0 \qquad (1.70)$$

bo'ladi, bunda t_1 nuqta eng yuqori holatga ko'tarilguncha o'tgan vaqt. (1.70) dan

$$t_1 = \frac{\vartheta_0 \sin\alpha}{g} \qquad (1.71)$$

Vaqt t_1 ning qiymatini (1.66)ga qo'yib, nuqtaning maksimal ko'tarilish balandligini aniqlaymiz:

$$h = y_{max} = \frac{\vartheta_0^2 \sin^2\alpha}{g} - \frac{g\vartheta_0^2 \sin^2\alpha}{2g^2} = \frac{\vartheta_0^2 \sin^2\alpha}{2g} \qquad (1.72)$$

Nuqta maksimal balandlikka ko'tarilganda boshlang'ich holatidan gorizontal yo'nalishda s siljishini aniqlash uchun vaqt t ning qiymatini (1.65)ga qo'yamiz:

$$s_1 = x_1 = \vartheta_0 \cos\alpha \cdot \frac{\vartheta_0 \sin\alpha}{g} = \frac{\vartheta_0^2 \sin 2\alpha}{2g} \qquad (1.73)$$

Nuqtaning maksimal uchish uzoqligi (qancha uzoqqa borishi) traektoriya tenglamasidan y=0 bo'lgan holatda (harakatlanayotgan jism yerga tushganda) aniqlanadi:

$$tg\alpha \cdot x - \frac{gx^2}{2\vartheta_0^2 cos^2\alpha} = 0 \qquad (1.74)$$

Bu tenglamadan x ning ikki qiymati

$$x_1 = 0, \quad x_2 = \frac{\vartheta_0^2 sin2\alpha}{g} \qquad (1.75)$$

aniq bo'ladi. Bunda x_1 nuqtaning boshlang'ich holatini, x_2 esa, nuqtaning gorizontal yo'nalishda uchish uzoqligini ifodalaydi. Binobarin, nuqtaning maksimal uchish uzoqligi quyidagiga teng bo'lar ekan:

$$x_2 = s_{max} = \frac{v_0^2 sin2\alpha}{g}$$

4-masala. Nuqta harakatining berilgan tenglamalariga qarab uning traektoriya tenglamasi topilsin; shuningdek, masofani nuqtaning boshlang'ich holatidan hisoblab, nuqtaning traektoriya bo'ylab harakatlanish qonuni ko'rsatilsin.

x=3sint, y=3cost

Yechish. Nuqta traektoriyasini aniqlash uchun harakatning berilgan tenglamalaridan vaqt t ni qisqartiramiz:

$$\left.\begin{aligned} \sin t &= \frac{x}{3} \\ \cos t &= \frac{y}{3} \end{aligned}\right\}$$

Tenglamalarning har ikki tomonlarini kvadratga ko'tarib qo'shsak, quyidagi ko'rinishdagi tenglamaga ega bo'lamiz:

$$\frac{x^2}{9} + \frac{y^2}{9} = 1 \text{ yoki } x^2 + y^2 = 9$$

Mazkur tenglama, radiusi R=3 bo'lgan aylana tenglamasini ifodalaydi. Nuqtaning tezligi quyidagicha aniqlanadi:

$$v_x = \frac{dx}{dt} = 3\cos t \quad v_y = -3\sin t \quad v = \sqrt{v_x^2 + v_y^2} = 3$$

Nuqtaning tezligini bilgan holda traektoriya bo'ylab harakatlanish qonunini aniqlaymiz:

$$v=\frac{ds}{dt};\ 3=\frac{ds}{dt}\ \text{bundan}\ \int_0^s ds=\int_0^t 3dt$$

Natijada nuqtaning traektoriya bo‘ylab harakatlanish qonuni uchun S=3t tenglamaga ega bo‘lamiz.

8-§ . Mustaqil o‘rgamish uchun talabalarga tavsiya etiladigan masalalar

Masala -1. Nuqtaning koordinata usulida berilgan harakat tenglamasiga ko‘ra uning traektoriya tenglamasi topilsin va rasmda harakat yo‘nalishi ko‘rsatilsin.

X=3t-5 y=4-2t

Masala -2. Nuqtaning koordinata usulida berilgan harakat tenglamasiga ko‘ra uning traektoriya tenglamasi topilsin va rasmda harakat yo‘nalishi ko‘rsatilsin.

X=5sin10t, y=3cos10t

Masala -3. Nuqta harakatining berilgan tenglamalariga qarab unung traektoriya tenglamasi topilsin; shuningdek, masofani nuqtaning boshlang‘ich holatidan hisoblab, nuqtaning traektoriya bo‘ylab harakatlanish qonuni ko‘rsatilsin.

$X=3t^2$ $y=4t^2$

Masala -4. Nuqta harakatining berilgan tenglamalariga qarab unung traektoriya tenglamasi topilsin; shuningdek, masofani nuqtaning boshlang‘ich holatidan hisoblab, nuqtaning traektoriya bo‘ylab harakatlanish qonuni ko‘rsatilsin.

$X=a\cos^2 t$ $y=a\sin^2 t$

Masala -5. Nuqtaning harakati $x=2a\cos^2\frac{kt}{2}$, y=asinkt tenglamalar bilan berilgan, bundagi *a* va *k* musbat o‘zgarmaslar. Masofani nuqtaning boshlang‘ich holatidan hisoblab, harakat traektoriyasi va traektoriya bo‘ylab harakat qonuni aniqlansin.

Masala -6. Moddiy nuqtaning harakati $S=(2t^2-8t+6)m$ tenglama orqali berilgan (t- sekundlarda o‘lchanadi). Qanday vaqt momentida nuqtaning tezligi nolga teng bo'ladi? Harakat boshlangan paytdan t=3s vaqt davomida bosib o'tgan yo'l aniqlansin. (1.23-rasm)

1.23-rasm

9-§ Nuqatаning tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtanig tezligi deb berilgan sanoq sistemasida har qanday vaqt onida nuqta harakatining qanchalik ildamligi va yo'nalishini ifodalovchi vektor kattalikka aytiladi:

$$\vec{\vartheta} = \frac{d\vec{r}}{dt} = v_x * \vec{i} + v_y * \vec{j} + v_z * \vec{k} \qquad (1.76)$$

Bunda $\vec{i}$, $\vec{j}$, $\vec{k}$ lar koorinata o'qlari birlik vektorlari.

Tezlik vektorining Dekart o'qlaridagi proyeksiyalari quyidagicha aniqlanadi:

$$\vartheta_x = \frac{dx}{dt} = x, \quad \vartheta_y = \frac{dy}{dt} = y, \quad \vartheta_y = \frac{dz}{dt} = z,$$

Tezlik moduli

$$\vartheta = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad (1.77)$$

formula asosida, uning yo'nalishi esa

$$\cos(\widehat{\vec{v} \wedge \vec{i}}) = \frac{v_x}{v}, \cos(\widehat{\vec{v} \wedge \vec{j}}) = \frac{v_y}{v}, \cos(\widehat{\vec{v} \wedge \vec{k}}) = \frac{v_z}{v} \qquad (1.78)$$

fo'rmulalar asosida aniqlanadi.

Ko'pincha masalalarda harakatdagi nuqtaning ma'lum vaqt oralig'idagi o'rtacha sur'atini aniqlash talab etiladi:

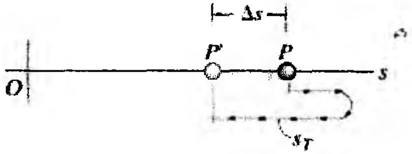
$$\vartheta_{ort} = \frac{\Delta S}{\Delta t}$$

Ba'zi hollarda harakatdagi nuqtaning "average speed" – o'rtacha tezligini topish ham ma'lum qiziqish uyg'otadi.

$$\Theta_{ort} = \frac{S_T}{\Delta t}$$

O‘rtacha tezlik har doim musbat kattalik hisoblanadi.

O‘rtacha sur’at va o‘rtacha tezlik quyidagi rasmdan yaqqol ko‘rinadi:



1.24-rasm

Agar nuqtaning harakati tabiiy usulda berilgan bo‘lsa, uning tezligi quyidagicha aniqlanadi:

$$\vec{v} = \frac{ds}{dt}\vec{\tau} = v_\tau \vec{\tau} \qquad (1.79)$$

Bunda $\vec{\tau}$- urinmaning birlik vektoru,u yoy koordinatasi S ning o‘sishi tomon yo‘naladi.

Tezlik moduli quyidagi formula yordamida aniqlanadi:

$$v = \frac{ds}{st} = s'$$

bunda: $v_\tau > 0$ bo‘lsa, nuqta yoy koordinatasining o‘sish tomoniga harakatlanadi.

$v_\tau < 0$ bo‘lsa, nuqta yoy koordinatasining kamayishi tomoniga harakatlanadi.

Nuqta kinematikasida nuqtaning tezligini aniqlashga doir masalalrni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinata o‘qlari sistemasi tanlab olinadi.

2. Tanlab olingan koordinata o‘qlari sistemasida nuqta harakatining tenglamalari tuziladi.

3. Nuqta harakatining tenglamalariga ko‘ra tezlik vektorining o‘qlaridagi proyeksiyalari aniqlanadi.

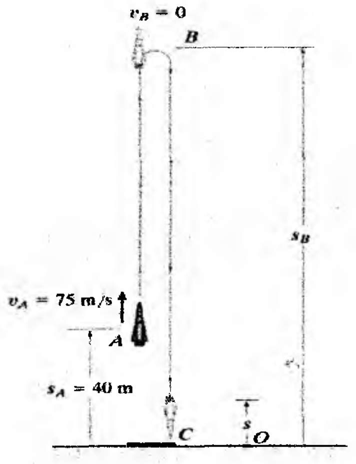
4. Nuqtaning tezligini o‘qlaridagi proyeksiyalariga ko‘ra uning miqdori va yo‘nalishi aniqlanadi.

10-§ Nuqtaning tezligini aniqlashga doir masalalar.

1. Masala.

Sinov paytida raketaning dvigateli u yerdan 40 m balandlikka ko'tarilganda ishdan chiqqan. U paytda raketa tezligi 75m/s bo'l-gan.Raketaning maksimal ko'tarilish balandligi va u qaytib yerga tushganda qanday tezlikka ega bo'lishi aniqlansin. Erkin tushish tezlanishi a_c=9.81m/s^2, u vertical payti yo'nalgan.

Havo qarshiligi e'tiborga olinmasin.



(1.25- rasm)

Yechilishi:

Koordinata boshi sifatida yer sirtidagi O nuqtani tanlab, koordinata o'qini raketa harakati tomon vertical va yuqoriga yo'naltiramiz.

Raketaning maksimal ko'tarilish balandligini aniqlaymiz. Raketa maksimal balandlik B nuqtaga yetganda uning tezligi quyidagicha ifodalanadi:

$$v_B^2 = v_A^2 + 2a_c(S_B - S_a)$$

Raketaning maksimal balandlikdagi tezligi ϑ_B bo'ladi. Shuning uchun

$$0=(75m/s)^2+2(-9.81m/s^2)(s_B-40m)$$

Bu ifodadan S_b=327m,

Raketa C nuqtaga tushganda uning tezligi quyidagiga teng bo'ladi:

$$v_C^2=v_B^2+2a_c(S_C-S_B)=0+2\left(-9.81\frac{m}{s^2}\right)(0-327)$$

Bu ifodadan

$$\vartheta c=-80.1m/s$$

$\overrightarrow{v_C}$ ning (-) ishorasi arametr pastga yo'nalganligidan darak beradi.

Raketaning yerga tushgandagi uning AC uchaskadagi harakatini o'rganishdan ham aniqlanadi.

$$v_C^2=v_A^2+2a_c(S_C-S_B)=\left(75\frac{m}{s}\right)^2+2\left(-9.81\frac{m}{s^2}\right)(0-40);$$

Bundan

$$v_C=-80.1\frac{m}{s},\quad |v_C|=80.1\frac{m}{s}.$$

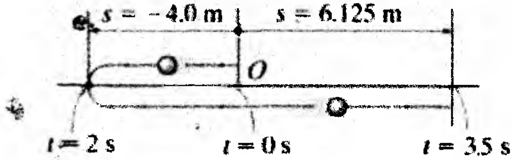
2.*masala.* Moddiy nuqta yo'lning qismida qismida

$\vartheta=(3t^2-6t)$ m/s tezlik bilan harakatlanmoqda,bunda t sekundlarda o'lchanadi.

Agar , dastlab nuqta O holatd abo'lsa, 3,5s. Davomida nuqta bosib o'tgan masofa va shu vaqt orasidagi o'rtacha sur'at va o'rtacha tezlik aniqlansin.

Yechish. 1. Koordinata o'qini nuqtaning to'g'ri chiziqli harakati trayektoriyasi bo'ylab o'ng tomon yo'naltiramiz.

Koordinata boshi sifatida nuqtaning boshlang'ich (t-0) holatini tanlaymiz (1.26-rasm).



1.26-rasm

2.Nuqtaning berilgan trayektoriyadagi o'rnini aniqlash usuli

$$\vartheta = ds/dt$$

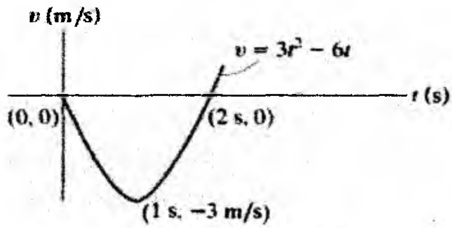
$$ds = \vartheta dt = (3t^2 - 6t)dt$$

$$\int_0^S dS = \int_0^t (3t^2 - 6t)dt$$

Tenglamani integrallasak va harakatning boshlang'ich shartlaridan foydalansak, nuqtaning istalgan vaqt momentida trayektoriyadagi o'rnini aniqlash uchun quyidagi tenglama(munbosabatga)ega bo'lamiz.

$$S = (t^3 - 3t^2)m$$

Avtomobilning t=3.5s vaqt onidagi trayektoriyada egallagan o'rnini aniqlash uchun harakat grafigini tuzamiz (1.27-rasm).



1.27-rasm

Harakat grafigidan ko'rinib turibdiki, 0<t<2 s vaqt oralig'ida , avtomobil tezligi manfiy ishoraga ega bo'lar ekan va avtomobil O nuqtadan chap tomonga harakatlanar ekan.

t>2 s dan boshlab,avtomobil tezligi musbat ishoraga ega bo'lib,u o'ng tomonga harakatlanar ekan. Avtomobil tezligi grafigida t=0, t=2 sva t=3,5s vaqt onlari uchun tezliklari ko'rsatilgan.

Avtomobil mazhur vaqt oralig'ida trayektoriyadagi o'rnini aniqlash uchun.

$$S = (t^3 - 3t^2)$$

Munosabatdan foydalanilamiz:

a). T=0 S=0

b). t=2.

v). t=3.5 s

Avtomobilning t=3.5 s. vaqt davomida bosib o'tgan **masofa** quyidagicha aniqlanadi:

$$S_r=4.0+4.0+6.125=14.125=14.12m$$

Avtomobil t=0 dan t=3,5 s. vaqt oralig'ida **ko'chishi** quyidagiga teng

$$\Delta S = S|_{t=3.5s} - S|_{t=0} = 6.125m - 0 = 6.125m.$$

Buni e'tiborga olsak, shu vaqt orasidagi o'rtacha sur'at (tezlikni o'zgarish jadalligi)quyidagiga teng bo'ladi:

$$v = \frac{\Delta S}{\Delta t} = \frac{6.125m}{3,5s-0} = 1,75\text{m/s}.$$

O'rtacha tezlik esa

$$v = \frac{S_T}{\Delta t} = \frac{14,125m}{3,5s-0} = 4,04\ m/s$$

3. ***masala.*** Nuqta harakati.

$$X=\vartheta_0 t\cos\alpha_0$$

$$y = v_0 t\sin\alpha_0 - \frac{1}{2}gt^2$$

Tenglamalar bilan berilgan; Ox o'q gorizontal, Oy o'q parametr bo'yicha yuqoriga yo'nalgan, vo, g va $\alpha_0 < \frac{\pi}{2}$ doimiy miqdorlar.

1. Nuqta trayektoriyasi,
2. Uning yuqori holatining koordinatalari
3. Nuqta Ox o'qda bo'lgan paytdagi tezligining koordinata o'qlaridagi proyeksiyalari topilsin.

Yechilishi:

1) Nuqtaning trayektoriyasini aniqlaymiz.

Masala shartiga nuata trayektoriyasining parametrik tenglamalari berilgan

Koordinatalar formasidagi trayektoriya tenglamasini tuzish uchun

berilgan tenglamalardan parameter "t"ni qisqartiramiz:

$$x=\vartheta_0 t\cos\alpha_0 \qquad (1)$$

$$y = v_0 t\sin\alpha_0 - \frac{1}{2}gt^2 \qquad (2)$$

(1) dan $t = \frac{x}{v_0\cos\alpha_0}$; (3)

(2) dan $t = \frac{v_0 \sin \alpha_0}{g} \pm \sqrt{\frac{v_0^2 \sin^2 \alpha_0}{2g} - \frac{2y}{g}}$ (4)

(3) va (4)larning o'ng tomonlarini tenglashtirsak, quyidagi tenglamaga ega bo'lamiz:

$$\frac{x}{v_0 \cos \alpha_0} = \frac{v_0 \sin \alpha_0}{g} \pm \sqrt{\frac{v_0^2 \sin^2 \alpha_0}{2g} - \frac{2y}{g}}$$

$$\left(\frac{x}{v_0 \cos \alpha_0} - \frac{v_0 \sin \alpha_0}{g}\right)^2 = \left(\sqrt{\frac{v_0^2 \sin^2 \alpha_0}{2g} - \frac{2y}{g}}\right)$$

4-masala. Nuqtaning harakati.

$$x = 2t, \quad y = t^2 \qquad (1)$$

tenglamalar bilan berilgan (t– sekundlarda, x va y – santimetrlarda o'lchanadi).

t=1 vaqt uchun tezlik va tezlanishning qiymati topilsin va shaklda ko'rsatilsin.

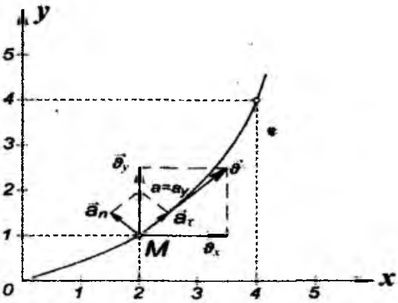
Yechish. Traektoriya tenglamasini tuzish uchun harakat tenglamalarining biridan vaqt t ni aniqlab, ikkinchisiga qo'yamiz:

$$t = \frac{x}{2}, \qquad y = \frac{x^2}{4}. \qquad (2)$$

Bu tenglama parabola tenglamasi. Binobarin, nuqtaning traektoriyasi paraboladan iborat ekan.

Traektoriyani chizish uchun (2) tenglamada x ga qiymatlar berib, unga mos y ning qiymatlarini topamiz (1.28-rasm).

x	0	2	4
y	0	1	4



1.28-rasm

t sekundda nuqtaning traektoriyada o‘rnini topamiz. Buning uchun berilgan harakat tenglamalaridagi t ning o‘rniga uning qiymatini qo‘yib, nuqtaning koordinatalarini topamiz.

t=1s.da x=2sm, y=1sm

Demak, t=1 sekundda nuqtaning koordinatalari (2,1) bo‘lar ekan

Nuqtaning tezligini koordinata o‘qlaridagi proeksiyalari orqali aniqlaymiz:

$$\vartheta_x = \dot{x} = 2\, sm/s, \quad (\vartheta_x = const).\vartheta_y = \dot{y} = 2t\, sm/s\,. \tag{3}$$

t=1s. da, ϑ_x=2m/s ϑ_y=2*1=2m/s

Natijada

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = 2\sqrt{2} \text{ sm/s.} \tag{4}$$

tezlik uchun masshtabni 1 smda 2 m/s deb tanlaymiz va chizmada ko‘rsatamiz (4- rasm).

11-§ Mustaqil o‘rgamish uchun talabalarga tavsiya etiladigan muammolar

Muammo -1. Moddiy nuqta to‘g‘ri chiziq bo‘ylab $v=(4t-3t^2)$ m/s tezlik bilan harakatlanmoqda, bu ifodada t sekundlarda o‘lchanadi. Agar $t=0$ da $S=0$ bo‘lsa, nuqtaning $t=4s$ da traektoriyadagi o‘rni aniqlansin (1.29-rasm).

1.29-rasm

Muammo -2. Shar vertikal holda yuqoriga Yerdan *15 m/s* tezlik bilan harakatlana boshlagan. Shar Yerga qancha vaqt o‘tgach qaytip tushadi? (1.30-rasm).

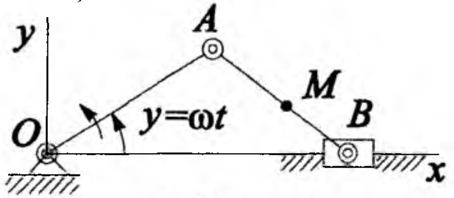
1.30-rasm

Muammo -3. Harakatdagi nuqtaning traektoriyadagi o‘rni $S=(2t^2-8t+6)$ m masofa orqali aniqlanadi. Harakat boshlangandan qanday vaqt o‘tgach nuqta tezligi *0* ga teng bo‘ladi? Nuqta $t=3$ *s* vaqt davomida qanday masofani bosib o‘tadi? (1.31-rasm)

1.31-rasm

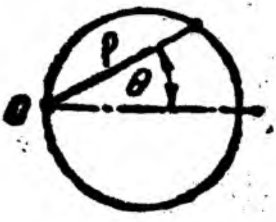
Muammo -4. Nuqta to‘g‘ri chiziq bo‘ylab $a=\left(12t-3t^{\frac{1}{2}}\right)$ m/s^2 tezlanish bilan harakatlanmoqda, bunda t sekundlarda o‘lchanadi. Nuqtaning tezligi va to‘g‘ri chiziqdagi holati (o‘rni) vaqt funksiyasi sifatida aniqlansin. $t=0$ da $v=0$, $S=15$ *m* bo‘lgan.

Muammo -5. OA krivoship ω o‘zgarmas burchak tezlik bilan aylanadi. Krivoship n-polzunli mexanizm shatunining o‘rtasidagi M nuqtaning tezligi va polzunning tezligi vaqt funksiyasi sifatida topilsin; OA=AB=a (1.32-rasm)



1.32- rasm.

Muammo -6. Elektrovozning tezligi $v_0=72$ *km/soat*; g‘ildiragining radiusi $R=1$ *m*; g‘ildirak to‘g‘ri chiziqli temir izda sirpanmasdan g‘ildirab boradi. G‘ildirak gardishidagi M nuqtaning radiusi v_0 tezlik yo‘nalishi bilan $\frac{\pi}{2}+\alpha$ burchak hosil qilgan paytda shu nuqta v tezligining miqdori va yo‘nalishi aniqlansin (1.33-rasm).



1.33-rasm.

12-§ Nuqataning tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning tezlanishi deb nuqta tezligining vaqt o'tishi bilan miqdor va yo'nalish jixatdan o'zgarishini ifodalovchi vektor kattalikka aytiladi:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \qquad (1)$$

Bu ifodada

$$a_x = \frac{dv_x}{dt} = x'', a_y = \frac{dv_y}{dt} = y'', a_z = \frac{dv_z}{dt} = z''$$

Tezlanishning koordinata o'qlaridagi pyeksiyalari mumkin bo'lsa tezlanish moduli quyidagicha aniqlanadi:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Tezlanish vektorining yo'nalishi esa uning yo'naltiruvchi qoidalari orqali aniqlanadi.

$$\cos(\vec{a}^\wedge\vec{i}) = \frac{a_x}{a}, \cos(\vec{a}^\wedge\vec{j}) = \frac{a_y}{a}, \quad \cos(\vec{a}^\wedge\vec{k}) = \frac{a_z}{a}$$

Ba'zan, masalalar yechishda nuqtaning ma'lum vaqt oralig'idagi o'rtacha tezlanishini aniqlash talab etiladi.

$$a_{o'r.} = \frac{\Delta v}{\Delta t},$$

Bunda $\Delta v = v' - v$ nuqataning tezlanishini Δt vaqt oraligida o'zgarishi.

Nuqtaning harakati tabiiy usulda berilganda uning tezlanishi

$$\vec{a} = \vec{a}_\tau + \vec{a}_n = \frac{dv}{dt}\vec{\tau}_0 + \frac{v^2}{\rho}\vec{n}_0 \ (2)$$

formula asosida aniqlanadi.

Bu ifodada $\vec{a}_\tau$ va $\vec{a}_n$lar nuqtaning urinma va normal tezlanishlarini ifodalaydi.

Bunday holda tezlanish moduli

$$a = \sqrt{a_\tau^2 + a_n^2}$$

formula asosida hisoblanadi.

Tezlanishning yo'nalishi esa quyidagi formulada aniqlanadi.

$$tg\mu = \frac{|a_\tau|}{a_n}$$

Nuqta kinematikasida nuqtaning tezlanishini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi.

1. Koordinata o'qlari sistemasi tanlab olinadi.
2. Tanlab olingan koordinata o'qlari sistemasida nuqta harakatining tenglamalari tuziladi.
3. Nuqta harakatining tenglamalariga ko'ra tezlanish vektorining o'qlardagi proyeksiyalari aniqlanadi.
4. Nuqtaning tezlanishini o'qlardagi proyeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.

Agar moddiy nuqtaning tezlanishi mavxum bo'lsa, u orqali nuqta harakatining tenglamalari va trayektoriyasini aniqlash mumkin.

Nuqta harakatining tezlanishi orqali uning harakati tenglamalarini va trayektoriyasini aniqlashda quyidagi amallarni bajarish tavsiya etiladi:

1. Koordinata o'qlari sistemasi tanlab olinadi.
2. Tezlanishning tanlab olingan o'qlardagi proyeksiyalari aniqlanadi.
3. Hosil bo'lgan tenglamani integrallab , nuqta tezligining o'qlardagi proyeksiyalari aniqlanadi.
4. Nuqta tezligining ma'lum vaqt oni uchun mumkin bo'lgan qiymatlaridan foydalanib hosil bo'lgan ifodalarda ishtirok etuvchi integrallash o'zgarmaslari aniqlanadi.
5. Hosil bo'lgan tezlikning o'qlardagi proyeksiyalari bo'lmish ifodalarni integrallab, nuqtaning harakat tenglamalari aniqlanadi.

6. Nuqtaning biror vaqt uchun ma'lum bo'lgan koordinatalaridan foydalanib , integrallash o'zgarmaslari aniqlanadi.

7. Hosil bo'lgan nuqtaning harakat tenglamalaridan vaqtni yo'qotib (qisqartirib), koordinatalar formasidagi trayektoriya tenglamasi tuziladi

13-§ Nuqtaning tezlanishini aniqlashga doir masalalar

1-Masala.

Samolyotdan h=320m balandlikdan tashlangan yuk

$$x=60t, \qquad y=5t^2 \tag{3}$$

tenglamalarga asosan harakatlanadi, bunda x,y lar metrlarda, *t* – sekundlarda o'lchanadi.

Yukning traektoriyasi, samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofa, yerga tushish paytidagi tezligi va tezlanishi, tushish nuqtasida traektoriyaning egrilik radiusi aniqlansin (1.34a-rasm).

Yechimi.

Yukning traektoriyasini aniqlaymiz. Buning uchun harakat tenglamalarining biridan t vaqtni topib ikkinchi tenglamaga qo'yamiz:

$$t=\frac{x}{60}; \quad y=5\left(\frac{x}{60}\right)^2=\frac{1}{720}x^2.$$

Natijada

$$y=\frac{1}{720}x^2 \tag{4}$$

ko'rinishdagi parabola tenglamasi hosil bo'ladi. Demak, yukning traektoriyasi y o'qiga simmetrik, uchi koordinata boshida bo'lgan parabola ekan (1.34a-rasm).

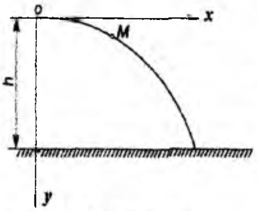
Yukning samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofani aniqlaymiz. Yukning M_1 tushish nuqtasidagi $y_1=h$, $x_1=l$ koordinatalarni aniqlash uchun yukning harakat tenglamalaridan foydalanamiz (1.34b-rasm):

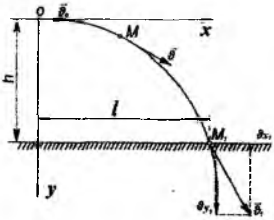
$$y=5t^2, \qquad t_1=\sqrt{\frac{y_1}{5}}=\sqrt{\frac{h}{5}}=8s. \tag{5}$$

Shuning uchun

$$l = x_l = 60t = 60*8 = 480m.$$

Demak, yukning samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofa 480 m ekan.





1.34a-rasm 1.34b-rasm

Yukning tushish nuqtasidagi tezligi va tezlanishini aniqlaymiz.

Yukning tezligi uning koordinata o‘qlaridagi proeksiyalari orqali aniqlanadi:

$$\vartheta_1 = \sqrt{\vartheta_{x_1}^2 + \vartheta_{y_1}^2} = \sqrt{60^2 + (10t)^2}. \qquad (6)$$

Yuk yerga tushganda t_1=8s shuning uchun

$$\vartheta_1 = \sqrt{3600 + 6400} = 100 \ m/s.$$

Yukning tezlanishi ham uning tezligi kabi aniqlanadi (1v-rasm).

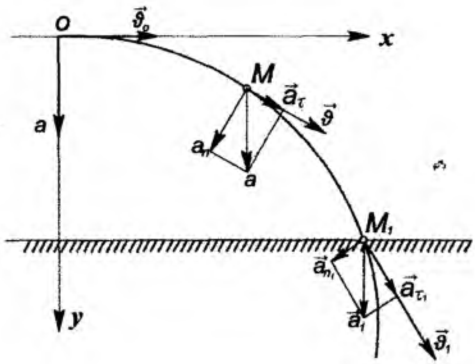
$$a_1 = \sqrt{a_{x_1}^2 + a_{y_1}^2},$$

$$a_{x_1} = \frac{d\vartheta_{x_1}}{at} = 0, \quad a_{y_1} = \frac{d\vartheta_{y_1}}{at} = 10.$$

Shuning uchun

$$a_1 = \sqrt{a_{x_1}^2 + a_{y_1}^2} = 10\,\mathrm{m/s^2}. \qquad (7)$$

Yukning yerga tushish nuqtasida traektoriyaning egrilik radiusini aniqlash uchun uning *urinma* va *normal* tezlanishini aniqlaymiz (1.34v-rasm).



1.34v-rasm

Yukning urinma tezlanishini quyidagi formula yordamida aniqlaymiz:

$$a_\tau = \left|\frac{d\vartheta}{at}\right| = \frac{\vartheta_x a_x + v_y a_y}{\vartheta} = 8\ \text{m/s}^2. \qquad (8)$$

Yukning normal tezlanishi quyidagicha aniqlanadi:

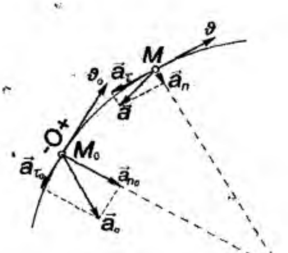
$$a^2 = a_\tau^2 + a_n^2,\ a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{100 - 64} = 6\text{m/s}^2. \qquad (9)$$

Traektoriyaning yuk tushgan a_0 nuqtasining egrilik radiusi

$$\rho = \frac{\vartheta^2}{a_n} = \frac{100^2}{6} = 1667\ \text{m}. \qquad (10)$$

2-Masala.

Poezd radiusi R=1 km bo'lgan aylana yoyi bo'ylab tekis sekinlanuvchan harakat qiladi va S=1 m. yo'l bosadi. Uning boshlang'ich tezligi $\vartheta_0 = 36\frac{km}{soat}$, boshlang'ich tezlanishi esa a_0=0.125m/s^2. Poezdning yoy oxiridagi tezligi va tezlanishi aniqlansin (1.35-rasm).



1.35-rasm

Yechimi.

Poezd nuqtalaridan birining, masalan, og'irlik markazining harakatini o'rganamiz.

Poezdning harakat tenglamasini yozish uchun yoy koordinatasining sanoq boshini tanlashimiz kerak. Bunday nuqta sifatida poezdning boshlangich holatini olamiz va poezdning harakat yo'nalishini musbat yunalish deb qabul qilamiz. Bu holda $S_0=0$

Nuqtaning tekis sekinlanuvchan harakatida uning harakat tenglamasi va tezligi quyidagi formulalar asosida ifodalanadi:

$$s = \vartheta_0 t - \frac{a_\tau t^2}{2}, \quad (11)$$

$$\vartheta = \vartheta_0 - a_\tau t, \quad (12)$$

bunda a_τ – urinma tezlanish moduli.

Masala shartidan harakatdagi M nuqtaning yoy oxiridagi yoy koordinatasi S=560 m, boshlang'ich tezligi $\vartheta_0 = 36\frac{km}{s} = 10\,m/s$, boshlang'ich tezlanish $a_0=0.125 m/s^2$, hamda traektoriyaning egrilik radiusi R=100m berilgan.

M nuqtaning yoy boshidagi normal tezlanishini quyidagi formula asosida aniqlaymiz:

$$a_{no} = \frac{v_0^2}{\rho} = \frac{100}{1000} = 0{,}1\ m/s^2.$$

M nuqtaning yoy boshidagi to'la tezlanishini bilgan holda, uning yoy boshidagi urinma tezlanishini aniqlaymiz:

$$a_0^2 = a_{n0}^2 + a_\tau^2; \qquad a_\tau = \sqrt{a_0^2 - a_{n0}^2} = \sqrt{0{,}125^2 - 0{,}1^2} = 0{,}075\ m/s^2.$$

Nuqtaning harakati tekis sekinlanuvchan bo'lganligi uchun

$$a_\tau = const.$$

(11) va (12) tenglamalarga aniqlangan kattaliklarning qiymatlarini qo'yamiz:

$$560=10t-0.075t^2, \quad (13)$$

$$\vartheta=10-0{,}075\ t. \quad (14)$$

Bu tenglamalardan harakatlanish vaqti t – aniqlanadi:

$$0.075t^2-20t+1120=0$$

$$t = \frac{10 \pm \sqrt{100 - 1120 \cdot 0{,}075}}{0{,}075} = \frac{10 \pm 4}{0{,}075}\ s.$$

Harakatlanish vaqti uchun kichik ildiz qiymatini tanlaymiz:

$$t = \frac{6}{0{,}075} = 80\ s, \qquad (15)$$

chunki katta ildiz qiymati (187s) nuqtaning to'xtashi uchun (ϑ=0) ketgan vaqtdan katta.

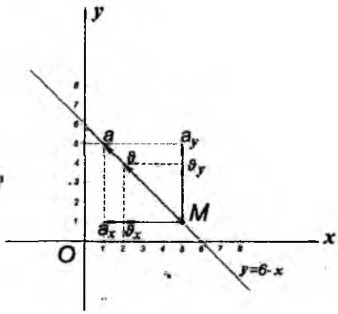
$$(t_{to'xtash} = \frac{\vartheta_0}{a} = \frac{10}{0{,}075} = 133\ \text{sek.}). \qquad (16)$$

(1.12) tenglamadan nuqtaning yoy oxiridagi tezligini aniqlaymiz:

$$\vartheta=\vartheta_0-at=10-0.075*80=4\ \text{m/s}. \qquad (17)$$

Nuqtaning yoy oxiridagi normal tezlanishi quyidagiga teng bo'-ladi:

$$a_n = \frac{\vartheta^2}{R} = \frac{\vartheta^2}{1000} = 0{,}016\ \text{m/s}^2. \qquad (18)$$



Nuqtaning yoy oxiridagi to'la tezlanishi quyidagi formuladan aniqlanadi:

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{(0{,}075)^2 + (0{,}016)^2} = 0{,}0767\ \text{m/s}^2. \qquad (19)$$

Aylana yoyi bo'ylab tekis sekinlanuvchan harakatda nuqtaning urinma tezlanishining moduli o'zgarmaydi, to'la tezlanish moduli esa, normal tezlanish modulining kamayishi tufayli kamayadi.

Aniqlangan tezlik va tezlanishlar 1.36-rasmda tasvirlangan.

3-Masala.

M nuqtaning berilgan harakat tenglamalariga ko'ra traektoriyasining ko'rinishi aniqlansin va t=t_1 vaqt oni uchun nuqtaning traekdagi o'rni, uning tezligi, to'la, urinma va normal tezlanishlari, hamda traektoriyaning egrilik radiusi topilsin.

$$x = 4\cos^2\left(\frac{\pi t}{3}\right) + 2\ \text{sm},$$

$$y = 4\sin^2\left(\frac{\pi t}{3}\right)\ \text{sm}.$$

$$t=1/2\ \text{s}.$$

Yechimi.

Nuqtaning traektoriyasini aniqlaymiz. Traektoriya tenglamasini tuzish uchun harakat tenglamalaridan t vaqtni yo'qotamiz. Buning uchun, berilgan masalada quyidagi ayniyatdan foydalanamiz:

$$sin^2\left(\frac{\pi t}{3}\right) + cos^2\left(\frac{\pi t}{3}\right) = 1. \qquad (20)$$

Masalada:

$$sin^2\left(\frac{\pi t}{3}\right) = \frac{y}{4}, \quad cos^2\left(\frac{\pi t}{3}\right) = \frac{x-2}{4}, \qquad (21)$$

(20) ni e'tiborga olsak

$$\frac{y}{4} + \frac{x-2}{4} = 1, \text{ yoki } \quad y = 6 - x.$$

Nuqtaning traektoriyasi to'g'ri chiziqdan iborat ekan.

Harakat tenglamalaridan foydalanib, nuqtaning t=1/2 ekunddagi koordinatalarini topamiz va shaklda ko'rsatamiz (1-36 rasm):

$$x = 4cos^2\frac{\pi}{6} + 2 = 4\left(\frac{\sqrt{3}}{2}\right)^2 + 2 = 5 \text{ sm},$$

$$y = 4sin^2\frac{\pi}{6} = 4 \cdot 0{,}25 += 1 \text{ sm} \qquad (22)$$

Demak, t=1/2 sekundda nuqtaning koordinatalari x=5,y=1 bo'lar ekan.

Nuqtaning tezligini uning koordinata o'qlaridagi proeksiyalari orqali aniqlaymiz:

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2}.$$

Buning uchun nuqta harakat tenglamalaridan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$\vartheta_x = \dot{x} = -\frac{8\pi}{3}\cos\left(\frac{\pi t}{3}\right)\sin\left(\frac{\pi t}{3}\right) = -\frac{4\pi}{3}sin\left(\frac{2\pi t}{3}\right) \qquad (23)$$

$$\vartheta_y = \dot{y} = \frac{8\pi}{3}\sin\left(\frac{\pi t}{3}\right)\cos\left(\frac{\pi t}{3}\right) = \frac{4\pi}{3}sin\left(\frac{2\pi t}{3}\right). \qquad (24)$$

t=1/2 s da,

$$\vartheta_x = -\frac{4\pi^2}{3}\sin\left(\frac{2\pi}{6}\right) = -\frac{4\pi}{3}\cdot\frac{\sqrt{3}}{2} = -3{,}6 \text{ m/s},$$

$$\vartheta_y = \frac{4\pi^2}{3}\sin\left(\frac{2\pi}{6}\right) = \frac{4\pi}{3}\cdot\frac{\sqrt{3}}{2} = 3{,}6 \text{ sm/s}.$$

Binobarin,

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2} = 5{,}1 \text{ m/s}.$$

Tezliklar uchun masshtab tanlab, ularni shaklda ko'rsatamiz (1.36 rasm).

Nuqtaning tezlanishini uning koordinata o'qlaridagi proeksiyalari orqali aniqlaymiz:

$$a = \sqrt{a_x^2 + a_y^2}. \qquad (25)$$

Buning uchun ϑ_x, ϑ_y lardan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$a_x = \vartheta_x' = \ddot{x} = -\frac{8\pi^2}{9}\cos\left(\frac{2\pi t}{3}\right),$$

$$a_y = \frac{8\pi^2}{9}\cos\left(\frac{2\pi t}{3}\right). \qquad (26)$$

t=1/2 s da,

$$a_x = -\frac{8\pi^2}{9}\cos\left(\frac{2\pi}{6}\right) = -\frac{8\pi^2}{9}\cdot 0{,}5 = -4{,}4\ \text{sm/s}^2,$$

$$a_y = \frac{8\pi^2}{9}\cos\left(\frac{2\pi}{6}\right) = \frac{8\pi^2}{9}\cdot 0{,}5 = 4{,}4\ \text{sm/s}^2.$$

Binobarin,

$$a = \sqrt{a_x^2 + a_y^2} = 6{,}2\ \text{m/s}^2.$$

Nuqtaning urinma tezlanishi quyidagiga teng bo'ladi:

$$a_\tau = \left|\frac{d\vartheta}{dt}\right| = \frac{\vartheta_x a_x + v_y a_y}{\vartheta} = 6{,}2\text{sm/s}^2. \qquad (27)$$

Nuqtaning normal tezlanishi quyidagicha aniqlanadi:

$$a^2 = a_\tau^2 + a_n^2;\ a_n = \sqrt{a^2 - a_\tau^2} = 0. \qquad (28)$$

Tezlanishlar uchun masshtab tanlab, ularni shaklda ko'rsatamiz (1.37- rasm).

Traektoriyaning egrilik radiusi quyidagi formula asosida aniqlanadi:

$$a_n = \frac{\vartheta^2}{\rho}; \qquad \rho = \frac{\vartheta^2}{a_n} = \infty. \qquad (29)$$

Masalada, nuqtaning traektoriyasi to'g'ri chiziq bo'lganligi uchun, egrilik radiusi ∞ ga teng.

Hisoblash natijalarini quyidagi jadvalda joylashtiramiz:

Nuqta koordina-talari (sm)		Nuqta tezligi (sm/s)			Nuqta tezlanishi (sm/s^2)					Egrilik radiusi (sm)
x	y	ϑ_x	ϑ_y	ϑ	a_x	a_y	a	a_τ	a_n	ρ
5	1	-3,6	3,6	5,1	-4,4	4,4	6,2	62	0	∞

Aniqlangan kattaliklar *1.36*-rasmda ko‘rsatilgan.

4-masala.

Avtomobil yo‘lning to‘g‘ri chiziqli uchastkasida ma’lum qisqa vaqt harakatlanib $\vartheta=(3t^2+2t)$m/s tezlikka ega bo‘ladi(ifodada t sekundda o‘lchanadi), harakat boshlangan vaqtda t_1=3s o‘tgach avtomobilning bisib o‘tgan yo‘li va tezlanishi aniqlansin. t=0 da S=0 bo‘lgan.

Yechimi.

1. Koordinata o‘qini avtomobil harakati tomon yo‘naltiramiz.

2. Avtomobilning t=3 s da bosib o‘tgan yo‘lini aniqlaymiz. Koordinata boshi sifatida avtomobilnbing boshlang‘ich holatini tanlaymiz.Nuqtaning trayektoriyadagi o‘rni $v=\frac{dS}{dt}$ formuladan aniqlanadi. t=0 da S=0 bo‘lgan. $v=\frac{dS}{dt}=(3t^2+2t)$

$\int_0^S dS=\int_0^t(3t^2+2t)dt$

$$\int_0^S S=(t^3+t^2)\int_0^t.$$

$$S=t^3+t^2$$

Harakat boshlangandan t= 3s vaqt o‘tgach avtomobil bosib o‘tgan yo‘l quyidagiga teng bo‘ladi.

$$S=(3)^3+(3)^2=36m$$

3. Avtomobilning t= 3s dagi tezlanishini aniqlaymiz.

$$a=\frac{dv}{dt}=\frac{d(3t^2+2t)}{dt}=6t+2$$

Harakat boshlangandan t= 3s vaqt o‘tgach

$$a=6(3)+2=20m/s$$

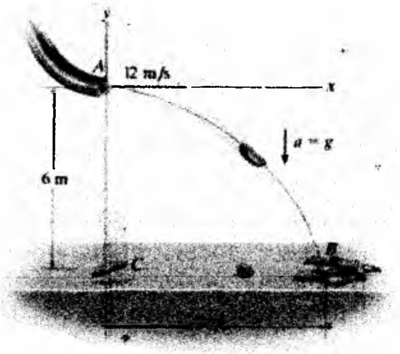
14-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo.

Moddiy nuqta to'g'ri chiziq bo'ylab $a=(10-0.25)m/s^2$ tezlanish bilan harakatlanadi(S-metrlarda o'lchanadi). Nuqta S= 10 m masofani bosib o'tgach qanday tezlikka ega bo'ladi. Nuqtaning boshlang'ich tezligi $\vartheta_0=5m/s$ ga teng.

2-muammo.

Moddiy nuqta to'g'ri chiziq bo'ylab $\vartheta=(20-0.05s^2)$ m/s tezlik bilan harakatlanmoqda (S- metrlarda o'lchanadi). Nuqta S= 15 m masofani bosib o'tgach qanday tezlanishga ega bo'ladi?



3-muammo.

Moddiy nuqtaning trayektoriyadagi holati to'g'ri chiziqli harakatida

$$S=(1.5t^3-13.5t^2+22.5t)$$

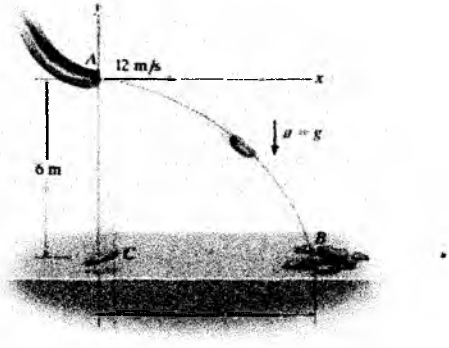
tenglama orqali aniqlanadi. Moddiy nuqtaning t= 6s bo'lganda trayektoriyadagi holati va 6 s vaqt intervalida bosib o'tgan yo'li aniqlansin.

4-muammo.

Moto to'g'ri chiziqli harakatida uning trayektoriyadagi holati rasmda ko'rsatilgan grafik asosida aniqlanadi.

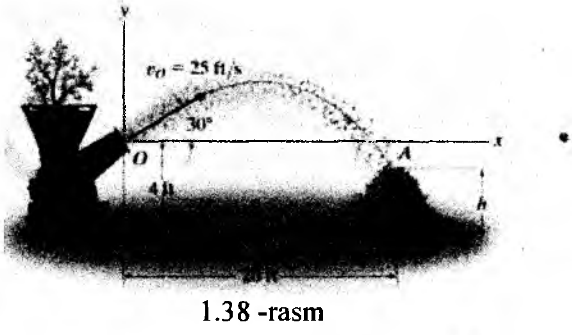
$0 \le t \le 30s$ vaqt intervali uchun ϑ-t va a-t garafiklari tuzilsin.

5-muammo. Qop nishablikda harakatlanib, A nuqtaga $\vartheta_A=12$m/s gorizontal tezlikka ega bo'ladi. Agar A nuqtaning poldan balandlikgi 6m bo'lsa, qopning B nuqtaga tushishi uchun ketadigan vaqt va B nuqtaning C nuqtadan qanday uzoqlikda yotishi aniqlansin (1.37-rasm).



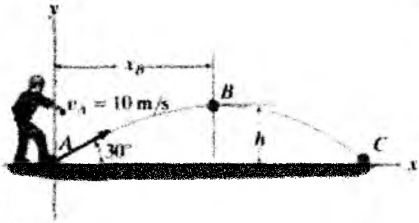
1.37 - rasm

6-muammo. Yog'och qirqadigan mashinaning a nuqtasidan yog'och qirindisi $\vartheta_0=25$m/s tezlik bilan otilib chiqadi. Agar qirindining otilib chiqish tezligi gorizontal bilan **90°** burchak tashkil qilsa, uning A tushish nuqtasining yerdan qanday h balandlikda bo'lishi aniqlansin. A nuqta qirindining otilib chiqishi nuqtasidan yer bo'ylab hisoblanganda 20 m uzoqlikda joylashgan (1.38 –rasm).



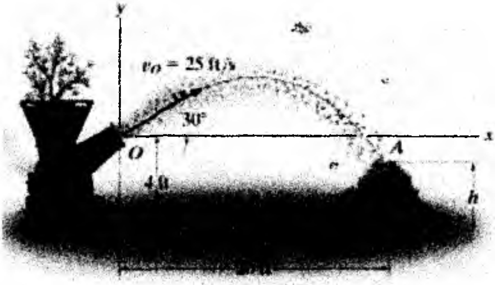
1.38 -rasm

7-muammo. Qop nishablikda harakatlanib, A nuqtaga ϑ_A=12m/s gorizontal tezlikka ega bo'ladi. Agar A nuqtaning poldan balandlikgi 6m bo'lsa, qopning B nuqtaga tushishi uchun ketadigan vaqt va B nuqtaning C nuqtadan qanday uzoqlikda yotishi aniqlansin (1.39-rasm).



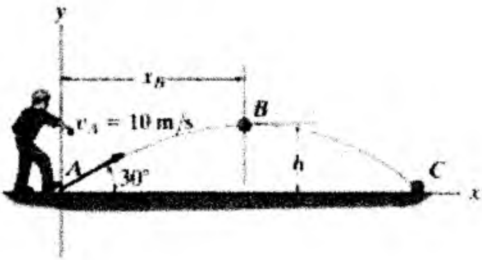
1.39- rasm

8-muammo. Yog'och qirqadigan mashinaning a nuqtasidan yog'och qirindisi ϑ_0=25m/s tezlik bilan otilib chiqadi. Agar qirindining otilib chiqish tezligi gorizontal bilan 90^0 burchak tashkil qilsa, uning A tushish nuqtasining yerdan qanday h balandlikda bo'lishi aniqlansin. A nuqta qirindining otilib chiqishi nuqtasidan yer bo'ylab hisoblanganda 20 m uzoqlikda joylashgan (1.40- rasm).



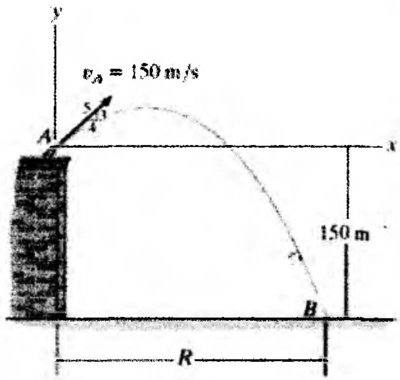
1.40 - rasm

9-muammo. Koptok A nuqtadan ϑ_A=10m/s tezlik bilan tepiladi. Koptokning maksimal ko'talish balandligini aniqlang (1.41 – rasm).



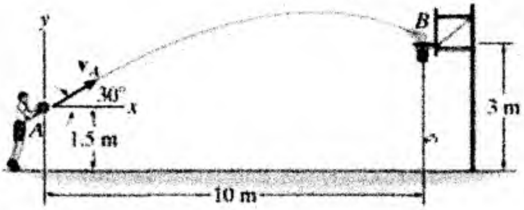
1.41 - rasm

10-muammo. Koptok A nuqtada ϑ_A=10m/s tezlik bilan tepiladi. Koptokning uchish uzoqligini va Yerga tushgandagi tezligi aniqlansin (1.42 – rasm).



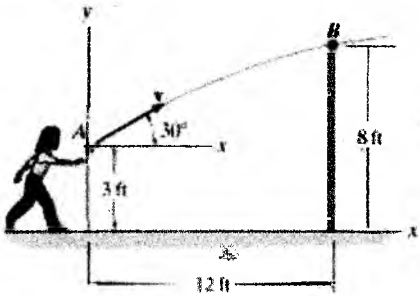
1.42- rasm

11-muammo. Basketbol to'pini A nuqtadan gorizont bilan $\alpha = 30°$ burchak hosil qiluvchi $\vec{v}_A$ tezlik bilan otilib, Yerda 3m balandlikda turuvchi basketbol setkasiga tushadi. Basketbol to'pining otilish tezligi ϑ_A aniqlansin (1.43 – rasm).



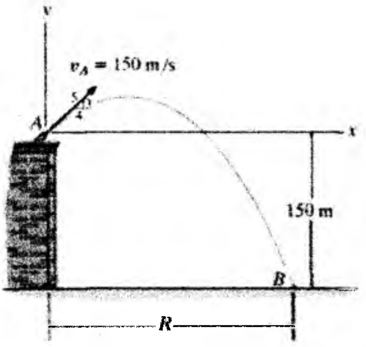
1.43 – rasm

12-muammo. To'p A nuqtadan otiladi. U Yerdan 8m, otilish nuqtasidan 12m masaofada joylashgan B nuqtaga tushish uchun, qanday ϑ_A tezlik bilan otiladi? (1.44 – rasm).



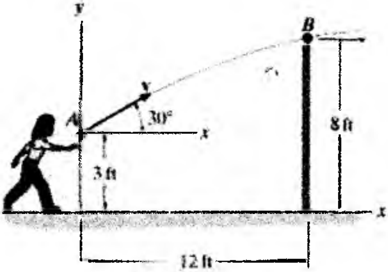
1.44 - rasm

13-muammo. Reaktiv snaryad A nuqtadan ϑ_A=250m/s tezlik bilan otiladi. Agar A nuqta Yerdan 150m balandlikda joylashgan bo'lsa , snaryadning uchish uzoqligi aniqlansin (1.45 – rasm).



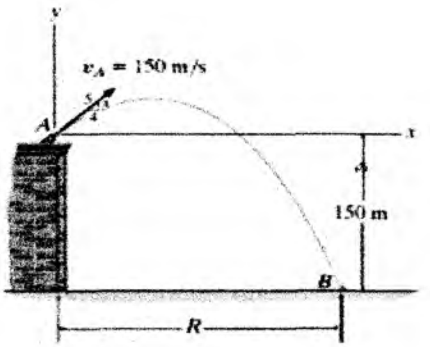
1.45 – rasm. Расм йўқ.

14-muammo. To‘p A nuqtadan otiladi. U Yerdan 8m, otilish nuqtasidan 12m masaofada joylashgan B nuqtaga tushishi uchun, qanday tezlikda otiladi? (1.46- rasm).



1.46 – rasm

15-muammo. Reaktiv snaryad A nuqtadan ϑ_A=250m/s tezlik bilan otiladi. Agar A nuqta Yerdan 150m balandlikda joylashgan bo‘lsa , snaryadning uchish uzoqligi aniqlansin (1.47 – rasm).



1.47 – rasm.

15-§ Talabalar mustaqil o'rganishi uchun keyslar

Nuqta harakatining berilgan tenglamalariga ko'ra uning tezligi va tezlanishini aniqlash.

M nuqtaning berilgan harakat tenglamalariga ko'ra traektoriyasining ko'rinishi aniqlansin va $t=t_1$(s) vaqt oni uchun nuqtaning traektoriyadagi o'rni, uning tezligi, to'la, urinma va normal tezlanishlari, hamda traektoriyaning egrilik radiusi topilsin.

Topshiriqni yechish uchun zarur bo'lgan ma'lumotlar quyidagi jadvalda keltirilgan.

Variantlar raqamlari	**Harakat tenglamalari**		t_1, s
	$x = x\,(t)$, *sm*	$y = y\,(t)$, *sm*	
1.	$x = 3t$	$y = 4t^2 + 1$	½
2.	$x = 7sin^2\left(\frac{\pi t}{6}\right) - 5$	$y = -7cos^2\left(\frac{\pi t}{6}\right)$	1
3.	$x = 1 + 3cos\left(\frac{\pi t^2}{3}\right)$	$y = 3\sin\left(\frac{\pi t^2}{3}\right) + 3$	1
4.	$x = -5t^2 - 4$	$y = 3t$	1

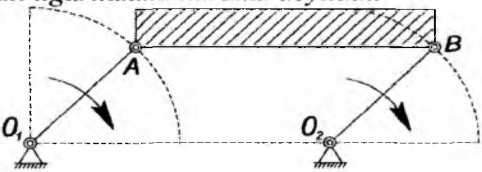
5.	$x = 2 - 3t - 6t^2$	$y = 3 - \frac{3}{2}t - 3t^2$	0
6.	$x = 6\sin\left(\frac{\pi t^2}{6}\right) - 2$	$y = 6\cos\left(\frac{\pi t^2}{6}\right) + 3$	1
7.	$x = 7t^2 - 3$	$y = 5t$	¼
8.	$x = 3 - 3t^2 + 1$	$y = 4 - 5t^2 + \frac{5}{3}t$	1
9.	$x = -4\cos\left(\frac{\pi t}{3}\right) - 1$	$y = -4\sin\left(\frac{\pi t}{3}\right)$	1
10.	$x = -6t$	$y = -2t^2 - 4$	1
11.	$x = 8\cos^2\left(\frac{\pi t}{6}\right) + 2$	$y = -8\sin^2\left(\frac{\pi t}{6}\right) - 7$	1
12.	$x = -4t^2 + 1$	$y = -3t$	1
13.	$x = 5t^2 + \frac{5}{3}t - 3$	$y = 3t^2 + t + 3$	1
14.	$x = 2\cos\left(\frac{\pi t^2}{3}\right) - 2$	$y = -2\sin\left(\frac{\pi t^2}{3}\right) + 3$	1
15.	$x = 4\cos\left(\frac{\pi t}{3}\right)$	$y = -3\sin\left(\frac{\pi t}{3}\right)$	1
16.	$x = -2t - 2$	$y = -\frac{2}{(t + 1)}$	2
17.	$x = 5\cos\left(\frac{\pi t^2}{3}\right)$	$y = -5\sin\left(\frac{\pi t^2}{3}\right)$	1
18.	$x = 5\sin^2\left(\frac{\pi t}{6}\right)$	$y = -5\cos^2\left(\frac{\pi t}{6}\right) - 3$	1
19.	$x = -4t^2 + 1$	$y = -3t$	½
20.	$x = -4\cos\left(\frac{\pi t}{3}\right)$	$y = -2\sin\left(\frac{\pi t}{3}\right) - 3$	1
21.	$x = -\frac{3}{(t + 2)}$	$y = 3t + 6$	2

22.	$x = 7\sin\left(\frac{\pi t^2}{6}\right) + 3$	$y = 2 - 7\cos\left(\frac{\pi t^2}{6}\right)$	1
23.	$x = 3t^2 - t + 1$	$y = 5t^2 - \frac{5}{3}t - 2$	1
24.	$x = 3t^2 + 2$	$y = -4t$	½
25.	$x = 2sin\left(\frac{\pi t}{3}\right)$	$y = -3cos\left(\frac{\pi t}{3}\right) + 4$	1
26.	$x = 4t + 4$	$y = -\frac{4}{(t+1)}$	2
27.	$x = -2t^2 + 3$	$y = -5t$	½
28.	$x = 4cos^2\left(\frac{\pi t}{3}\right) + 2$	$y = 4sin^2\left(\frac{\pi t}{3}\right)$	1
29.	$x = -cos\left(\frac{\pi t^2}{3}\right) + 3$	$y = \sin\left(\frac{\pi t^2}{3}\right) - 1$	1
30.	$x = -3 - 9\sin\left(\frac{\pi t^2}{6}\right)$	$y = -9\cos\left(\frac{\pi t^2}{6}\right) + 5$	1

II-BOB. QATTIQ JISMNING ILGARILANMA VA QO‘ZG‘ALMAS O‘Q ATROFIDA AYLANMA HARAKATI

16-§ Qattiq jismning ilgarilanma harakati

Jismda olingan har qanday kesma jism harakati davomida doimo o‘zining boshlang‘ich holatiga parallel qolsa, jismning bunday harakati ilgarilanma harakat deyiladi.



2.1-rasm

Jismning ilgarilanma harakatini uning to‘g‘ri chiziqli harakati bilan aralashtirib bo‘lmaydi. Ilgarilanma harakatdagi jism nuqtasining traektoriyasi egri chiziqdan iborat bo‘lishi ham mumkin. Masalan, 2.1-rasmda ko‘rsatilgan AB sparnikning harakati davomida O_1A va O_2A kripovshiplar O_1,O_2 nuqtalardan o‘tuvchi o‘qlar atrofida aylanadi, ***AB*** sparnik esa hamma vaqt o‘z-o‘ziga parallel qoladi, ya’ni ilgarilanma harakatda bo‘ladi.

Ilgarilanma harakatda bo‘lgan qattiq jismning kinematik xarakteristikalari quyidagi teoremada o‘z ifodalarini topgan:

Teorema.*Ilgarilanma harakatdagi jismning hamma nuqtalari bir xil traektoriya chizadi va har onda bir xil tezlik, hamda bir xil tezlanishga ega bo‘ladi*

Teoremani isbotlash uchun jismning berilgan Oxyz qo‘zg‘almas koordinatalar sistemasiga nisbatan ilgarilanma harakatini o‘rganamiz (2.2-rasm). Jismda ixtiyoriy A va B nuqtalarni olib, ularning radius vektorlarini $\vec{r}_A$ va $\vec{r}_B$ bilan belgilaymiz. Rasmdan:

2.2-rasm

$$\vec{r}_B = \vec{r}_A + \overrightarrow{AB} \qquad (2.1)$$

Jism harakatlanganda $\vec{r}_A, \vec{r}_B$ o'zgaradi, ammo AB kesmaning uzunligi va yo'nalishi o'zgarmaydi.

B nuqtaning tezligini aniqlash uchun (2.1) dan vaqt t bo'yicha hosila olamiz:

(2.2)

bunda, $\frac{d\overline{AB}}{dt} = 0$ bo'lgani uchun,

(2.3)

yoki $\vec{\vartheta}_B = \vec{\vartheta}_A$ bo'ladi.

Bu tenglik ilgarilanma harakatdagi jism barcha nuqtalarining har ondagi tezliklari bir xil bo'lishini ifodalaydi.

Agar (2.3) dan vaqt t bo'yicha hosila olsak:

(2.4)

yoki $\vec{a}_B = \vec{a}_A$ bo'ladi.

(2.4) tenglik ilgarilanma harakatdagi jism barcha nuqtalarining har ondagi tezlanishlari bir xil bo'lishini ifodalaydi.

Shunday qilib teorema isbotlandi.

Isbotlangan teoremadan jismning ilgarilanma harakati uning biror nuqtasininig harakati bilan aniqlanishi mumkinligi ma'lum bo'ladi. Odatda bunday nuqta sifatida jismning og'irlik markazi C nuqta olinadi.

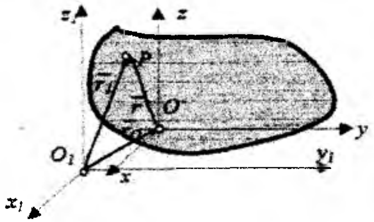
Olingan nuqtaning harakat tenglamalarini koordinata usulida quyidagicha yozish mumkin:

$$x_c=f_1(t), \quad y_c=f_2(t), \quad z_c=f_3(t) \qquad (2.5)$$

(2.5) tenglama C nuqtaning harakat tenglamasi bo'lib, jismning ilgarilanma harakat tenglamasini ham ifodalaydi.

Jismning ilgarilanma harakatida hamma nuqtalari uchun bir xil bo'lgan tezlik jismning ilgarilanma harakat tezligi deyiladi, tezlanish *esa – jismning ilgarilanma harakat tezlanishi deyiladi.* Ilgarilanma harakat tezlik vektori $\vec{v}$ va tezlanish vektori $\vec{a}$ larni jismning ixtiyoriy nuqtasiga qo‘yilgan holda ko‘rsatish mumkin. Bu hol qattiq jismning faqat ilgarilanma harakatida o‘rinli bo‘ladi. Boshqa harakatlarda jismning turli nuqtalari turlicha tezlik va turlicha tezlanishga ega bo‘ladi.

Qattiq jismning ilgarilanma harakatida jism O nuqtasining tezligi $\vec{\vartheta}_0 \neq 0$ bo‘lib, burchak tezlik $\vec{\omega}_0 = 0$ bo‘ladi (2.3-rasm).



2.3-rasm

Rasmdan

$$\vec{r}_1 = \vec{r}_0 + \vec{r}$$

Ilgarilanma harakat ta’rifidan,

$$\vec{r} = const.$$

Shuning uchun, $\frac{d\vec{r}}{dt} = 0.$

$$\frac{d\vec{r}}{dt} = 0; \frac{d\vec{r_1}}{dt} = \frac{d\vec{r_0}}{dt} \text{ bo‘ladi.}$$

Bundan,

$$\vec{v}_p = \vec{v}_0$$

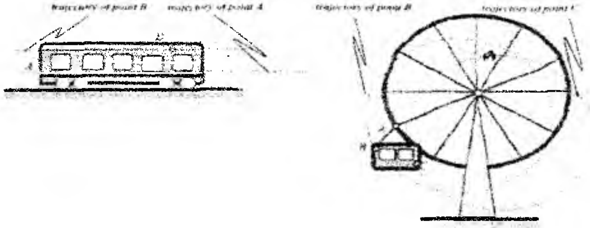
natija kelib chiqadi.

Yuqoridan qattiq jismning ilgarilanma harakatida uning barcha nuqtalarining tezlanishini ham bir xil miqdor va yo‘nalishga ega bo‘lishi ma’lum bo‘ladi:

$$\vec{a}_p = \vec{a}_0$$

2.4a-rasmda tramvayning ilgarilanma harakatida uning A va B nuqtalarining trayektoriyasi ko‘rsatilgan.

2.4b-rasmda charxpalak A,B,C nuqtalarining trayektoriyasi ko‘rsatilgan.



2.4 a, b-rasmlar

17-§ Qattiq jismning ilgarilanma harakatiga doir masalalarni yechish uchun uslubiy ko‘rsatmalar

Qattiq jismning ilgarilanma harakatiga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinatalar sistemasi tanlab olinadi, bunda koordinata o‘qlaridan birini jismning ilgarilanma harakati yo‘nalishida o‘tkazish maqsadga muvofiq bo‘ladi.

2. Masala shartidan ilgarilanma harakatda bo‘ladigan jism tanlab olinadi.

3. Tanlab olingan koordinata o‘qlari sistemasida jismning ilgarilanma harakati tenglamasi tuziladi.

4. Jismning ilgarilanma harakat tenglamalariga ko‘ra tezlik vektorining o‘qlardagi proeksiyalari aniqlanadi.

5. Jism ilgarilanma harakat tezligining o‘qlardagi proeksiyalariga ko‘ra uning miqdori va yo‘nalishi aniqlanadi.

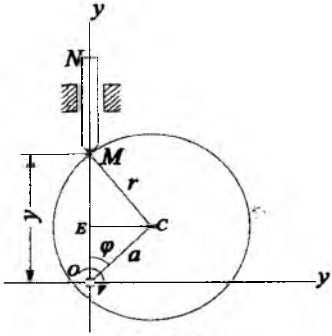
6. Jismning ilgarilanma harakat tenglamalari yoki jism tezligining o‘qlardagi proeksiyalariga ko‘ra uning tezlanishini o‘qlardagi proeksiyalari aniqlanadi.

7. Jism tezlanishining o‘qlardagi proeksiyalariga ko‘ra uning miqdori va yo‘nalishi aniqlanadi.

18-§ Qattiq jismning ilgarilanma harakatiga doir masalalar

1-masala. Diametri d=2r bo'lgan ekssentrik *O* nuqata atrofidan o'tuvchi o'q aylanadi; bunda φ burchak $\varphi = \frac{\pi}{2}t$ qonunga muvofiq o'zgaradi. Ekssentrik geometrik markazi bo'lgan *C* va *O* nuqtalar orasidagi masofa $OC = a = \frac{r}{3}$. Vertikal yo'nalishda harakatlanuvchi *MN* sterjen *M* nuqtasining to'g'ri chiziqli harakat tenglamasi tuzilsin (2.3- rasm) hamda t_1=1s vaqt oni uchun mazkur nuqtaning tezligi va tezlanishi aniqlansin.

Yechimi. Masala shartiga ko'ra *MN* sterjen O nuqtadan o'tuvchi vertikal chiziq bo'ylab to'g'richiziqli harakatda bo'ladi, yani MN sterjen ilgarilanma harakat sodir etadi. Shuning uchun sterjenning M nuqtasi ham O nuqtasidan o'tuvchi to'g'ri chiziq bo'ylab harakatlanadi .



2.3- rasm

Mazkur to'g'ri chiziq bo'ylab *Oy* koordinata o'qini o'tqazamiz; koordinata boshi sifatida *O* nuqta olinadi; *Ox* o'qi gorizontal holda yo'naltiriladi. Rasmdan *OM=y;* Bu masofa vaqtga bog'liq holda o'zgaradi. Bu holni aniqlash uchun *OM* masofani *φ* burchak orqali ifodalash lozim. Buning uchun C nuqtadan mos uchburcφhakning *OM* tomoniga CE balandlikni o'takazamiz.Natijada OM=OE+EM Lekin OE=a cosφ, EC=a sinφ.

$$EM = \sqrt{(MC)^2 - (EC)^2} = \sqrt{r^2 - a^2 sin^2\varphi},$$

Shuning uchun

$$y = OM = a\cos\varphi + \sqrt{r^2 - a^2 sin^2\varphi}$$

Hosil bo'lgan ifodaga φ burchak qiymatini qo'yib, r=3a ekanligini e'tiborga olsak, M nuqtaning harakat tenglamasi uchun quyidagi ifoda kelib chiqadi:

$$y = a\left(cos\frac{\pi}{2}t + \sqrt{9} - sin^2\frac{\pi}{2}t\right).$$

M nuqtaning tezligini quyidagicha aniqlaymiz:

$$v = \frac{dy}{dt} = \frac{d}{dt}a\left(\cos\frac{\pi}{2}t + \sqrt{9 + sin^2\frac{\pi}{2}t}\right) = -\frac{\pi}{2}a\sin\frac{\pi}{2}t\cdot\left(1 + \frac{\cos\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t)}}\right)$$

$$= \frac{\pi a\ \sin\frac{\pi}{2}t}{2\sqrt{9 - \sin^2\frac{\pi}{2}t)}}\cdot\left(\cos\frac{\pi}{2}t + \sqrt{9 - sin^2\frac{\pi}{2}t}\right),$$

yoki

$$v = -\frac{\pi}{2}\cdot\frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t)}}\cdot y$$

M nuqtaning tezlanishini aniqlaymiz:

$$a = \frac{dv}{dt} = \frac{d^2y}{dt^2} = -\frac{\pi}{2}\left[y\frac{d}{dt}\left(\frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\right) + \frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\frac{dy}{dt}\right].$$

Agar $\frac{dy}{dt} = v$ va $\frac{d}{dt}\left(\frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\right) = \frac{9\pi\cos\frac{\pi}{2}t}{2\left(9 - \sin^2\frac{\pi}{2}t\right)^{\frac{3}{2}}},$

ekanligini etiborga olsak,

$$a = -\frac{\pi}{2}\left[y\frac{9\cos\frac{\pi}{2}t}{2\left(9 - \sin^2\frac{\pi}{2}t\right)^{\frac{3}{2}}} + v\frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\right]$$

Shunday qilib,

$$a = -\frac{\pi^2}{4}y\frac{\left[9\cos\frac{\pi}{2}t - \sin^2\frac{\pi}{2}t\sqrt{9 - \sin^2\frac{\pi}{2}t}\right]}{\left(9 - \sin^2\frac{\pi}{2}t\right)^{\frac{3}{2}}}$$

M nuqtaning tezligit t_1=1s da quyidagi qiymatlarga ega bo‘ladi:

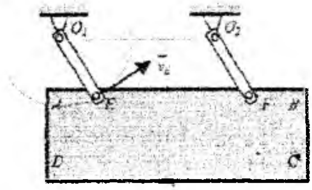
$$v_1 = \frac{\pi}{2} a \quad sm/s,$$

$$a_1 = \frac{\pi^2}{8\sqrt{2}} a \quad sm/s.$$

$\vartheta_1>0$ va $a_1>0$ bo‘lganligi uchun, M nuqtaning tezligi va tezlanishi O nuqtadan vertikal holda yuqoriga yo‘nalgan bo‘ladi, yani M nuqta tezlanuvchan harakatda bo‘ladi.

2-masala. O‘lchamlari l_{AB}3lva l_{BC}=30sm=l bo‘lgan to‘g‘ri burchakli plastina O_1 va O_2 nuqtalardagi sharnirlarga sterjenlar yordamida biriktirilgan plastinka E nuqtasining tezligi V_E =0.6m/s. Boshlang‘ich paytda sterjenlar vertikal xolatda bo‘lgan plastina ABC va D nuqtalarining $t=t_1$=0.5sdagi tezliklari va tezlanishlari aniqlansin (2.4- rasm).

$$O_1O_2 = EF = 2l, \; O_1E = O_1F = l.$$



2.4- rasm

Yechimi.

Masalaga *ABCD* plastina ilgarilanma harakat sodir etiladi, chunki *y O_1E va O_2E* strjenlarning aylanma harakatlari natijasida har doim o‘zining boshlang‘ich holatiga parallel qolgan holda harakatlanadi. Boshlang‘ich paytda sterjenlar vertikal holatda bo‘lgan. Plastinkaning $t=t_1$vaqt momentidagi vaziyati E nuqtaning burchak tezligi orqali quydagicha aniqlanadi:

$$\omega_E = \frac{v_E}{l} = \frac{0,6}{0,3} = 2rad/s$$

Demak; O_1E va O_2E sterjenlar tekis aylanma harakatda bo‘ladi

Sterjenlarning boshlang‘ich holatiga nisbatan (vertikal holati) og‘ish burchagi quydagicha topiladi:

$$\Theta = \frac{\omega E}{l}.$$

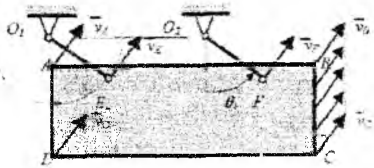
Bunday t=t vaqt momenti uchun sterjenlarning boshlang'ich holatidan og'ish burchagi quydagiga teng boladi:

$$\theta=\theta(t_1)=2\ 0.5=1\ pag=57°32.$$

Demak, t-=t_1 vaqt momentiga boshlang'ich holati vertikal bo'lgan sterjenlar θ=57°32 burchakga burilar ekan.

Plastinka ilgarilanma harakatda bo'lishi tufayli, uning barcha nuqtalaring tezligi E nuqtaning tezligiga teng boladi va $\vec{v}_E$ biron bir xil yo'nalishda bo'ladi:

$$v_A = v_B = v_C = v_D = v_E = 0.6\text{m/s}.$$



2.5-rasm

Plastinka nuqtalari tezliklarining taqsimoti (2.5rasm)da ko'rsatilgan.

Plastina E nuqtasining tezlanishini aniqlaymiz . E nuqtaning tezlanishi urinma va normal tashkil etuvchilardan iborat bo'ladi;

$$a_n= a_{Er}+a_{En}$$

Bunda

$$a_{Er}=\vartheta_E,$$

$$a_{En} = \frac{v_E^2}{0.E} = \frac{(0.6)^2}{0.3} = 1.2\ m/s^2.$$

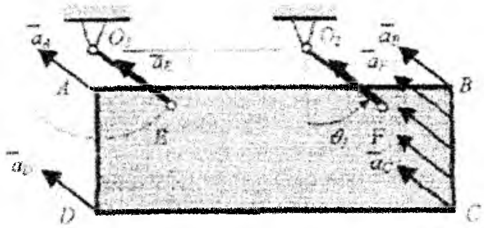
Demak, E nuqtaning tezlanishi y tekis aylanma harakatda bo'lishi sababli, normal markazga intilma tezlanishdan iborat bo'lar ekan:

$$a_n=a_{En}=1.2\text{m/s}^2$$

Bu tezlanish E nuqta chizadigan aylanma radiusi bo'ylab, aylanma markazi tomon yo'naladi .Plastina ilgarilanma harakatda bo'lishi tufayli, uning barcha nuqtalarining tezlanishlari ham o'zaro teng bo'ladi va $a_{\vec{E}}$ yo'nalishi bilan bir xil bo'ladi:

$$a_A=a_B=a_C=a_D$$

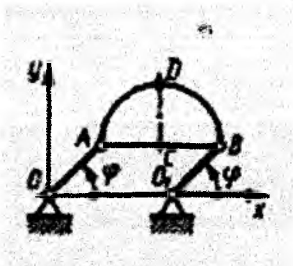
Plastina nuqtalari tezlanishlarining taqsimoti (2.6-rasm)da ko'rsatilgan.



2.6-rasm

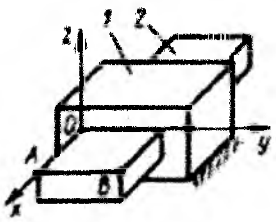
19-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan masalalar

1-masala. Uzunliklari $OA=O_1B=0,16$m bo'lgan ikki krivoship-larning harakat qonuni $\varphi=\pi t$ bo'lib, yarim aylana shaklidagi ABD jis-mni ilgarilanma harakatga keltiradi. Agar $AB=0,25$m bo'lsa, t=2s da jismning D nuqtasi trayektoriyasining egrilik radiusini toping (2.7 – rasm).



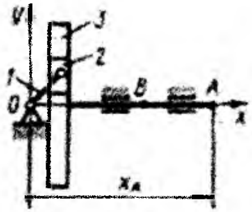
2.7- rasm

2-masala. 1 g'ilof ichida 2 polzun harakat qiladi. Agar polzun-ning ilgarilanma harakat qonuni $x_A=0,1$ cost, $y_A=0,1$, z_A 0 bo'lsa t=π (sek) paytda B nuqtaning tezligini aniqlang. Bunda masofa $AB=0,3$m (2.8- rasm).



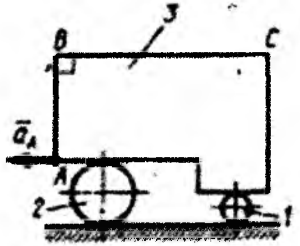
2.8 – rasm

3-masala. Krivoship 1 va polzun 2 yordamida ilgarilanma harakatga keluvchi 3 kulisali mexanizm x_A=0,4-0,1sin t^2 qonun asosida siljisa, t=2s dagi *B* nuqtaning tezligini aniqlang (2.9 – rasm).



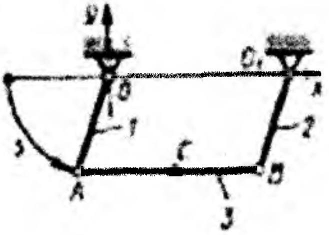
2.9 – rasm.

4-masala. Ikkita 1 va 2 silindirik o'qlarga o'rnatilgan 3 jism ilgarilanma harakat qiladi. Agar masofalar *BC*=2*AB*=1m bo'lib, jismning *A* nuqtasi 2m/s^2 tezlanishga ega bo'lsa, C nuqtasining tezlanishini hisoblang (2.10 – rasm).



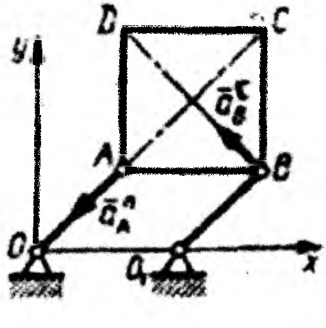
2.10 - rasm

5-masala. Bir xil uzunlikdagi $OA=O_1B=0{,}2m$ 1 va 2 krivoshiplarga o'rnatilgan 3 sterjen *Oxy* tekisligida ilgarilanma harakat qiladi. Uning A nuqtasining harakat qonuni $s=0{,}2\pi t$ bo'lsa, t=0 paytdagi sterjen o'rtasidagi C nuqtaning tezlanishini aniqlang. Bunda masofa AB=0,36m (2.11 – rasm).



2.11- rasm

6-masala. ABCD kvadrat plastina *Oxy* tekisligida ilgarilanma harakat qiladi. Agar uning A nuqtasi $a_a^n = 4m/s^2$ normal tezlanishga va B nuqtasi $a_B^\tau = 3m/s^2$ urinma tezlanishga ega bo'lsa, C nuqtasining tezlanishini toping (2.12 – rasm).



2.12 -rasm

20-§ Qattiq jismning qo‘zg‘almas o‘q atrofidagi aylanma harakati

Qattiq jismning harakatida ikki nuqtasi doimo qo‘zg‘almasdan qolsa, uning bunday harakati qo‘zg‘almas o‘q atrofidagi aylanma harakat, qo‘zg‘almas nuqtalardan o‘tuvchi o‘q esa aylanish o‘qi deyiladi.

Qattiq jismning qo‘zg‘almas o‘q atrofidagi aylanma harakatida uning aylanish o‘qida yotuvchi barcha nuqtalari qo‘zg‘almas bo‘ladi. Aylanish o‘qida yotmaydigan boshqa barcha nuqtalar aylanish o‘qiga perpendikulyar tekisliklarda yotuvchi, markazi aylanish o‘qida bo‘lgan aylanalar bo‘ylab harakatlanadi.

Qattiq jismning aylanma harakatini o‘rganish uchun aylanish o‘qi orqali o‘tuvchi qo‘zg‘almas Π_0 va jismga mahkam biriktirilgan, u bilan birga harakatlanadigan Π tekisliklarni o‘tkazamiz.

Jism aylanish o‘qi Az atrofida harakatlanganda Π tekislik Π_0 tekislikka nisbatan φ burchakka buriladi. Bu burchak aylanish burchagi deyiladi va Π tekislik jism bilan mahkam biriktirilganligidan jismning holati φ burchak bilan aniqlanadi.

Jism Az o‘q atrofida aylanganda uning aylanish burchagi φ vaqtning uzluksiz, bir qiymatli funksiyasi sifatida o‘zgaradi:

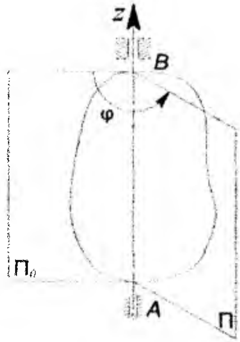
$$\varphi=f(t) \qquad (2.6)$$

Bu tenglama jismning qo‘zg‘almas o‘q atrofida aylanma harakatining kinematik tenglamasi deyiladi. Aylanish burchagi radianlarda o‘lchanadi.

Qo‘zg‘almas o‘q atrofida aylanma harakatda bo‘lgan jismning asosiy kinematik xarakteristikalari uning burchak tezligi va burchak tezlanishi hisoblanadi.

Qo‘zg‘almas o‘q atrofida aylanma harakatda bo‘lgan jismning asosiy kinematik xarakteristikalari uning burchak tezligi va burchak tezlanishi hisoblanadi.

21-§ Qo‘zg‘almas o‘q atrofida aylanma harakatda bo‘lgan jismning burchak tezligi. Tekis aylanma harakat.



2.13-rasm

Burchak tezlik aylanma harakatda bo‘lgan jism aylanish burchagining o‘zgarishini ifodalovchi kattalik bo‘lib, u aylanish burchagidan vaqt bo‘yicha olingan birinchi tartibli hosilaga teng:

$$\omega = \varphi' = \frac{d\varphi}{dt}. \qquad (2.7)$$

Burchak tezlik φ burchakning o‘zgarish qonuniga mos ravishda musbat yoki manfiy qiymatga ega bo‘lishi mumkin.

Agar $\varphi' = \frac{d\varphi}{dt} > 0$ bo‘lsa, aylanma harakat aylanish o‘qining musbat yo‘nalishidan qaraganda soat milining aylanishiga teskari yo‘nalishda yuz beradi; $\varphi' = \frac{d\varphi}{dt} < 0$ bo‘lsa, jism soat milining aylanish yo‘nalishida aylanma harakatda bo‘ladi.

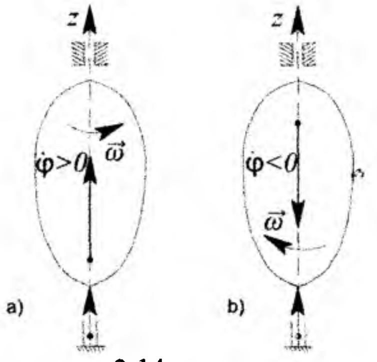
Burchak tezlik vektori aylanish o‘qi bo‘ylab yo‘naladi va uning musbat yo‘nalishidan qaraganda, aylanish soat mili harakatiga teskari yo‘nalishda ko‘rinadi (2.13-rasm). Burchak tezlik vektori aylanish o‘qining ixtiyoriy nuqtasiga qo‘yiladi. Shuning uchun ham u erkin vektor hisoblanadi. Burchak tezlik vektorining moduli

$$\omega = \left|\frac{d\varphi}{dt}\right| \qquad (2.8)$$

formula yordamida aniqlanadi.

Burchak tezlik SI birliklar sistemasida rad/s yoki 1/s da o‘lchanadi.

Jism harakati davomida $\omega=\omega_0$=cont bo‘lsa, u tekis aylanma harakatda bo‘ladi.



2.14-rasm

Bu holda $\frac{d\varphi}{dt} = \omega_0 = const$, shuning uchun

$$d\varphi=\omega_0 dt \tag{2.9}$$

Vaqt 0 dan *t* gacha o'zgarganda aylanish burchagi φ_0 dan φ gacha o'zgarishini e'tiborga olib, (2.9) ni integrallasak

$$\varphi=\varphi_0+\omega_0 t \text{ bo'ladi.} \tag{2.10}$$

(2.10) ifoda jismning tekis aylanma harakat tenglamasini ifodalaydi.

Texnikada tekis aylanma harakatda burchak tezlik ko'pincha bir minutdagi aylanishlar soni bilan o'lchanadi.

Jism bir marta to'la aylanganda $\varphi=2\pi$, bo'ladi. Agar jism bir minutda n marta aylansa, tekis aylanma harakatning burchak tezligi quyidagiga teng bo'ladi:

$$\omega = \frac{2\pi n}{60} = \frac{\pi n}{30} \text{ rad/s.} \tag{2.11}$$

22-§ Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning burchak tezlanishi. Tekis o'zgaruvchan aylanma harakat

Burchak tezlanishi aylanma harakatda bo'lgan jism burchak tezligining o'zgarishini ifodalovchi kattalik bo'lib, u burchak tezligidan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki aylanish *o'qi atrofidagi aylanish burchagidan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi:*

$$\varepsilon = \frac{d\varphi}{at} = \frac{d^2\varphi}{at^2} \cdot \qquad (2.12)$$

Burchak tezlanish ham, burchak tezlik kabi vektor kattalik hisoblanadi.

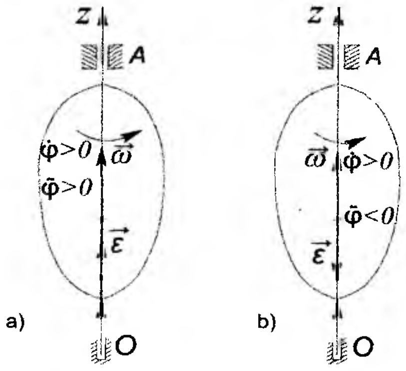
Agar $\frac{d\varphi}{at}$ va $\frac{d^2\varphi}{at^2}$ bir xil ishorali bo'lsa, ya'ni aylanma harakat tezlanuvchan bo'lsa, burchak tezlik va burchak tezlanish vektorlari aylanish o'qi bo'ylab bir tomonga (2.15a-rasm), turli ishorali bo'lsa, ya'ni aylanma harakat sekinlanuvchan bo'lsa, qarama-qarshi tomonlarga yo'naladi (2.15b-rasm). Burchak tezlanish vektorining moduli

$$\varepsilon = \left|\frac{d^2\varphi}{at^2}\right| = \left|\frac{d\varphi}{at}\right| \qquad (2.13)$$

formula yordamida aniqlanadi. Burchak tezlanish SI birlilar sistemasida rad/s^2 yoki 1/s^2 larda o'lchanadi.

Agar aylanma harakat davomida $\frac{d\varphi}{at} > 0$ bo'lsa, φ orta boradi va bunday harakat ***tezlanuvchan aylanma harakat*** deyiladi; $\frac{d\varphi}{at} < 0$ bo'lsa, ω kamaya boradi va bunday harakat ***sekinlanuvchan aylanma harakat*** deyiladi.

Agar aylanma harakat davomida $\varepsilon = \varepsilon_0 = const$ bo'lsa, jismning harakati tekis o'zgaruvchan aylanma harakat bo'ladi.



2.15a,b-rasm

Bunday holda

$$\frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \varepsilon = \varepsilon_0 = const. \qquad (2.14)$$

Vaqt 0 dan t gacha o'zgarganda, burchak tezlik ω dan ω_0 gacha o'zgarishini e'tiborga olib, (2.14) ni integrallasak,

$$\omega = \omega_0 + \varepsilon t \qquad (2.15)$$

bo'ladi. (2.15) tenglik yordamida tekis o'zgaruvchan aylanma harakat burchak tezligi aniqlanadi.

Tekis o'zgaruvchan aylanma harakat tenglamasini ifodalash uchun (2.15) ni quyidagicha yozamiz:

$$\frac{d\varphi}{dt} = \omega_0 + \varepsilon t. \qquad (2.16)$$

Bundan,

$$d\varphi = (\omega_0 + \varepsilon t)dt. \qquad (2.17)$$

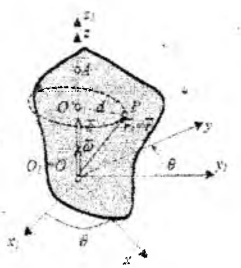
(2.10) ni e'tiborga olib, (2.17) ni integrallasak,

$$\varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2} \qquad (2.18)$$

ko'rinishdagi tekis o'zgaruvchan aylanma harakat tenglamasi kelib chiqadi.

Qattiq jismning qo'zg'almas o'q atrofida aylanma harakatini quyidagi rasm orqali soddaroq holda tushuntirish mumkin (2.16-rasm)

Rasmda θ burchak orqali jismning OZ aylanish o'qi atrofidagi aylanma harakatida burilish burchagi ko'rsatilgan (OXYZ – koordinata o'qlari sistemasi qo'zg'aluvchan sistema, u jism bilan bog'langan).



2.16-rasm

Chizmada $\vec{r}_i$ vektor orqali jism ixtiyoriy nuqtasining radius vektori ko'rsatilgan. Shuning uchun, $\vec{r}_i = \vec{r}$ bo'lib, jismning aylanma harakat burchak tezligi va burchak tezlanishlari quyidagicha aniqlanadi:

$$\omega = \omega_z = \theta'$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{d\omega_t}{dt} = \theta''$$

23-§ Qo'zg'almas o'q atrofida aylanuvchi jism nuqtasining chiziqli tezligi

Qo'zg'almas *Oz* o'qi atrofida ω burchak tezlik bilan aylanuvchi qattiq jismning aylanish o'qidan *R* masofada joylashgan *M* nuqtasining tezligini aniqlaymiz (2.17a-rasm). Biror t vaqtda mazkur nuqta *M* holatda bo'lib, dt vaqt oralig'ida jism $d\varphi$ burchakka aylansin. Bunda *M* nuqta aylanish o'qiga perpendikulyar tekislikda aylana bo'ylab harakatlanib, $d\varphi = Rd\varphi$ yoyni bosib o'tadi. *M* nuqta tezligining algebraik qiymati quyidagi formulaga muvofiq aniqlanadi:

$$\vartheta_\tau = \frac{ds}{dt} = R\frac{d\varphi}{dt} = R\omega \qquad (2.19)$$

Tezlikning moduli esa,

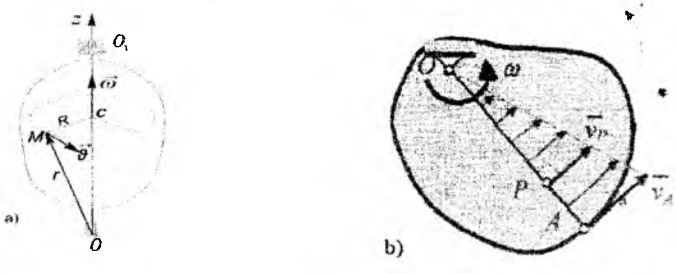
$$\vartheta = \left|\frac{ds}{dt}\right| = R\left|\frac{d\varphi}{dt}\right|$$

formula bilan aniqlanadi.

(2.19) formula bilan aniqlanadigan tezlik qo'zg'almas o'q atrofida aylanuvchi jism nuqtasining chiziqli tezligi deyiladi.

Shunday qilib, qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jism ixtiyoriy nuqtasining chiziqli tezligi miqdor jihatdan jism burchak tezligining mazkur nuqtadan aylanish o'qigacha bo'lgan masofaga ko'paytmasiga teng bo'ladi.

Jism barcha nuqtalarining burchak tezliklari berilgan onda bir hil qiymatga ega bo'lgani uchun, (2.19) dan qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jism nuqtalarining chiziqli tezliklari mazkur nuqtalardan aylanish o'qigacha bo'lgan masofaga to'g'ri proportsional holda o'zgari shi ma'lum bo'ladi (2.17b-rasm).



2.17a,b-rasm

Nuqtaning chiziqli tezligi vektori $\vec{\vartheta}$ nuqta chizgan aylanaga harakat yo'nalishi bo'yicha o'tkazilgan urinma bo'ylab yo'naladi.

Chiziqli tezlik vektori burchak tezlik vektori bilan, mazkur nuqtaning aylanish o'qidagi O nuqtaga nisbatan radius – vektorining vektor ko'paytmasiga teng bo'ladi:

$$\vec{\vartheta} = \vec{\omega} \times \vec{r} \qquad (2.20)$$

Chunki, mazkur vektor ko'paytmaning moduli

$$|\vec{\omega} \times \vec{r}| = \omega \cdot r\sin(\vec{\omega} \,\hat{}\, \vec{r}) = \omega \cdot R$$

tezlikning moduliga teng bo'ladi. $\vec{\omega} x \vec{r}$ vektori, $\vec{\omega}$ va $\vec{r}$ yotgan tekislikka perpendikulyar holda jismning aylanish yo'nalishi bo'yicha yo'naladi, ya'ni $\vec{\omega} \times \vec{r}$ ning yo'nalishi $\vec{\vartheta}$ yo'nalishi bilan bir xil bo'ladi.

Aylanma harakatdagi jism ixtiyoriy P nuqtasining tezligini quyidagicha aniqlash ham mumkin(2.17a-rasm).

$$\vec{v_p} = \vec{\omega} * \vec{r} = \omega\vec{k} * \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \omega \\ x & y & z \end{vmatrix} = -\omega y\vec{i} + \omega x\vec{j}$$

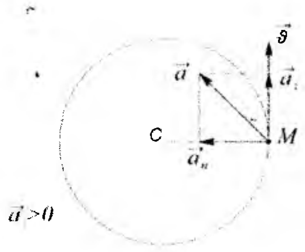
P nuqta tezligining moduli esa quyidagicha aniqlanadi:

$$v_p = \omega\sqrt{x^2 + y^2} = \omega * d$$

24-§ Qo'zg'almas o'q atrofida aylanuvchi jism nuqtasining tezlanishi

Qo'zg'almas o'q atrofida aylanma harakatdagi jism nuqtalari aylanish o'qiga perpendikulyar tekislikda aylanalar bo'ylab harakatlanishi tufayli M nuqtaning tezlanishi urinma va normal tezlanishlardan tashkil topadi (2.18-rasm):

$$\vec{a} = \vec{a}_\tau + \vec{a}_n. \qquad (2.21)$$



2.18-rasm

Agar ko'rilayotgan holda ρ=R va ϑ=Rω ekanligini e'tiborga olsak,

$$a_\tau = \frac{d\vartheta_\tau}{dt} = \frac{d}{dt}(R\omega) = R\varepsilon, \quad (2.22)$$

$$a_n = \frac{\vartheta^2}{\rho} = \frac{R^2\omega^2}{R} = \omega^2 R \qquad (2.23)$$

bo'ladi.

Urinma tezlanish $\vec{a}_\tau$ aylanma harakat tezlanuvchan bo'lganda, traektoriyaga o'tkazilgan urinma bo'ylab harakat yo'nalishida, sekinlanuvchan aylanma harakatda esa, unga teskari yo'naladi. Normal tezlanish $\vec{a}_n$ doimo bosh normal bo'yicha aylanish o'qi tomon yo'naladi (2.19a-rasm). Ba'zan $\vec{a}_\tau$ *aylanma tezlanish*, $\vec{a}_n$ esa *markazga intilma tezlanish* deb ham yuritiladi.

M nuqta tezlanishining moduli:

$$a = \sqrt{a_\tau^2 + a_n^2} = R\sqrt{\varepsilon^2 + \omega^4} \qquad (2.24)$$

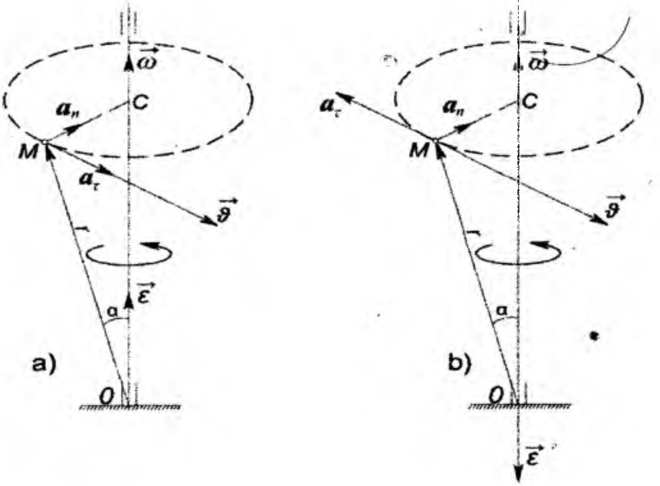
formula orqali aniqlanadi.

Aylanma harakatdagi qattiq jism ixtiyoriy M nuqtasining tezlanishini quyidagicha aniqlash ham mumkin (2.19b-rasm)

$$\vec{a_p} = \varepsilon\vec{k} * \vec{r} + \omega\vec{k} * (\omega\vec{k} * \vec{r}) = (-\varepsilon y - \omega^2 x)\vec{i} + (\varepsilon x - \omega^2 y)\vec{j}$$

Tezlanish moduli esa quyidagicha aniqlanadi:

$$a_p = \sqrt{(x^2 + y^2)(\varepsilon^4 + \omega^4)} = d * \sqrt{\varepsilon^2 + \omega^4}.$$



2.18- rasm

M nuqta tezlanishining yo'nalishi bosh normal bilan $\vec{a}$ tezlanish vektori orasidagi μ burchak orqali aniqlanadi (2.18- rasm):

$$tg\mu = \frac{|a_\tau|}{a_n} = \frac{\varepsilon}{\omega^2}. \quad (2.25)$$

Aylanma harakatdagi jismning barcha nuqtalari uchun ω va ε lar bir xil bo'lganidan, jism nuqtalarining tezlanishi aylanish o'qidan mazkur nuqtalargacha bo'lgan masofalarga proportsional ravishda o'zgaradi. Berilgan onda jismning barcha nuqtalari uchun μ burchak ham bir xil bo'ladi.

Urinma va normal tezlanishlarning vektorli ifodalarini aniqlash uchun (2.20) dan vaqt bo'yicha hosila olamiz:

$$\vec{a} = \frac{d\vec{\vartheta}}{dt} = \frac{d\vec{\omega}}{dt} x \vec{r} + \vec{\omega} x \frac{d\vec{r}}{dt}. \quad (2.26)$$

Bunda,

$$\frac{d\vec{\omega}}{dt} = \vec{\varepsilon}, \qquad \frac{d\vec{r}}{dt} = \vec{\vartheta}. \quad (2.27)$$

Shuning uchun,

$$\vec{a} = \vec{\varepsilon} x \vec{r} + \omega x \vec{\vartheta}. \quad (2.28)$$

Bu formulada,

$$\vec{\varepsilon} x \vec{r} = \vec{a}_\tau \quad (2.29)$$

urinma tezlanish vektorini ifodalaydi.

Ko'rinib turibdiki, qo'zg'almas o'q atrofida aylanma harakatdagi jism ixtiyoriy nuqtasining urinma tezlanishi jismning burchak tezlanishi vektori bilan mazkur nuqtaning aylanish o'qidagi ixtiyoriy O nuqtaga nisbatan radius – vektorining vektorli ko'paytmasiga teng bo'lar ekan.

(2.28) formulada

$$\vec{a}_n = \vec{\omega} x \vec{\vartheta} \quad (2.30)$$

normal tezlanish vektorini ifodalaydi.

Demak, qo'zg'almas o'q atrofida aylanma harakatdagi jism ixtiyoriy nuqtasining normal yoki markazga intilma tezlanishi jismning burchak tezlik vektori bilan mazkur nuqta chiziqli tezligining vektorli ko'paytmasiga teng bo'lar ekan

Takrorlash uchun savollar

1. Qattiq jismning qanday harakatiga ilgarinlanma harakat deyiladi va bu harakatning asosiy xususiyatlari?

2. Qattiq jismning qanday harakatiga qo‘zg‘almas o‘q atrofidagi aylanma harakat deyiladi?

3.Aylanma harakat tenglamasi.

4. Aylanma harakat qilayotgan qattiq jismning burchak tezlik va burchak tezlanish modullari qanday formula bilan aniqlanadi?

5. Qo‘zg‘almas o‘q atrofidagi aylanma harakat qilayotgan qattiq jism burchak tezlik va burchak tezlanish vektorlari qanday yo‘nalgan bo‘ladi?

6. Aylanma harakat qilayotgan nuqtaning chiziqli tezligi qanday formula orqali ifodalanadi?

7. Aylanma harakat qilayotgan nuqtaning chiziqli tezlanishi qanday formula orqali ifodalanadi?

8. Eyler formulasi qanday ko‘rinishda bo‘ladi?

9. Aylanma harakat qilayotgan nuqtaning tezlik vektori qanday ifodalanadi?

10. Aylanma harakat qilayotgan nuqtaning tezlanish vektori qanday ifodalanadi?

25-§ Qattiq jismning ilgarilanma va qo‘zg‘almas o‘q atrofida aylanma harakatiga doir masalalarni yechish uchun uslubiy ko‘rsatmalar

Qattiq jismning qo‘zg‘almas o‘q atrofida aylanishiga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinatalar sistemasi tanlab olinadi, bunda koordinata o‘qlaridan birini (z o‘qini) aylanish o‘qi bo‘ylab yo‘naltirish maqsadga muvofiq bo‘ladi.

2. Qattiq jismning aylanma harakati tenglamasi tuziladi.

3. Qattiq jismning aylanish burchagidan vaqt bo‘yicha birinchi tartibli hosila hisoblab, burchak tezlikning aylanish o‘qidagi proeksiyasi aniqlanadi.

4. Qattiq jismning aylanish burchagidan vaqt bo'yicha ikkinchi tartibli hosila hisoblab, burchak tezlanishning aylanish o'qidagi proeksiyasi aniqlanadi.

5. Aylanma harakat burchak tezligini bilgan holda, jism nuqtasining chiziqli tezligi va normal tezlanishi aniqlanadi.

6. Aylanma harakat burchak tezlanishini bilgan holda, jism nuqtasining urinma tezlanishi aniqlanadi.

7. Aniqlangan normal va urinma tezlanishlar orqali jism nuqtasining to'la tezlanishi aniqlanadi.

Jismda olingan har qanday kesma jism harakati davomida doimo o'zining boshlang'ich holatiga parrallel qolsa, jismning bunday harakati ilgarilanma harakati deyiladi. Ilgarilanma harakatda jismning barcha nuqtalari bir xil traektoriya bo'ylab harakatlanadi va har yerda bir xil tezlik, hamda bir xil tezlanishga ega bo'ladi. Binobarn jismning ilgarilanma harakati uning biror nuqtasining harakati bilan aniqlanishi mumkin. Nuqta kinematikasida uning kinematik xarakteristikalarini aniqlash bilan tanishgan edik. Nuqtaning harakat tenglamalari trayektoriyasi, tezligi,tezlanishi va h.k larni aniqlash yo'llari bilan batafsil tanishdik. Shuning uchun mazkur amaliy mashg'ulotda qattiq jismning ilgarilanma harakatiga doir masalalar o'rganilmaydi.

Qattiq jismning harakatida ikki nuqtasi doimo qo'zg'almasdan qolsa, uning bunday harakati qo'zg'almas o'q atrofidagi aylanma harakat, qo'zg'almas nuqtalardan o'tuvchi o'q esa aylanish o'qi deyiladi. Agar masalada qattiq jismning burchak tezlanishi yoki burchak tezligi berilgan bo'lib, aylanma harakat tenglamasini qattiq jism nuqtalarining tezligi va teshlanishini aniqlash talab etilsa, masalani quyidagi tartibda yechish maqsadga muvofiq bo'ladi.

1. Qattiq jism burchak tezlanishining aylanish o'qidagi proyeksiyasini ifodalovchi tenglamani integrallab, burchak tezlikning aylanish o'qidagi proyeksiyasini aniqlaymiz. Bundan integrallash doimiylari –o'zgarmaslari boshlang'ich kattaliklar orqali aniqlanadi.

2. Burchak tezlikning aylanish o'qidagiproyeksiyasini ifodalovchi tenglamani integrallab, jismning aylanma harakat tenglamasi-

ni aniqlaymiz. Bunda ham integrallash o‘zgarmaslari boshlang‘ich kattaliklar orqali aniqlanadi.

3. Burchak tezlikning aylanish o‘qidagi proyeksiyasi ifodasidan foydalanib, jism nuqtalarining tezligini va normal tezlanishini aniqlaymiz.

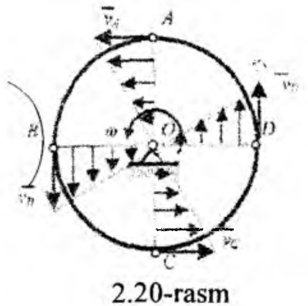
4. Burchak tezlanishning aylanish o‘qidagi proyeksiyasi ifodasidan foydalanib, jism nuqtalarining urinma tezlanishlarini aniqlaymiz.

5. nuqtalarining normal va urinma tezlanishlarini bilgan holda uning to‘la tezlanishi aniqlanadi.

26-§ Qattiq jismning qo‘zg‘almas o‘q atrofidagi aylanma harakatiga doir masalalar

1-masala.

Radius R=40sm bo‘lgan disk qo‘zg‘almas O nuqta atrofida qo‘zg‘almas ω=5 radius tezlik bilan aylanadi. (2.20-rasm) Disk gorizontal va vertikal deametrlari uchlaridagi nuqtalarning tezligi va tezlanishi aniqlansin va mazkur diametrlar nuqtalari tezliklarining taqsimoti ko‘rsatilsin.



2.20-rasm

Yechish: Disk qo‘zg‘almas O nuqtadan o‘tuvchi o‘q atrofida o‘zgarmas burchak tezlik bilan aylanma harakatda bo‘lishi sababli, disk gorizontal va vertikal diametrlari uchlaridagi nuqtalarning tezliklari quydagicha aniqlanadi:

v_A=OA*ω=40*0.5=20sm/s; v_B=OB*ω=40*0.5=20sm/s;

v_C=OC*ω=40*0.5=20sm/s; v_D=OD*ω=40*0.5=20sm/s;

Mazkur tezliklar nuqtalar radiuslariga perpendikulyar holda ω yo'nalishi tomon yo'naladi:

$$\vec{V_A} \perp \vec{OA};\ \vec{D_B} \perp \vec{OB};\ \vec{V_{CA}} \perp \vec{OC}\ \vec{V_{AD}} \perp \vec{OD}.$$

Diskning garizontal va vertikal diametridagi nuqtalar tezliklarining taqsimoti 1-rasmda ko'rsatilgan.

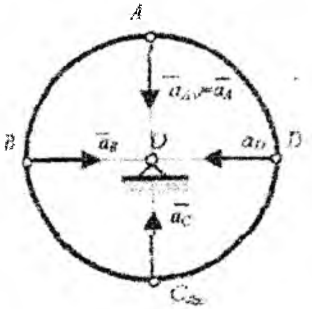
Diskning garizontal va vertikal deametrlari uchlaridagi nuqtalar tezlanishlarining normal va urinma tashkil etuvchilarini aniqlaymiz.

A nuqtaning tezlanishi:

$$a_{A\tau}=OA*\varepsilon=OA*\omega=0$$

$$a_{An}=OA*\omega^2=40*(0.5)^2=10sm/s^2$$

Nuqta normal tezlanishi uchun radiusi bo'ylab, disk markazi tomon yo'naladi.



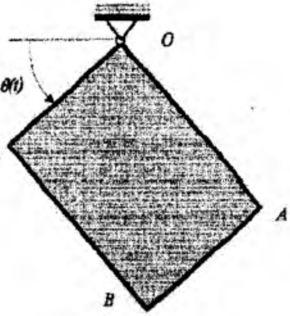
2.21-rasm

Disk boshqa B,C,D nuqtalarning tezlanishlari ham miqdor jihatdan A nuqtasining tezlanishiga teng bo'lib , nuqtalar radiuslari bo'ylab, disk markazi tomon yo'naladi (2.21-rasm).

2-masala. Ko'rsatilgan *OABS* plastina chizma tekisligida O nuqta atrofida aylanadi. Agar plastinaning aylanma harakat tenglamasi

$$\theta(t)=\sin t(rad)$$

bo'lib, *OA=40 sm, AB=30sm* bo'lsa, plastina *A,B* va *C* nuqtalarining tezligi, tezlanishi aniqlansin. Plastina OA va AB tomonlari nuqtalarining $t_1=1s$ vaqat onidagi tezliklarining taqsimoti ham ko'rsatilsin (2.22-rasm).



2.22-rasm

Yechimi. 1.Plastinaning t_1=1s vaqt onida egallagan o'rnini aniqlaymiz.

$$\theta_1=\theta(t_1)=\sin(1rad)=9841rad$$

Bundan

$$\theta_1=48^{\circ}23!$$

Plastina nuqtalarining tezliklarini aniqlash uchun dastlab uning burchak tezlligini aniqlaymiz:

$$\omega_1=\frac{d\theta_1}{dt}=cost_1=0,54\frac{1}{s}$$

Burchak tezlik burilish tezlik o'sish tomoni qarab yo'naladi (ularning ishoralari bir xil).

Plastina A nuqtasining tezligini aniqlaymiz:

$$w_B=w_1\cdot OB=w_1\cdot\sqrt{(OA)^2+(AB)^2}=0.54\cdot 50=27\frac{sm}{s}$$

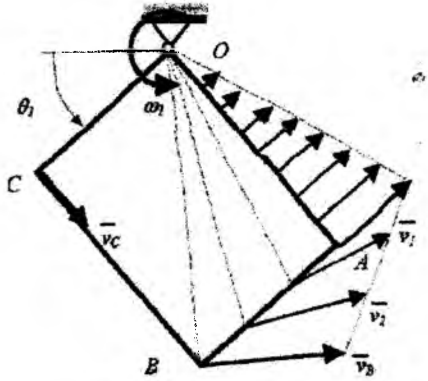
$\vec{\vartheta}_B$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OB radiusga perpendikulyar holda ω_1 tomon yo'naladi

Plastina C nuqtasining tezligi quyidagiga teng bo'ladi:

$$\vartheta_c=\omega_1\cdot OC=0.54\cdot 30=16.2\frac{sm}{s}$$

$\vec{\vartheta}_C$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OC radiusga perpendikulyar holda ω_1 tomon yo'naladi.

Plastina OA tomoni nuqtalari tezliklarining taqsimoti 2.23-rasmda ko‘rsatilgan



2.23-rasm.

Plastina OA tomoni nuqtalarining aylanish nuqtasiga bo‘lgan masofalarga to‘g‘ri proporsoinal holda o‘sib boradi.

Plastina AB tomoni nuqtalari tezliklarining taqsimoti ham 2.23-rasmda ko‘rsatilgan.

Plastina nuqtalari tezliklarining miqdorlari nuqtalardan aylanish markazigacha bo‘lgan masofalarga to‘g‘ri proporsional bo‘ladi. Mazkur vektorlar uchlari $\vec{\vartheta}_B$ va $\vec{\vartheta}_A$ vektorlar uchlarini bir kesishtiruvchi to‘g‘ri chiziq kesmasida yotadi.

2.24-rasmda Plastina AB tomoni 1 va 2 nuqtalari tezliklarining vektorlari ham ko‘rsatilgan.

Plastina nuqtalarining tezlanishlarini aniqlash uchun plastinaning aylanma harakat burchak tezlanishini aniqlaymiz:

$$\varepsilon_1 = \frac{d\omega_1}{dt} = -\sin t_1 = -0.841\,\frac{1}{s}$$

Burchak tezlanishining “manfiy” ishorasi plastinaning aylanma harakati tekis sekinlashuvlar ekanligidan dalolat beradi. $\varepsilon_1$va O_1 yo‘nalishlari qarama qarshi bo‘ladi.

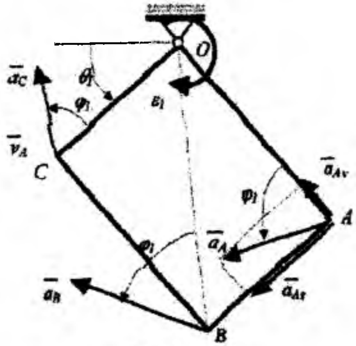
Plastina A nuqtaning urinma tezlanishi quyidagiga teng bo‘ladi:

$$a_{A\tau} = OA \cdot \varepsilon_1 = 40 \cdot 0.841 = 33.64 sm/s^2$$

Plastina A nuqtasining normal tezlanishi esa quyidagiga teng bo‘ladi:

$$a_{An} = OA \cdot \omega_1^2 = 40 \cdot (0.54)^2 = 11.64 sm/s^2$$

$\vec{a}_{A\tau}$ vektor OA radiusga perpendikulyar holda ε_1 tomon $\vec{a}_{An}$ vektor esa A nuqtadan OA radius bo‘ylab aylanish markazi tomon yo‘naladi.



2.24-rasm

A nuqta tezlanishining moduli quyidagiga teng:

$$a_A = \sqrt{(a_{A\tau})^2 + (a_{An})^2} = \sqrt{(33,64)^2 + (11,64)^2} = 35,59 sm/s^2$$

$\vec{a}_A$ ning yo‘nalishi quyidagi formula asosida aniqlanadi:

$$tg\varphi_1 = \frac{a_{A\tau}}{a_{An}} = \frac{33,64}{11,64} = -2,89$$

$$\varphi_1 = arctg 2,89 = 70°91!$$

Plastina B nuqtasi tezlanishining miqdori va yo‘nalishi quyidagicha topiladi:

$$a_B = OB \cdot \sqrt{\varepsilon_1^2 + \omega_1^4} = 50\sqrt{(0,84)^2 + (0,54)^2} = 50 \cdot 0,89 = 44,5 sm/s^2$$

$$tg\varphi_1 = \frac{\varepsilon_1}{\omega_1^2} = \frac{0,841}{(0,54)^2} = 2,89$$

Plastina C nuqtasi tezlanishi miqdorini va yo‘nalishi quyidagicha aniqlanadi:

$$a_C = OC \cdot \sqrt{\varepsilon_1^2 + \omega_1^4} = 30 \cdot 0{,}89 = 26{,}7 sm/s^2$$

$\vec{a}_C$ vektorning plastina OC tomoni bilan hosil qilgan burchagi ham φ ga teng bo‘ladi.

Plastina B va C nuqtalarning tezlanishlari ham 2.24-rasmda ko‘rsatilgan.

3-Masala.

Radiusi R m bo‘lgan maxovik tinch holatdan boshlab tekis tezlanish bilan aylanadi, gardishida yotuvchi nuqtalar t=10s dan keyin ϑ=100m/s chiziqli tezlikka ega bo‘ladi. G‘ildirak gardishida yotgan nuqtaning t_1=15 bo‘lgan vaqtdagi tezligi, urinma va normal tezlanishlari topilsin.

Yechimi.

Maxovik tinch holatdan boshlab, tekis tezlanish bilan aylanadi. Shuning uchun, ω_0=0. t=10s vaqt oni uchun maxovikning burchak tezligini aniqlaymiz:

$$\vartheta=\omega*R$$

Bundan,

$$\omega = \frac{\vartheta}{R} = \frac{100 m/s}{2m} = 50\frac{rad}{s}.$$

Maxovikning shu ondagi burchak tezlanishi esa quyidagicha aniqlanadi:

$$\omega=\varepsilon t$$

Bundan,

$$\varepsilon = \frac{\omega}{t} = \frac{50\, rad/s}{10\, s} = 5\frac{rad}{s^2} = const.$$

Maxovik gardishida yotgan nuqtaning $t_1 = 15s$ dagi tezligi quyidagiga teng bo‘ladi:

$$\vartheta_1=\omega_1*R$$

Bunda,

$$\omega_1=\varepsilon t_1 \text{ rad/s.}$$

Shuning uchun,

$$\vartheta_1=150 \text{ m/s.}$$

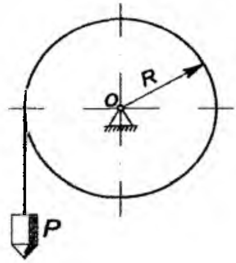
Maxovik gardishida yotgan nuqtaning urinma va normal tezlanishlarini aniqlaymiz:

$a_\tau = \varepsilon R = 5*2 = 10$ m/s^2,

$a_n = \vartheta_1^2/R = 11250$ m/s^2

3-masala.

Radiusi R=10 sm bo‘lgan val unga ipda osilgan P tosh bilan aylantiriladi. Toshning harakati x=100t tenglama bilan ifodalanadi, bunda x – toshdan qo‘zg‘almas OO_1 gorizontalgacha bo‘lgan, santimetrlar hisobida ifodalangan masofa, t vaqt (sekundlar hisobida). t paytida valning burchak tezligi va burchak tezlanishi, shuningdek, val sirtidagi M nuqtaning tezligi va to‘la tezlanishi aniqlansin (2.25-rasm).



2.25-rasm

Yechimi.

Toshning tezligi uning harakat tenglamasidan vaqt bo‘yicha olingan birinchi tartibli hosilaga teng:

$$\vartheta = x = (100t^2)' = 200.$$

Tosh osilgan ipni cho‘zilmaydi deb faraz qilsak, val O_1 nuqtasining chiziqli tezligi tosh tezligiga teng bo‘ladi ($\vartheta = \vartheta_0$) Shuning uchun valning burchak tezligini quyidagicha aniqlash mumkin:

$$\vartheta = \vartheta_0 = \omega * R\cdot$$

Bundan,

$$\omega = \frac{v_0}{R} = v/R = 20t \text{ rad/s}.$$

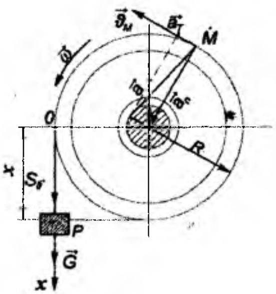
Valning burchak tezlanishi uning burchak tezligidan vaqt bo‘yicha hisoblangan birinchi tartibli hosilaga teng:

$$\varepsilon = \frac{d\omega}{dt} = (20t)' = 20 \text{ rad/s}^2.$$

ω va ε lar yo‘nalishlari $\vec{\vartheta}$ yo‘nalishi orqali aniqlanadi, $\vec{\vartheta}$ esa toshning harakati tomon yo‘naladi (2.25rasm).

Val sirtidagi nuqtaning to‘la tezlanishi quyidagicha aniqlanadi:

$$a = \sqrt{a_\tau^2 + a_n^2}.$$



Bunda,

$a_\tau = \varepsilon * R = 20*10 = 200$ sm/s^2,

$$a_n=\omega^2R=400t^2*-4000t^2 \text{ sm/s}^2.$$

Shuning uchun,

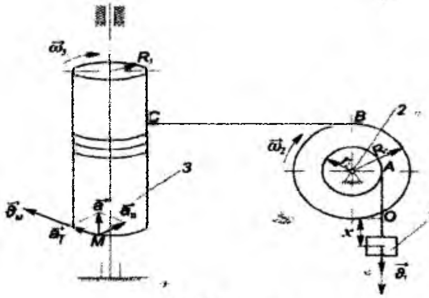
$$a=\sqrt{(200)^2+(4000t^2)}=200\sqrt{1+(4000t^4)} \text{ sm/s}^2.$$

27-§ Jismlarning ilgarilanma va aylanma harakatlarini mexanizmlarda qo'llanilishiga doir masalalar

5-masala.

1-jismning harakat tenglamasi $x=10+30t^2$ ga ko'ra, uning vertikal o'q bo'ylab, S=0.3 m yo'lni o'tgan vaqt momentida, 3 jism *M* nuqtasining tezligi, urinma, normal va to'la tezlanishi topilsin (2.25-rasm).

Shkiv va tsilindr o'lchamlari quyidagicha: $R_2=30, r_2=25$ sm, $R_3=30$sm



2.26-rasm

Yechimi:

Birinchi jismning s=0.3m=30sm yo'lni o'tish vaqti τ ni topamiz:

$$s=x_{t=\tau}-x_{t=0}=10+30\tau^2-10=30\tau^2$$

Bundan,

$$\tau=\sqrt{s/30}=\sqrt{s/30}=1 \ s$$

Birinchi jism tezligini aniqlash uchun uning harakat tenglamasidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$\vartheta_1=\frac{dx}{dt}=(10+30t^2)'=60t.$$

Agar birinchi jism osilgan, hamda 2- va 3- jismlarni biriktiruvchi tasmalarni cho'zilmaydi deb olsak, $\vec{\vartheta}_A=\vec{\vartheta}_1$ bo'ladi. U vaqtda

2- jismning burchak tezligi

$$\omega_2 = \frac{\vartheta_A}{r_2} = \frac{\vartheta_1}{r_1} = \frac{60t}{20} = 3t$$ bo'ladi.

Bu jism B nuqtasining tezligi:

$$\vartheta_B=\omega_2*R_2=3t*30=90$$

BC tasmani ham cho'zilmaydi, deb olsak,

$$\vartheta_B=\vartheta_C=90t$$ bo'ladi.

Bu holatda 3-jismning burchak tezlig*i* quyidagi formula bilan aniqlanadi:

$$\omega_3 = \frac{\vartheta_c}{R_3} = \frac{90t}{30} = 3t\frac{1}{s}$$

3-jismning burchak tezlanishi esa uning burchak tezligidan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng:

$$\varepsilon_3 = \frac{d\omega_3}{at} = (3t)' = 3\frac{1}{c^2}$$

M va *C* nuqtalar t silindrning sirtida, ya'ni aylanish o'qidan bir xil masofada yotganligi uchun, ularning tezliklari, urinma, normal va to'la tezlanishlari o'zaro teng bo'ladi. Shuning uchun:

$$\vartheta_M=\vartheta_C=\omega_3*R_3=3t*30=90t,$$

$$a_\tau=\varepsilon_3 R_3=3*30=90$$

$$\vartheta_n = \omega_3^2 \cdot R_3 = 9t^2 \cdot 30 = 270t^2,$$

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{90^2 + (270t^2)^2} = \sqrt{8100 + 72900t^4}.$$

t=τ=1 sekundda:

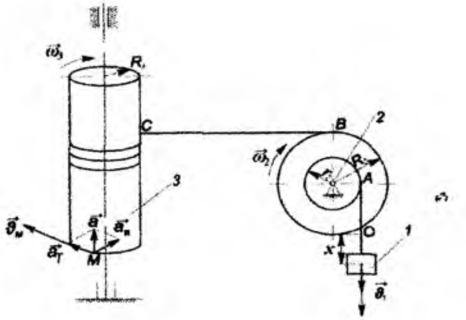
$$\vartheta_M=90*1=90 sm/s,$$

$$a_\tau=90 sm/s^2,$$

$$a_n=270*1=270 sm/s$$

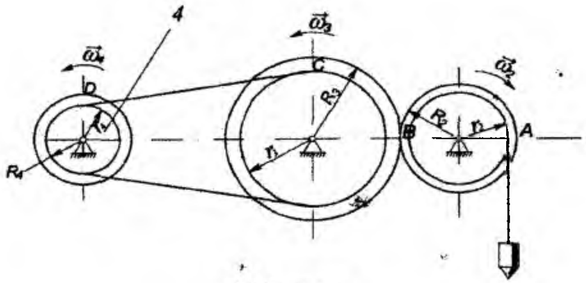
$$a = \sqrt{8100 + 72900} = 284{,}6 \text{ sm/s}^2.$$

M nuqtaning tezligi, urinma, normal va to'la tezlanishi (2.27-rasmda ko'rsatilgan).



2.27-rasm

6-masala. 1-jismning harakat tenglamasi $x=5+8t^2$ ga koʻra, uning vertikal oʻq boʻylab, s=0.32m yoʻlni bosib oʻtgan vaqt momentida, 4-jism M *nuqtasining* tezligi, urinma, normal va toʻla tezlanishlari topilsin. 2,3,4 Jismlar radiuslari r_2=16sm, R_2=20sm, r_3=20sm, R_3=25sm teng (2.28-rasm).



2.28-rasm

Yechimi:

1-jismning s=0.32m yoʻlni bosib oʻtish vaqti τ ni aniqlaymiz:

$$s=x_{t=\tau}-x_{t=0}=5+8\tau^2-5=8\tau^2$$

Bundan,

$$\tau=\sqrt{S/8}=\sqrt{32/8}=2\,s.$$

Birinchi jism tezligini topamiz. Buning uchun uning harakat tenglamasidan vaqt boʻyicha birinchi tartibli hosila hisoblaymiz:

$$\vartheta_1=\frac{dx}{dt}=16t$$

Yuk osilgan arqonni cho'zilmaydi deb hisoblasak, 2 jismning burchak tezligi quyidagicha aniqlanadi:

$$\omega_2 = \frac{\vartheta_A}{r_2} = \frac{\vartheta_1}{r_2} = \frac{16t}{16} = t;$$

bunda, $\vartheta_a=\vartheta_1$ ekanligi e'tiborga olindi.

2-jism B nuqtasining tezligi esa $\vartheta_B=\omega_2*R_2$ bo'ladi.

B nuqtani 2- va 3- jismlar uchun umumiy deb olib, 3 jismning burchak tezligini topamiz:

$$\omega_3 = \frac{\vartheta_B}{R_3} = \frac{20t}{25} = 0{,}8t.$$

3- jism *C* nuqtasining tezligi esa quyidagiga teng bo'ladi:

$$\vartheta_c=\omega_3*r_3=0.8t*20=16t\ .$$

Agar 3- va 4- jismlarni biriktiruvchi tasmani cho'zilmaydi deb hisoblasak,

$$\vartheta_c=\vartheta_D$$

bo'ladi.

Shuning uchun 4- jismning burchak tezligi

$\omega_4 = \frac{\vartheta_d}{r_4} = \frac{16t}{8} = 2t$ bo'ladi.

4-jismning burchak tezlanishini aniqlash uchun burchak tezligidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$\varepsilon_4 = \frac{d\omega_4}{at} = (2t)' = 2$$

4-jism *M* nuqtasining tezligi, urinma, normal va to'la tezlanishlari quyidagicha aniqlanadi:

$$\vartheta_M=\omega_4*R_4=2t*12=24t,$$

$$a_\tau=\varepsilon_4*R_4=2*12=24,$$

$$a_n = \omega_4^2 \cdot R_4 = 4t^2 \cdot 12 = 48t^2.$$

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{576 + 2304t^2}.$$

t=τ=2 sekundda:

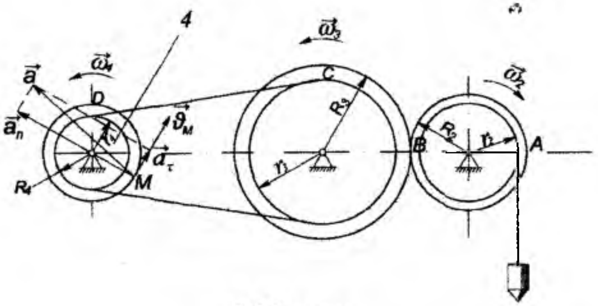
$$\vartheta_m=24*2=48 sm/s^2,$$

$$a_\tau=24 sm/s^2\ ,$$

$$a_n=48*4=192\ sm/s^2,$$

$$a = \sqrt{576 + 36864} = 193{,}4\ sm/s^2.$$

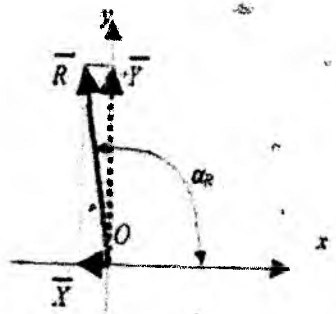
Tezlik va tezlanishlar uchun masshtab tanlab, ularni chizmada ko'rsatamiz: *M* nuqtaning tezlik vektori nuqtadan traektoriyaga o'tkazilgan urinma bo'ylab, to'la tezlanish vektori esa urinma va normal tezlanishlarga qurilgan parallelogramning diagonali bo'ylab yo'naladi (2.29-rasm).



2.29-rasm

28-§ Mustaqil o'rgamish uchun talabalarga tavsiya etiladigan muammolar

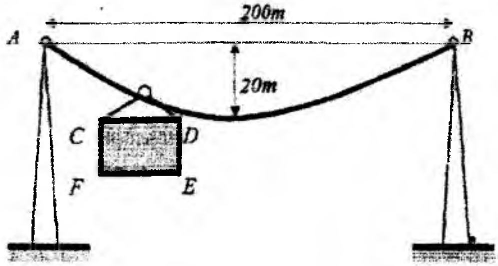
Muammo-1. To'g'ri burchakli prizma gorizontal tekislikda harakatlanmoqda (2.30-rasm).



2.30-rasm

Agar A nuqta o'zgarmas a_A=230sm/s^2 tezlanishga ega bo'lsa. t_1=1s vaqt momenti uchun prizma A,B,C nuqtalarining tezligi va tezlanishi aniqlansin.

Muammo -2. Lift kabinasi parabola qismi bo'ylab *A* nuqtadan *B* nuqtaga tortilgan arqonga bog'langan holda harakatlanmoqda. Kabina harakati *A* nuqtadan gorizontal ϑ_{tor}=1m/s o'zgarmas tezlik bilan boshlangan. Kabina *AB* arqon o'rtasida bo'lgan paytda t_1=3s vaqt momenti uchun uning *CDFE* nuqtalarining tezligi va tezlanishi aniqlansin.(2.31-rasm)



2.31-rasm

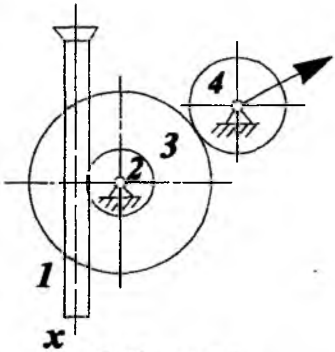
Muammo -3. Kvadrat plastina chizma tekisligida o'zgarmas ε=1rad/s^2 burchak tezlanish bilan aylanma harakat qilmoqda. Agar *t=0* vaqt onida plastina *OA* tomoni gorizontal holatni egallagan bo'lib, B nuqtaning tezligi ϑ_{BO}=1 ga teng bo'lsa, kvadrat plastina uchlarining tezligi va tezlanishi aniqlansin va t=1s vaqt oni uchun kvadrat plastina dioganallarida yotuvchi nuqtalartezliklarining taqsimoti ko'rsatilsin. Kvadrat plastina tomoni *l=0,5m* ga teng.

Muammo -4. Jism qo'zg'almas o'q atrofida ε=5rad/s^2 burchak tezlanish bilan aylanadi. Boshlang'ich paytda, t_0=0 da, jismning burchak tezligi ω_0=0 bo'lsa, t=2s da uning aylanish o'qidan r=0,2m masofadagi nuqtasining tezligini aniqlang

Muammo -5. Qo'zg'almas o'q atrofida aylanayotgan jismning aylanish o'qidan r=0,2m masofadagi nuqtasining tezligi $\upsilon=4t^2$ qonun bo'yicha o'zgarsa, t=2s dagi jismning burchak tezlanishini toping.

Muammo -6. Jismning burchak tezligi ω=1+t qonun bo'yicha o'zgarsa, t=1s paytda uning aylanish o'qidan r=0,2m masofadagi nuqtasining tezlanishini toping.

Muammo -7. Strelkani indikator mexanizmida harakat o'lchov shtiftining 1 reykasidan 2 shesternyaga uzatiladi; 2 shesterniyaning o'qiga 3 tishli g'ildirak o'rnatilgan, 3 g'ildirak esa strelka biriktirilgan 4 shesternya bilan tishlashadi. Agar shtiftning harakati x=asin kt tenglama bilan berilgan bo'lsa va tishli g'ildiraklarning radiuslari tegishlicha r_2, r_3 va r_4 bo'lsa, strelkaning burchak tezligi aniqlansin (2.32 – rasm).

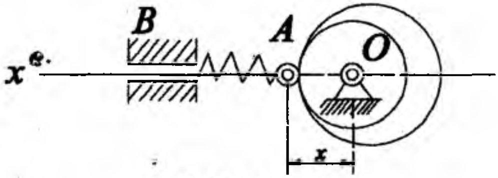


2.32 – rasm

Muammo -8. Kulak O o'q atrofida tekis aylanib, AB sterjenni teng o'lchovchi ilgarilamma-qaytma harakatga keltiradi. Kulakning bir marta to'liq aylanish vaqti 8 c, sterjenning shu vaqt ichidagi harakati tenglamasi:

$$x = \begin{cases} 30 + 5t, & 0 < t < 4, \\ 70 - 5t, & 4 < t < 8 \end{cases}$$

ko'rinishga ega (x-santimetrlar, t-sekundlar hisobida). Kulak konturining tenglamasi topilsin va sterjen harakatining grafigi chizilsin (2.33 – rasm).



2.33 - rasm

29-§ Talabalar mustaqil bajarishi uchun ko'p variantli keyslar (hisob chizma ishlari uchun)

Ilgarilanma va aylanma harakatlarda qattiq jism nuqtalarining tezliklari va tezlanishlarini aniqlash.

1-yukning harakati:

$$x=c_2t_2 + c_1t+c_0$$

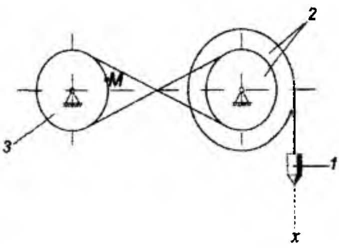
tenglama bilan tavsiflanishi kerak. Bu yerda t-vaqt, s; s_0; s_1; s_2; - doimiylar.

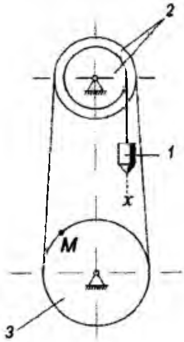
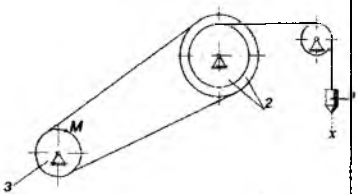
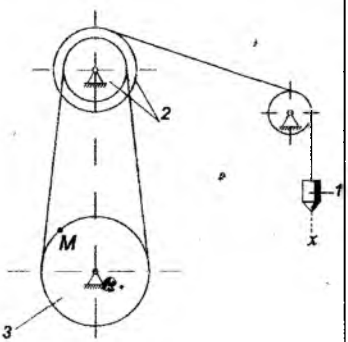
Vaqtning boshlang'ich onida $(t=0)$ yukning koordinatasi x_0, tezligi esa ϑ_0 bo'lishi kerak.

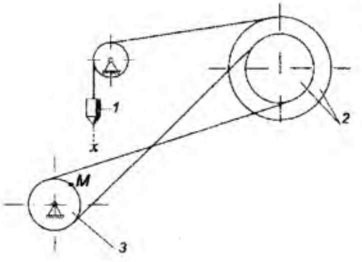
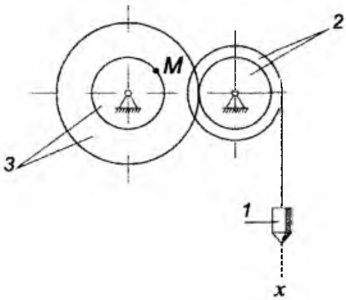
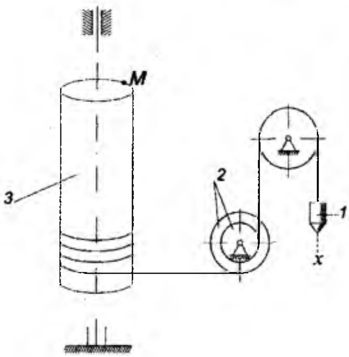
Undan tashqari $t=t_2$ vaqt onida yukning koordinatasi x_2 ga teng bo'lishi lozim.

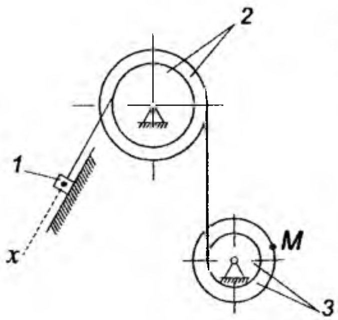
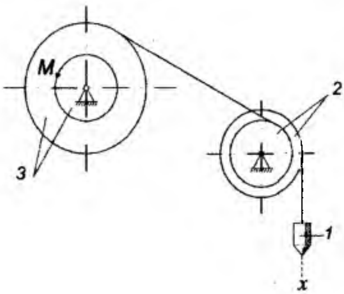
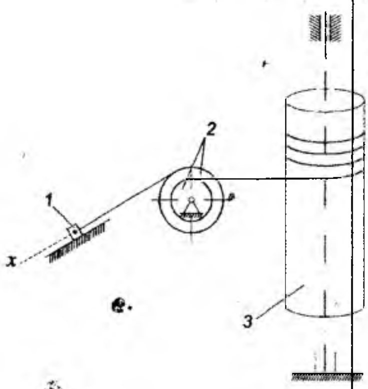
s_0, s_1 s_2 koeffitsientlar shunday aniqlansinki, bunda yuk 1- ning talab qilgan harakati amalga oshsin. Shuningdek, $t=t_1$ vaqt onida yukning hamda mexanizm g'ildiraklaridan birining M nuqtasining tezligi va tezlanishi aniqlansin.

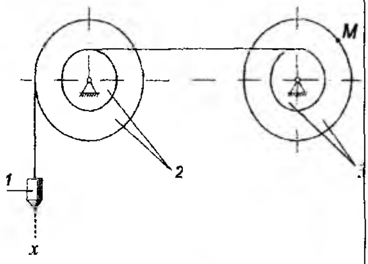
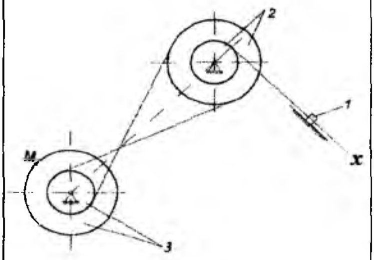
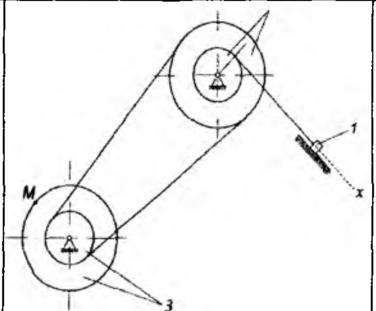
Mexanizmlarning sxemalari, hisoblash uchun kerakli ma'lumotlar jadvalda keltirilgan.

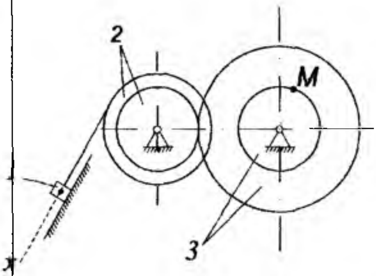
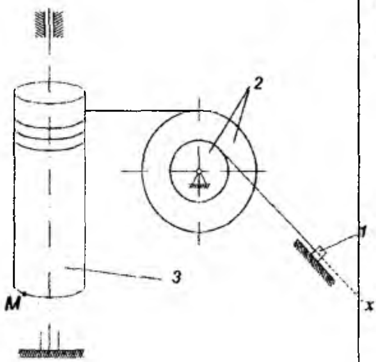
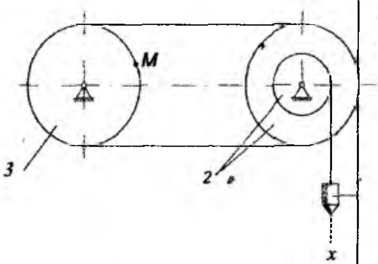
Variant raqamlari	Mexanizmlarning sxemalari	Radiuslar, sm	1 yukning koordinatalari va tezliklari	Hisob uchun vaqt onlari
1.	2.	3.	4.	5.
1.		$R_2 = 20\,sm$ $r_2 = 15\,sm$ $R_3 = 15\,sm$	$x_0 = 4\,sm$ $\vartheta_0 = 6\,sm/s$ $x_2 = 220\,sm$	$t_2 = 4s$ $t_1 = 3s$

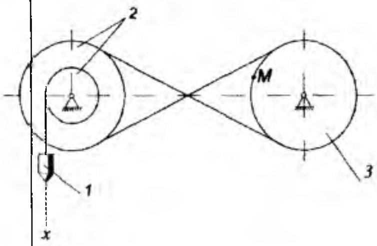
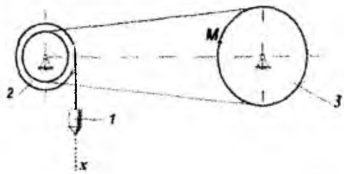
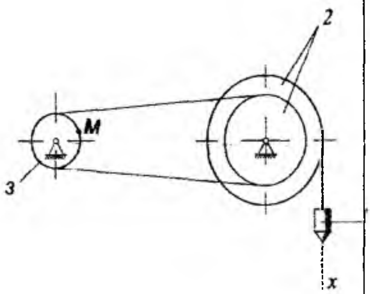
2.		$R_2 = 20\,sm$ $r_2 = 15\,sm$ $R_3 = 25\,sm$	$x_0 = 8\,sm$ $\vartheta_0 = 4\,sm/s$ $x_2 = 44\,sm$	$t_2 = 2s$ $t_1 = 1s$
3.		$R_2 = 25\,sm$ $r_2 = 20\,sm$ $R_3 = 15\,sm$	$x_0 = 3\,sm$ $\vartheta_0 = 12\,m/s$ $x_2 = 211\,sm$	$t_2 = 4s$ $t_1 = 1s$
4.		$R_2 = 20\,sm$ $r_2 = 15\,sm$ $R_3 = 25\,sm$	$x_0 = 5\,sm$ $\vartheta_0 = 10\,sm/s$ $x_2 = 505\,sm$	$t_2 = 5s$ $t_1 = 3s$

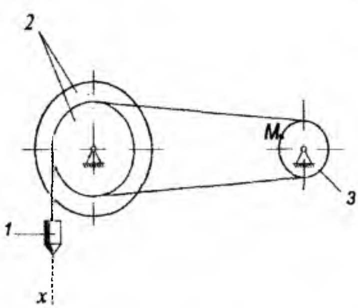
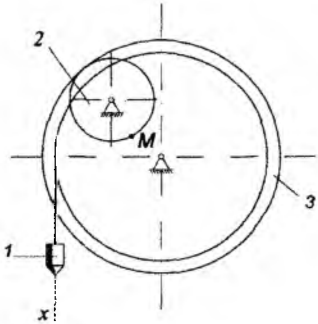
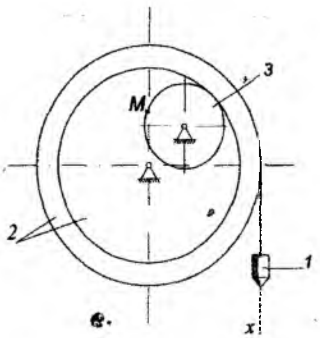
5.		$R_2 = 30\,sm$ $r_2 = 20\,sm$ $R_3 = 15\,sm$	$x_0 = 10\,sm$ $\vartheta_0 = 8\,sm/s$ $x_2 = 277\,sm$	$t_2 = 3s$ $t_1 = 1s$
6.		$R_2 = 25\,sm$ $r_2 = 15\,sm$ $R_3 = 35\,sm$ $r_3 = 15\,sm$	$x_0 = 6\,sm$ $\vartheta_0 = 5\,sm/s$ $x_2 = 356\,sm$	$t_2 = 5s$ $t_1 = 2s$
7.		$R_2 = 50\,sm$ $r_2 = 25\,sm$ $R_3 = 40\,sm$	$x_0 = 7\,sm$ $\vartheta_0 = 6\,sm/s$ $x_2 = 103\,sm$	$t_2 = 2s$ $t_1 = 1s$

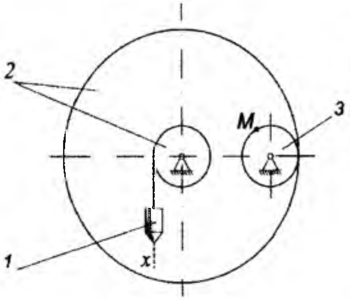
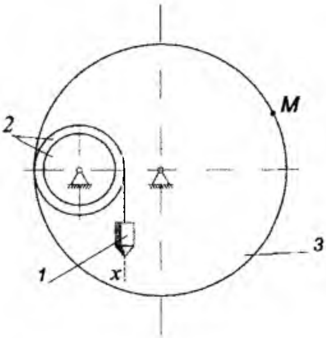
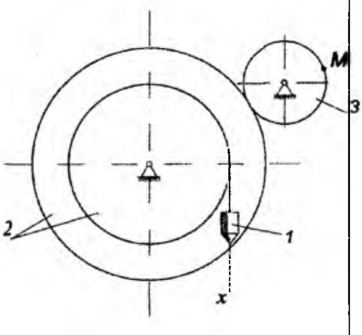
8.		$R_2 = 50\,sm$ $r_2 = 40\,sm$ $R_3 = 40\,sm$ $r_3 = 20\,sm$	$x_0 = 5\,sm$ $\vartheta_0 = 9\,sm/s$ $x_2 = 194\,sm$	$t_2 = 3s$ $t_1 = 2s$
9.		$R_2 = 40\,sm$ $r_2 = 20\,sm$ $R_3 = 50\,sm$ $r_3 = 25\,sm$	$x_0 = 9\,sm$ $\vartheta_0 = 8\,sm/s$ $x_2 = 105\,sm$	$t_2 = 4s$ $t_1 = 2s$
10.		$R_2 = 60\,sm$ $r_2 = 25\,sm$ $R_3 = 70\,sm$	$x_0 = 8\,sm$ $\vartheta_0 = 4\,sm/s$ $x_2 = 119\,sm$	$t_2 = 3s$ $t_1 = 2s$

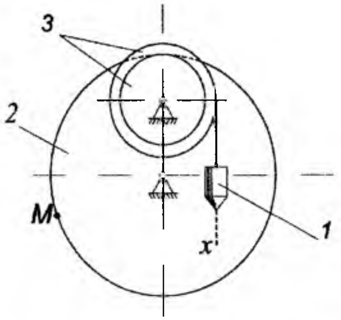
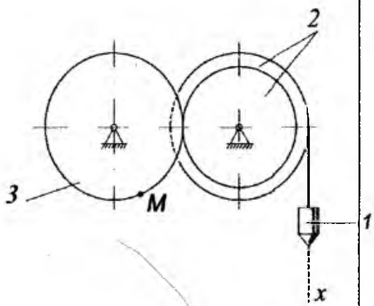
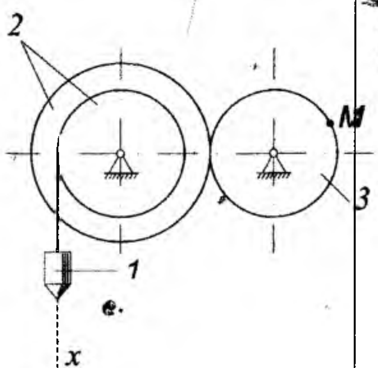
11.		$R_2 = 40\,sm$ $r_2 = 20\,sm$ $R_3 = 40\,sm$ $r_3 = 20\,sm$	$x_0 = 6\,sm$ $\vartheta_0 = 14\,sm/s$ $x_2 = 862\,sm$	$t_2 = 4s$ $t_1 = 2s$
12.		$R_2 = 50\,sm$ $r_2 = 20\,sm$ $R_3 = 50\,sm$ $r_3 = 20\,sm$	$x_0 = 5\,sm$ $\vartheta_0 = 10\,sm/s$ $x_2 = 193\,sm$	$t_2 = 2s$ $t_1 = 1s$
13.		$R_2 = 50\,sm$ $r_2 = 25\,sm$ $R_3 = 50\,sm$ $r_3 = 20\,sm$	$x_0 = 8\,sm$ $\vartheta_0 = 5\,sm/s$ $x_3 = 347\,sm$	$t_2 = 3s$ $t_1 = 2s$

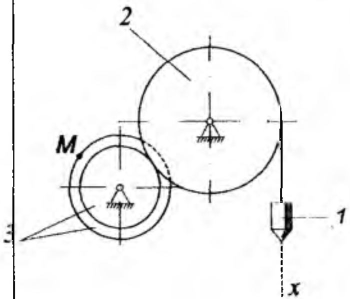
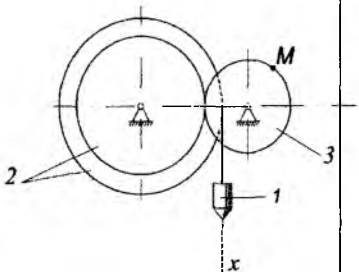
14.		$R_2 = 30\ sm$ $r_2 = 22\ sm$ $R_3 = 60\ sm$ $r_3 = 30\ sm$	$x_0 = 4\ sm$ $\vartheta_0 = 6\ sm/s$ $x_2 = 32\ sm$	$t_2 = 2s$ $t_1 = 1s$
15.		$R_2 = 40\ sm$ $r_2 = 20\ sm$ $R_3 = 25\ sm$	$x_0 = 10\ sm$ $\vartheta_0 = 7\ sm/s$ $x_2 = 128\ sm$	$t_2 = 2s$ $t_1 = 1s$
16.		$R_2 = 30\ sm$ $r_2 = 20\ sm$ $R_3 = 30\ sm$	$x_0 = 5\ sm$ $\vartheta_0 = 2\ sm/s$ $x_2 = 189\ sm$	$t_2 = 4s$ $t_1 = 2s$

17.		$R_2 = 30\, sm$ $r_2 = 20\, sm$ $R_3 = 30\, sm$	$x_0 = 6\, sm$ $\vartheta_0 = 3\, sm/s$ $x_2 = 80\, sm$	$t_2 = 2s$ $t_1 = 1s$
18.		$R_2 = 40\, sm$ $r_2 = 25\, sm$ $R_3 = 50\, sm$	$x_0 = 7\, sm$ $\vartheta_0 = 0\, sm/s$ $x_2 = 557\, sm$	$t_2 = 5s$ $t_1 = 2s$
19.		$R_2 = 40\, sm$ $r_2 = 30\, sm$ $R_3 = 20\, sm$	$x_0 = 5\, sm$ $\vartheta_0 = 10\, sm/s$ $x_2 = 179\, sm$	$t_2 = 3s$ $t_1 = 2s$

20.		$R_2 = 50\,sm$ $r_2 = 30\,sm$ $R_3 = 25\,sm$	$x_0 = 9\,sm$ $\vartheta_0 = 8\,sm/s$ $x_2 = 65\,sm$	$t_2 = 2s$ $t_1 = 1s$
21.		$R_2 = 30\,sm$ $R_3 = 80\,sm$ $r_3 = 70\,sm$	$x_0 = 5\,sm$ $\vartheta_0 = 3\,sm/s$ $x_2 = 129\,sm$	$t_2 = 4s$ $t_1 = 3s$
22.		$R_2 = 40\,sm$ $r_2 = 30\,sm$ $R_3 = 15\,sm$	$x_0 = 10\,sm$ $\vartheta_0 = 7\,sm/s$ $x_2 = 48\,sm$	$t_2 = 2s$ $t_1 = 1s$

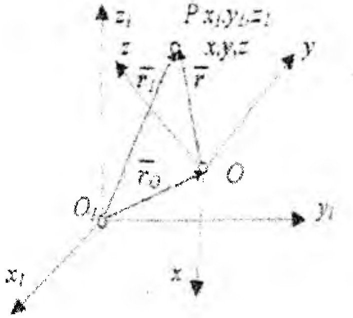
23.		$R_2 = 40\,sm$ $r_2 = 15\,sm$ $R_3 = 15\,sm$	$x_0 = 6\,sm$ $\vartheta_0 = 2\,sm/s$ $x_2 = 111\,sm$	$t_2 = 3s$ $t_1 = 2s$
24.		$R_2 = 60\,sm$ $r_2 = 45\,sm$ $R_3 = 130\,sm$	$x_0 = 8\,sm$ $\vartheta_0 = 5\,sm/s$ $x_2 = 124\,sm$	$t_2 = 4s$ $t_1 = 3s$
25.		$R_2 = 120\,sm$ $r_2 = 72\,sm$ $R_3 = 36\,sm$	$x_0 = 7\,sm$ $\vartheta_0 = 16\,sm/s$ $x_2 = 215\,sm$	$t_2 = 4s$ $t_1 = 2s$

26.		$R_2 = 80\ sm$ $R_3 = 45\ sm$ $r_3 = 30\ sm$	$x_0 = 3\ sm$ $\vartheta_0 = 15\ sm/s$ $x_2 = 102\ sm$	$t_2 = 3s$ $t_1 = 2s$
27.		$R_2 = 58\ sm$ $r_2 = 45\ sm$ $R_3 = 60\ sm$	$x_0 = 4\ sm$ $\vartheta_0 = 4\ sm/s$ $x_2 = 172\ sm$	$t_2 = 4s$ $t_1 = 3s$
28.		$R_2 = 120\ sm$ $r_2 = 72\ sm$ $R_3 = 90\ sm$	$x_0 = 8\ sm$ $\vartheta_0 = 6\ sm/s$ $x_2 = 40\ sm$	$t_2 = 4s$ $t_1 = 2s$

29.		$R_2 = 100\ sm$ $r_2 = 75\ sm$ $R_3 = 60\ sm$	$x_0 = 5\ sm$ $\vartheta_0 = 10\ sm/s$ $x_2 = 41\ sm$	$t_2 = 2s$ $t_1 = 1s$
30.		$R_2 = 60\ sm$ $r_2 = 45\ sm$ $R_3 = 36\ sm$	$x_0 = 2\ sm$ $\vartheta_0 = 12\ sm/s$ $x_2 = 173\ sm$	$t_2 = 3s$ $t_1 = 2s$

III-BOB. NUQTANING MURAKKAB HARAKATI

30-§ Nuqtaning nisbiy, ko‘chirma va absolyut harakatlari



3.1-rasm

Nuqta bir vaqtning o‘zida ikki yoki undan ortiq harakatda ishtirok etsa, bunday harakat murakkab harakat deyiladi.

Nuqtaning murakkab harakatini o‘rganish uchun qo‘zg‘almas $O_1x_1y_1z_1$ va unga nisbatan ixtiyoriy ravishda harakatlanadigan oxyz koordinatalar sistemasini tanlab olamiz (3.1-rasm).

M nuqtaning qo‘zg‘aluvchi Oxyz koordinatalar sistemasiga nisbatan harakati nisbiy harakat deyiladi.

Nuqtaning bunday harakatdagi tezlik va tezlanishi mos ravishda nisbiy tezlik va nisbiy tezlanish deyiladi hamda $\vec{\vartheta}_n$ va $\vec{a}_n$ bilan belgilanadi.

M nuqtaning qo‘zg‘aluvchi koordinatalar sistemasi bilan birgalikda qo‘zg‘almas koordinatalar sistemasiga nisbatan harakati ko‘chirma harakat deyiladi. Qo‘zg‘aluvchi koordinatalar sistemasining berilgan onda M nuqta bilan ustma – ust tushuvchi nuqtasining tezligi va tezlanishi ko‘chirma tezlik va ko‘chirma tezlanish deyiladi hamda $\vec{\vartheta}_k$ *va* $\vec{a}_k$ *bilan belgilanadi.*

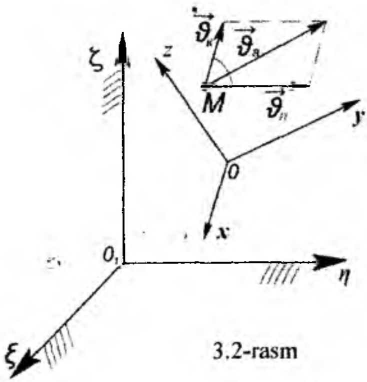
M nuqtaning qo‘zg‘almas koordinatalar sistemasiga nisbatan harakati *absolyut harakat* deyiladi. Nuqtaning absolyut harakati o‘z navbatida nisbiy va ko‘chirma harakatlardan tashkil topgani tufayli nuqtaning absolyut harakatini murakkab deb atash mumkin. Absolyut harakatdagi nuqtaning tezlik va tezlanishi mos ravishda absolyut tezlik va absolyut tezlanish deyiladi hamda $\vec{\vartheta}_a$ va $\vec{a}_a$ bilan belgilanadi.

Nuqtaning nisbiy va ko‘chirma harakatini bilgan holda uning absolyut harakatini, binobarin, absolyut harakat tezligi va tezlanishini aniqlash nuqta murakkab harakati kinematikasining asosiy masalasi hisoblanadi.

31-§ Murakkab harakatdagi nuqtaning tezliklarini qo‘shish haqidagi teorema

Faraz qilaylik *M* nuqta qo‘zg‘almas O_1,x_1,y_1,z_1 koordinatalar sistemasiga nisbatan murakkab harakatda bo‘lsin (3.2-rasm). [4]

Nuqtaning qo‘zg‘almas va qo‘zg‘aluvchan koordinatalar sistemasiga nisbatan holatini aniqlovchi radius – vektorlarni $\vec{r}_1$ va $\vec{r}_0$ deb, qo‘zg‘aluvchan sistemani qo‘zg‘almas sistemaga nisbatan holatini aniqlovchi radius –vektorni $\vec{r}$ deb belgilasak, 18.1-rasmdan:



3.2-rasm

$$\vec{r}_1 = \vec{r}_0 + \vec{r} \qquad (3.1)$$

Nuqtaning tezligi uning holatini aniqlovchi radius – vektordan vaqt bo‘yicha hisoblangan birinchi tartibli hosilaga teng:

$$\vec{\vartheta} = \frac{d\vec{r}_1}{dt} = \frac{d\vec{r}_0}{dt} + \frac{d\vec{r}}{dt}. \qquad (3.2)$$

$\vec{\rho}$ – vector *M* nuqtaning ko‘zg‘almas koordinatalar sistemasiga nisbatan holatini aniqlovchi radius – vektor bo‘lgani uchun $\frac{d\vec{r}}{dt}$ hosila nuqtaning absolyut tezligi $\vec{\vartheta}_a$ ni, $\vec{r}$ – vector *M* nuqtaning qo‘zg‘aluvchi koordinatalar sistemasiga nisbatan holatini aniqlovchi radius – vektor bo‘lgani uchun $\frac{d\vec{r}}{dt}$ hosila nuqtaning nisbiy harakat tezligi $\vec{\vartheta}_n$ ni, $\vec{r}_0$ – vektor qo‘zg‘aluvchi koordinatalar sistemasining qo‘zg‘almas sistemaga nisbatan holatini aniqlovchi radius – vektor bo‘lgani uchun, $\frac{d\vec{r}_0}{dt}$ hosila nuqtaning ko‘chirma harakat tezligi $\vec{\vartheta}_k$ ni ifodalaydi.

Shuning uchun (3.2) dan:

$$\vec{\vartheta}_a = \vec{\vartheta}_n + \vec{\vartheta}_k.\ \vec{v}_k = \vec{v}_0 + \vec{w} * \vec{r} \tag{3.3}$$

Agar ekvivalent e'tiborga olsak,nuqtaning absolyut tezlanishi quyidagicha aniqlanadi:

$$\vec{v}_a = \vec{v}_0 + \vec{w} * \vec{r} + \frac{d\vec{r}}{dt}$$

Binobarin, murakkab harakatdagi nuqtaning absolyut tezligi nisbiy va ko'chirma harakat tezliklarining geometrik yig'indisiga teng. (3.3) tenglama murakkab harakatdagi nuqtaning tezliklarini qo'shish haqidagi teoremani ifodalaydi.

Absolyut tezlik vektori nisbiy va ko'chirma harakat tezliklariga qurilgan parallelogramning diagonali bo'ylab yo'nalgan bo'lib, moduli quyidagi formula asosida aniqlanadi (3.2-rasm):

$$v = \sqrt{\vartheta_n^2 + \vartheta_k^2 + 2\vartheta_n\vartheta_k cos\alpha}\,. \tag{3.4}$$

Bunda:

a) Agar α=90°, ya'ni $\vec{\vartheta}_n \perp \vec{\vartheta}_k$ bo'lsa, absolyut tezlik moduli

$$\vartheta_a = \sqrt{\vartheta_n^2 + \vartheta_k^2}$$

formula yordamida hisoblanadi.

b) Agar α=0° bo'lsa, ya'ni $\vec{\vartheta}_n$ *va* $\vec{\vartheta}_k$ bir to'g'ri chiziq bo'ylab bir tomonga yo'nalsa, absolyut tezlik moduli

$$\vartheta_a = \sqrt{\vartheta_n^2 + \vartheta_k^2 + 2\vartheta_n\vartheta_k} = (\vartheta_n + \vartheta_k)$$

formula orqali aniqlanadi.

v) Agar α=180° bo'lsa, ya'ni $\vec{\vartheta}_n$ bilan $\vec{\vartheta}_k$ bir to'g'ri chiziq bo'ylab qarama – qarshi tomonga yo'nalsa, absolyut tezlik moduli

$$\vartheta_a = \sqrt{\vartheta_n^2 + \vartheta_k^2 - 2\vartheta_n\vartheta_k} = (\vartheta_n - \vartheta_k) \tag{3.5}$$

formuladan aniqlanadi.

Absolyut tezlik modulini proeksiyalash usuli yordamida ham aniqlash mumkin. Buning uchun koordinata o'qlari o'tqaziladi va (3.3) tenglik koordinata o'qlariga proeksiyalanadi:

$$\vartheta_{ax} = \vartheta_{nx} + \vartheta_{kx}\,,$$
$$\vartheta_{ay} = \vartheta_{ny} + \vartheta_{ky}\,. \tag{3.6}$$

Absolyut tezlik moduli va yo'nalishi quyidagi formulalar asosida aniqlanadi:

$$\vartheta_a = \sqrt{\vartheta_{ax}^2 + \vartheta_{ay}^2},$$

$$\cos(\vec{\vartheta}_a{}^\wedge x) = \frac{v_{ax}}{\vartheta_a}, \quad \cos(\vec{\vartheta}_a{}^\wedge y) = \frac{\vartheta_{ay}}{\vartheta_a}. \qquad (3.7)$$

Shuni ta'kidlash lozimki, nuqtaning nisbiy tezligini aniqlash uchun ko'chirma harakat xayolan to'xtatiladi.

Ko'chirma harakat tezligini aniqlash uchun nisbiy harakat xayolan to'xtatiladi va berilgan onda qo'zg'aluvchan sanoq sistemasinning *M* nuqta bilan ustma – ust tushuvchi nuqtasining tezligi aniqlanadi.

32-§ Koriolis tezlanishi

Nisbiy tezlikning yo'nalishini ko'chirma harakatda, ko'chirma tezling miqdori va yo'nalishini nisbiy harakatda o'zgarishlarini xarakterlovchi kattalik Koriolis tezlanishi deyiladi.

Koriolis tezlanishi murakkab harakatdagi nuqtaning ko'chirma harakat burchak tezligi vektori bilan nisbiy harakat tezligi vektorining vektr ko'paytmasining ikkilanganiga teng:

$$\vec{a}_c = 2(\vec{\omega}_k \times \vec{\vartheta}_n) \qquad (3.8)$$

Agar $\vec{\omega}_k$ bilan $\vec{\vartheta}_n$ orasidagi burchakni α bilan belgilasak, Koriolis tezlanishining moduli

$$a_c = 2\omega_k \vartheta_n \sin\alpha \qquad (3.9)$$

formuladan aniqlanadi.

Koriolis tezlanishining yo'nalishini aniqlash uchun nuqtaning nisbiy tezligini ko'chirma harakat aylanish o'qiga perpendikulyar tekislikka proeksiyalab, bu proeksiyani mazkur tekislikda ko'chirma harakat aylanishi yo'nalishida 90° burchakka burish kerak (3.3-rasm).

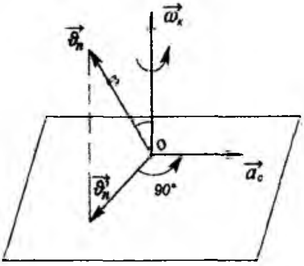
Bu usul Jukovskiy qoidasi deyiladi.

Agar $\vec{\omega}_k \perp \vec{\vartheta}_n$ bo'lsa (3.4-rasm), sinα=1 bo'ladi.

U holda

$$a_c = 2\omega_k \cdot \vartheta_n. \qquad (3.10)$$

Koriolis tezlanishining yo'nalishini $(\vec{\omega}_k \times \vec{\vartheta}_n)$ *vektor ko'paytma qoidasiga muvofiq aniqlash ham mumkin.* Qoidaga ko'ra, Koriolis tezlanishi $\vec{\omega}_k$ va $\vec{\vartheta}_n$ vektorlar joylashgan tekislikka perpendikulyar holda shunday tomonga qarab yo'nalgan bo'ladiki, u tomondan qaraganda $\vec{\omega}_k$ vektorni $\vec{\vartheta}_n$ vektor bilan kichik burchak orqali ustma-ust tushirish uchun qilinadigan aylanma harakat soat mili harakatiga qarama-qarshi yo'nalishda ro'y beradi (3.4- rasm).



3.4-rasm

Koriolis tezlanishi quyidagi hollarda nolga teng bo'ladi:

a) agar ω_k ya'ni ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa;

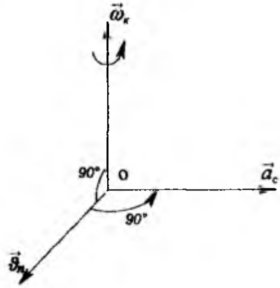
b) agar $\vartheta_n=1$ ya'ni nisbiy harakat tezligi biror onda nolga teng bo'lsa;

v) agar $\alpha=0$ yoki $\alpha=180°$, ya'ni nisbiy harakat ko'chirma harakat aylanish o'qiga parallel ravishda sodir bo'lsa, yoki berilgan onda nisbiy harakat tezligi mazkur o'qqa parallel bo'lsa.

33-§. Murakkab harakatdagi nuqtaning tezlanishlarini qo'shish haqidagi Koriolis teoremasi

Murakkab harakatdagi nuqtaning tezlanishi nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisiga teng:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k + \vec{a}_c \qquad (3.11)$$



(3.11) tenglik ko'chirma harakati ilgarilanma harakat bo'lmagan holda nuqtaning tezlanishlarini qo'shish haqidagi Koriolis teoremasini ifodalaydi. Nuqtaning nisbiy tezlanishini aniqlash uchun ko'chirma harakat xayolan to'xtatiladi va nisbiy tezlanish aniqlanadi.

Nuqtaning ko'chirma tezlanishini aniqlash uchun nisbiy harakat xayolan to'xtatiladi va berilgan onda qo'zg'aluvchan sanoq sistemasining M nuqta bilan ustma – ust tushuvchi nuqtasinnig tezlanishi aniqlanadi.

Agar qo'zg'aluvchan koordinatalar sistemasining nuqtalari egri chiziqli harakatda bo'lsa, hamda nuqtaning nisbiy harakat traektoriyasi egri chiziqdan iborat bo'lsa, ko'chirma va nisbiy tezlanishlarni normal va urinma tezlanishlarning geometrik yig'indisidan iborat deb qarash mumkin. U vaqtda (3.11) quyidagicha yoziladi:

$$\vec{a}_a = \vec{a}_n^n + \vec{a}_n^\tau + \vec{a}_k^n + \vec{a}_k^\tau + \vec{a}_c \qquad (3.12)$$

bunda,

a_n^n; a_k^n – nisbiy va ko'chirma harakatlardagi normal tezlanishlar.

$\vec{a}_n^\tau$, $\vec{a}_k^\tau$ – nisbiy va ko'chirma harakatlardagi urinma tezlanishlar.

M nuqtaning murakkab harakatida absolyut tezlanishning modulini aniqlash uchun proeksiyalash usulidan foydalanish ham mumkin. Buning uchun koordinata o'qlari o'tkaziladi va (18.9) tenglik koordinata o'qlariga proeksiyalanadi:

$$\begin{aligned} a_{ax} &= a_{nx}^n + a_{nx}^\tau + a_{kx}^n + a_{kx}^\tau + a_{cx}, \\ a_{ay} &= a_{ny}^n + a_{ny}^\tau + a_{ky}^n + a_{ky}^\tau + a_{cy}, \\ a_{az} &= a_{nz}^n + a_{nz}^\tau + a_{kz}^n + a_{kz}^\tau + a_{cz}. \end{aligned} \qquad (3.13)$$

Absolyut tezlanish moduli va yo'nalishi quyidagi formulalar asosida aniqlanadi:

$$\begin{aligned} a_a &= \sqrt{a_{ax}^2 + a_{ay}^2 + a_{az}^2}. \\ \cos(\vec{a}_a \wedge x) &= \frac{a_{ax}}{a}, \cos(\vec{a}_a \wedge y) = \frac{a_{ay}}{a}, \\ \cos(\vec{a}_a \wedge z) &= \frac{a_{az}}{a}. \end{aligned} \qquad (3.14)$$

Ko'chirma harakati ilgarilanma harakatdan iborat bo'lgan nuqtaning absolyut tezlanishi uning nisbiy va ko'chirma tezlanishlarining geometrik yig'indisiga teng:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k \qquad (3.15)$$

Bunday xolda absolyut tezlanish nisbiy tezlanish $\vec{a}_n$ va ko'chirma tezlanish $\vec{a}_k$ larga qurilgan parallelogramning diagonali bilan ifodalanadi. Absolyut tezlanish moduli quyidagicha hisoblanadi:

$$a_a = \sqrt{a_n^2 + a_k^2 + 2a_n a_k \cos(\vec{a}_n \wedge \vec{a}_k)} \qquad (3.16)$$

Nuqtaning absolyut tezlanishini(3.1-rasm) quyidagi usul bilan ham aniqlash mumkin:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}_0}{dt} + \frac{d\vec{r}}{dt} = \vec{v}_0 + (w \times r) + \frac{dr}{dt}$$

Nuqtaning absolyut tezlanishi absolyut tezlikdan vaqt bo'yicha hisoblashgan birinchi tartibli hosilaga teng:

$$\vec{a}_a = \frac{dv_a}{dt} = \frac{dv_0}{dt} + \left(\frac{d\vec{w}}{dt} \times \vec{r}\right) + \vec{w} \times \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right)$$

Agar

$$\frac{dv_a}{dt} = a_a;\ \frac{d\vec{v}_0}{dt} = \vec{a}_0;\ \frac{d\vec{w}}{dt} = \vec{\varepsilon};$$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dt} + \vec{w} \times \vec{r};\ \frac{d}{dt}\left(\frac{d^2\vec{r}}{dt^2}\right) = \frac{d^2\vec{r}}{dt^2} + \vec{w} \times \frac{d\vec{r}}{dt}$$

ekanligini e'tiborga olsak

$$\vec{a}_a = \vec{a}_0 + \vec{\varepsilon} * \vec{r} + \vec{w} \times \left(\frac{d\vec{r}}{dt} + \vec{w} * \vec{r}\right) + \frac{d^2\vec{r}}{dt^2} + \vec{w} \times \frac{d\vec{r}}{dt}$$

$$= \vec{a}_0 + \vec{\varepsilon} \times \vec{r} + \vec{w} * (\vec{w} \times \vec{r}) + \frac{d^2r}{dt^2} + 2\vec{w} \times \frac{dr}{dt}$$

Bu ifodada

$\vec{a}_0 + \vec{\varepsilon} \times \vec{r} + \vec{w} * (\vec{w} \times \vec{r}) = \vec{a}_k$- nuqtaning ko'chirma tezlanishi.

$\frac{d^2r}{dt^2} = \vec{a}_n$ -nuqtaning nisbiy tezlanishi

$2\vec{w} \times \frac{d\vec{r}}{dt} = 2(\vec{w}_k \times \vec{v}_n) = \vec{a}_c$- Koraolis tezlanishi.

Yozilganlarni e'tiborga olsak, murakkab harakatdagi nuqtaning absolyut tezlanishini ifodalovchi (18.8) tenglamasi hosil bo'ladi.

$$\vec{a}_a = \vec{a}_k + \vec{a}_n + \vec{a}_c$$

Takrorlash uchun savollar

1. Nuqtaning qanday harakatga nisbiy harakat deyiladi?
2. Nuqtaning qanday harakatiga ko'chirma harakat deyiladi?
3. Nuqtaning qanday harakatiga absolyut harakat deyiladi?
4. Nuqtaning nisbiy tezligi qanday topiladi?
5. Nuqtaning ko'chirma tezligi qanday topiladi?
6. Nuqtaning absolyut tezligi qanday topiladi?
7. Tezliklarni qo'shish haqidagi teoremani aytib bering.
8. Absolyut tezlik modulini topish formulasini yozib bering.
9. Nuqtaning nisbiy tezlanishi qanday aniqlanadi?
10. Nuqtaning ko'chirma tezlanishi qanday aniqlanadi?
11. Nuqtaning absalyut tezlanishi qanday aniqlanadi?
12. Koriolis tezlanishining yuzaga kelish shartlarini aytib bering.
13. Koriolis tezlanishi qanday yo'naladi?
14. Koriolis tezlanishining moduli qanday ifodalanadi?
15. Koriolis tezlanishining nolga teng bo'lishi shartlarini aytib bering.

34-§ Nuqtaning nisbiy va absalyut harakatlarida uning traektoriyasi va harakat tenglamalarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Mazkur mavzuga doir masalalarni ikki asosiy turga ajratish mumkin.

1) Nuqtaning nisbiy va ko'chirma harakatlarini bilgan holda absalyut harakat traektoriyasining tenglamalarini aniqlash.

2) Nuqtaning absalyut va ko'chirma harakatlarini bilgan holda nisbiy harakat taektoriyasining tenglamalarini aniqlash.

Birinchi turdagi masalalarni yechishda nuqtaning nisbiy va ko'chirma harakatlarini qo'shish lozim.

Ikkinchi turdagi masalalarni yechishda nuqtaning berilgan absalyut harakatini masala shartida ma'lum bo'lgan ko'chirma va noma'lum nisbiy harakatlarga ajratish talab etiladi.

Masalalarni yechishda, dastavval qo'zg'almas va qo'g'aluvchan sanoq sistemalari tanlab olinadi va qo'zg'aluvchan sanoq sistemalari bog'langan jism harakati yani ko'chirma harakat o'rganiladi.

Natijada nuqtaning absalyut va nisbiy harakatlarining xususiyatlarini oson aniqlash imkoni tug'iladi.

Birinchi turdagi masalalarni quyidagi tartibda yechish maqsadga muvofiq bo'ladi:

1) Masala shartidan ma'lum bo'lgan nuqta absalyut harakati ko'chirma va nisbiy harakatlarga ajratiladi.

2) Shartli ravishda qo'zg'almas deb qabul qilingan absalyut va harakatdagi jism bilan bog'langan nisbiy sanoq sistemalari tanlab olinadi.

3) Nuqta nisbiy harakakatining tenglamalari tuziladi.

4) Nuqta absalyut harakatining parametric ko'rinishdagi tenglamalari tuziladi.

Ikkinchi turga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1) Nuqtaning masala shartidan ma'lum bo'lgan absalyut harakati ko'chirma va nisbiy harakatlarga ajratiladi.

2) Shartli ravishda qo'zg'almas deb qabul qilingan absalyut va harakatdagi jism bilan bog'langan nisbiy sanoq sistemalari tanlab olindi.

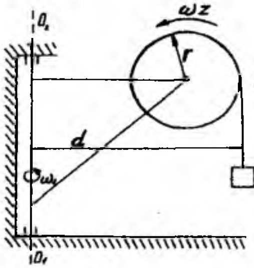
3) Nuqta absalyut harakatining tenglamalari tuziladi.

4) Nuqta nisbiy harakatining parametrik formadagi tenglamalari tuziladi.

5) Nisbiy harakatning parametrik tenglamalaridan parametr-vaqtni qisqartirib, koordinatalar ko'rinishidagi nisbiy harakat tenglamalari tuziladi.

35-§ Nuqta absalyut harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar

1-masala. Aylanuvchi kranning O_1O_2 o'q atrofida ω_1 o'zgarmas burchak tezlik bilan aylanishida A yuk B barabanga o'ralgan kanat yordamida yuqoriga ko'tariladi. *r* radiusli *B* baraban ω_2 o'zgarmas burchak tezlik bilan aylanadi. Agar kranning qulochi *d* ga teng bo'lsa, yukning absalyut harakati traektoriyasi aniqlansin (3.5 – rasm).



3.5- rasm

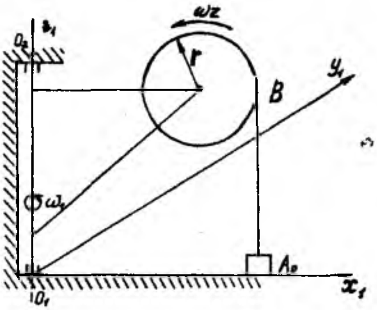
Yechimi: Aylanuvchi kranning O_1O_2 o'q atrofida ω_1 o'zgarmas burchak tezlik bilan aylanishi ko'chirma harakat hisoblanadi. A nuqtaning B barabanga o'ralgan kanat yordamida yuqoriga ko'tarilishi nisbiy harakat hisoblanadi.

Aylanuvchi kran asosi bilan bog'langan $O_1x_1y_1z_1$ koordinata o'qlari sistemasi qo'zg'almas sanoq sistemasini tashkil etadi. Aylanuvchi kran bilan bog'lashgan va u bilan birga aylanuvchi O_2xyz koordinata o'qlari sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi (3.6 – rasm).

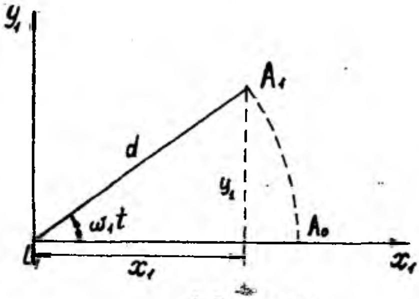
Bunda x_1 o'q O_1O_2 o'q va yukning boshlang'ich holatidan o'tadi, z_1 o'q esa kran aylanish o'qi bo'ylab yo'naladi.

A yukning holati uning absalyut harakatida quyidagi koordinatalar orqali aniqlanadi (3.7 – rasm)

$$\begin{cases} x_1 = d\cos\omega_1 t \\ y_1 = d\sin\omega_1 t \\ z_1 = r\,\omega_2 t \end{cases}$$



3.6 – rasm



3.7 – rasm

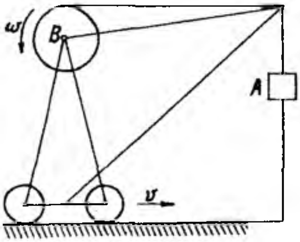
3.8

Hosil bo'lgan tenglamalarni A yukning absalyut harakat traektoriyasining parametrik ko'rinishidagi tenglamalari sifatida qarash mumkin.

Koordinatalar fermasidagi traektoriya tenglamasini tuzish uchun yuqoridagi tenglamalardan parametr-vaqtni qisqartiramiz. Natijada A yuk absalyut harakati traektoriyasining tenglamalari hosil bo'ladi:

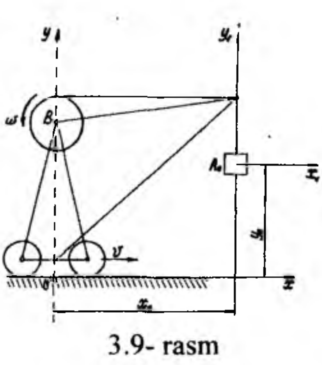
$$x_1 = d\cos\frac{\omega_1 z_1}{\omega_2 r}, \; y_1 = d\sin\frac{\omega_1 z_1}{\omega_2 r}.$$

2-masala. Yukni ko'tarish va kranni siljitish mexanizmlarining ishlarini birlashtirishda A yuk gorizontal va vertical yo'nalishlarda siljiydi. $r=0,5$ *m* radiusli B barabanga o'raglan kanat vositasida A yuk ushlab turiladi. B baraban ishga tushirilishida $\omega=2\pi$ *rad/s* burchak tezlik bilan aylanadi. Kran gorizontal yo'nalishda $\upsilon=0,5$ *m/s* doimiy tezlik bilan siljiydi. Agar yukning boshlang'ich koordinatalari $x_0=10$ *m*, $y_0=6$ *m* bo'lsa, uning absalyut traektoriyasi aniqlansin (3.8- rasm).



3.8- rasm

Yechimi: Kranning gorizontal yo'nalishda $v=0.5$ *m/s* doimiy tezlik bilan siljishi ko'chirma harakat deyiladi.A nuqtaning vertikal yo'nalishda siljishi nisbiy harakat sifatida qaraladi. Yer bilan bog'langan *Oxy* koordinata o'qlari sistemasi shartli ravishda qo'zg'almas sanoq sistemasi sifatida qaraladi. Gorizontal yo'nalishda $v=0,5$ *m/s* doimiy tezlik bilan siljuvchi kran bilan bog'langan Aox_1y_1 koordinata o'qlari sistemasi qo'zg'luvchan sanoq sistemasini tashkil etadi, bunda qo'zg'luvchan sanoq sistemasining boshi *A* yukning boshlang'ich holati bilan ustma ust tushadi (3.9 – rasm).



3.9- rasm

A yukning qo‘zg‘almas – absalyut sanoq sistemasidagi holati quyidagi koordinatalar orqali aniqlanadi:

$$\begin{cases} x = x_0 + Vt, \\ y = y_0 + \omega \cdot rt. \end{cases}$$

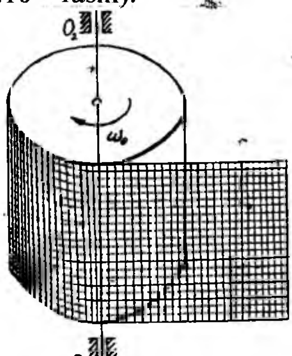
Hosil bo‘lgan tenglamalar sistemasini *A* yuk absalyut harakati traektoriyasining parametrik tenglamalari sifatida qarash mumkin. *A* yuk absalyut harakati traektoriyasining koordinatalar formasidagi tenglamalarini tuzish uchun yuqoridagi tenglamalardan parametr – vaqtni qisqartiramiz. Natijada *A* yuk absalyut harakati traektoriyasining tenglamalari hosil bo‘ladi:

$$t = \frac{x - x_0}{V};$$

$$y = y_0 + \omega r\left(\frac{x - x_0}{V}\right) = 6 + 2\pi \cdot 0{,}5\left(\frac{x - 10}{0{,}5}\right) = 6{,}28x - 56{,}8 \text{ м}.$$

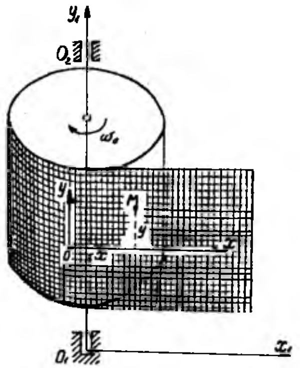
36-§ Nuqta nisbiy harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar

1-masala. Yozib oluvchi moslamaning barabani ω_0 burchak tezlik bilan bir tekis aylanadi. Barabanning radiusi *r*. O‘ziyozar, vertikal yo‘nalishda $y = a\sin\omega_1 t$ qonun bilan harakatlanuvchi detal bilan birlashtirilgan. Qozg‘oz lentada pero yozib olgan egri chiziqning tenglamasi topilsin (3.10 – rasm).



3.10 - rasm

Yechimi. Yozib oluvchi moslama barabanning ω_0 burchak tezlik bilan aylanishi ko'chirma harakat hisoblanadi. O'ziyozar apparat perosining harakati nisbiy harakat sifatida qaraladi. Yer bilan bog'langan va shartli ravishda qo'zg'almas deb qabul qilingan $O_1x_1y_1$ koordinata o'qlari sistemasi qo'zg'almas – absalyut sanoq sistemasini tashkil etadi. Aylanuvchi baraban bilan bog'langan Oxy koordinata o'qlari sistemasi qo'zg'aluvchan sanoq sistemasi sifatida qaraladi (3.11 –rasm).



3.11 –rasm.

Faraz qilaylik t vaqt onida o'zi yozar apparat perosi M holatda bo'lsin. Pero M holatining koordinatalari quyidagicha aniqlanadi:

$$x = vt = \omega_0 rt$$

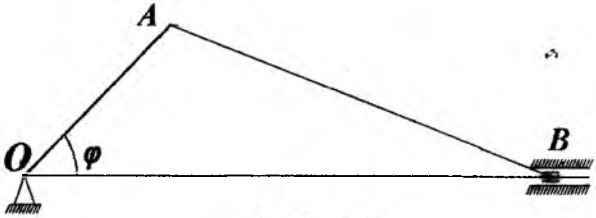
$$y = a \sin\omega_1 t$$

Yozilgan tenglamalar o'ziyozar apparat perosi nisbiy harakatining parametrik tenglamalarini ifodalaydi. Pero nisbiy harakatining koordinatalar ko'rinishidagi tenglamasini yozish uchun yuqoridagi tenglamalardan parametr – vaqtni qisqartiramiz. Natijada pero nisbiy harakatining quyidagi ko'rinishdagi tenglamasiga ega bo'lamiz:

$$t = \frac{x}{\omega_0 r}; \quad y = a\sin\frac{\omega_1 x}{\omega_0 r}.$$

2-masala. Krivoship shatunli mexanizmda uzunligi $OA=r$ bo'lgan privoship O nuqtadan chizma tekisligiga perpendikulyar holda o'tuvchi o'q atrofida o'zgarmas ω_0 burchak tezlik bilan aylanadi, bunda $\varphi=\omega_0 t$.

Shatun uzunligi $AB=l$, B polzun O nuqtadan o'tuvchi gorizontal chiziq bo'ylab harakatlanadi. B polzunni moddiy nuqta sifatida qarab, uning nisbiy harakati tenglamasi tuzilsin (3.12 – rasm).



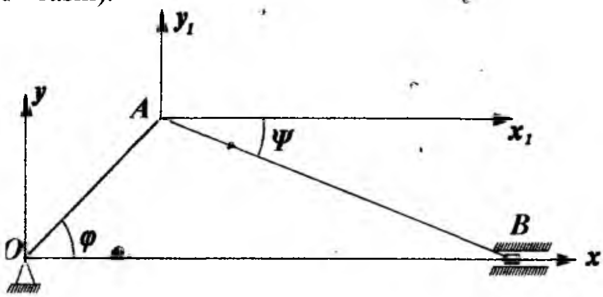
3.12 - rasm

Yechimi. AB shatun murakkab harakatini ikki soda harakatlarga ajratamiz:

a) O nuqta atrofida ω_0 burchak tezlik bilan yuz beruvchi ko'chirma harakat

b) A nuqta atrofida notekis yuz beruvchi aylanma harakat – nisbiy harakat.

Qo'zg'almas sanoq sistemasi sifatida Yer bilan bog'langan o nuqtadan o'tuvchi *Oxy* koordinata o'qlari sistemasini tanlaymiz. Qo'zg'luvchan sanoq sistemasi sifatida Krivoship va shatun birlashadigan A nuqtadan o'tuvchi Ax_ly_l koordinata o'qlari sistemasi olinadi (3.13 –rasm).



3.13 - rasm

AB shatunning mazkur sanoq sistemasidagi holati

$\Psi = \angle x_1AB = \angle ABO$

Burchak orqali aniqlanadi. Ψ burchak qiymati sinuslar teoremasi orqali aniqlanadi:

$$\frac{r}{\sin\Psi} = \frac{l}{\sin\varphi},$$

bundan

$$\sin\Psi = \frac{r\sin\varphi}{l} = \frac{r\sin\omega_0 t}{l};$$

yoki

$$\Psi = \arcsin\left(\frac{r}{l}\sin\omega_0 t\right)$$

Hosil bo'lgan tenglama AB shatun nisbiy harakatining tenglamasini ifodalaydi.

B polzun nisbiy harakati tenglamalarini tuzish uchun uning nisbiy koordinatalari $x_l y_l$ larni yuqorida aniqlangan Ψ burchak qiymati orqali ifodalashimiz lozim.

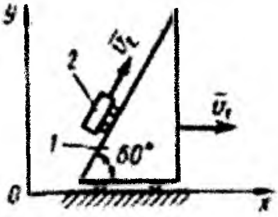
$$x_1 = l\cos\Psi = l\sqrt{1 - \frac{r^2}{l^2}\sin^2\omega_0 t}$$

$$y_1 = l\sin\Psi = -r\sin\varphi = -r\sin\omega_0 t\ (m).$$

37-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

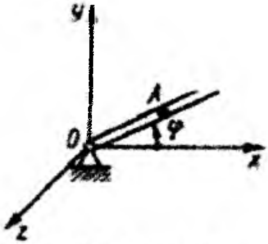
1-muammo. Platforma gorizontal yo'l bo'ylab 1m/s tezlik bilan tekis harakatlanadi. Platforma ichidagi moddiy nuqta ham shu yo'nalish bo'yicha unga nisbatan s=0,5t qonun asosida siljisa, boshlang'ich paytda t=0 va $x=0$ deb, t=4s paytdagi nuqtaning x koordinatasini hisoblang.

2-muammo. 1 jism o'zgarmas ϑ_1=2 m/s tezlik bilan gorizontal tekislik bo'ylab harakat qiladi. Uning ustida esa 2 jism o'zgarmas ϑ_2=2 m/s tezlik bilan yuqoriga ko'tarilmoqda. Agar boshlang'ich paytda, t=0 s. da x_2=0 bo'lsa, t=0,5s paytdagi 2 jismning x_2 koordinatasini aniqlang. 2 jism moddiy nuqta deb qaralsin (3.14-rasm).



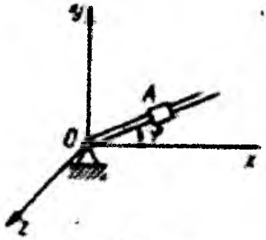
3.14 -rasm

3-muammo. *Oz* o'qi atrofida φ=4t qonun bo'yicha aylanayotgan naycha ichidagi A sharcha $OA=5t^2$ tenglama asosida harakat qilsa, $t=0,25$s paytdagi A nuqtaning x_A koordinatasini toping (3.15 - rasm)



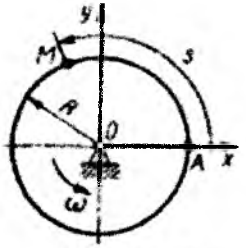
3.15 - rasm

4-muammo. *Oz* o'qi atrofida φ=2t qonun bo'yicha aylanayotgan sterjen bo'ylab A polzun $OA=3t^2$ tenglama asosida harakat qilsa, polzunning o'lchamlarini hisobga olmay, t=0,5s paytdagi uning y_A koordinatasini hisoblang (3.16-rasm).



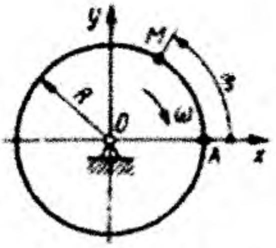
3.16-rasm

5-muammo. Radiusi $R=0,5m$ bo'lgan disk o'zgarmas burchak tezlik ω=2rad/s bilan aylanadi. Diskning gardishi esa $s=2t^2$ tenglama asosida *M* nuqta *Ox* o'qida bo'lgan bo'lsa, t=0,5s paytdagi nuqtaning yoy koordinatasi s ni aniqlang. (3.17-rasm)



3.17 - rasm

6-muammo. Radiusi $R=0,5m$ bo'lgan disk o'zgarmas burchak tezlik ω=2rad/s bilan aylanadi. *M* nuqta esa diskning gardishi bo'ylab $s=2t^2$ qonun asosida harakat qiladi. Agar boshlang'ich paytda *M* nuqta *Ox* o'qida bo'lgan bo'lsa, $t=1s$ paytda nuqtaning yoy koordinatasi s ni aniqlang (3.18 - rasm).



3.18-rasm

38-§. Nuqtaning nisbiy, ko'chirma va absalyut tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning murakkab harakatida absalyut tezligini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1) Masala shartiga ko'ra nuqtaning nisbiy, ko'chirma va absalyut harakatlari aniqlanadi.

2) Qo'zg'almas va qo'zg'aluvchan sanoq sistemalari tanlab olinadi.

3) Ko'chirma harakat xayolan to'xtatiladi va nuqtaning nisbiy harakat tezligi aniqlanadi.

4) Nisbiy harakat xayolan to'xtatiladi va nuqtaning ko'chirma tezligi aniqlanadi.

5) Murakkab harakatda nuqtaning tezliklarini qo'shish haqidagi teoremadan foydalanib, nuqtaning absalyut tezligi aniqlanadi.

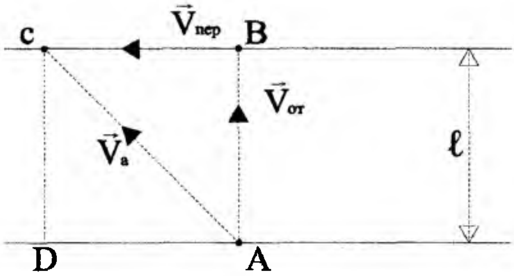
39-§. Murakkab harakatda nuqtaning nisbiy, ko'chirma va absalyut tezligini aniqlashga doir masalalar

1-masala. Daryo qirg'oqlari parallel; qayiq A nuqtadan chiqib, qirg'oqlarga tik kurs oldi va jo'naganidan 10 minut keyin narigi qirg'oqqa borib yetdi. Bunda u, A nuqtadan daryoning oqimi bo'ylab hisoblaganda 120 m pastdagi C nuqtaga keldi. A nuqtadan chiqib, qirg'oqqa tik bo'lgan AB to'g'ri chiziqqa nisbatan qandaydir burchak ostida va oqimga qarshi kurs olishi kerak; bu holda qayiq narigi qirg'oqqa, 12,5 minutda yetadi. Daryo kengligi l, qyiqning suvga nisbatan nisbiy tezligi u va daryo oqimining tezligi v aniqlansin.

Yechimi. Masalada qayiqning daryo oqimiga nisbatan harakati nisbiy harakat deyiladi. Daryoning qirg'oqqa nisbatan harakati (qirg'oq qo'zg'almas sanaladi) ko'chirma harakat sifatida qaraladi.

Qayiqning qirg'oqqa nisbatan harakati absalyut harakat hisoblanadi.

a) Qayiqning A nuqtadan qirg'oqqa perpendikulyar yo'nalishdagi harakatini o'rganamiz. Bunda qayiq qarama – qarshi qirg'oqning C nuqtasiga borib yetadi.(3.19- rasm).



3.19-rasm

Bunday harakat $t_1 = 10$ minutda amalga oshadi.

Masalada qayiqning daryo oqimiga nisbatan harakatidagi tezligi nisbiy tezlik hisoblanadi va u $\vec{\vartheta}_n$ vektor orqali belgilanadi. Daryoning qayiq turgan nuqtasining qirg‘oqqa nisbatan oqimi tezligi hisoblanadi va u $\vec{\vartheta}_k$ vektor orqali belgilanadi. 3.19- rasmdan

$$V = V_{пер} = \frac{AD}{t_1} = \frac{120}{10} = 12\ m/min.$$

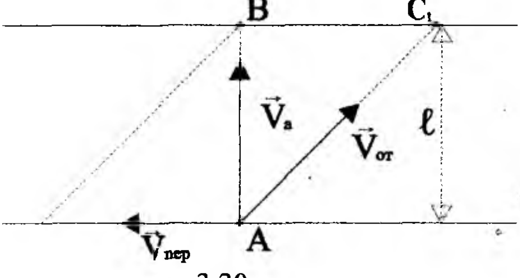
Bunda daryo kengligi quyidagi formula asosida aniqlanadi:

$$l = \vec{\vartheta}_n \cdot t_1,$$

bundan,

$$\vec{\vartheta}_n = \frac{l}{t_1}, \qquad (1)$$

b) Qayiqning AB tog‘ri chiziqqa ma’lum burchak ostida daryo oqimiga qarshi yo‘nalishdagi harakatini o‘rganamiz.



3.20- rasm

Bunda qayiq qarama – qarshi qirg‘oq $t_2=12,5$ minut vaqt o‘tgach yetadi 3.20- rasmdan:

$$l=\vartheta a * t_2.$$

Bundan

$$\vartheta_a = \frac{l}{t_2}. \qquad (2)$$

ΔABC_1dan

$$\vartheta_n^2 = \vartheta_a^2 + \vartheta_k^2. \qquad (3)$$

(1) va (2) ifodalarni (3) ifodaga qo'ysak:

$$\frac{l^2}{t_1^2} = \frac{l^2}{t_2^2} + \vartheta_k^2,$$

yoki

$$\frac{l^2}{10^2} = \frac{l^2}{(12{,}5)^2} + 144$$

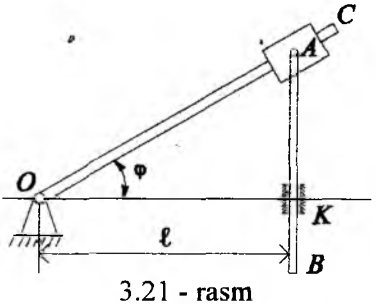
Hosil bo'lgan ifodadan daryo kengligi aniqlanadi:

$$l = \sqrt{\frac{144}{0{,}0036}} = 200m.$$

Bunday holda qayiqning daryo oqimiga nisbatan tezligi quyidagicha aniqlanadi:

$$\vartheta_n = \frac{l}{t_1} = \frac{200}{10} = 20\,m/s.$$

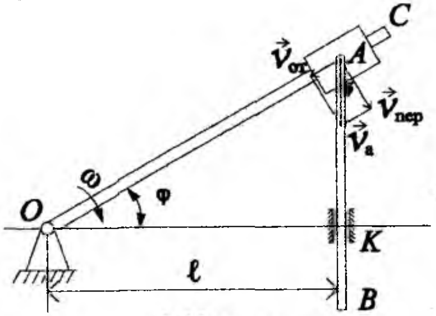
2-masala. Kulisali mexanizmda OC krivoshipning rasm tekisligiga perpendikulyar bo'lgan O o'q atrofida tebranishi natijasida, A polzun OC krivoship bo'ylab surilib, vertikal k yo'naltiruvchilarda harakatlanuvchi AB sterjenni harakatga keltiradi. Masofa OK=l A polzunning OC krivoshipga nisbatan harakatidagi tezligi krivoshipning burchak tezligi ω va aylanish burchagi φ funksiayasi sifatida aniqlansin.



3.21 - rasm

Yechilishi. Masalada A polzun uchun, OC krivoshipning chizma tekisligiga perpendikulyar holda O nuqtadan o'tuvchi o'q atrofidagi tebranishi, ko'chirma harakat hisoblanadi. Polzunning OC krivoshipga nisbatan harakati esa nisbiy harakat deb qaraladi.

OC krivoshipning qaralayotga vaqt momentida A polzun bilan ustma ust tushuvchi nuqtaning tezligi A polzun uchun ko'chirma tezlik hisoblanadi.



3.22-rasm

Shuning uchun

$$\vartheta_k = \omega * OA$$

$\vec{\vartheta}_k$ vektor OC krivoshipga perpendikulyar holda krivoshipning A nuqtasidan uning aylanishi tomon yo'naladi.

(3.22- rasm) dan

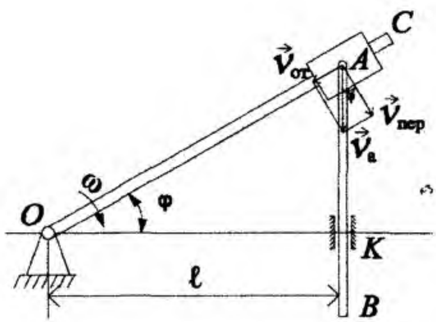
$$OA = l/\cos\varphi$$

Shuning uchun

$$V_{пер} = \frac{\omega l}{\cos\varphi}, \ (1)$$

A polzunning nisbiy tezligi $\vec{\vartheta}_n$ OC krivoship bo'ylab yo'naladi.

Mexanizmning A nuqtasida tezliklar parallelogramni chizamiz



3.23 - rasm

Tezliklar parallelogramidan

$$V_{от}=V_{пер}*tg\varphi \qquad (2)$$

(1) ni (2) ga qo'ysak, A polzunning nisbiy tezligi uchun quyidagi ifodaga ega bo'lamiz.

$$(2) \quad V_{от}=\frac{\omega l}{cos\varphi}\cdot tg\varphi.$$

3-masala. Gorizontal yo'lda 72 *km/soat* tezlik bilan borayotgan avtomobildagi passajir kabinaning yon oynasiga tushgan yomg'ir tomchisining vertikalga nisbatan 40^0 ga teng burchakka og'gan traektoriyasini kuzatadi. Vertikal tushayotgan yomg'ir tomchisining absalyut tezligini aniqlansin. Tomchi bilan oyna orasidagi ishqalanish hisobga olinmasin.

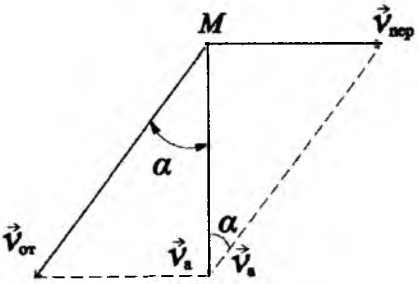
Yechilishi: Avtomobilning gorizontal yo'ldagi harakati pasajjir uchun ko'chirma harakat hisoblanadi.

Yomg'ir tomchisining avtomobil oynasida vertikalga nisbatan 40^0 burchakka og'gan traektoriyasi nisbiy harakatni ifodalaydi. Vertikal tushayotgan yomg'ir tomchisining harakati absalyut harakat hisoblanadi.

Murkkab harakatda tezliklarni qo'shish teoremasiga asosan yomg'ir tomchisining absalyut tezligi uning nisbiy va ko'chirma tezliklarining geometrik yig'indisiga teng bo'ladi:

$$\vec{\vartheta}_a=\vec{\vartheta}_n+\vec{\vartheta}_k$$

Ko'chirma, nisbiy va absalyut tezliklarning yo'nalishini bilgan holda tezliklar paralelogramini chizamiz (3.24-rasm).



3.24-rasm

Chizilgan paralelogramdan

$$\frac{\vartheta_k}{\vartheta_a} = \tan 40^0.$$

Agar

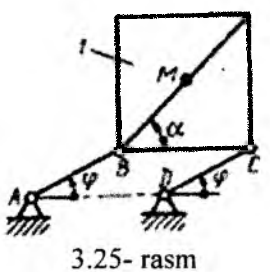
$$\vartheta_k = 72\frac{km}{s} = 20m/s$$

Ekanligini etiborga olsak

$$\vartheta_a = \frac{\vartheta_k}{\tan 40^0} = \frac{20m/s}{0.839} = 23{,}8\,m/s.$$

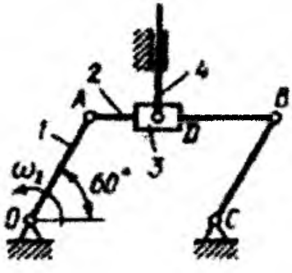
40-§. Murakkab harakatda nuqtaning nisbiy, ko'chirma va absalyut tezligini aniqlashga doir mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. O'zaro teng krivoshiplar AB=CD=0,5m, φ=0,25πt qonun bo'yicha aylanadi. Krivoshiplarga o'rnatilgan kvadrat plastina diagonali bo'ylab harakatlanayotgan M nuqtaning tenglamasi BM=0,1 t^2 bo'lsa, t=1s paytdagi M nuqtaning absolyut tezligini aniqlang (3.25- rasm).



3.25- rasm

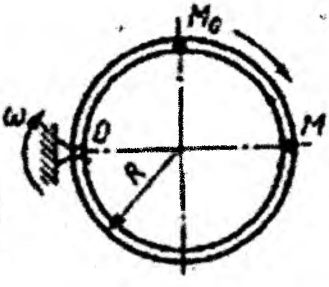
2-muammo. OABC sharnirli paralelogramning 2 shatuni bo'ylab 3 haqasimon polzun (vtulka) harakat qiladi. O'z o'rnida 3 polzun 4 sterjenni harakatga keltiradi. Mexanizmning berilgan holati uchun 1 krivoship A nuqtasining tezligini 2m/s deb olib, 4 sterjenning tezligini toping (3.26 – rasm).



3.26 – rasm

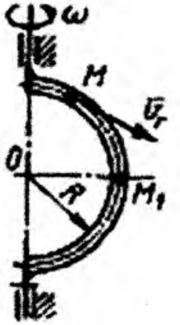
3.27

3-muammo. Radiusi R=0,1m bo'lgan halqa shakl tekisligida O nuqta atrofida o'zgarmas ω=4rad/s burchak tezlik bilan aylanadi. Halqadagi M shar esa M_0M=0,1t qonun bo'yicha nisbiy harakat qilsa, ko'rsatilgan holat uchun M sharning absalyut tezligini toping (3.27 – rasm).



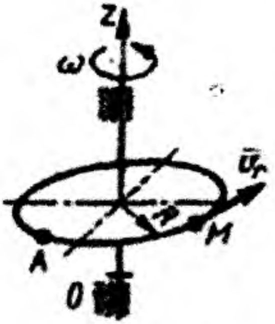
3.27 – rasm

4-muammo. Radiusi R=1m bo'lgan yarim doira shaklidagi naycha ω=3rad/s burchak tezlik bilan aylanadi. Naycha ichidagi M sharcha o'zgarmas nisbiy tezlik υ_r=3m/s bilan harakatlansa, M sharchaning M_1 holatga kelgan paytdagi absalyut tezligini aniqlang (3.28 – rasm).



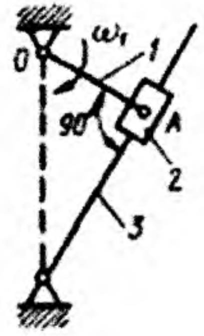
3.28 - rasm

5-muammo. Radiusi R=1m bo'lgan disk *Oz* o'qi atrofida φ=4sin3t qonun bilan aylanadi. M nuqta esa diskning gardishi bo'ylab AM=0,66sin6t+4 tenglama bo'yicha harakatlanadi. Vaqtning t=0,35s paytda M nuqtaning absalyut tezligini toping (3.29- rasm).



3.29 – rasm

OA=0,1m bo'lgan 1 krivoshipning O o'q atrofida ω_1=5rad/s burchak tezlik bilan aylanadi. Shaklda ko'rsatilgan holat uchun 2 polzunning 3 kulisaga nisbatan tezligini aniqlang (3.30 – rasm).



3.30 – rasm

41-§. Murakkab harakatda ko'chirma harakat ilgarinma harakat bo'lgan hol uchun nuqtaning absalyut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Murakkab harakatda ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa, nuqtaning absalyut tezlanishi nisbiy va ko'chirma tezlanishlarining geometrik yig'indisidan iborat bo'ladi.

$$\vec{a}_a = \vec{a}_n + \vec{a}_k \qquad (1)$$

yoki

$$\vec{a}_a = \vec{a}_n^{mi} + a_n^{ayl} + \vec{a}_k^n + \vec{a}_k^\tau. \qquad (2)$$

Bu ifodada:

$\vec{a}_n^{mi}$ *va* $\vec{a}_n^{ayl}$ – nuqtaning nisbiy harakatida markazga intilma va aylanma tezlanishlar.

$\vec{a}_k^k$ *va* $\vec{a}_k^\tau$ – ko'chirma harakatda nuqtaning normal va urinma tezlanshlari.

Agar murakkab harakatda nuqtaning nisbiy va ko'chirma harakatlari to'g'ri chiziqli harakatlardan iborat bo'lsa, nuqtaning nisbiy markazga intilma va ko'chirma normal tezlanishlar nolga teng bo'ladi.

Agar murakkab harakatda nuqtaning nisbiy va ko'chirma harakatlari egri chiziqli tekis harakatlardan iborat bo'lsa, nuqtaning nisbiy aylanma va ko'chirma urinma tezlanishlari nolga teng bo'ladi.

Mavzuga doir masalalarni ikki usulda yechish tavsiya etiladi: geometrik va analitik usullar.

Masalalarni geometrik usulda yechishda tanlangan masshtabda tezlanishlar parallelogram yoki ko'p burchagi chiziladi.

Masalalarni analitik usulda yechishda proeksiyalar metodidan foydalanish tavsiya etiladi. Buning uchun koordinata o'qlari o'tkaziladi va (2) tenglamani chap va o'ng tomonlari tanlab olingan koordinata o'qlariga proeksiyalanadi:

$$(a_a)_x = \left(a_n^{mi}\right)_x + \left(a_n^{ayl}\right)_x + (a_k^n)_x + (a_k^\tau)_x,$$

$$(a_a)_y = \left(a_n^{mi}\right)_y + \left(a_n^{ayl}\right)_y + (a_k^n)_y + (a_k^\tau)_y,$$

$$(a_a)_z = \left(a_n^{mi}\right)_z + \left(a_n^{ayl}\right)_z + (a_k^n)_z + (a_k^\tau)_z.$$

Bunda absolyut tezlanishning moduli

$$a_a = \sqrt{(a_a)_x^2 + (a_a)_y^2 + (a_a)_z^2}$$

formula yordamida, yo'nalishi esa

$$\cos(\vec{a}_a \wedge x) = \frac{(a_a)_x}{a_a}$$

$$\cos(\vec{a}_a \wedge y) = \frac{(a_a)_y}{a_a}$$

$$\cos(\vec{a}_a \wedge z) = \frac{(a_a)_z}{a_a}.$$

formulalar asosida aniqlanadi.

Mavzuga doir masalalarni quyidagi tartibda yechish maqsadga muvofiq bo'ladi.

1) Masala shartidan nuqtaning nisbiy, ko'chirma va absolyut harakatlari aniqlab olinadi;

2) Qo'zg'almas va qo'zg'aluvchan koordinata o'qlari sistemasi tanlab olinadi;

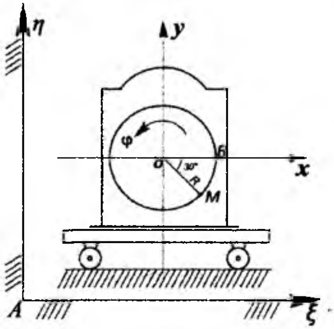
3) Ko'chirma harakat hayolan to'xtatilib, nuqtaning nisbiy tezligi va nisbiy tezlanishi aniqlab olinadi;

4) Nisbiy harakat xayolan to'xtatilib, nuqtaning ko'chirma harakat tezligi va tezlanishi aniqlab olinadi;

5) Masalani geometrik usulda yechishda tezlanishlar parallelogram yoki ko'p burchagi chiziladi va ulardan noma'lum tezlanish aniqlanadi.

6) Masalani analitik usulda yechishda proeksiyalar usulidan foydalanish tavsiya etiladi yani absalyut tezlanishning o'qlardagi proeksiyalari aniqlanadi

7) Absolyut tezlanishning o'qlardagi proeksiyalariga ko'ra uning moduli va yo'nalishi topiladi.



3.31-rasm

42-§. Ko'chirma harakat ilgarilanma harakatdan iborat bo'lganda nuqtaning absolyut tezlanishini aniqlashga doir masalalar

1-masala.

O'ng tomonga gorizontal yo'nalishda $x_k=t^3+4t$ m. qonunga muvofiq harakat qiluvchi aravachaga elektr motori o'rnatilgan. Uning rotori harakatga keltirish vaqtida $\varphi_n=t^2$ tenglamaga muvofiq aylanadi, bunda φ_n burchak radianlarda o'lchanadi. Rotor gardishidagi M nuqtaning t=1 bo'lgandagi absolyut tezligi va absolyut tezlanishi aniqlansin. Rotorning radiusi 0,2m.ga teng. Shu paytda M nuqta rasmda ko'rsatilgan holda turadi (3.31-rasm).

Masalada:

$X_k=t^3+4t$ m.

$\Phi_n=t^2$ rad, R=0.2m, t=1s

Yechimi:

Rasmda ko‘rsatilgan $A\xi\eta$ o‘qlar sistemasi qo‘zg‘almas sanoq sistemasini, aravacha bilan bog‘langan va u bilan birga harakatlanuvchi Oxy o‘qlar sistemasi qo‘zg‘aluvchan sanoq sistemasini tashkil etadi.

Rotor gardishidagi M nuqtaning motor korpusi-aravachaga bog‘langan Oxy sanoq sistemasiga nisbatan harakati nisbiy, rotorning qo‘zg‘aluvchan O, x, y, z sanoq sistemasi bilan birgalikda qo‘zg‘almas $A\xi\eta$ sanoq sistemasiga nisbatan harakati M nuqta uchun ko‘chirma va M nuqtaning bevosita qo‘zg‘almas $A\xi\eta$ sanoq sistemasiga nisbatan harakati murakkab harakat hisoblanadi.

M nuqtaning absolyut tezligini nuqtaning murakkab harakatida tezliklarni qo‘shish teoremasiga asosan aniqlaymiz.

Teoremaga ko‘ra:

$$\vec{\vartheta}_M = \vec{\vartheta}_n + \vec{\vartheta}_k. \quad (1.1)$$

Nisbiy tezlikning moduli

$$\vartheta_n = R*\omega_n \quad (1.2)$$

bu yerda R – rotorning radiusi,

ω_n– rotor burchak tezligining moduli

$$\omega_n = [\tilde{\omega}_n], \tilde{\omega}_n = \frac{d\varphi_n}{dt} = 2t.$$

t=1 sekundda

$$\tilde{\omega}_n = 2rad/s, \qquad \omega_n = 2\, rad/s.$$

$\tilde{\omega}_n$kattalikning oldidagi musbat ishora rotorning aylanishi φ_n burchakning o‘sish tomoniga qarab ro‘y berishini ko‘rsatadi.

Nisbiy tezlikning moduli (1.2) formula asosida aniqlanadi:

$$\vartheta_n = 0.2*2 = 0.4 m/s.$$

$\vec{\vartheta}_n$ vektor, M nuqta nisbiy harakatda chizgan aylanaga urinma bo‘ylab, rotorning aylanish tomoniga qarab yo‘naladi (3.31a-rasm).

M nuqtaning ko‘chirma tezligi qaralayotgan vaqt momentida motor korpusi-aravachaning M nuqta bilan ustma – ust tushuvchi nuqtasining tezligiga teng bo‘ladi:

$$v_k = |\tilde{x}'_k| = |3t^2 + 4| \quad (1.3)$$

t=1 sekundda

$$\tilde{x}'_k = 7sm/s, \quad x'_k = 7sm/s.$$

Demak, ϑ_k=7m/s

$\vec{\vartheta}_k$ vektor, $\dot{x}'_k$ kattalik oldidagi ishora musbat bo'lganligi uchun, x ning o'sish tomoniga, ya'ni aravachaning harakat yo'nalishi tomon yo'naladi (3.31a-rasm).

M nuqtaning absolyut tezligi uning nisbiy va ko'chirma harakat tezliklaridan ko'rilgan parallelogramning diagonali orqali ifodalan adi.

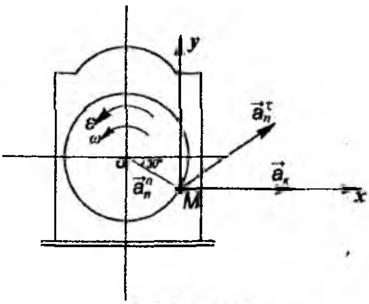
Uning moduli:

$$\vartheta_m = \sqrt{\vartheta_n^2 + \vartheta_k^2 + 2\vartheta_n\vartheta_k\cos60^0} = 7{,}21\ sm/s.$$

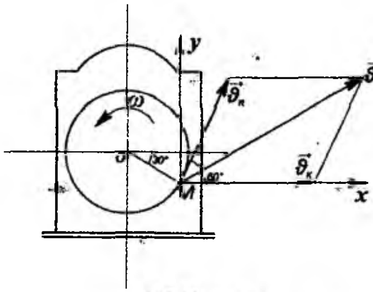
M nuqtaning absolyut tezlanishini nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasidan aniqlaymiz. Ko'chirma harakat ilgarilanma harakat bo'lganligi uchun

$$\vec{a}_M = \vec{a}_n + \vec{a}_k, \qquad (1.4)$$

yoki, yoyilgan ko'rinishda



3.31a-rasm



3.31b-rasm

$$\vec{a}_M = \vec{a}_n^\tau + \vec{a}_n^n + \vec{a}_k. \qquad (1.5)$$

Nisbiy urinma tezlanishning moduli:

$$a_n^\tau = R\varepsilon_n, \qquad (1.6)$$

bu yerda $\varepsilon_n = |\vec{\varepsilon}_n|$ – rotor burchaktezlanishining moduli.

$$\vec{\varepsilon}_n = \frac{d^2\varphi_n}{dt^2} = 2\frac{rad}{s^2}, \quad \varepsilon_n = \frac{2rad}{s^2}.$$

$\vec{\varepsilon}_n$ va $\vec{\omega}_n$ larning ishoralari bir xil. Demak, $\vec{a}_n^\tau$ va $\vec{\vartheta}_n$ vektorlar bir xil yo'nalishga ega bo'ladi (3.31a,b-rasmlar).(3.22)ga asosan

$$a_n^{\tau} = 0{,}2 \cdot 2 = 0{,}4\ sm/s^2.$$

Nisbiy normal tezlanishning moduli:

$$a_n^n = R\omega_n^2 = 0{,}2 \cdot 4 = 0{,}8\ sm/s^2.$$

$\vec{a}_n^n$ vektor rotor M nuqtasining nisbiy harakatda chizgan aylanasining markazi O nuqta tomon yo'naladi (3.31b-rasm).

M nuqtaning ko'chirma tezlanishi qaralayotgan vaqt momentida motor korpusi – aravachaning M nuqtasi bilan ustma – ust tushuvchi nuqtasining tezlanishiga teng bo'ladi:

$$a_k = |\ddot{x}_k''| = |6t|.$$

t=1 sekundda

$$\ddot{x}_k'' = 6sm/s^2, \quad x'' = 6sm/s^2.$$

Demak, a_k=6sm/s^2

$\dot{x}'$ va $\ddot{x}''$ kattaliklarning ishoralari bir xil bo'lganligi uchun $\vec{\vartheta}_k$ va $\vec{a}_k$ vektorlarning yo'nalishlari ustma – ust tushadi (3.31a,b-rasmlar).

M nuqta absolyut tezlanishining modulini proeksiyalash usuli yordamida topamiz:

$$a_{Mx} = a_k - a_n^n cos30^0 + a_n^{\tau} cos60^0 = 5{,}52\ sm/s^2,$$

$$a_{My} = a_n^n cos60^0 + a_n^{\tau} cos30^0 = 0{,}74\ sm/s^2,$$

$$a_M = \sqrt{a_{Mx}^2 + a_{My}^2} = 5{,}6\ sm/s^2.$$

Hisob natijalari jadvalda ko'rsatilgan.

$\tilde{\omega}_n$, rad/s.	Tezlik, sm/s.			$\tilde{\varepsilon}$, rad/s^2.	Tezlanish, sm/s^2.					
	ϑ_n	ϑ_k	ϑ_M		a_n^{τ}	a_n^n	a_k	a_{Mx}	a_{My}	a_M
2	0,4	7	7,21	2	0,4	0,8	6	5,52	0,74	5,6

2-masala. *M* nuqta *D* jismga nisbatan OM=S_n=$6\pi t^2$ tenglama bo'yicha harakatlanadi. *D* jism O_1OAO_2 sharnirli to'rt zvenolikga mahkamlangan. To'rt zvenolikning O_1O va O_2A sterjenlari O_1 va O_2 nuqtalar atrofida $\varphi = \frac{\pi t^3}{6}$ qonunga muvofiq aylanadi. *M* nuqtaning $t=t_1$ vaqt onidagi absolyut tezligi va absolyut tezlanishi aniqlansin (3.32a-rasm).

Masalada:

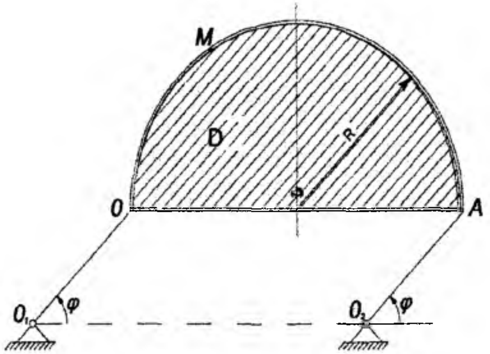
$OM = s_n = 6\pi t^2$

$\varphi = \frac{\pi t^3}{6}$(rad),

$t_1 = 1s$

$R = 18sm$

$o_1o = o_2A = 20sm.$



3.32a-rasm

Yechimi: Masalada to'rt zvenolikning O_1O va O_2A sterjenlari O_1 va O_2 sharnirlar atrofida aylanadi, OA sterjen esa ilgarilanma harakatda bo'ladi. Yarim doira ham OA sterjenga mahkamlanganligi tufayli ilgarilanma harakatda bo'ladi. M nuqta uchun D yarim doiraning harakati ko'chirma harakat hisoblanadi. Shuning uchun masalada ko'chirma harakat ilgarilanma harakat bo'ladi. M nuqtaning D jismga nisbatan harakati esa nisbiy harakat hisoblanadi.

Berilgan vaqt momentida M nuqtaning D jismdagi o'rni $\alpha = \frac{s_n}{R}$ burchak orqali aniqlanadi:

$t_1 = 1s.$ da

$$\alpha = \frac{6\pi t_1^2}{18} = \frac{\pi}{3} = 60^0.$$

D jismning tekislikdagi holati φ burchak orqali aniqlanadi:

$t_1 = 1s.$ da

$$\varphi = \frac{\pi t_1^3}{6} = \frac{\pi}{6} = 30^0.$$

Nuqtaning murakkab harakatida tezliklarni qo'shish haqidagi teoremaga asosan M nuqtaning absolyut tezligi uning nisbiy va ko'chirma harakat tezliklarining geometrik yig'indisiga teng bo'ladi:

$$\vec{\vartheta}_a = \vec{\vartheta}_n + \vec{\vartheta}_k.$$

Nisbiy tezlikning miqdorini aniqlaymiz:

$$\vartheta_n = s_n = (6\pi t^2) = 12\pi t$$

$t_1 = 1s$. da

$$\vartheta_n = 12*\pi*1 = 37.68 sm/s$$

Ilgarilanma harakatdagi jismning barcha nuqtalari bir xil traektoriya bo‘ylab harakatlanadi va har onda miqdor va yo‘nalishlari bir xil bo‘lgan tezlik va tezlanishga ega bo‘ladi. Shuning uchun *M* nuqtaning ko‘chirma tezligi *O* nuqtaning ko‘chirma tezligiga teng bo‘ladi:

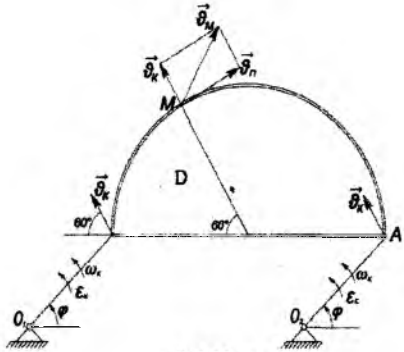
$$\vartheta_k = \vartheta_0 = \omega * 0_1 0_1$$

Bunda:

$$\omega_K = \varphi' = \left(\frac{\pi t^3}{6}\right)' = \frac{\pi t^2}{2};$$

$t_1 = 1s$ da

$\omega_k 1.57$ rad/s



3.32b-rasm

Binobarin,

$$\vartheta_k = \omega * 0_1 0 = 1.57*20 = 3{,}14 rad/s$$

$\vartheta_n = \vartheta_k$ vektorlar o‘zaro perpendikulyar yo‘nalgan (3.32b-rasm). Shuning uchun *M* nuqta absolyut tezligining miqdori quyidagicha aniqlanadi:

$$\vartheta_M = \sqrt{\vartheta_r^2 + \vartheta_K^2} = \sqrt{(37{,}68)^2 + (3{,}14)^2} = 49{,}05\, sm/s.$$

M nuqtaning absolyut tezlanishini aniqlaymiz. Ko‘chirma harakat ilgarilanma harakat bo‘lganligi uchun kariolis tezlanishi $\vec{a}_c = 2(\vec{\omega}_k \times \vec{\vartheta}_n) = 0$

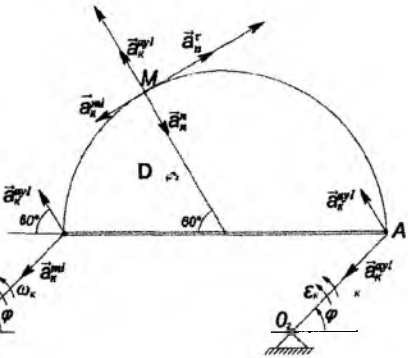
Shuning uchun

$$\vec{a}_2 = \vec{a}_n + \vec{a}_k = \vec{a}_n^n + \vec{a}_n^\tau + \vec{a}_k^{mi}$$

M nuqta nisbiy tezlanishlarining miqdorlari:

$$a_n^n = \frac{\vartheta_n^2}{R} = \frac{(37,68)^2}{18} = 78,88\,sm/s^2,$$

$$a_n^\tau = \frac{d\vartheta_n}{dt} = 12 = 37,68\,sm/s^2.$$



3.32v -rasm

M nuqtaning ko'chirma tezlanishlarining miqdorlari:

$$a_k^{mi} = \omega^2 \cdot O_1O = (1,57)^2 \cdot 20 = 49,30\,sm/s^2;$$

$$a_k^{ayl} = \varepsilon \cdot O_1O.$$

Bunda: $$\varepsilon = \varepsilon_k = \frac{d\omega_k}{dt}.$$

t = 1 s. da , $$\varepsilon_k = \frac{d\omega_k}{dt} = \pi t = 3,14\,rad/s^2.$$

Shuning uchun,

$$a_k^{ayl} = 3,14 \cdot 20 = 62,8\,sm/s^2.$$

M nuqtaning nisbiy va ko'chirma tezlanishlari 3.32v-rasmda ko'rsatilgan.*M* nuqtaning absolyut tezlanishining miqdorini proeksiyalash usulidan foydalanib aniqlaymiz

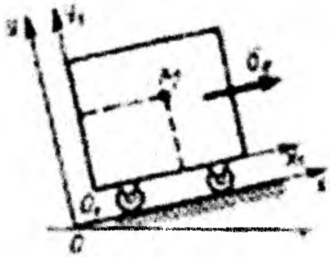
$$(a_M)_x = a_n^\tau - a_k^{mi} = 37,68 - 49,30 = -11,62,$$

$$(a_M)_y = a_n^n - a_k^{ayl} = -78,88 + 62,8 = -16,08,$$

$$a_M = \sqrt{(a_M)_x^2 = (a_M)_y^2} = 19,84\,sm/s^2.$$

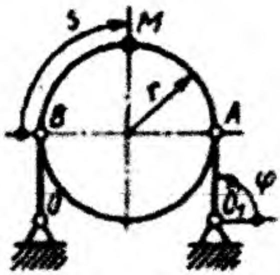
43-§. Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar

1-muammo. Arava qiya tekislikda a_e=2m/s^2 tezlanish bilan harakat qiladi. Aravadagi M nuqta esa shakil tekisligida $x_1=3t^2$ *va* $y_1=4t^2$ tenglamalar bo'yicha harakatlanadi. Nuqtaning absolut tezlanishin toping (3.33-rasm).



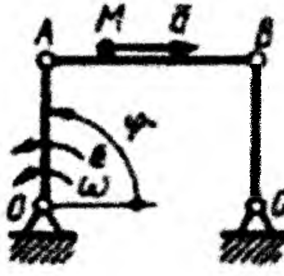
3.33-rasm

2-muammo. O_1A zveno φ=2t qonun bilan aylanib, radiusi r=0,5m li diskni harakatga keltiradi. Diskning gardishi bo‘ylab esa M nuqta s=2rt tenglama asosida aylanadi. Nuqtaning t=0,25π paytdagi absolyut tezlanishi miqdorini toping (3.34-rasm).



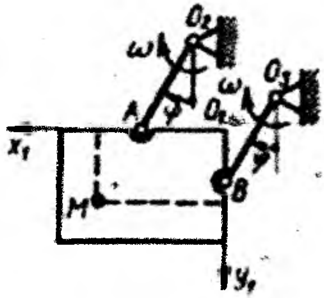
3.34-rasm

3-muammo. Uzunligi OA=0,1m bo‘lgan sterjen ω=4rad/s burchak tezlik va ε=0,4rad/s^2 burchak tezlanish bilan aylanib, $OABO_1$ sharnirli parallelogramni harakatga keltiradi. M nuqta AB sterjen bo‘ylab $a=0,4m/s^2$ tezlanish bilan harakat qiladi. M nuqtaning absolyut tezlanish modulini φ=0,5 holat uchun aniqlang. (3.35-rasm).



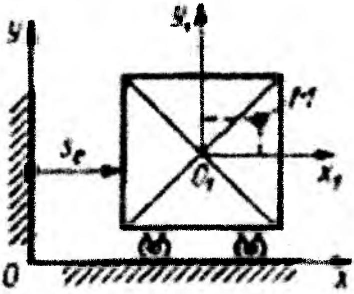
3.35-rasm

4-muammo. To'rtburchak shakildagi plastina uzunliklari $AO_2=BO_3=1$m bo'lgan krivoshiplar yordamida harakatga keltiriladi. M nuqta esa plastina bo'ylab $x_l=0,2t^3$ va $y_l=0,3t^2$ tenglama asosida harakatlanadi. Agar krivoshiplar o'zgarmas $\omega=2\pi$ burchak tezlik bilan aylansa, t=1s paytda $\varphi=30^0$ holat uchun M nuqtaning absolyut tezlanishini hisoblang (3.36-rasm).



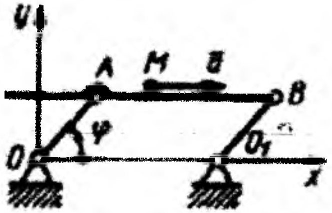
3.36-rasm

5-muammo. Arava gorizontal yo'lda $s_e=0,5t^3$ qonun bilan harakatlanadi. Aravadagi M nuqta esa vertikal shakil tekisligida $x_l=0,3t$ va $y_l=0,1t^2$ tenglamalar asosida harakat qiladi. t=1s paytdagi nuqtaning absolyut tezlanishini toping (3.37-rasm).



3.37-rasm

6-muammo. Uzunligi OA=2m bo'lgan sterjen φ=t qonun bilan aylanib, $OABO_1$ sharnirli parallelogrammni harakatga keltiradi. M nuqta AB sterjen bo'ylab esa $a=\cos t$ tezlanish bilan harakat qilsa, M nuqtaning $t=\pi$ paytdagi absolyut tezlanish miqdorini toping (3.38-rasm).



3.38-rasm

44-§. Koriolis tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Kariolis tezlanishini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi.

1. Masala shartiga ko'ra murakkab harakatdagi nuqtaning nisbiy harakati tenglamasi aniqlanadi.

2. Nuqtaning nisbiy harakat tenglamasiga ko‘ra uning nisbiy tezligining miqdori va yo‘nalishi aniqlanadi.

3. Masala shartiga ko‘ra ko‘chirma harakat tenglamasi aniqlanadi.

4. Ko‘chirma harakat tenglamasiga ko‘ra nuqta ko‘chirma harakatining burchak tezligining miqdori va yo‘nalish aniqlanadi.

5. Nuqtaning aniqlangan nisbiy tezligi va ko‘chirma harakati burchak tezligining miqdori va yo‘nalishiga asosan uning Koriolis tezlanishi aniqlanadi.

45-§. Murakkab harakatda nuqtaning Koriolis tezlanishini aniqlashga doir masalalar

1-masala. Eni 500 m bo‘lgan daryo janubdan shimolga qarab 1,5 m/s tezlik bilan oqadi. 60^0 shimoliy kenglikda suv zarrasining ω_c koriolis tezlanishi aniqlansin. Keyin, suv daryoning qaysi qirg‘og‘ida ekanligi va qancha baland ekanligi aniqlansin; suv sathi, koriolis tezlanishiga teng va unga qarama-qarshi yo‘nalgan vektor bilan og‘irlik kuchining tezlanishi ***g*** vektorning yig‘indisiga teng bo‘lgan vektor yo‘nalishiga perpendikulyar.

Yechilishi: Daryo suv zarrasining koriolis tezlanishi quyidagi formula asosida aniqlanadi:

$$\vec{a}_c = 2(\vec{\omega}_k \text{x}^\wedge \vec{v}_n).$$

Uning moduli esa

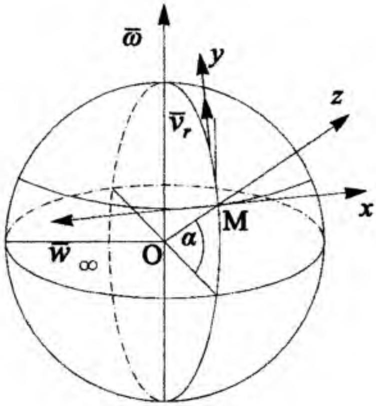
$$a_c = 2\omega_k \vartheta_n \sin(\vec{\omega}_k {}^\wedge \vartheta_n)$$

formula yordamida hisoblanadi.

Agar

$$\vartheta_n = \frac{1.5m}{s}, \quad \vartheta_k = \frac{2\pi}{24\cdot60\cdot60} = 0{,}000073\frac{1}{s}; \quad \sin(\vec{\omega}_k {}^\wedge \vec{\vartheta}_n) = \sin 60^0 = 0{,}87$$

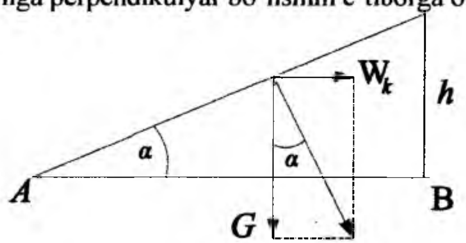
Ekanligini e’tiborga olsak, daryo suv zarrasining Koriolis tezlanishi quyidagi miqdorga teng bo‘ladi.



3.39-rasm

$a_c=2*0.000073*1.5*0.87=1.89*10^{-4}m/s^2$.

Yerning shimoliy yarim sharida, Yer aylanishi tufayli, Yer sirtida harakatlanayotgan har qanday jism o'ng tomonga og'adi. Binobarin, suv daryoning o'ng qirg'og'ida baland bo'ladi. Daryo suvining o'ng qirg'og'i qancha baland bo'lishini aniqlash uchun suv sathi, koriolis tezlanishiga teng va unga qarama –qarshi yo'nalgan vektor bilan og'irlik kuchining tezlanishi $\vec{g}$ vektorning yig'indisiga teng bo'lgan vektor yo'nalishiga perpendikulyar bo'lishini e'tiborga olamiz.



3.40- rasm

3.40-rasmdan h=tanα Ikkinchi tomondan $\tan\alpha = \frac{a_k}{g}$.

Shuning uchun

$$h = 500 \cdot \tan\alpha = 500 \cdot \frac{1{,}89 \cdot 10^{-4}}{9{,}81} = 0{,}0096\, m.$$

2-masala. Meridian bo'yicha harakatlanuvchi elektrovoz ekvatorni kesib o'tayotgan paytda uning g'ildiragidagi M_1, M_2, M_3 *va* M_4 nuqtalarning koriolis tezlanishlari aniqlansin. Elektrovoz g'ildiragi markazining tezligi υ_0=40 m/s.

Yechimi. Elektrovoz g'ildiragi Yerning meridian bo'ylab harakatlanib ekvatorni kesib o'tadi. Elektrovoz g'ildiragining shu holatida M_1 nuqta g'ildirak nuqtalarining Yer sirtidagi nisbiy harakati tezliklarining oniy markazi bo'ladi (3.41- rasm).

Rasimdan $M_1M_2 = M_1M_4 = r\sqrt{2}$ bunda r- elektrovoz g'ildiragining radiusi.

Elektovoz g'ildiragining oniy burchak tezligini aniqlaymiz:

$$\omega = \frac{\vartheta_0}{r} = \frac{\vartheta_2}{r\sqrt{2}} = \frac{\vartheta_4}{r\sqrt{2}}$$

bundan $\vartheta_2 - \vartheta_4 = \vartheta_0\sqrt{2}$

ϑ_2 va ϑ_4 – lar g'ildirak 2va 4 nuqtalarining nisbiy tezliklari Yerning aylanishi elektrovoz uchun ko'chirma harakat hisoblanadi.

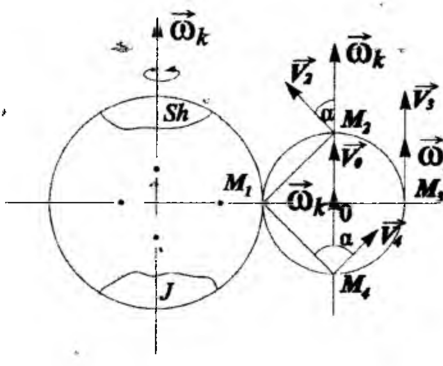
Uning burchak tezligi

$$\omega_k = \frac{2\pi}{24\cdot 60\cdot 60} = 0{,}000073\frac{1}{c}.$$

Ma'lumki, elektrovoz g'ildiragi nuqtalarining koriolis tezlanishi quyidagi formula asosida aniqlanadi.

$a_c=2*\omega_k*\vartheta_n\sin\alpha$.

Bu ifodada $\alpha - \vec{\omega}_k$ va $\vec{\vartheta}_n$ vektorlar orasidagi burchak.



Elektrovoz g'ildiragi nuqtalarning koriolis tezlanishlarini aniqlaymiz:

$a_{c1}=2*\omega_k*\vartheta_1\sin\alpha=0$ chunki $\vartheta_1=0$

$$a_{\vartheta_2} = 2\cdot\omega_k\cdot\vartheta_2\sin 45 = 2\cdot 0{,}000073\cdot 40\cdot\sqrt{2}\cdot 0{,}71 = 5{,}81\cdot 10^{-3}\ m/s^2.$$

$a_{c3}=2*\omega_k*\vartheta_3\sin 0=0$ chunki $\sin 0^0=0$

$a_{c4}=a_{c2}=5.81*10^{-3}m/s^2$

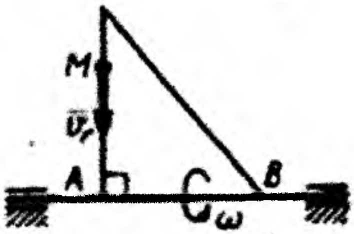
3-masala. Shimoliy kenglik paralleli bo‘ylab o‘tkazilgan temir yo‘lda teplovoz g‘arqdan sharqqa qarab ϑ_r=20m/s tezlik bilan harakat qiladi. teplovozning koriolis tezlanishi ω_c topilsin.

Yechimi: Masala shartiga ko‘ra shimoliy kenglik paralleli bo‘ylab, o‘tkazilgan temir yo‘lda teplovoz g‘arbdan sharqqa qarab ϑ_n=20m/s tezlik bilan harakat qiladi. Yerning aylanishi teplovoz uchun ko‘chirma harakat hisoblanadi. Uning burchak tezlik vektori $\vec{\omega}_k$ yerning aylanish o‘qi bo‘ylab yuqoriga yo‘nalgan. Shuning uchun $\vec{\omega}_k \perp \vec{\vartheta}_n$. Natijada teplovozning koriolis tezlanishi quyidagiga teng bo‘ladi:

$$a_c=2*\omega_k*\vartheta_n=2*0/000073*20=2.91*10^{-3}m/s$$

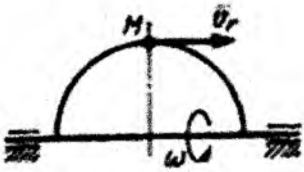
46-§. Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar

1-muammo. Uchburchak shaklidagi jism AB tomoni atrofida ω=8rad/s burchak tezlik bilan aylanadi. M nuqta esa uchburchakning AB ga perpendikulyar tomoni bo‘ylab υ=4m/s nisbiy tezlik bilan harakat qiladi. M nuqtaning Koriolis tezlanishini toping (3.42- rasm).



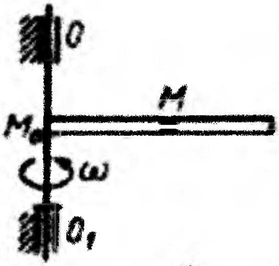
3.42- rasm

2-muammo. Yarim doira shaklidagi jism ω=4rad/s burchak tezlik bilan aylanadi. M nuqta esa uning yoyi bo‘ylab $\vec{\vartheta}_r$ tezlik bilan harakat qilsa, M nuqtaning Koriolis tezlanishini toping (3.43- rasm).



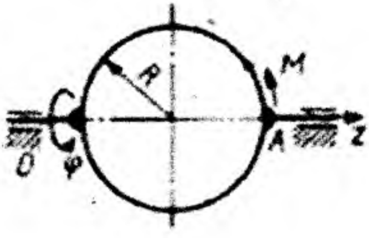
3.43- rasm

3-muammo. Naycha OO_1 o‘q atrofida ω=1.5 rad/s burchak tezlik bilan ayanadi. Uning ichida M nuqta $M_0M=4t$ qonun bo‘yicha harakat qilsa, nuqtaning Koriolis tezlanishini aniqlang (3.44- rasm).



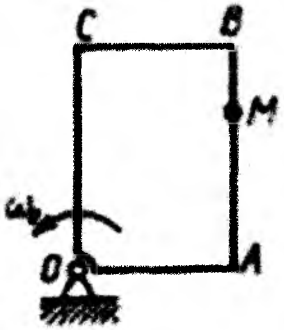
3.44- rasm

4-muammo. Radiusi R=0,4m bo‘lgan disk *Oz* o‘qi atrofida $\varphi=4\sin0.25\pi t$ qonun bo‘yicha aylanadi. Uning gardishi bo‘ylab M nuqta $AM=0,25\pi Rt^2$ tenglama bilan harakatlansa, t=1s paytda nuqtaning Koriolis tezlanishini toping. (3.45- rasm).



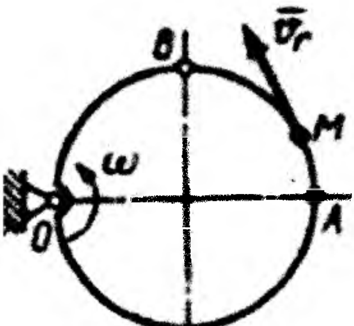
3.45- rasm

5-muammo. To'rtburchak shaklidagi plastina shakl tekisligida O nuqta atrofida aylanadi. M nuqta AB qirrasi bo'ylab $AM=3sin(\pi/3)t$ qonun bo'yicha harakatlanadi. Agar t=2sek da M nuqtaning Koriolis tezlanishi 4π(m/s) bo'lsa, plastinaning ω_B ko'chirma burchak tezligini toping (3.46- rasm).



3.46- rasm

6-muammo. Shakil tekisligida ω=2rad/s burchak tezlik bilan aylanuvchi diskning gardishi bo'ylab M nuqta ϑ=0.2m/s nisbiy tezlik bilan harakat qiladi. M nuqta A holatdan B holatga o'tgan bo'lsa, uning Koriolis tezlanishining miqdori o'zgaradimi? (3.47- rasm).



3.47- rasm

47-§. Ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolyut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning murakkab harakatida uning absolyut tezlanishini aniqlashga doir masalalarni yechishda ko'chirma harakatning ko'rinishi muhim ahamiyat kasb etadi.

Agar nuqtaning murakkab harakatida ko'chirma harakat ilgarilanma harakat bo'lmasa, yani qo'zg'aluvchi koordinatalar sistemasining berilgan ondagi burchak tezligi ω_k ma'lum bo'lsa, nuqtaning absolyut tezlanishi uning nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisidan iborat bo'ladi:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k + 2(\vec{\omega}_k x^{\wedge} \vec{\vartheta}_n) = \vec{a}_n + \vec{a}_k + \vec{a}_c.$$

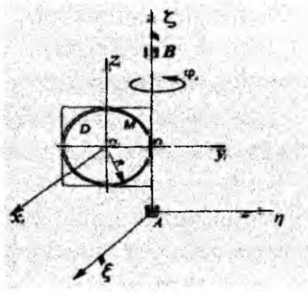
Murakkab harakatda ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolyut tezlanishini aniqlashda quyidagi tartibga rioya etish tavsiya etiladi:

1. Masala shartiga ko'ra nuqtaning nisbiy, ko'chirma va absolyut harakatlari aniqlab olinadi
2. Qo'zg'almas va qo'zg'aluvchan sanoq sistemalari tanlab olinadi.
3. Ko'chirma harakat hayolan to'xtatilib, nuqtaning nisbiy tezlanishi aniqlab olinadi.
4. Nisbiy harakat hayolan to'xtatilib, nuqtaning ko'chirma tezlanishi aniqlab olinadi.
5. Nuqtaning nisbiy tezligi va ko'chirma harakatning burchak tezliklarini bilgan holda nuqtaning Koriolis tezlanishi aniqlanadi.
6. Koriolis teoremasiga asosan nuqtaning absolyut tezlanishi aniqlanadi.

48-§. Ko'chirma harakat ilgarilanma harakat bo'lmagan hol uchun nuqtaning absolyut tezlanishini aniqlashga doir masalalar

1-masala.

To'g'ri burchakli ramka AB qo'zg'almas o'q atrofida $\varphi_k=3t-0.5t^3$rad. qonun bo'yicha aylanadi. M nuqta to'g'ri burchakli ramkaga nisbatan unda chizilgan radiusi R=40sm bo'lgan aylana bo'ylab O nuqtadan $OM = s_n = 40\pi cos\frac{\pi t}{3}$sm. qonun bo'yicha harakatlanadi. M nuqtaning t=1 sekunddagi absolyut tezligi va abolyut tezlanishi topilsin (3.48-rasm).



3.48-rasm

$\varphi_k=3t-0.5t^3$rad., $OM = s_n = 40\pi cos\frac{\pi t}{3}$sm.,R=40sm;t=1s

Yechish:

Berilgan vaqt onida chizma tekisligi to'g'ri burchakli ramkaning tekisligi bilan ustma – ust tushadi deb hisoblaymiz.

Shaklda ko'rsatilgan $A\xi\eta\zeta$ o'qlar sistemasi qo'zg'almas sanoq sistemasini, to'g'ri burchakli ramka bilan bog'langan va u bilan birga aylanuvchi $O_1x_1y_1z_1$o'qlar sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi.

M nuqtaning to'g'ri burchakli ramka bilan bog'langan $O_1x_1y_1z_1$ sanoq sistemasiga nisbatan harakati nisbiy, to'g'ri burchakli ramkaning va u bilan bog'langan $O_1x_1y_1z_1$ sanoq sistemasining qo'zg'almas $A\xi\eta\zeta$ sanoq sistemasiga nisbatan harakati ko'chirma va nuqtaning qo'zg'almas $A\xi\eta\zeta$ sanoq sistemasiga nisbatan harakati murakkab harakat hisoblanadi.

M nuqtaning to'g'ri burchakli ramkada chizilgan aylanadagi holatini uning aylana bo'ylab harakat qonunidan foydalanib quyidagi α burchak orqali aniqlaymiz:

$$\alpha = \frac{s_n}{R} = \frac{40\pi cos\frac{\pi t}{3}}{40};$$

T=1 sekundda, $\alpha=90^0$

M nuqtaning absolyut tezligini nuqtaning murakkab harakatida tezliklarni qo'shish haqidagi teoremaga asosan nisbiy va ko'chirma tezliklarning geometrik yig'indisi kabi topamiz:

$$\vec{\vartheta}_M = \vec{\vartheta}_n + \vec{\vartheta}_k. (1)$$

Nisbiy tezlikning moduli:

$$\vartheta_n = |\tilde{\vartheta}_n|, (2)$$

bu yerda,

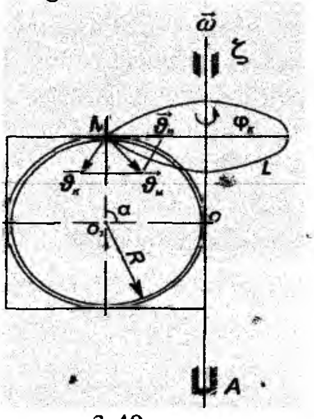
$$\tilde{\vartheta}_n = \frac{ds_n}{at} = -\frac{40\pi^2}{3} sin\frac{\pi t}{3}.$$

t=1 sekundda, $\tilde{\vartheta}_n = \frac{-40\cdot(3.14)^2}{3} \cdot 0{,}86 = -113{,}06$ sm/s,

$$\vartheta_n = 113.06 sm/s$$

$\tilde{\vartheta}_n$ kattalikning oldidagi manfiy ishora M nuqtaning nisbiy tezligi s_n ning kamayish tomoniga qarab aylanaga urinma holda yo'nalishini bildiradi (6-rasm).

Ko'chirma tezlikning moduli:



3.49-rasm

$$\vartheta_k = R_k \omega_k, \quad (3)$$

bu yerda R_k to'g'ri burchakli ramkaning qaralayotgan vaqt onida M nuqta bilan ustma – ust tushuvchi nuqtasi tomonidan $A\xi$ o'q atrofida chizadigan L aylanasining radiusi, R_k=R=40sm

ω_k– to‘g‘ri burchakli ramka burchak tezligining moduli:

$$\omega_k = |\tilde{\omega}_k|,\ \tilde{\omega}_k = \frac{d\varphi_k}{at} = 3 - 1{,}5t^2.$$

t=1sekundda

$$\tilde{\omega}_k = 1{,}5 \text{ rad/s},\ \omega_k = 1.5\text{rad/s}$$

$\vec{\omega}_k$ kattalikning musbat ishorasi to‘g‘ri burchakli ramkaning $A\xi$ o‘q atrofidagi aylanishi φ_k burchakning o‘sish tomoniga ro‘y berishini ko‘rsatadi. Shuning uchun $\vec{\omega}_k$ Ko‘chirma tezlikning moduli (3) formula bo‘yicha hisoblanadi:

$$\vartheta_k = 40*1.5 = 60\text{sm/s}.$$

$\vec{\vartheta}_k$ vektor L aylanaga urinma bo‘ylab, to‘g‘ri burchakli ramkaning aylanish tomoniga qarab yo‘nalgan. $\vec{\vartheta}_k$ va $\vec{\vartheta}_n$ vektorlar o‘zaro perpendikulyar bo‘lgani uchun M nuqta absolyut tezligining moduli:

$$\vartheta_M = \sqrt{\vartheta_n^2 + \vartheta_k^2} = 128\, sm/s.$$

M nuqtaning absolyut tezlanishini nuqtaning murakkab harakatida tezlanishlarni qo‘shish teoremasidan aniqlaymiz. Masalada, ko‘chirma harakat ilgarilanma bo‘lmagan murakkab harakat bo‘lganligi uchun, absolyut tezlanish nisbiy, ko‘chirma va Koriolis tezlanishlarining geometrik yig‘indisiga teng:

$\vec{a}_M = \vec{a}_n + \vec{a}_k + \vec{a}_s$,(4)

yoki, yoyilgan ko‘rinishda

$\vec{a}_M = \vec{a}_n^\tau + \vec{a}_n^n + \vec{a}_k^{ayl} + \vec{a}_k^{mi} + \vec{a}_c.$ (5)

Nisbiy urinma tezlanishning moduli

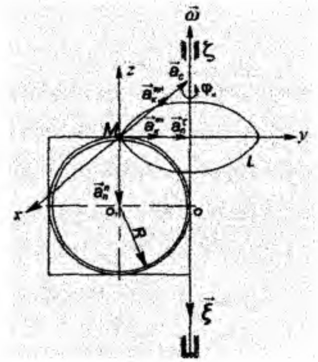
$$a_n^\tau = |\,\tilde{a}_n^\tau| \qquad (6)$$

bu yerda,

$$\tilde{a}_n^\tau = \frac{d^2 s_n}{at^2} = -\frac{40\pi^2}{9}\cos\frac{\pi t}{3}.$$

$t = 1$ sekundda,

$$\tilde{a}_n^\tau = -21{,}91 \text{ sm/s}^2,$$



3.50-rasm

$$a_n^\tau = 21{,}91 \text{ sm/s}^2.$$

$\bar{a}_n^\tau$ning manfiy ishorasi $\bar{a}_n^\tau$ vektorning S_n ning kamayish tomoniga qarab yo'nalganligini ko'rsatadi. $\bar{a}_n^\tau$ va $\bar{v}_n$ ishoralari bir xil. Demak, $\bar{a}_n^\tau$ va $\bar{v}_n$ vektorlari bir xil yo'nalishga ega. Nisbiy normal tezlanish:

$$a_n^n = \frac{\vartheta_n^2}{R} = 319{,}56 \text{ sm/s}^2,$$

a_n^n vektor O_1 nuqtaga qarab yo'nalgan. Ko'chirma aylanma tezlanishning moduli

$$a_k^{ayl} = R_k \mathcal{E}_k; \qquad (7)$$

bu yerda $\mathcal{E}_k = |\vec{\mathcal{E}}_k|$ – to'g'ri burchakli ramkaning burchak tezlanishining moduli

$$\tilde{\mathcal{E}}_k = \frac{d^2\varphi_k}{at^2} = -3t. \qquad (8)$$

sekundda,

$$\tilde{\mathcal{E}}_k = -3\, rad/s^2, \quad \mathcal{E}_k = 3\, rad/s^2.$$

$\tilde{\mathcal{E}}_k$ va $\tilde{\omega}_k$ larning ishoralari har xil. Demak, to'g'ri burchakli ramkaning aylanishi sekinlanuvchan, $\vec{\omega}_k$ va $\vec{\mathcal{E}}_k$ vektorlarning yo'nalishlari qarama – qarshi bo'ladi.

(7) ga asosan,

$$a_k^{ayl} = 40 \cdot 3 = 120\, sm/s^2.$$

$\vec{a}_k^{ayl}$ va $\vec{\vartheta}_k$ vektorlar qarama-qarshi tomonlarga yo'nalgan (3.49 - 3.50-rasmlar)

Ko'chirma markazga intilma tezlanishning moduli

$$a_k^{mi} = R_k\omega_k^2 = 40\cdot(1{,}5)^2 = 90\, sm/s^2. \qquad (9)$$

$\vec{a}_k^{mi}$ vektor L aylananing markazi tomon yo'nalgan.

Koriolis tezlanishining moduli

$$a_c = 2\omega_k\vartheta_n \sin(\widehat{\vec{\omega}_k\ \vec{\vartheta}_n}) \qquad (10)$$

bu yerda,

$$\sin(\widehat{\vec{\omega}_k\ \vec{\vartheta}_n}) = sin90^0 = 1.$$

ω_n va ϑ_n larning yuqorida topilgan qiymatlarini hisobga olgan holda a_c uchun quyidagi natijaga ega bo'lamiz:

$$a_c = 2*1.5*113.06 = 339.18 sm/s^2.$$

$\vec{a}_c$ vektor $(\vec{\omega}_k \mathrm{x} \vec{\vartheta}_n)$ vektor ko'paytma qoidasiga muvofiq yo'nalgan (3.50-rasm).

M nuqta absolyut tezlanishining modulini proeksiyalash usuli orqali aniqlaymiz:

$$a_{Mx} = -a_k^{ayl} - a_c = 459{,}18\, sm/s^2;$$

$$a_{My} = a_n^{\tau} + a_k^{mi} = 111{,}91\frac{sm}{s^2};$$

$$a_{Mz} = -a_n^{n} = -319{,}56\, sm/s^2;$$

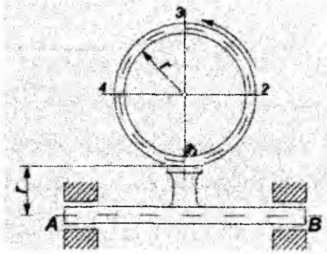
$$a_M = \sqrt{a_{mx}^2 + a_{my}^2 + a_{mz}^2} = 570{,}5\, sm/s^2.$$

Hisob natijalari quyidagi jadvalda keltirilgan:

ω_k, rad /s.	Tezlik, sm/s.			ε_k, rad/ s^2	Tezlanish, sm/s^2								
	ϑ_k	ϑ_n	ϑ_M		a_k^{mi}	a_k^{ayl}	a_n^{τ}	a_n^{n}	a_c	a_{Mx}	a_{My}	a_{Mz}	a_M
115	60	113 ,06	1 2 8	-3	90	120	21, 91	319, 56	339, 18	459, 18	111 ,91	31 9,5 6	57 0,5

2-masala.

Radius r bo‘lgan kovak xalqa AB val bilan mahkam biriktirilgan, bunda valning o‘qi halqa o‘qining tekisligida joylashgan. Xalqa rasmda ko‘rsatilgan sterlka yo‘nalishida o‘zgarmas u nisbiy tezlik bilan harakat qiluvchi suyuqlik bilan to‘ldirilgan.



3.51a-rasm

Agar aylanish o‘qi bo‘yicha A dan B ga qaralsa, AB val soat strelkasi aylanadigan tomonga aylanadi. Valning ω burchak tezligi o‘zgarmas. 1, 2, 3 va 4 nuqtalardagi suyuqlik zarralarining absolyut tezlanishlari miqdorlari aniqlansin (3.51-rasm).

Yechimi: Masalada suyuqlik zarralarining halqa ichidagi harakati nisbiy harakat, halqaning esa, AB val bilan birgalikda soat strelkasi aylanadigan tomonga aylanishi ko‘chirma harakat hisoblanadi.

Nuqtaning murakkab harakatida tezlanishlarni qo‘shish teoremasiga asosan 1,2,3 va 4 nuqtalardagi suyuqlik zarralarining absolyut tezlanishlari quyidagi formula asosida aniqlanadi:

$$\vec{a} = \vec{a}_n + \vec{a}_k + \vec{a}_c = \vec{a}_n^n + \vec{a}_n^\tau + \vec{a}_k^{mi} + \vec{a}_k^{ayl} + \vec{a}_c.$$

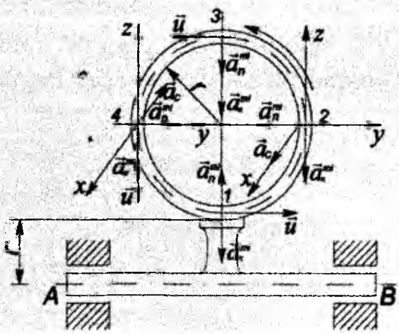
Masala shartiga ko‘ra:

$$\vartheta_n = u = const$$

Shuning uchun barcha nuqtalarda

$$\vartheta_n^\tau = 0, \ \vartheta_k^{ayl} = 0.$$

1-nuqtada suyuqlik zarralarining absolyut tezlanishining miqdorini aniqlaymiz (3.51b-rasm).



3.51b-rasm

$$a_n^n = \frac{u^2}{r};\quad a_k^{mi} = \omega^2 \cdot r;\quad a_c = 2\omega_k \vartheta_n \cdot \sin(\widehat{\vec{\omega}_n \vartheta_n}) = 0:$$

Shuning uchun

$$a_1 = a_k^{mi} - a_n^n = \omega^2 r - \frac{u^2}{r};$$

2-nuqtada suyuqlik zarralarining absolyut tezlanishining miqdorini aniqlaymiz (3.51b-rasm).

$$a_n^n = \frac{u^2}{r};\quad a_k^{mi} = \omega^2 \cdot 2r.$$

$$a_c = 2 \cdot \omega_k \vartheta_n \sin(\widehat{\vec{\omega}_k \vec{\vartheta}_n}) 2 \cdot \omega_k \cdot \vartheta_n = 2\omega \cdot u.$$

Bularni e'tiborga olsak,

$$\sqrt{(a_c)^2 + (-a_n^n)^2 + (-a_k^{mi})^2} = \sqrt{4\omega^2 u^2 + \frac{u^4}{r^2} + 4\omega^4 r^2} =$$

$$= \sqrt{\frac{4\omega^2 u^2 r^2 + u^4 + 4\omega^4 \cdot 2^4}{2^2}} = \frac{1}{2}\sqrt{(u^2 + 2\omega^2 2^2)^2} = .$$

$$= \frac{1}{2}(u^2 + 2\omega^2 2^2) = \frac{u^2}{r} + 2\omega^2 r.$$

3-nuqtada suyuqlik zarralarining absolyut tezlanishining miqdori quyidagiga teng bo'ladi (3.51b-rasm).

$$a_n^n = \frac{u^2}{r};\quad a_k^{mi} = \omega^2 \cdot 3r.$$

Shuning uchun $a_c = 2 \cdot \omega_k \vartheta_n \sin(\widehat{\vec{\omega}_k \vec{\vartheta}_n}) = 0.$

$$a_3 = a_n^{mi} + a_k^{mi} = \frac{u^2}{r} + 3\omega^2 2.$$

4- nuqtada suyuqlik zarralarining absolyut tezlanishining miqdorini aniqlaymiz. (3.51b-rasm).

$$a_n^n = \frac{u^2}{r};\quad a_k^{mi} = \omega^2 \cdot 2r.$$

$$a_c = 2 \cdot \omega_k \vartheta_n \sin(\widehat{\vec{\omega}_k \vec{\vartheta}_n}) 2\omega_k \vartheta_n = 2\omega u.$$

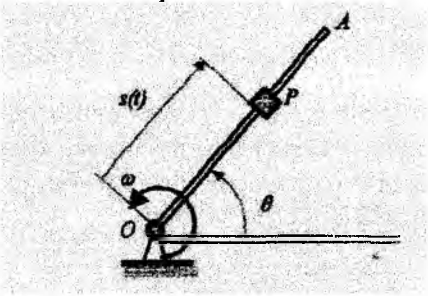
Shuning uchun ϑ

$$a = \sqrt{(-a_c)^2 + (-a_n^{mi})^2 + (-a_k^{mi})^2}$$

$$= \sqrt{(-2\omega u)^2 + \left(\frac{u^2}{r}\right)^2 + (2\omega^2 r)^2} = \frac{u^2}{r} + 2\omega^2 r.$$

3-masala. Uzunligi l=1m bo'lgan sterjen chizma tomoniga perpendikulyar holda O nuqtadan o'tuvchi o'q atrofida $w = \frac{\pi t}{3} \frac{1}{s}$ burchak tezlik`bilan aylanmoqda. Shu vaqtning o'zida sterjen bo'ylab P polzun $S(t)=OP=12,5t^2$ qonunga muvofiq harakatlanadi.

Sterjen harakati gorizontal holatdan boshlanadi deb faraz qilib, P polzunning , u sterjenning yarmida bo'lgan holatida, absolyut tezligi va absolyut tezlanishi aniqlansin.



3.52-rasm

Yechimi: Sterjenning O nuqtadan o'tuvchi o'q atrofidagi aylanma harakati P polzun uchun ko'chirma harakat, P polzunning sterjen bo'ylab O nuqtadan sterjenga nisbatan harakati nisbiy harakat hisoblanadi.

Agar t_1 vaqt onida polzun sterjenning yarmida bo'ladi deb faraz qilsak,

$$S(t_1)=l/2;\ 12.5t^2=50 \text{ bo'ladi.}$$

Bundan t_1=2s

Shu vaqt onida sterjenning boshlang'ich holati (gorizontal holat)bilan tashkil etgan burchak quyidagicha aniqlanadi:

$$w = \frac{d\theta}{dt};\ d\theta=wdt$$

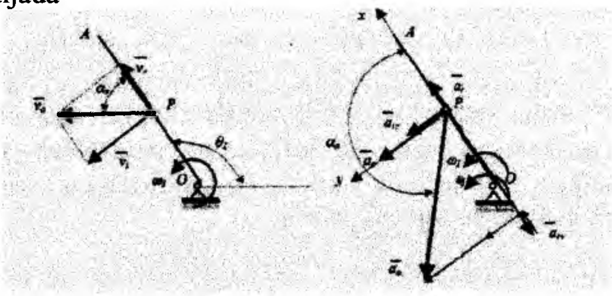
$$\theta(t)=\int wdt+C$$

yoki $$\theta(t) = \frac{\pi t^2}{6} + C$$

Bu ifoda C-integrallash doimiysi.

Agar boshlang'ich paytda θ=0 ekanligini e'tiborga olsak C=0 bo'ladi.

Natijada



3.53-rasm

Polzunning nisbiyli tezlik va nisbiy tezlanishini aniqlash uchun sterjenning aylanma harakatini-ko'chirma harakatini hayolan to'xtatamiz. Natijada P polzunnung nisbiy tezligi va nisbiy tezlanishi uchun quyidagi kattaliklarga ega bo'lamiz:

$$v_n = \frac{dS(t_1)}{dt} = 25t_1 = \frac{50sm}{s}$$

$$a_n = \frac{d^2S(t_1)}{dt^2} = \frac{dv_n}{dt} = \frac{25sm}{s^2}$$

P polzun ko'chirma harakat tezligi va tezlanishini aniqlash uchun P polzunning nisbiy harakatini hayolan to'xtatamiz. Natijada P polzun ko'chirma tezligi va ko'chirma tezlanishlari uchun quyidagi ifodalarga ega bo'lamiz:

$$v_k = OP(t_1) * w(t_1) = \frac{l}{2} * \frac{\pi t_1}{3} = 104{,}7\frac{cm}{s}$$

$\vec{v}_k$ vektor OP kesmaga perpendikulyar holda $w(t_1)$ tomon yo'naladi.

P polzunning ko'chirma tezlanishi uning urinma va normal tezlanishlaridan tashkil topadi:

$$\vec{a}_k = \vec{a}_{k\tau} + \vec{a}_{ku}$$

Ko'chirma urinma tezlanishi:

$$a_{k\tau} = OP(t_1) * \varepsilon(t_1) = OP(t_1) * \frac{dw(t_1)}{dt} = \frac{1}{2} * \frac{\pi}{3} = 52{,}34\frac{sm}{s^2}$$

Ko'chirma normal tezlanishi:

$$a_{kn} = OP(t_1)w^2(t_1) = \frac{1}{2}\frac{\pi^2 t^2{}_1}{9} = 219{,}1\frac{sm}{s^2}$$

P polzunning Koriolisma tezlanishini aniqlaymiz:

$$\vec{w}_{1k} = w_1(t_1) * \vec{k} = 2{,}09\vec{k}$$

$$\vec{v}_n = v_n\vec{i} = 50\vec{i}$$

Bulardan foydalansak, P polzun Koriolis tezlanishini quyidagiga teng bo'ladi:

$$\vec{a}_s = 2\vec{w}_1(t_1) * \vec{v}_n(t_1) = 2 * 2{,}09\vec{k} * 50\vec{i} = 209{,}4\vec{j}$$

Yuqorida hisoblangan kattaliklarnin gqiymatlarini e'tiborga olsak, P polzunning absolyut tezligi quyidagiga teng bo'ladi:

$$v_a = \sqrt{v^2{}_n + v^2{}_k} = \sqrt{(104{,}7)^2 + (261{,}74)^2} = 325{,}82\frac{sm}{s^2}$$

Absolyut tezlikning yo'nalishini quyidagicha topiladi;

$$tg\alpha_\lambda = \frac{v_k}{v_n} = \frac{104{,}7}{50} = 2{,}09$$

$$a_\lambda = \text{arctg}\alpha_\lambda = 64^\circ 47'$$

P polzunning absolyut tezlanishini aniqlaymiz:

$$\vec{a}_n = \vec{a}_k + \vec{a}_n + \vec{a}_s$$

Absolyut tezlanishi moduli va yo'nalishini aniqlash uchun proyeksiyalar usulidan foydalanilamiz:

$$a_{ax} = a_n - a_{kt} = 25 - 219{,}1 = -194{,}1\frac{sm}{s^2}$$

$$a_{ay} = a_{k\tau} + a_s = 52.34 + 209.4 = 261.74 sm/s^2$$

Natijada P polzun absolyut tezlanishi quyidagiga teng bo'ladi:

$$a_a = \sqrt{a^2{}_{ax} + a^2{}_{ay}} = \sqrt{(-194{,}1)^2 + (261{,}74)^2} = 325{,}82\frac{sm}{s^2}$$

Absolyut tezlanish yo'nalishi esa uning Px o'qi bilan hosil qilgan burchak orqali aniqlanadi:

$$cos\alpha_\lambda = \frac{Q_x}{Q_a} = \frac{-194{,}1}{325{,}82} = -0{,}595$$

$$\alpha_\lambda = arccos\alpha_\lambda = 180^\circ - 53^\circ 44' = 126^\circ 56'$$

Absolyut tezlik vektori, absolyut tezlanish vektorri va ularning yo'nalishlari 3.53-rasmda ko'rsatilgan.

Masalani yechishda quyidagi belgilashlar kiritiladi:

$$\vec{v}_r \to \vec{v}_n \; \vec{a}_r = \vec{a}_n \; w_1 \to w_k$$

$$\vec{v}_1 \rightarrow \vec{v}_k \; \vec{a}_{i\tau} = \vec{a}_{k\tau} \; \varepsilon_1 \rightarrow \varepsilon_k$$

4-masala. M nuqta *D* jismga (doiraga) nisbatan $OM = s_n = 20\sin\frac{\pi t}{6}$ qonun bo'yicha harakat qiladi. *D* jism (doira) chizma tekisligiga perpendikulyar o'q atrofida $\varphi_k = 2t - 0.5t^2$ qonun bo'yicha aylanadi. M nuqtaning t=1 sekunddagi absolyut tezligi va absolyut tezlanishi topilsin (3.54-rasm).

Masalada: $OM = s_n = 20\sin\frac{\pi t}{6}$sm., $\varphi_k = 2t - 0.5t^2$rad. t=1s, a=20sm

Yechimi:

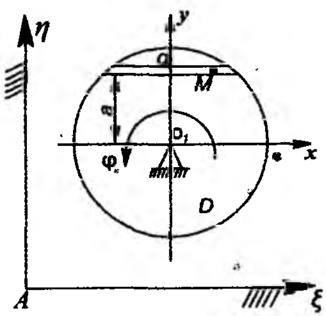
Chizmada ko'rsatilgan $A\xi\eta$ o'qlar sistemasi qo'zg'almas sanoq sistemasini, doira bilan bog'langan va u bilan birga aylanuvchi O_1xy sanoq sistemasi quzg'aluvchan sanoq sistemasini tashkil etadi.

M nuqtaning qo'zg'aluvchan O_1xy sanoq sistemasiga nisbatan harakati nisbiy, D jismning qo'zg'aluvchan

O_1xy sanoq sistemasi bilan birgalikda qo'zg'almas $A\xi\eta$ sanoq sistemasiga nisbatan harakati M nuqta uchun ko'chirma va M nuqtaning bevosita qo'zg'almas sanoq sistemasiga nisbatan harakati murakkab harakat hisoblanadi.

Nuqtaning t=1 sekunddagi holatini uning to'g'ri chiziq bo'ylab harakat qonunidan foydalanib topamiz va 3.54-rasm chizmada ko'rsatamiz (3.54a-rasm).

t=1 sekundda $OM = s_n = 20\sin\frac{\pi t}{6} = 10$ sm.



M nuqtaning absolyut tezligini nuqtaning murakkab harakatida tezliklarni qo'shish haqidagi teoremaga asosan, nisbiy va ko'chirma tezliklarning geometrik yig'indisi kabi topamiz:

$$\vec{\vartheta}_M = \vec{\vartheta}_n + \vec{\vartheta}_k. \qquad (1.1)$$

Nisbiy tezlikning moduli

$$\vartheta_n = |\vec{\vartheta}_n|; \qquad (1.2)$$

bu yerda,

$$\tilde{\vartheta}_n = \frac{dS_n}{dt} = \frac{20\pi}{6}\cos\frac{\pi t}{6}.$$

t=1 sekundda,

$$\tilde{\vartheta}_n = 9\, sm/s,\ \vartheta_n = 9\, sm/s.$$

$\tilde{\omega}_k$oldidagi musbat ishora $\vec{\vartheta}_n$ vektorning S_n ning o‘sish tomoniga qarab yo‘nalganligini ko‘rsatadi.

Ko‘chirma tezlikning moduli

$$\vartheta_k = R_k\omega_k \tag{1.3}$$

bu yerda R_k – doiraning berilgan onda M nuqta bilan ustma – ust tushuvchi nuqtasi tomonidan chiziladigan *L* aylananing radiusi, ω_k – doira burchak tezligining moduli.

$$R_k = O_1M = \sqrt{a^2 + (oM)^2} = 22{,}36 \text{ sm}. \tag{1.4}$$

$$\omega_k = |\tilde{\varphi}_k|,\ \tilde{\omega}_k = \frac{d\varphi_k}{dt} = 2 - t. \tag{1.5}$$

t=1 sekundda,

$$\tilde{\omega}_k = 1\frac{rad}{s},\ \omega_k = 1 rad/s.$$

(L aylana rasmda ko‘rsatilmagan).

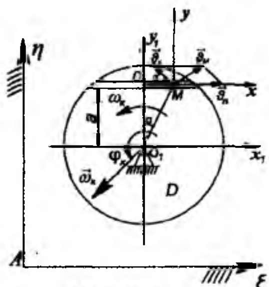
$\tilde{\varphi}_k$ kattalikning oldidagi musbat ishora doiraning O_1 nuqta atrofida aylanishi φ_k burchakning o‘sish tomoniga qarab ro‘y berishini ko‘rsatadi. Shuning uchun $\vec{\omega}_k$ vektor chizma tekisligiga perpendikulyar holda O_1 nuqtadan o‘tkazilgan aylanish o‘qi bo‘ylab tepaga qarab yo‘nalgan (3.7a-rasm).

Ko‘chirma tezlikning moduli (1.3)3.35 formula bo‘yicha topiladi:

$$\vartheta_k = 22.36 sm/s$$

ϑ_k vektor *L* aylanaga urinma bo‘ylab, doiraning aylanish tomoniga qarab yo‘nalgan.

M nuqtaning absolyut tezligi uning nisbiy va ko‘chirma harakat tezliklaridan qurilgan parallelagramning dioganali orqali ifodalanadi. Uning modulini proeksiyalash usuli orqali aniqlaymiz:



3.54a- rasm

$$\vartheta_{Mx} = \vartheta_n - \vartheta_k\cos\alpha;\ \cos\alpha = 0.89.$$

$$\vartheta_y = \vartheta_k\sin\alpha;\ \sin\alpha = 0.45$$

Shuning uchun,

$$\vartheta_{Mx}=-10.9 sm/s;$$
$$\vartheta_{My}=10.06\ sm/s$$
$$\vartheta_M=\sqrt{\vartheta_{Mx}^2+\vartheta_{My}^2}=14{,}83\ sm/s.$$

M nuqtaning absolyut tezlanishi nuqtaning murakkab harakatida tezlanishlarni qo‘shish teoremasidan aniqlanadi. Masalada, ko‘chirma harakat ilgarilanma bo‘lmagan murakkab harakat bo‘lganligi uchun, absolyut tezlanish nisbiy, ko‘chirma va Koriolis tezlanishlarining geometrik yig‘indisiga teng:

$$\vec{a}_M=\vec{a}_n+\vec{a}_k+\vec{a}_c\,, \quad (1.6)$$

yoki, yoyilgan ko‘rinishda:

$$\vec{a}_M=\vec{a}_n^{\tau}+\vec{a}_n^{n}+\vec{a}_k^{ayl}+\vec{a}_k^{mi}+\vec{a}_c \quad (1.7)$$

Nisbiy urinma tezlanishning moduli

$$a_n^{\tau}=|\tilde{a}_n^{\tau}| \quad (1.8)$$

bu yerda ,

$$\tilde{a}_n^{\tau}=\frac{d^2s_n}{dt^2}=-\frac{20\pi^2}{36}\sin\frac{\pi t}{6}.$$

sekundda

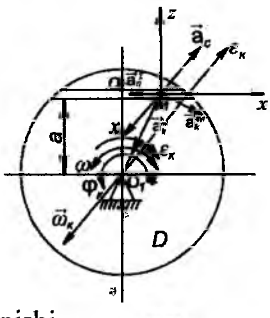
$$\tilde{a}_n^{\tau}=-2{,}74\ sm/s^2,\ a_n^{\tau}=2{,}74\ sm/s^2.$$

$\tilde{a}_n^{\tau}$ ning manfiy ishorasi $\vec{a}_n^{\tau}$ vektorning S_n ning kamayish tomoniga qarab yo‘nalganligini ko‘rsatadi. $\tilde{a}_n^{\tau}$ va $\tilde{\vartheta}_n$ larning ishoralari har xil. Demak, $\vec{a}_n^{\tau}$ va $\vec{\vartheta}_n$ vektorlar qarama-qarshi tomonlarga yo‘nalgan bo‘ladi.

M nuqtaning nisbiy harakatdagi normal tezlanish

$$a_n^n=\frac{\vartheta_n^2}{\rho}=0,$$

chunki nisbiy harakat traektoriyasi to‘g‘ri chiziq ($\rho=\infty$).



3.54b-rasm

Ko‘chirma aylanma tezlanishning moduli

$$a_k^{ayl}=R_k\mathcal{E}_k; \quad (1.9)$$

bu yerda: $\varepsilon_k=|\vec{\varepsilon}_k|$ – doira burchak tezlanishining moduli

$$\tilde{\varepsilon}_k = \frac{d^2\varphi_k}{dt^2} = -1 \text{ rad/s}^2, \ \varepsilon_k = 1 \text{rad/s}^2.$$

$\tilde{\varepsilon}_k$va $\tilde{\omega}_k$ larning ishoralari har xil. Demak, doira sekinlanuvchan aylanma harakatda bo'lar ekan. $\vec{\omega}_k$ va $\vec{\varepsilon}_k$ vektorlar qarama – qarshi tomonlarga yo'naladi (3.54a,b-rasmlar).

(1.9) ga asosan,

$$a_k^{ayl} = 22{,}36\ sm/s^2.$$

$\vec{a}_k^{ayl}$ vektor $\vec{\vartheta}_k$ vektorga qarama-qarshi holda yo'nalgan.

M nuqtaning ko'chirma harakat markazga intilma tezlanishning moduli:

$$a_k^{mi} = R_k\omega_k^2 = 22{,}36\ sm/s^2 \qquad (1.10)$$

$\vec{a}_k^{mi}$ vektor *L* aylananing markaziga (O_1 nuqtaga) yo'nalgan.

Koriolis tezlanishi:

$$\vec{a}_c = 2(\vec{\omega}_k x\, \vec{\vartheta}_n) \qquad (1.11)$$

Koriolis tezlanishining moduli

$$a_c = 2\omega_k\vartheta_n \sin(\vec{\omega}_k \ \hat{}\ \vec{\vartheta}_n).$$

masalada,

$$\sin(\vec{\omega}_k \ \hat{}\ \vec{\vartheta}_n) = \sin 90^0 = 1.$$

ω_k va ϑ_n larning yuqorida topilgan qiymatlarini hisobga olsak,

$$a_c = 18 sm/s^2$$

$\vec{a}_c$ vektor $(\vec{\omega}_k x \vec{\vartheta}_n)$vektor ko'paytma qoidasiga muvofiq yo'naladi (3.54b-rasm).

M nuqtaning absolyut tezlanishining modulini proeksiyalash usuli yordamida aniqlaymiz:

$$a_{Mx} = -a_c = -18 sm/s^2$$

$$a_{My} = -a_n^\tau - a_k^{mi}\sin\alpha + a_k^{ayl}\cos\alpha = 7{,}1\ sm/s^2;$$

$$a_{Mz} = -a_k^{mi}\cos\alpha - a_k^{ayl}\sin\alpha = 30{,}5\ sm/s^2.$$

$$a_M = \sqrt{a_{mx}^2 + a_{my}^2 + a_{mz}^2} = 36{,}12\ sm/s^2$$

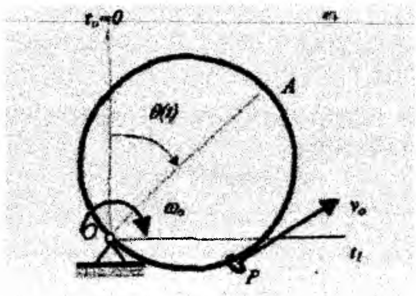
Hisoblash natijalari quyidagi jadvalda keltirilgan.

$\tilde{\omega}_k$, rad/s	Tezlik, sm/s			$\tilde{\varepsilon}_k$, rad/s^2	Tezlanish, sm/s^2								
	ϑ_k	ϑ_n	ϑ_M		a_k^{mi}	a_k^{ayl}	a_n^{τ}	a_n^{n}	a_c	a_{Mx}	a_{My}	a_{Mz}	a_M
1	22,36	9	14,83	-1	22,36	22,36	2,74	0	18	18	7,1	30,5	36,12

49-§. Talabalarga mustaqil o'rganish uchun tavsiya etiladigan muammolar

Muammo-1.

Radiusi R=30sm bo'lgan doira chizma tekisligiga O nuqtadan o'tuvchi o'q atrofida w0=1rad/s o'zgarmas tezlik bilan aynalmoqda. Shu vaqtning o'zida P nuqta O nuqtadan v0=25sm/s tezlik bilan doira gardishi bo'ylab harakatlana boshlaydi.Boshlang'ich paytda vertical holatda bo'lgan doira diametrik gorizontal holatini egallaganda P nuqtaning absolyut tezligi va absolyut tezlanishi aniqlanadi (3.55-rasm).

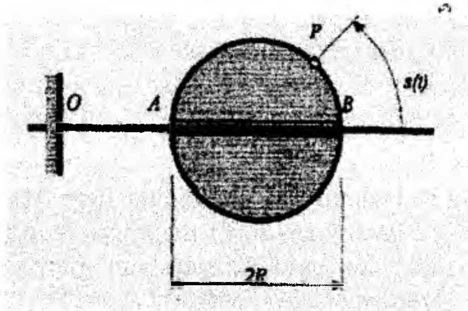


3.55-rasm

Muammo-2.

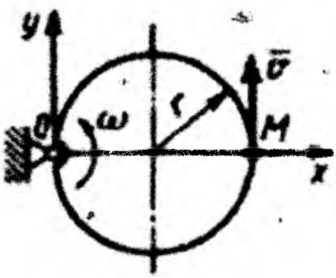
Disk uning diametridan o'tuvchi gorizontal to'g'ri chiziq bo'ylab OA=at^2 qonungamuvofiq ilgarilanma harakatlanmoqda. Shu vaqtning o‘zida disk gardishi bo‘ylab P nuqta $BP = S(t) = \frac{R\pi t^2}{12}$ qonunga muvofiq harakatlanadi.

Agar a=10sm/s^2, R=25sm bo‘lsa,t1=2s vaqt oni uchun P nuqtaning absolyut tezligi va absolyut tezlanishi aniqlansin (3.56- rasm).



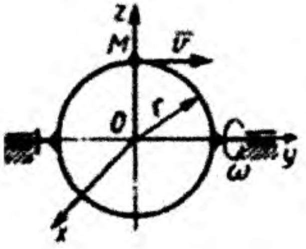
3.56-rasm

Muammo-3. Radiusi r=0,5m bo‘lgan halqa shakl tekisligida o‘zgarmas ω=4rad/s burchak tezlik bilan aylanadi. M nuqta esa halqa bo‘ylab o‘zgarmas υ=2m/s tezlik bilan harakat qiladi. Ko‘rsatilgan holat uchun M nuqtaning absoyut tezlanishini toping (3.57- rasm).



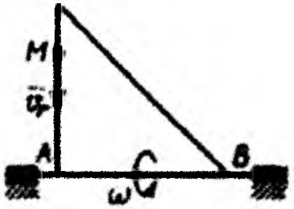
3.57- rasm

Muammo-4. *Oy* o‘qi atrofida o‘zgarmas ω=4rad/s burchak tezlik bilan aylanayotgan halqa bo‘ylab M nuqta o‘zgarmas υ=2m/s tezlik bilan harakat qiladi. Agar halqaning radiusi r=0,5m bo‘lsa, ko‘rsatilgan holat uchun M nuqtaning absolyut tezlanishini toping (3.58- rasm).



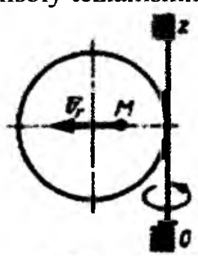
3.58- rasm

Muammo-5. Uchburchak shaklidagi jism AB tomoni atrofida ω burchak tezlik bilan aylanadi. M nuqta esa uning tomoni bo'ylab $\upsilon_r=3t^2$ nisbiy tezlik bilan harakatlanadi. Nuqtaning t=2s paytdagi nisbiy tezlanishini aniqlang (3.59- rasm).



3.59- rasm

Muammo-6. Disk *Oz* o'qi atrofida aylanadi. M nuqta esa $\upsilon_r=4t^3$ nisbiy tezlik bilan diskning diametric bo'ylab harakatlanadi. Nuqtaning t=1s paytdagi nisbiy tezlanishini toping (3.60- rasm).

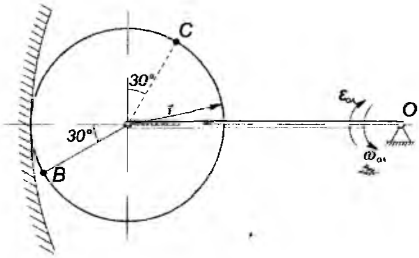


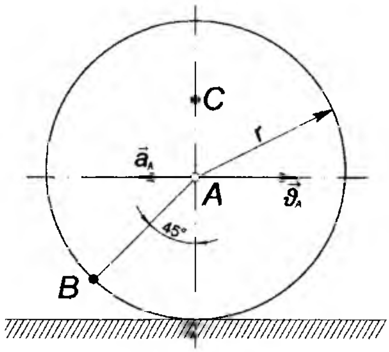
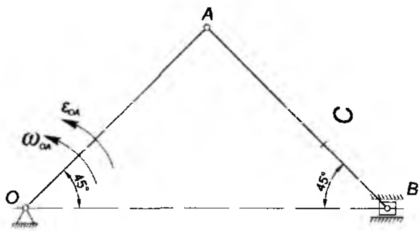
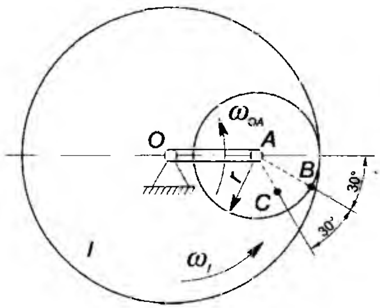
3.60- rasm

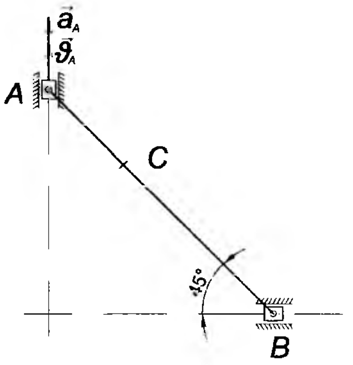
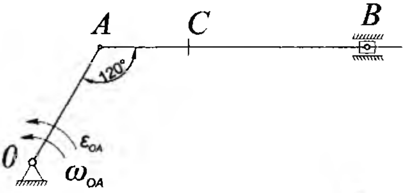
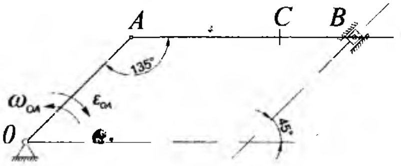
50-§. Talabalar tomonidan mustaqil bajariladigan masalalar variantlari (keys, hisob chizma ishlari) Tekis mexanizmning kinematika tahlili

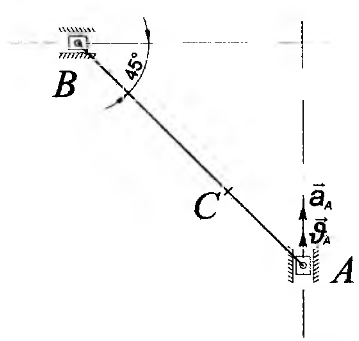
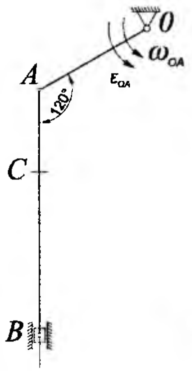
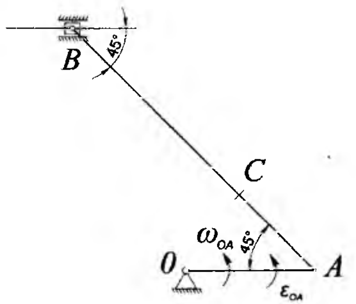
Mexanizmning berilgan holati uchun *B* va *C* nuqtalarning tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zvenoning burchak tezligi va burchak tezlanishi topilsin.

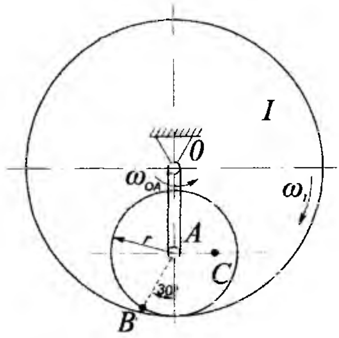
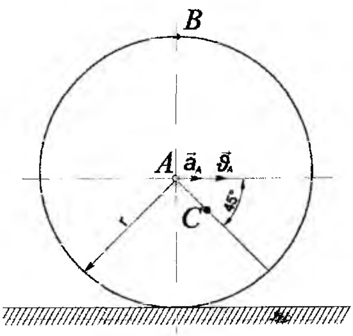
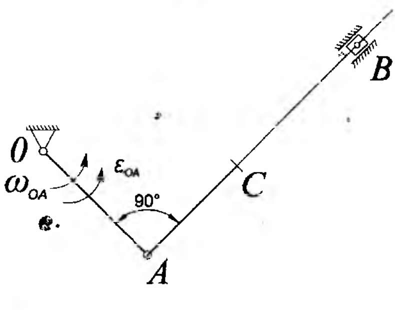
Mexanizmlarning sxemalari va hisoblash uchun kerakli ma'lumotlar quyidagi jadvalda keltirilgan.

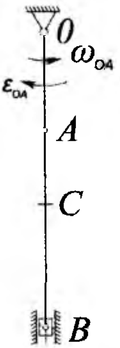
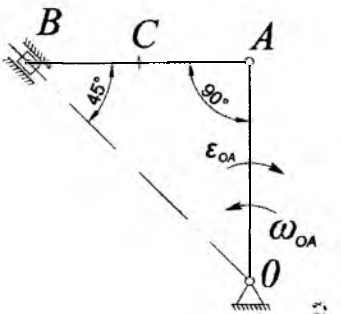
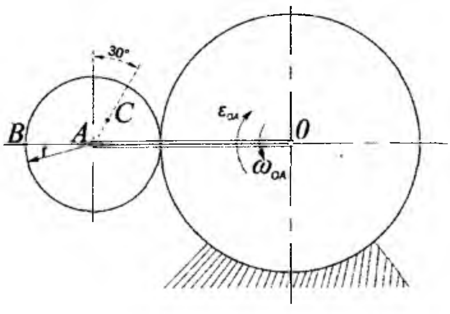
Variant raqamlari	Mexanizmlarning sxemalari	Hisoblash uchun kerak ma'lumotlar
1.		OA=60 sm r=20 sm ω_{OA}=2 rad/s ε_{OA}=4 rad/s^2

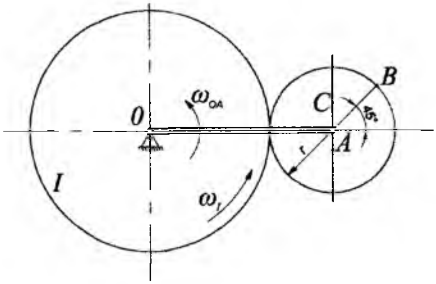
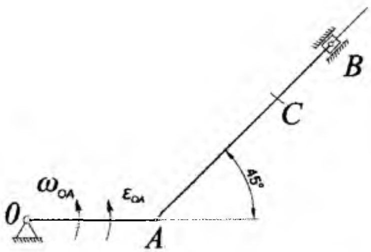
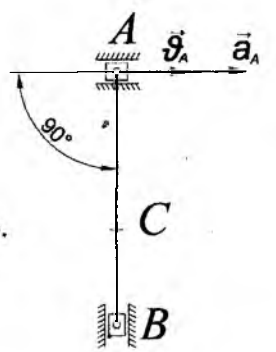
<table>
<tr><td>2.</td><td>
</td><td>r=45 sm
AS=15 sm
$\vec{\vartheta}_A$=100 sm/s
a_A=50sm/s^2</td></tr>
<tr><td>3.</td><td>
</td><td>OA=20 sm
AB=20 sm
AC=10 sm
ω_{OA}=2 rad/s
ε_{OA}=6 rad/s^2</td></tr>
<tr><td>4.</td><td>
</td><td>OA=30 sm
r=20 sm
AC=15 sm
ω_{OA}=1rad/s
ω_I=2,5rad/s
ε_{OA}=0 rad/s^2</td></tr>
</table>

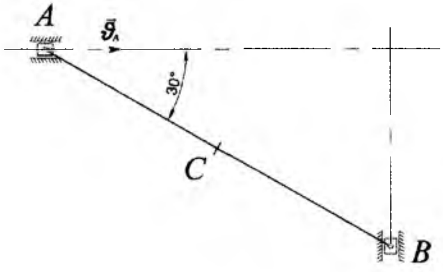
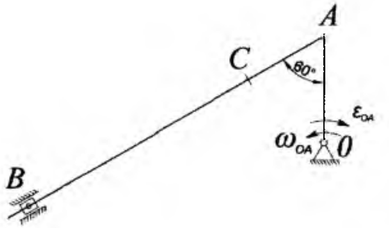
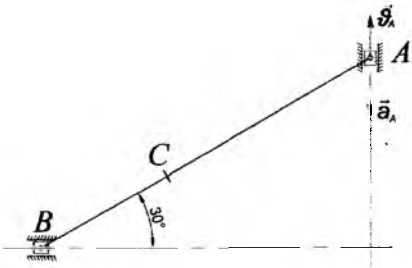
5.		AB=30 sm AC=10 sm $\vec{\vartheta}_A$=10sm/s a_A=15sm/s^2
6.		OA=30 sm AB=60 sm AC=20 sm ω_{OA}=2rad/s ε_{OA}=6rad/s^2
7.		OA=40 sm AB=60 sm AC=40 sm ω_{OA}=3rad/s ε_{OA}=8 rad/s

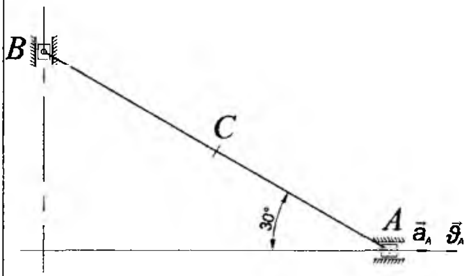
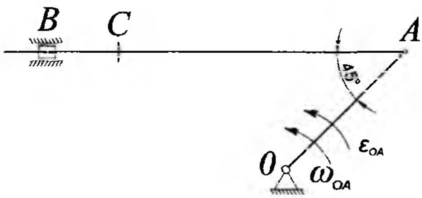
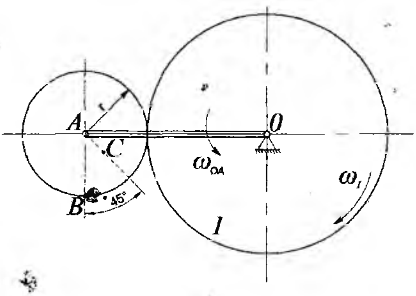
8.		AB=60 sm AC=20 sm $\vec{\vartheta}_A$=5sm/s a_A=10sm/s^2
9.		OA=30 sm AB=40 sm AC=15 sm ω_{OA}=3rad/s ε_{OA}=3 rad/s^2
10.		OA=30 sm AB=80 sm AC=25 sm ω_{OA}=1rad/s ε_{OA}=2 rad/s^2

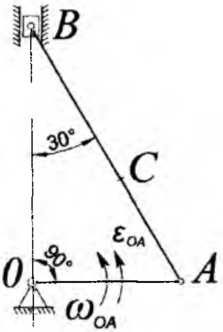
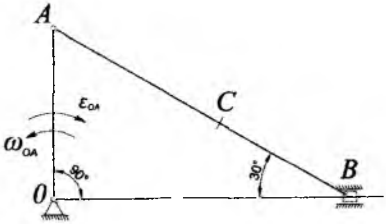
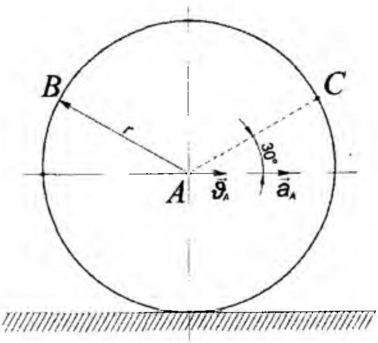
11.		OA=20 sm r=15 sm AC=10 sm ω_{OA}=7,0rad/s ω_I=1,2rad/s ε_{OA}=0
12.		r=20 sm AC=10 sm $\vec{\vartheta}_A$=60 sm/s a_A=30 sm/s^2
13.		OA=30 sm AB=60 sm AC=25 sm ω_{OA}=1rad/s ε_{OA}=1 rad/s^2

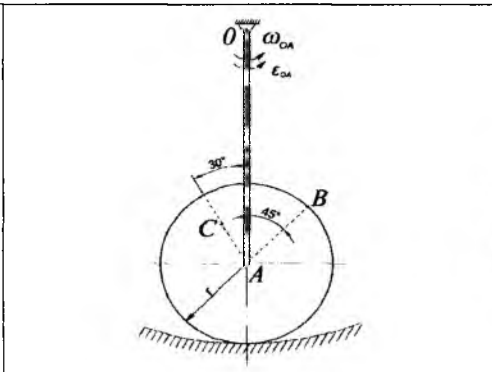
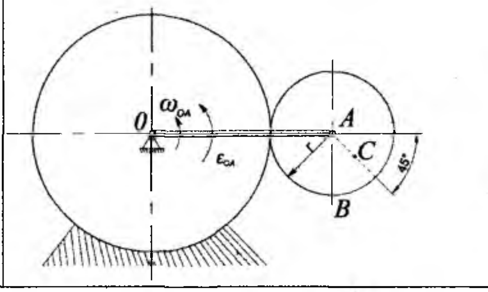
<table>
<tr><td>14.</td><td></td><td>OA=20 sm
AB=40 sm
AC=15 sm
ω_{OA}=4rad/s
ε_{OA}=6 rad/s^2</td></tr>
<tr><td>15.</td><td>
</td><td>OA=40 sm
AC=20 sm
ω_{OA}=4rad/s
ε_{OA}=8 rad/s^2</td></tr>
<tr><td>16.</td><td>
</td><td>OA=50 sm
r=20 sm
AC=10 sm
ω_{OA}=1rad/s
ε_{OA}=8 rad/s^2</td></tr>
</table>

17.		OA=60 sm r=25 sm ω_{OA}=3rad/s AC=10 sm ω_I=12rad/s
18.		OA=30 sm AB=60 sm AC=40 sm ω_{OA}=2rad/s ε_{OA}=4 rad/s^2
19.		AB=40 sm AC=25 sm $\vec{\vartheta}_A$=20sm/s a_A=20sm/s^2

20.		AB=40 sm AC=20 sm $\vec{\vartheta}_A$=10 sm/s a_A=0
21.		OA=25 sm AB=80 sm AC=20 sm ω_{OA} =2rad/s ε_{OA}=2 rad/s^2
22.		AB=50 sm AC=30 sm $\vec{\vartheta}_A$=20sm/s $\vec{a}_A$=10 sm /s^2

23.		AB=30 sm AC=15 sm $\vec{\vartheta}_A$=40sm/s $\vec{a}_A$=20 sm/s^2
24.		OA=35 sm AB=75 m AC=60 sm ω_{OA}=4rad/s ε_{OA}=10 rad/s^2
25.		OA=60 sm r=15 sm AC=6 sm ω_{OA}=1rad/s ω_1=1rad/s ε_{OA} =0

<table>
<tr><td>26.</td><td>
</td><td>OA=25 sm
AC=20 sm
ω_{OA} =1 rad/s^2
ε_{OA} =1 rad/s^2</td></tr>
<tr><td>27.</td><td>
</td><td>OA=40 sm
AC=50 sm
ω_{OA}=4 rad/s
ε_{OA} =8 rad/s^2</td></tr>
<tr><td>28.</td><td>
</td><td>r=50 sm
$\vec{\vartheta}_A$=50 sm/s
a_A=100 sm/s^2</td></tr>
</table>

29.		OA=40 sm r=20 sm AC=10 sm ω_{OA}=3rad/s ε_{OA} =2 rad/s^2
30.		OA=40 sm r=15 sm AC=8 sm ε_{OA}=1rad/s ε_{OA} =1 rad/s^2

Eslatma. ε_{OA}, ε_{OA}-OA krivoship mexanizmning berilgan vaziyatidagi burchak tezligi va burchak tezlanishi; ω_1-1 g'ildirakning burchak tezligi (doimiy); v_A-va a_A – A nuqtaning tezligi va tezlanishi. G'ildiraklar sirpanishsiz aylanadi.

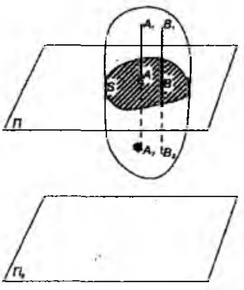
IV BOB. QATTIQ JISMNING TEKISLIKKA PARALLEL HARAKATI

Agar jismning barcha nuqtalari berilgan qo‘zg‘almas tekislikka parallel tekisliklarda harakatlansa, uning bunday harakati tekislikka parallel harakat deyiladi.

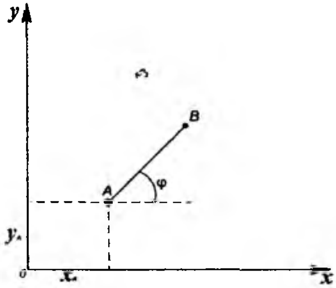
Jismning tekislikka parallel harakatiga misol tariqasida to‘g‘ri chiziqli relisda g‘ildirakning dumalashini,bir tekislikda hara-katlanuvchi mashina va mexanizm qismlarining harakatini va hokazolarini keltirish mumkin.

51-§. Tekis shakil harakatini qutb bilan birgalikda oniy ilgarilanma va qutb atrofida oniy aylanma harakatlarga ajratish

Jismning tekislikka parallel harakatini o‘rganish uchun uni qo‘zg‘almas π_0 tekislikka parallel bo‘lgan π tekislik bilan fikran kesamiz. Kesish natijasida hosil bo‘lgan kesimni S bilan belgilab, uni *tekis shakl* deb ataymiz. Jismning tekislikka parallel harakati ta’rifiga ko‘ra, jismning harakati davomida bu tekis shakl doimo qo‘zg‘almas π tekislikka parallel bo‘lgan π tekislikda harakatlanadi. Tekislikka parallel harakatdagi jismda π tekislikka perpendikulyar qilib olingan A_1A_2 kesma o‘ziga parallel holda ko‘chadi, ya’ni, kesma ilgarilanma harakatda bo‘ladi. Shu sababli jismning bu kesmada yotgan barcha nuqtalarining harakatini o‘rganish o‘rniga, ulardan birining, masalan, S tekis shakl A nuqtasining harakatini o‘rganish yetarli bo‘ladi. π tekislikka perpendikulyar B_1B_2 kesmaning harakatini o‘rganishda ham xuddi shunday xulosaga kelish mumkin. Shunday qilib, qattiq jismning tekislikka parallel harakatini o‘rganish uchun π_0 (4.1 – rasm) qo‘zg‘almas tekislikka parallel bo‘lgan tekis shaklning π tekislikdagi harakatini o‘rganish kifoya bo‘lar ekan. Tekis shakl harakatlanadigan π tekislik tekis shaklning harakat tekisligi deyiladi (4.1-rasm).



Tekis shakl harakatini undagi kinetik holati aniq bo'lgan nuqta harakatiga bog'lab o'rganish qulay bo'ladi. Bunday nuqta qutb deb ataladi. Tekis shaklning harakat tekisligidagi har qanday ko'chishi quyidagi teorema orqali ifodalanadi: *tekis shaklning harakat tekisligidagi har qanday ko'chishi qutb bilan birgalikdagi ilgarilanma ko'chish, hamda qutb atrofidagi aylanma ko'chishdan tashkil topadi.* Qutb atrofidagi aylanish burchagi qutbni tanlashga bog'liq bo'lmaydi, ilgarilanma harakat qutbni tanlashga bog'liq bo'ladi.



4.2-rasm

52-§. Qattiq jismning tekislikka parallel harakati tenglamalari

Tekis shaklning harakati quyidagi tenglamalar bilan ifodalanadi:

$$x_A=f_1(t),\quad y_A=f_2(t),\quad \varphi_\varphi=a_3(t). \qquad (4.1)$$

Bunda qutb A nuqtaning harakatini aniqlaydigan

$$x_A=f_1(t),\quad y_A=f_2(t), \qquad (4.2)$$

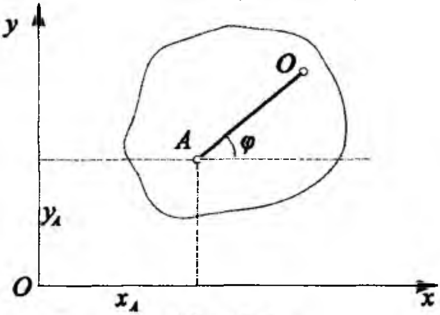
tenglamalar tekis shaklning ilgarilanma harakatini ifodalaydi. Tekis shaklda olingan ixtiyoriy AB kesmani x o'qi bilan tashkil qilgan φ burchagining o'zgarishini ifodalovchi

$$\Phi=f_3(t) \qquad (4.3)$$

tenglama tekis shaklning aylanma harakatini ifodalaydi (4.2-rasm).

Agar qattiq jismning tekislikka parallel harakatida φ=const bo'lsa, tekis shakilda olingan AB kesma doimo o'ziga parallel ravishda harakatlanib boradi va tekis shakil yoki tekis shakil taluqli bo'lgan qattiq jism ilgarilanma harakatda bo'ladi.

Agar qattiq jismning harakati davomida A nuqta (qutb) koordinatalari x_A, y_a lar o'zgarmas holda qolib, φ burchak o'zgarsa, u holda jismning A nuqtasi qo'zg'almasdan qoladi va tekis shakil A nuqta atrofida aylanma harakatda bo'ladi, yani qattiq jism A nuqtadan o'tuvchi va shakl tekisligiga perpendikulyar bo'lgan o'q atrofida aylanma harakatda bo'ladi (4.3- rasm).



4.3- rasm

53-§. Tekis shakilning burchak tezligi va burchak tezlanishi

Tekis shakl qutb atrofida aylanganda uning barcha nuqtalari har onda bir xil burchak tezlik va bir xil burchak tezlanishga ega bo'ladi:

Tekis shakilning aylanish burchagidan vaqt bo'yicha olingan hosila tekis shaklning burchak tezligi deyiladi.

$$\omega = \frac{dt}{at}.$$

Tekis shaklning burchak tezligidan vaqt bo'yicha olingan birinchi tartibli hosila yoki tekis shakl aylanish burchagidan vaqt bo'yicha olingan tartibli hosila tekis shaklning burchak tezlanishi deyiladi.

Tekis shaklning burchak tezligi va burchak tezlanishi qutbning tanlab olinishiga bog'liq bo'lmaydi, chunki tekis shaklning qutb atrofida aylanish burchagi qutbni tanlashga bog‘liq bo‘lmasligini yuqorida aytib o‘tgan edik.

$$\varphi = \frac{d\varphi}{at}, \qquad \varepsilon = \frac{d\omega}{at} = \frac{d^2\varphi}{at^2}. \tag{4.4}$$

Burchak tezlik $\vec{\omega}$ va burchak tezlanish $\vec{\varepsilon}$ vektorlari tekis shakl tekisligiga A qutb orqali perpendikulyar xolda o‘tgan o‘qda yotadi. Agar tekis shaklning qutb atrofidagi aylanma harakati tezlanuvchan bo‘lsa, $\vec{\omega}$ va $\vec{\varepsilon}$ lar bir tomonga, sekinlanuvchan bo‘lsa, qarama – qarshi tomonga yo‘naladi

Takrorlash uchun savollar

1. Qattiq jismning tekislikka parallel harakatini tariflang.
2. Qanday tekislik shaklning harakat tekisligi deyiladi?
3. Qanday nuqta qutb sifatida tanlanadi?
4. Tekis shaklning harakat tekisligidagi har qanday ko‘chishi qanday harakatlardan tashkil topadi?
5. Tekis shaklning tekislikka parallel harakati tenglamalarini yozing.
6. Agar tekis shaklning harakat tekisligida φ=const bo‘lsa tekis shakl qanday harakatda bo‘ladi?
7. Tekis shaklning burchak tezligini ta’riflang.
8. Tekis shaklning burchak tezlanishini ta’riflang.
9. Tekis shakl qutb atrofida tezlanuvchan harakatda bo‘lish shartini ta’riflang.
10. Tekis shakl qutb atrofida sekinlashuvchan harakatda bo‘lish shartini ta’riflang.

54-§. Tekis shaklning harakat tenglamalari, tekis shakl nuqtasining harakat tenglamalari, tekis shakl burchak tezligi va burchak tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko‘rsatmalar

Tekis shaklning harakat tekisligida har qanday ko‘chishi qutb bilan birgalikdagi ilgarilanma harakati hamda qutbdan harakat tekisligiga perpendikulyar ravishda o‘tuvchi o‘q atrofidagi aylanma harakatidan tashkil topadi.

Tekis shaklning harakat tekisligida qo‘zg‘alamas O nuqtani tanlab, *Oxy* qo‘zg‘almas sanoq sistemasini o‘tkazamiz.

Qo‘zg‘almas sanoq sistemasi sifatida markazi qutb deb tanlangan O_1 nuqtada bo‘lgan va tekis shakl bilan bog‘langan $O_1x_1y_1$ o‘qlar sistemasi olinadi.

Bunday holda tekis shakilning harakat tenglamasi quyidagi ko‘rinishda yoziladi:

$$x_{01} = f_1(t), \quad y_{01} = f_2(t), \quad \varphi = f_3(t) \qquad (1)$$

Yozilgan ifodalarda x_{01}, y_{01} - qutb sifatida tanlangan O_1 nuqtaning koordinatalari, φ– qo‘zg‘aluvchan o‘qlar sistemasining qo‘zg‘almas o‘qlar sistemasiga nisbatan burilish burchagi.

(1) tenglamalar sistemasi tekis shaklning ixtiyoriy vaqt onidagi holatini aniqlashga imkon beradi.

Tekis shaklda olingan ixtiyoriy M nuqtaning harakat tenglamalari quyidagi ko‘rinishda yoziladi:

$$\begin{cases} x = x_{o_1} + x_1 \cos\varphi - y_1 \sin\varphi \\ y = y_{o_1} + x \sin\varphi - y_1 \cos\varphi \end{cases} \qquad (2)$$

Bu tenglamalarda: *x,y* – M nuqtaning qo‘zg‘almas o‘qlar sistemasidagi koordinatalari; x_{o_1}, y_{o_1} - M nuqtaning tekis shakil bilan bog‘langan qo‘zg‘luvchan o‘qlar sistemasidagi koordinatalari; φ – qo‘zg‘aluvchan o‘qlar sistemasining burilish burchagi. Shuni ta’kidlash lozimki x_1, y_1 koordinatalar tekis shakilning harakati davomida doimo o‘zgarmas miqdor sifatida saqlanadi, qolgan barcha kattaliklar esa vaqtning funksiyalari hisoblanadi va (1) ifodalarda o‘z aksini topadi.

Tekis shaklning harakat tenglamalari, tekis shakl nuqtasining harakat tenglamasi va traektoriyasini burchak tezligi va burchak tezlanishini aniqlashda quyidagi tartibda amal qilish tavsiya etiladi:

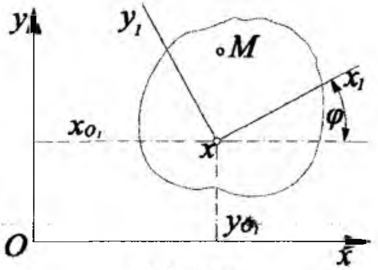
1. Qo‘zg‘almas va qo‘zg‘luvchan o‘qlar sistemalari tanlab olinadi.
2. Tekis shaklning harakat tenglamalari yoziladi.
3. Tekis shakl nuqtasining harakat tenglamasi tuziladi.
4. Tuzilgan harakat tenglamalaridan vaqtni qisqartirib, nuqta traektoriyasining tenglamasi tuziladi.

5.Tekis shakl burilish burchagidan birinchi tartibli hosila olinib, burchak tezlik aniqlanadi.

6.Tekis shakil burchak tezligidan birinchi tartibli xosila hisoblanib, burchak tezlanish aniqlanadi.

55-§. Tekis shakilning harakat tenglamalari, burchak tezligi va burchak tezlanishini aniqlashga doir masalalar

1-masala. Krivoship - polzunli mexanizmda krivoshipning aylanish markazi B polzun traektoriyasidan *a* masofa uzoqlikda joylashgan. Krivoship O nuqta atrofida $\Psi = kt$ qonunga muvofiq aylanadi, bunda *k*- doimiy koeffisient. Krivoship uzunligi OA=r, shatun uzunligi AB=l ekanligini e'tiborga olib (4.4-rasm),



4.4-rasm

AB shatun harakat tenglamasini aniqlang.

Yechilishi. Qo'zg'almas sanoq sistemasi sifatida markazi O nuqtada bo'lgan, *Ox* o'qi gorizontal holda o'ng tomon, *Oy* o'qi vertikal yuqoriga yo'nalgan o'qlar sistemasini tanlaymiz. Qo'zg'aluvchan sanoq sistemasi sifatida markazi A nuqtada bo'lgan, Ax_1 *o'qi AB* shatun bo'ylab, Ay_1 o'qi unga perpendikulyar holda yo'nalgan o'qlar sistemasi olinadi.

Shatun A nuqtasi qutb sifatida tanlaymiz. Qutbning harakat tenglamasi quyidagi ko'rinishda yoziladi:

$$x_A = OA\cos\psi = e\cos kt,$$

$$y_A = OA\sin\psi = r\sin kt.$$

Qutb A nuqtaning aylanma harakatini ifodalovchi tenglamani tuzish uchun AB kesmani *Oy* o‘qiga proeksiyalaymiz .

Agar Ax_1 va *Ox* o‘qlar orasidagi burchakni φ orqali belgilasak 4.4-rasmdan

$$AB\sin\varphi = OA\psi + a.$$

Agar AB=l, OA=r, ψ=kt ekanligini e’tiborga olsak,

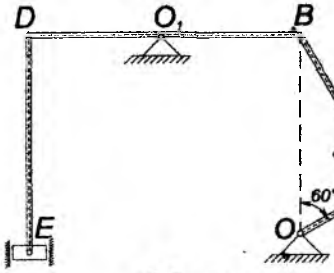
$$\sin\varphi = \frac{r}{l}\, sinkt + \frac{a}{l}.$$

Bu ifodadan AB polzun harakat tenglamalaridan uchinchisi quyidagi ko‘rinishda yozilishi ma’lum bo‘ladi:

$$\varphi = arcsin\left(\frac{r}{l}\, sinkt + \frac{a}{l}\right).$$

Shunday qilib, AB polzunning quyidagi ko‘rinishdagi harakat tenglamalariga esa bo‘lamiz.

2-masala. *OA* krivoship *2 rad/s* burchak tezlik bilan bir tekis aylanadi. Agar *OA=20sm*, $O_1B=O_1D$ bo‘lsa, rasmda ko‘rsatilgan holat uchun nasosning uzatmali mexanizmi *E* porshenining tezligi aniqlansin (4.5a-rasm).

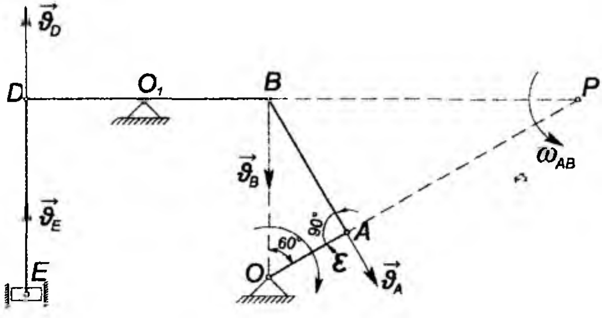


4.5a-rasm

Yechimi: *A* nuqtaning tezligi $\vec{\vartheta}_A$ *OA* krivoshipga perpendikulyar holda ω_{OA} tomon yo‘naladi ($\vec{\vartheta}_A \perp \overline{OA}$).

BD krivoship O_1 nuqta atrofida aylanishi tufayli *D* nuqtaning tezligi ϑ_D O_1D krivoshipga perpendikulyar holda, *B* nuqtaning tezligi $\vec{\vartheta}_B$ esa O_1B kesmaga perpendikulyar holda ω_{BD} tomon yo‘naladi:

$$\vec{\vartheta}_D \perp O_1D, \vec{\vartheta}_B \perp O_1\vec{B}$$



4.5b-rasm

Shuning uchun:

$$\omega_{BD} = \frac{\vartheta_B}{O_1B} = \frac{\vartheta_D}{O_1D}.$$

Lekin $O_1B = O_1D$ bo'lganligi uchun, $\vartheta_B = \vartheta_D$.

Rasmda *VD* krivoship gorizontal holatda bo'lganligi uchun

$$\vec{\vartheta}_D \parallel \vec{\vartheta}_E \; ; \vartheta_D = \vartheta_E \, .$$

Binobarin, $\vartheta_E = \vartheta_B$.

Demak, *E* nuqtaning tezligini aniqlash uchun *B* nuqtaning tezligini aniqlash yetarli ekan.

B nuqtaning tezligini aniqlash uchun berilgan mexanizm *AB* qismining harakatini o'rganamiz. *AB* qism nuqtalari tezliklarining oniy markazi $\vec{\vartheta}_A$ va $\vec{\vartheta}_B$ vektorlarga o'tkazilgan perpendikulyarlarning kesishish nuqtasi P hisoblanadi (4.5b-rasm).

Shuning uchun:

$$\omega_{AB} = \frac{\vartheta_A}{AP} = \frac{\vartheta_B}{BP}.$$

Bundan,

$$\vartheta_B = \frac{\vartheta_A}{AP} \cdot BP$$

Agar,

$$\vartheta_A = \omega_{OA} \cdot OA = 2 \cdot 20 = 40 sm/s;$$

$$OB = \frac{AO}{\cos 60} = 2 \cdot AO = 40 sm;$$

$$OP = \frac{OB}{\cos 60} = 2 \cdot OB = 80 sm;$$

$$AP = OP - OA = 80 - 20 = 60\ sm;$$

$$BP = OBtg60^0 = 40\sqrt{3} = 69{,}3 sm$$

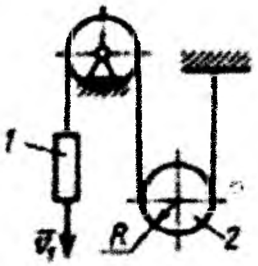
ekanligini e'tiborga olsak,

$$\vartheta_B = \frac{\vartheta_A}{AP} \cdot BP = \frac{40}{60} \cdot 69{,}3 = 46{,}2\ sm/s.$$

Demak,

$$\vartheta_E = \vartheta_B = 46{,}2\ sm/s.$$

3-masala. Agar 1 yukning tezligi ϑ=0.5m/s bo'lsa, radiusi *R=0,1m* bo'lgan qo'zg'luvchan 2 blokning burchak tezligi qancha bo'ladi? (4.6- rasm).



4.6 – rasm

Yechlishi: 2 qo'zg'almas blok A nuqtasining tezligi 1 yuk tezligiga teng bo'ladi, chunki masalada arqon cho'zilmas deb faraz qilinadi.

$$\vartheta_A = \vartheta_1$$

Bunday holda 2 qo'zg'almas blokning burchak tezligi quyidagicha aniqlanadi (2 qo'zg'almas blok B nuqtasining tezligi ϑ_B=0)

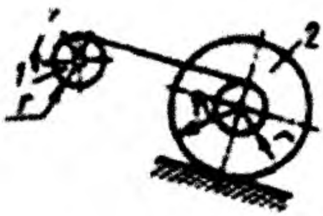
$$\vartheta_A = \omega * 2R$$

Bundan

$$\omega_2 = \frac{\vartheta_A}{2R} = \frac{\vartheta_1}{2R} = \frac{0{,}5}{2 \cdot 0{,}1} = 2{,}5\frac{1}{s}.$$

Demak ω_2=2.5 1/s

4-masala. Radiusi r=0,1m bo'lgan baraban 1 $\varphi=0.5t^2$ qonun bo'yicha aylanib, radiusi R=0,3 m li pog'onali 2 g'ildirak tortadi. 2 g'ildirakning burchak tezlanishini toping (4.7 – rasm).



4.7 – rasm

Yechilishi. 1 – barabanning burchak tezligini aniqlaymiz:

$$\omega_1 = \frac{dy}{at} = 1 \cdot t\frac{1}{s}.$$

U paytda 1 baraban to'g'inida yotgan nuqtasining tezligi quyidagicha topiladi:

$$\vartheta = \omega_1 * r = t * r$$

2- pog'onali g'ildirak A nuqtasining tezligi 1- baraban to'g'inida yotgan nuqtasining tezligiga teng bo'ladi (masalada arqon cho'zilmas deb faraz qilinadi).

$$\vartheta_A = \omega_1 * r = t * r.$$

2- pog'onali g'ildirak burchak tezligini aniqlaymiz:

$$\omega_2 = \frac{\vartheta_A}{R+r} = \frac{t \cdot r}{(R+r)}.$$

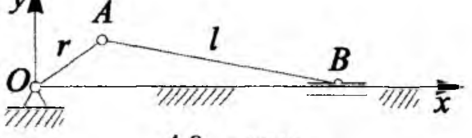
Mazkur g'ildirak burchak tezlanishi uning burchak tezligidan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng bo'ladi:

$$\varepsilon_2 = \frac{d\omega_2}{at} = \frac{r}{R+r} = \frac{0,1}{0,3+0,1} = 0,25\frac{1}{s}.$$

Demak $\varepsilon_2 = 0,25\frac{1}{s}$.

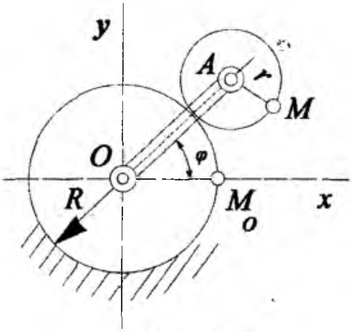
56-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Agar krivoship bir tekis aylansa, shatunning harakat tenglamalari topilsin; krivoship palesining o'qidagi A nuqta qutb deb olinsin; r- krivoship uzunligi; l- shatun uzunligi, ω_o-krivoshipning burchak tezligi. t= 0 bo'lganda α=0 (4.8 - rasm).



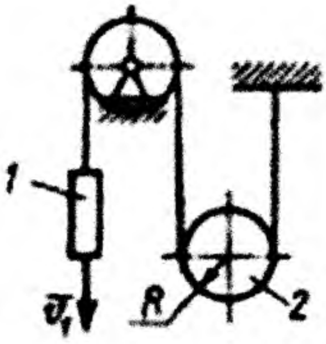
4.8 - rasm

2-muammo. R radiusli qo'zg'almas tishli g'ildirak bo'ylab dumalovchi r radiusli tishli g'ildirak OA krivoship bilan harakatga keltiriladi; krivoship qo'zg'almas tishli g'ildirakning O o'qi atrofida ε_o burchak tezlanish bilan tekis tezlanuvchan aylanma harakat qiladi. Agar t=0 da krivoshipning burchak tezligi ω_o va boshlang'ich aylanish burchagi ω_o=0 bo'lsa, qo'zg'aluvchan tishli g'ildirakning harakat tenglamalari tuzilsin; uning A markazi qutb deb qabul qilinsin (4.9 – rasm).



4.9- rasm

3-muammo. Agar 1 yukning tezligi ϑ=0.5m/s bo'lsa, radiusi *R=0,1m* bo'lgan qo'zg'luvchan 2 blokning burchak tezligi qancha bo'ladi? (4.10- rasm).

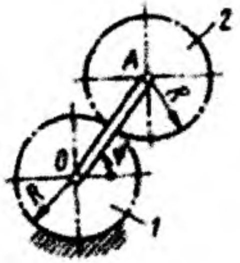


4.10-rasm

4-muammo. Uzunligi 80 sm bo‘lgan AB sterjen shakl tekisligida harakat qilib, A va B nuqtalari $\vartheta_A=0.2$m/s va $\vartheta_B=0.6$m/s tezlikka ega bo‘lsa, sterjenning burchak tezligini aniqlang (4.11- rasm).

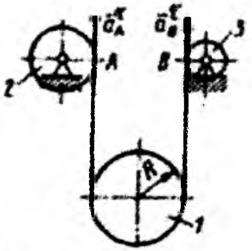
4.11-rasm

5-muammo. OA krivoship $0.5t^2$ qonun bo‘yicha aylansa, 2 g‘ildirakning burchak tezlanishini aniqlang (4.12- rasm).



4.12- rasm

6-muammo. Qo'zg'almas 2 va 3 bloklarning A va B nuqtalari $a_A^\tau = 5m/s^2$ va $a_B^\tau = 10m/s^2$ tangensial tezlanishga ega bo'lsa, radiusi R=0,5m li 1 qo'zg'luvchan blokning burchak tezlanishini toping (4.13- rasm).

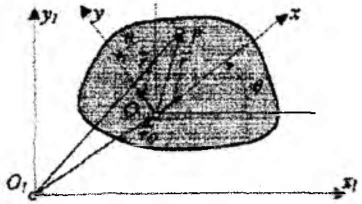


4.13- rasm

57-§. Tekis shakl nuqtasining tezligini qutb usulida aniqlash

Tekis shakl nuqtalarining tezliklari orasidagi bog'lanish quyidagi teorema yordamida ifodalanadi.

Teorema. Ekis shakl ixtiyoriy P nuqtasining tezligi qutb sifatida olingan O nuqtaning tezligi bilan mazkur nuqtaning qutb atrofidagi aylanma bo'ylab harakatidagi chiziqli tezlikning geometrik yig'indisidan iborat bo'ladi (4.14- rasm).



4.14-rasm

Isboti. Agar tekis shaklning o‘z tekisligidagi *Ox* va O_1x_1 o‘qlar orasidagi burchakni θ orqali belgilasak, qo‘zg‘aluvchan *Oxy* o‘qlar sistemasining birlik vektorlari quyidagicha ifodalanadi:

$$\vec{i} = \cos\theta \vec{i}_1 + \sin\theta \vec{j}_1$$

$$\vec{j} = \sin\theta \vec{i}_1 + \cos\theta \vec{j}_1$$

Tekis shaklning qutb atrofidagi aylanma harakati qutbdan shakl tekisligiga perpendikulyar bo‘lgan *Oz* o‘q atrofida yuzaga kelishini e’tiborga olsak, aylanma harakatning burchak tezligini quyidagicha aniqlash mumkin:

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \dot{\theta}\vec{k}_1.$$

Bunday holda qutb sifatida olingan O nuqtaning tezligi quyidagi formula asosida yoziladi:

$$\vec{\vartheta}_0 = \vartheta_{Ox_1} \cdot \vec{i}_1 + \vartheta_{Oy_1} \cdot \vec{j}_1 = \vartheta_{Ox} \cdot \vec{i} + \vartheta_{Oy} \cdot \vec{j}.$$

Natijada rekis shakldа olingan ixtiyoriy P nuqtaning tezligi teoremaga asosan quyidagicha ifodalanadi:

$$\vec{\vartheta}_P = \vec{\vartheta}_o + \vec{\omega} \times \vec{r} = \vartheta_{Ox} \cdot \vec{i} + \vartheta_{Oy} \cdot \vec{j} + \omega\vec{k} \times (x \cdot \vec{i} + y \cdot \vec{j}) =$$

$$= \vec{i}(\vartheta_{Ox} - \omega y) + \vec{j}(\vartheta_{Oy} - \omega x) \quad (4.5)$$

Bu ifodadan P nuqta tezligining qo‘zg‘luvchan O_x va O_y o‘qlardagi proeksiayalari quyidagi ifodalar orqali aniqlanishi ma’lum bo‘ladi:

$$\vartheta_{Px} = \vartheta_{ox} + \omega_y,$$

$$\vartheta_{Py} = \vartheta_{oy} + \omega_x.$$

Tekis shakl nuqtasining tezligini (4.5) formula asosida aniqlash qutb usulida aniqlash deyiladi.

58-§. Tekis shakl ikki nuqtasi tezliklarining proeksiayalariga oid teorema

Tekis shakl nuqtalarining tezliklari orasidagi bog‘lanish quyidagi ikkita teorema orqali aniqlanadi:

1-Teorema. Tekis shakl ixtiyoriy nuqtasining tezligi qutb tezligi bilan, nuqtaning qutb atrofidagi aylana bo‘ylab harakatidagi chiziqli tezligining geometrik yig‘indisiga teng bo‘ladi.

Isboti: Tekis shakl harakatini qo‘zg‘almas *Oxy* koordinatalar sistemasiga nisbatan o‘rganamiz. Agar A nuqta deb qutb sifatida olinsa, A va B nuqtalar radius vektorlari quyidagicha bog‘lanadi:

$$\vec{r}_B = \vec{r}_A + \overline{AB}. \qquad (4.6)$$

Tezlik tarifiga ko‘ra:

(4.7)

Bunda,

$$\frac{d\vec{r}_A}{at} = \vec{\vartheta}_A, \quad \frac{d\overline{AB}}{at} = \vec{\vartheta}_{BA} = \vec{\omega}\times\overrightarrow{AB}. \qquad (4.8)$$

Shuning uchun,

$$\vec{\vartheta}_B = \vec{\vartheta}_A + \vec{\vartheta}_{BA} = \vec{\vartheta}_A + \vec{\omega}\times\overline{AB}. \qquad (4.9)$$

Tekis shakl biror nuqtasining tezligi va tekis shakl aylanma harakatining burchak tezligi berilganda, tekis shakl boshqa biror nuqtasining tezligini (4.9) formula vositasida aniqlash, uni qutb usulida aniqlash deyiladi (4.15b- rasm).

Teorema. Tekis shaklning ikkita nuqtasi tezliklarining shu nuqtalardan o‘tuvchi o‘qdagi proeksiyalari o‘zaro teng bo‘ladi.

(4.9) dan $\vec{\vartheta}_B = \vec{\vartheta}_A + \vec{\vartheta}_{BA}$ (4.10)

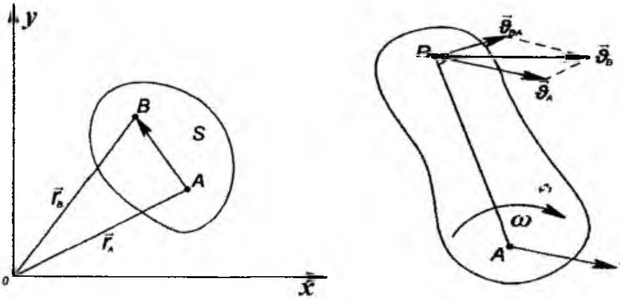
Teoremaga ko‘ra (4.10) ni har ikki tomonini A_x o‘qiga proeksiyalaymiz:

$$(\vec{\vartheta}_B)_x = (\vec{\vartheta}_A)_x + (\vec{\vartheta}_{BA})_x \qquad (4.11)$$

Lekin, $(\vartheta_{BA})_x = 0$, chunki $\vec{\vartheta}_{BA} \perp Ax$.

Shuning uchun 4.16 – rasmdan:

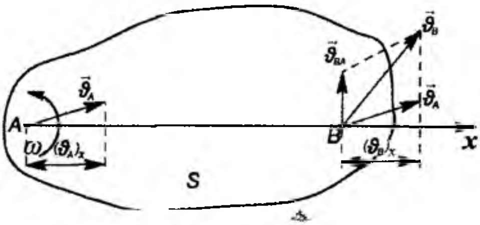
$(\vartheta_B)_x=(\vartheta_A)_x$(4.12)



a) b)

4.15-rasm

Mazkur teoremani isbotlashda 1-teoremadan foydalandik.



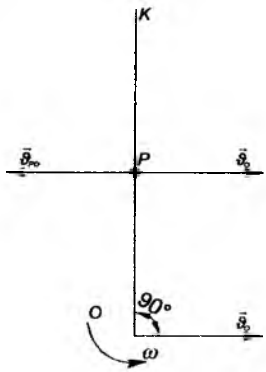
4.16-rasm

59-§. Tezliklarning oniy markazi

Tekis shaklning berilgan onda tezligi nolga teng bo'lgan nuqtasi tezliklar oniy markazi yoki aylanish oniy markazi deyiladi.

Agar tekis shaklning burchak tezligi noldan farqli bo'lsa, albatta tezliklar oniy markazi mavjud bo'ladi (4.17-rasm).

Tekis shakl biror O nuqtasining tezligi $\vec{\vartheta}_O$ va shu O nuqta atrofidagi aylanma hara-



4.17-rasm

katning burchak tezligi ω berilgan bo'lsin. P nuqtaning tezligini topamiz:

$$\vec{\vartheta}_P = \vec{\vartheta}_O + \vec{\vartheta}_{PO}. \tag{4.13}$$

Bunda,

$$\vartheta_{PO} = \omega \cdot OP,\ OP = \frac{\vartheta_O}{\omega}$$

bo'lgani uchun,

$$\vartheta_{PO} = \omega \cdot \frac{\vartheta_O}{\omega} = \vartheta_O,\quad \vec{\vartheta}_{PO} = -\vec{\vartheta}_O.$$

U holda, (4.13) dan $\vec{\vartheta}_p = 0$ bo'ladi. Demak, P nuqta tekis shakl tezliklarining oniy markazi ekan.

Berilgan onda tekis shakl nuqtalari tezliklarining oniy markazini qutb deb olsak, (4.9) formulaga asosan, tekis shakl A, B, C nuqtalarining tezliklari quyidagicha aniqlanadi:

$$\vec{\vartheta}_A = \vec{\vartheta}_P + \vec{\vartheta}_{AP};\ \vec{\vartheta}_B = \vec{\vartheta}_P + \vec{\vartheta}_{BP};\ \vec{\vartheta}_C = \vec{\vartheta}_P + \vec{\vartheta}_{CP}. \tag{4.14}$$

Lekin $\vec{\vartheta}_P = 0.$

Shuning uchun,

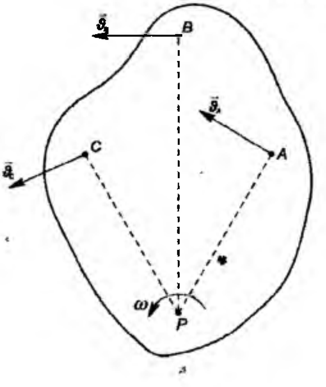
$$\vec{\vartheta}_A = \vec{\vartheta}_{AP};\ \vec{\vartheta}_B = \vec{\vartheta}_{BP};\ \vec{\vartheta}_C = \vec{\vartheta}_{CP}. \tag{4.15}$$

Bunda,

$$\vartheta_A = \omega * AP,\ \vartheta_B = \omega * BP,\ \vartheta_C = \omega * CP \tag{4.16}$$

va

$$\vec{\vartheta}_A \perp \overline{AP},\quad \vec{\vartheta}_B \perp \overline{BP},\quad \vec{\vartheta}_C \perp \overline{CP}.$$



4-18rasm

Demak, biror onda tezliklarining oniy markazi ma'lum bo'lgan tekis shakl nuqtalarining tezliklarini, oniy markaz atrofida aylanma harakatdagi nuqtalarining tezliklari kabi topish mumkin ekan.

Agar tezliklar oniy markazi tekis shakl konturidan tashqarida yotsa, tezliklar oniy markazi uchun tekis shaklga biriktirilgan tekislikning nuqtasi olinadi. (4.16) dan tekis shakl nuqtalarining ayni paytdagi tezliklari orasidagi quyidagi munosabatni aniqlash mumkin:

$$\frac{\vartheta_A}{AP}=\frac{\vartheta_B}{BP}=\frac{\vartheta_C}{CP}. \qquad (4.17)$$

Demak, tekis shakl nuqtalarining tezliklari, shu nuqtalardan tezliklar oniy markazigacha bo‘lgan masofalarga to‘g‘ri proportsional bo‘lar ekan (4.6-rasm).

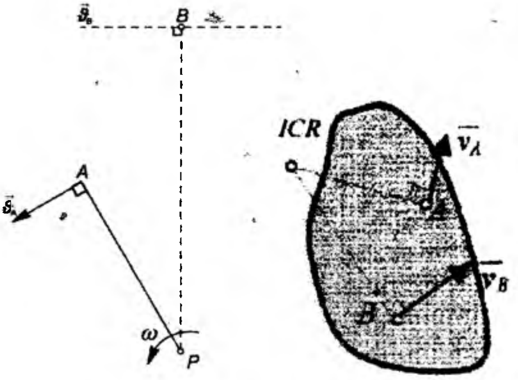
60-§. Bazi hollarda tezliklarning oniy markazini aniqlash

1) Tekis shakl biror A nuqtasining tezligi $\vec{\vartheta}_A$ va B nuqtasining tezligini yo‘nalishi ma’lum bo‘lsin. Bunday holda tekis shakl nuqtalari tezliklarining oniy markazi A va B nuqtalar tezliklariga o‘tkazilgan perpendikulyarlarning kesishgan nuqtasida bo‘ladi (4.19-rasm).

A nuqta tezligining moduli ma’lum bo‘lgani uchun (4.16) dan tekis shaklning burchak tezligini aniqlaymiz:

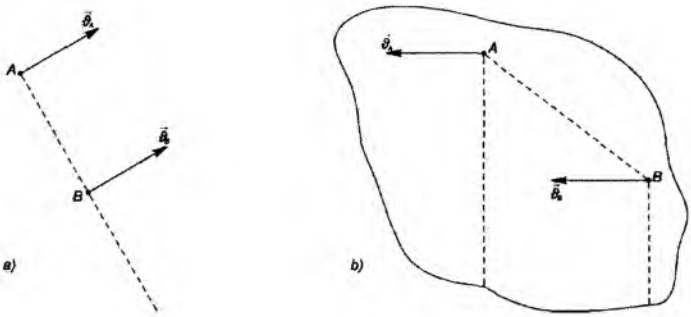
$$\omega=\frac{\vartheta_A}{AP}. \qquad (4.18)$$

AP masofa chizmadan aniqlanadi.



U paytda ϑ_B nuqtaning tezligi quidagicha teng bo‘ladi:

$$\vartheta_B=\omega*BP$$

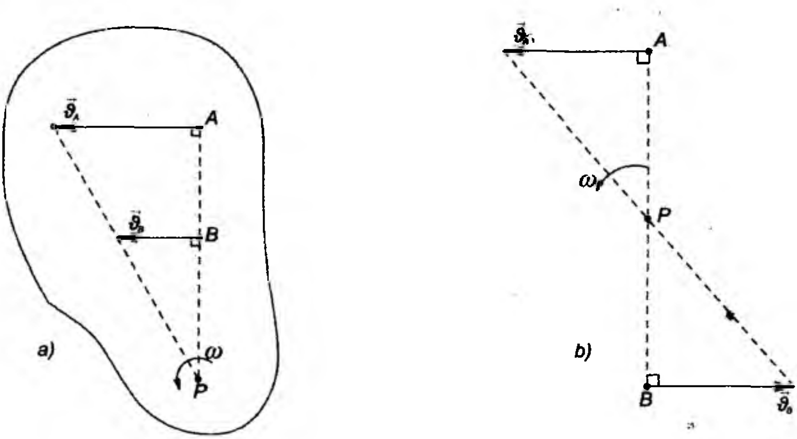


4.20a,b-rasmlar

2) Tekis shakl A va B nuqtalarining tezliklari parallel va AB ga perpendikulyar bo‘lsa, tezliklarning oniy markazini aniqlash uchun tezliklar moduli ham ma’lum bo‘lishi kerak.
(4.17) ga ko‘ra :

$$\frac{\vartheta_B}{\vartheta_A} = \frac{BP}{AP}. \quad (4.19)$$

Shuning uchun ham, A va B nuqtalar tezliklarining uchi oniy markaz orqali o‘tuvchi chiziqda yotadi. Shu chiziqning AB chiziq bilan kesishgan nuqtasi tezliklar oniy markazini ifodalaydi



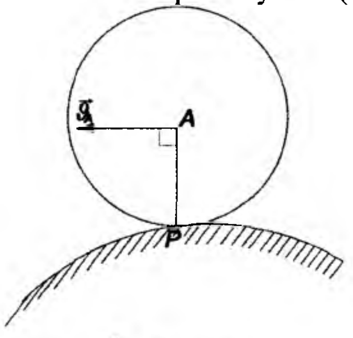
4.21a,b-rasmlar

Agar tekis shakl A va B nuqtalarining tezliklari o‘zaro teng va parallel yo‘nalgan bo‘lsa, u holda tezliklar oniy markazi cheksizlikda bo‘ladi ($AP = \infty$).

Tekis shakl burchak tezligi bunday holda nolga teng bo‘lib, u berilgan onda ilgarilanma harakatda bo‘ladi (4.21a,b-rasmlar):

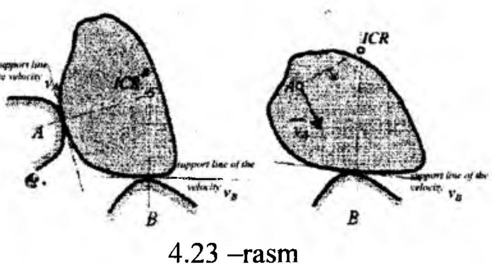
$$\omega = \frac{\vartheta_A}{AP} = \frac{\vartheta_A}{\infty} = 0$$

3) Tekis shakl konturi biror qo‘zg‘almas chiziq ustida sirpanmasdan dumalasa, tekis shakl konturining qo‘zg‘almas chiziqqa tegib turgan nuqtasining tezligi nolga teng bo‘ladi. Shuning uchun oniy markaz shu urinish nuqtasida yotadi (4.22-rasm).



4.22-rasm

4) Tekis shakl konturi A va B qo‘zg‘almas b) rasm yoki B qo‘zg‘almas c) rasm chizma ustida sirpanmasdan dumalasa (4.23-rasm).

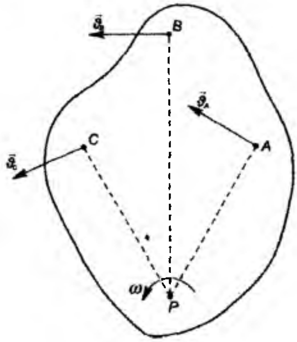


4.23 –rasm

Shakl tezliklarining oniy markazi A va B nuqtalar tezliklariga o‘tkazilgan perpendikulyarlarning kesishgan nuqtasida bo‘ladi.

61-§. Tekis shakl nuqtalarining tezliklarini tezliklarning oniy markazida foydalanib aniqlash

Berilgan onda tekis shakl nuqtalari tezliklarining oniy markazini qutb deb olsak, (4.9) formulaga asosan, tekis shakl A,B,C nuqtalarining tezliklari quyidagicha aniqlanadi:



4-24rasm

$$\vec{\vartheta}_A = \vec{\vartheta}_P + \vec{\vartheta}_{AP};\ \vec{\vartheta}_B = \vec{\vartheta}_P - \vec{\vartheta}_{BP};\ \vec{\vartheta}_C = \vec{\vartheta}_P + \vec{\vartheta}_{CP}$$

(420)

Lekin $\vec{\vartheta}_P = 0$.

Shuning uchun,

$$\vec{\vartheta}_A = \vec{\vartheta}_{AP};\ \vec{\vartheta}_B = \vec{\vartheta}_{BP};\ \vec{\vartheta}_C = \vec{\vartheta}_{CP}. \quad (4.21)$$

Bunda,

$\vartheta_A=\omega*AP$, $\vartheta_B=\omega*BP$, $\vartheta_C=\omega*CP$ (4.22)

va $\vec{\vartheta}_A \perp \overline{AP}$, $\vec{\vartheta}_B \perp \overline{BP}$, $\vec{\vartheta}_C \perp \overline{CP}$.

Demak, biror onda tezliklarining oniy markazi ma'lum bo'lgan tekis shakl nuqtalarining tezliklarini, oniy markaz atrofida aylanma harakatdagi nuqtalarining tezliklari kabi topish mumkin ekan.

Agar tezliklar oniy markazi tekis shakl konturidan tashqarida yotsa, tezliklar oniy markazi uchun tekis shaklga biriktirilgan tekislikning nuqtasi olinadi. (4.22) dan tekis shakl nuqtalarining ayni paytdagi tezliklari orasidagi quyidagi munosabatni aniqlash mumkin:

$$\frac{\vartheta_A}{AP} = \frac{\vartheta_B}{BP} = \frac{\vartheta_C}{CP} \quad (4.23)$$

Demak, tekis shakl nuqtalarining tezliklari, shu nuqtalardan tezliklar oniy markazigacha bo'lgan masofalarga to'g'ri proportsional bo'lar ekan (4.6-rasm).

62-§. Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Umuman, tekis shakl nuqtalarining tezliklarini quyidagi 3 usullarda aniqlash mumkin:

1. Analitik usul.
2. Grafik usul.
3. Grafoanalitik usul.

Mazkur o'quv qo'llanmada tekis shakl nuqtalarining tezliklarini aniqlashni grafoanalitik usuli bilan tanishamiz.

Grafoanalitik usulning o'zi ham ikki yo'ldan iborat.

a) Tekis shakl nuqtalarining tezliklarini qutb usulida aniqlash.

Bu usulda tekis shakl nuqtalarining tezliklari quyidagicha aniqlanadi.

1. Tezligi ma'lum yoki masala shartiga ko'ra aniqlanishi mumkin bo'lgan tekis shakl nuqtasi qutb sifatida tanlanadi.

2.Tekis shaklda tezligining yo'nalishi ma'lum bo'lgan boshqa nuqta aniqlanadi.

3. Bu nuqtaning tezligi tekis shakl nuqtalarining tezliklari haqidagi teorema asosida hisoblanadi.

4. Tekis shaklning shu vaqt onidagi burchak tezligi aniqlanadi.

5. Tekis shakl burchak tezligini bilgan holda, yuqorida bayon etilgan tekis shakl nuqtalarining tezliklari haqidagi teoremadan, tekis shaklning so'ralgan nuqtasining tezligi aniqlanadi.

b) Tekis shakl nuqtalarining tezliklarini tezliklarning oniy markazi orqali aniqlash.

Bu usulda tekis shakl nuqtalarining tezliklari quyidagicha aniqlanadi:

1. Tekis shakl nuqtalari tezliklarining oniy markazi aniqlanadi.

2. Tekis shaklning tezligi ma'lum bo'lgan nuqtasining oniy radiusi aniqlanadi va tezlik modulini oniy radiusga bo'lib, tekis shaklning burchak tezligi topiladi.

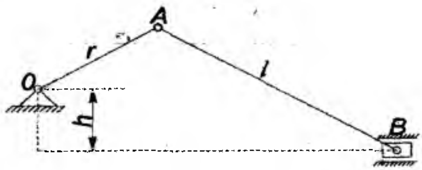
3.Tekis shaklning burchak tezligini bilgan holda, so'ralgan nuqtaning tezligi aniqlanadi.

Takrorlash uchun savollar

1. Tekis shakl nuqtalarining tezliklari orasidagi bog'lanishni ta'riflang.
2. Tekis shakl nuqtasining tezligini qutb usulida aniqlash deb qanday usulga aytiladi?
3. Tekis shakl ikki nuqtasi tezliklarining proeksiyalariga oid teoremani ta'riflang.
4. Tezliklar oniy markazi deb qanday nuqtaga aytiladi?
5. Tekis shakl nuqtalari tezliklarining oniy marazini aniqlash hollarini ko'rsating.
6. Agar tekis shakl A va B nuqtalarining tezliklari teng va parallel yo'nalgan bo'lsa, tezliklar oniy markazi qayerda joylashadi?

63-§. Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalar

1-masala. *O* val atrofida $\omega=1,5$ *rad/s* burchak tezlik bilan aylanuvchi krivoshipning ikkita gorizontal va ikkita vertikal holatida, markaziy

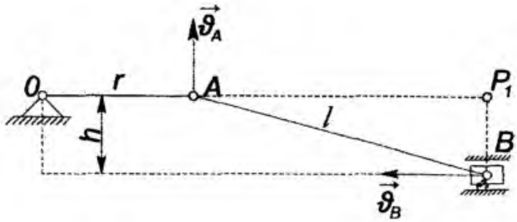


4.25a-rasm

bo'lmagan krivoship mexanizmi *B* polzuni tezligining qancha bo'lishi topilsin; $OA=40$ *sm*, $AB=200$ *sm* (4.25a-rasm).

Yechimi:

1. Krivoshipning birinchi gorizontal holatida *B* polzunning tezligini qancha bo'lishini aniqlaymiz (4.25b-rasm).



4.25b-rasm

Krivoship A nuqtasining tezligi:

$$\vartheta_A = \omega \cdot OA = 1{,}5 \cdot 40 = 60\, sm/s\,; \qquad \vec{\vartheta}_A \perp \vec{OA}$$

B polzun tezligini aniqlash uchun, *AB* shatun nuqtalari tezliklarining oniy markazini aniqlaymiz. Buning uchun $\vec{\vartheta}_A$ va $\vec{\vartheta}_B$ yo‘nalishlariga perpendikulyar chiziqlar o‘tkazib, ularning kesishish nuqtasi P_1 ni topamiz. P_1 nuqta *AB* shatun nuqtalari tezliklarining oniy markazini ifodalaydi. P_1 nuqta qutb sifatida qabul qilinsa, krivoship *A* nuqtasining tezligi quyidagicha yoziladi:

$$v_A = \omega_{AB} * AP_1\,.$$

Bundan,

$$\omega_{AB} = \frac{v_A}{AP_1}.$$

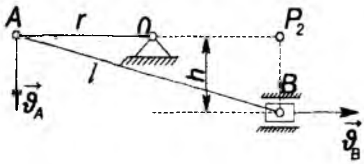
Bunday holda,

$$v_B = \omega_{AB} * BP_1$$

Agar $BP_1 = h$ ekanligini e’tiborga olsak:

$$v_{B_1} = \frac{v_A}{AP_1} \cdot h = \frac{60}{\sqrt{\ell^2 - h^2}} \cdot h = 6{,}03\, sm/s\,.$$

2. Xuddi shunday mulohazalar asosida, krivoshipning ikkinchi gorizontal holatida *B* polzunning tezligini aniqlaymiz (4.25v-rasm).



4.25v-rasm

4.25v-rasmdan ko‘rinib turibdiki, krivoshipning ikkinchi gorizontal holatida ham *B* polzun tezligining miqdori 1 gorizontal holatidagi tezligiga teng bo‘lar ekan:

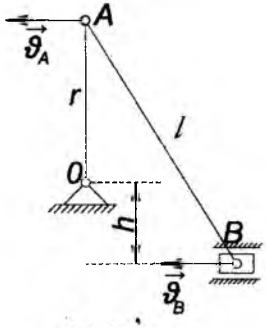
$\vartheta_{B2}=\vartheta_{B1}=6.03 sm/s$

3. Yuqorida keltirilgan mulohazalar asosida, krivoshipning birinchi vertikal holatida *B* polzun tezligining miqdorini aniqlaymiz (4.25g-rasm).

Krivoshipning bunday vertikal holatida *B* polzun tezligining miqdori krivoship *A* nuqtasining tezligiga teng bo'ladi:

$$\vec{\vartheta}_{B_3} = \vec{\vartheta}_A = 60 sm/s.$$

Chunki krivoshipning bunday holatida *AB* polzun oniy ilgarilanma harakatda bo'ladi:

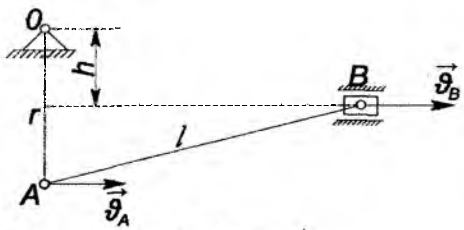


4.25g-rasm

$$AP_3 = \infty , \quad \omega_{AB} = \frac{v_A}{AP_3} = 0 .$$

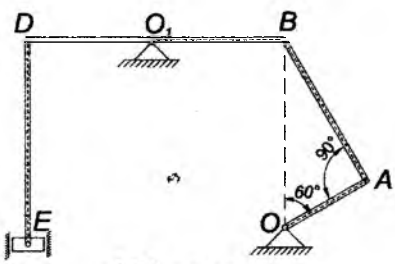
4. Xuddi shunday hol krivoshipning ikkinchi vertikal holatida ham yuzaga keladi (4.25d-rasm). Shuning uchun bu holatda ham:

$$AP_3 = \infty , \quad \omega_{AB} = 0 , \; \vartheta_{B4} = \vartheta_{B3} = 60 \, sm/s .$$



4.25d-rasm

2-masala. *OA* krivoship *2 rad/s* burchak tezlik bilan bir tekis aylanadi. Agar $OA=20sm$, $O_1B=O_1D$ bo‘lsa, rasmda ko‘rsatilgan holat uchun nasosning uzatmali mexanizmi *E* porshenining tezligi aniqlansin (4.26a-rasm).

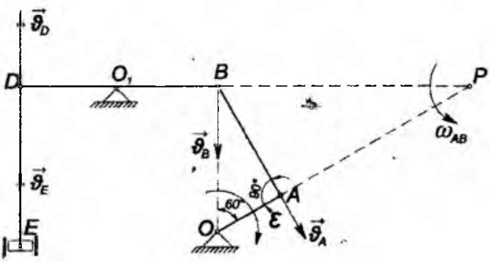


4.26a-rasm

Yechimi: *A* nuqtaning tezligi $\vec{\vartheta}_A$ *OA* krivoshipga perpendikulyar holda ω_{OA} tomon yo‘naladi ($\vec{\vartheta}_A \perp \overline{OA}$).

BD krivoship O_1 nuqta atrofida aylanishi tufayli *D* nuqtaning tezligi ϑ_D O_1D krivoshipga perpendikulyar holda, *B* nuqtaning tezligi $\vec{\vartheta}_B$ esa O_1B kesmaga perpendikulyar holda ω_{BD} tomon yo‘naladi:

$$\vec{\vartheta}_D \perp O_1D, \vec{\vartheta}_B \perp O_1\vec{B}$$



4.26b-rasm

Shuning uchun:

$$\omega_{BD} = \frac{\vartheta_B}{O_1B} = \frac{\vartheta_D}{O_1D}.$$

Lekin $O_1B = O_1D$ bo‘lganligi uchun, $\vartheta_B = \vartheta_D$.

Rasmda *VD* krivoship gorizontal holatda bo‘lganligi uchun

$$\vec{\vartheta}_D \text{ II } \vec{\vartheta}_E ; \vartheta_D = \vartheta_E .$$

Binobarin, $\vartheta_E=\vartheta_B$.

Demak, *E* nuqtaning tezligini aniqlash uchun *B* nuqtaning tezligini aniqlash yetarli ekan.

B nuqtaning tezligini aniqlash uchun berilgan mexanizm *AB* qismining harakatini o‘rganamiz. *AB* qism nuqtalari tezliklarining oniy markazi $\vec{\vartheta}_A$ va $\vec{\vartheta}_B$ vektorlarga o‘tkazilgan perpendikulyarlarning kesishish nuqtasi P hisoblanadi (4.26b-rasm).

Shuning uchun:

$$\omega_{AB}=\frac{\vartheta_A}{AP}=\frac{\vartheta_B}{BP}.$$

Bundan,

$$\vartheta_B=\frac{\vartheta_A}{AP}\cdot BP\ .$$

Agar,

$$\vartheta_A=\omega_{OA}\cdot OA=2\cdot 20=40\,sm/s\ ;$$

$$OB=\frac{AO}{\cos 60}=2\cdot AO=40sm;$$

$$OP=\frac{OB}{\cos 60}=2\cdot OB=80sm;$$

$$AP=OP-OA=80-20=60\ sm;$$

$$BP=OBtg60^0=40\sqrt{3}=69{,}3\,sm$$

ekanligini e’tiborga olsak,

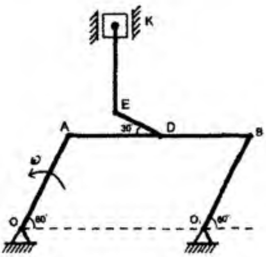
$$\vartheta_B=\frac{\vartheta_A}{AP}\cdot BP=\frac{40}{60}\cdot 69{,}3=46{,}2\ sm/s\ .$$

Demak,

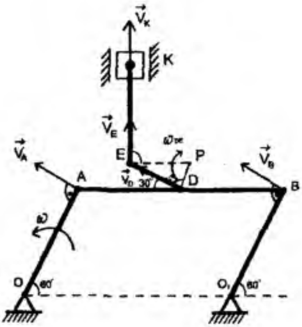
$$\vartheta_E=\vartheta_B=46{,}2\text{sm/s}$$

3-masala. Sharnirli *$OABO_1$* parallelogramning AB sterjeni o‘rtasida D nuqtaga K polzunni ilgarilanma-qaytma harakatga keltiruvchi DE sterjen sharnir yordamida birlashtirilgan. Agar $OA=O_1B=2DE=20$ sm bo‘lsa, mexanizmning rasmda tasvirlangan holati uchun K polzunning tezligi va DE sterjenning burchak tezligi aniqlansin; OA zvenoning berilgan paytdagi burchak tezligi 1 rad/s.

Yechilishi: Chizmada sharnirli parallelogramning A va B nuqtalari tezliklarining yo‘nalishlarini ko‘rsatamiz (4.28- rasm):



4.27 – rasm



4.28 – rasm

$\vec{V}_A \perp OA$; $\vec{V}_A \perp O_1B$.

Sharnirli parallelogramda

$$OA = O_1B$$

Shuning uchun sharnirli parallelogramning harakati davomida OA va O_1B krivoshiplarning burilish burchaklari o‘zaro teng bo‘ladi. Keltirilgan mulohazalar sharnirli parallelogramda AB serjen tekislikka parallel harakatda bo‘lishini e’tirof etadi.

Bu hol $\vartheta_A = \vartheta_B$ bo‘lishini taqozo etadi.

D nuqta ham AB sterjinga taluqli. Shuning uchun

$$\vec{\vartheta}_D = \vec{\vartheta}_A = \vec{\vartheta}_B;$$

$$\vec{\vartheta}_D \parallel \vec{\vartheta}_A \parallel \vec{\vartheta}_B.$$

Agar <AED=30° ekanliginni e’tiborga olsak, D nuqtaning tezligi $\vec{\vartheta}_D$ DE shatun bo‘ylab yo‘nalishi ma’lum bo‘ladi.

Mexanizmda K polzun vertikal yo‘nalishda harakatlanadi. K va E nuqtalar KE sterjinga taaluqli. KE sterjin vertikal yo‘nalishda ilgarilanma- qaytalanma harakatda bo‘lishi tufayli K nuqtaning tezligi E nuqta tezligiga teng bo‘ladi.

$$\vec{\vartheta}_K = \vec{\vartheta}_E.$$

KE sterjin E nuqtasining tezligini aniqlash uchun DE shatunning harakatini o‘rganamiz. DE shatun shakl tekisligida harakatlanadi. DE shatun nuqtalari tezliklarining vektorlariga perpendikulyar chizmalar tushiramiz. Ularning kesishish nuqtasi tezliklarning oniy markazini ifodalaydi (4.28-rasm):

Bunday holda DE shatun burchak tezligi quyidagicha aniqlanadi:

$$\omega_{DE} = \frac{V_D}{DP} = \frac{V_E}{EP}.$$

Bunda

$$V_D = V_A = \omega_{OA} = 1*20 = 20rad/s$$

$$DP = DE \tan 30^0 = \frac{10\sqrt{3}}{3} = 5{,}8\,sm.$$

$$EP = \frac{DE}{\cos 30^0} = \frac{10 \cdot 2 \cdot 10}{\frac{\sqrt{3}}{2}\sqrt{3}} = 11{,}5\,sm.$$

Aniqlangan kattaliklarni hisobga olsak

$$\omega_{ED} = \frac{V_D}{DP} = \frac{20}{5{,}8} = 3{,}5\frac{1}{s}.$$

4-masala. 1 qo‘zg‘aluvchi va 2 qo‘zg‘almas bloklar cho‘zilmaydigan ip bilan bog‘langan. Ipning uchiga biriktirilgan K yuk $x=2t^2$ m qonun bilan vertikal bo‘ylab pastga tushadi. t= 1 s bo‘lgan paytda rasmda tasvirlangan holat uchun harakatlanuvchi blok gardishida yotuvchi C, D, B va E nuqtalarning tezliklari topilsin; qo‘zg‘aluvchi 1 blok radiusi 0,2 m ga teng, CD⊥BE. Shuningdek, 1 blokning burchak tezligini ham toping.

Yechilishi: K yuk tezligini aniqlaymiz

$$\vartheta_K = \frac{Dx_K}{dt} = (2t^2)^I = 4t,$$

$$t_1 = 1\,s.da$$

$$\vartheta_k = 4*1 = 4m/s.$$

$\vec{\vartheta}_K$ K nuqtaga qo‘yilgan bo‘lib, vertikal pastga yo‘nalgan.

Chizmadan harakatlanuvchan 1- blok D nuqtasining tezligi K yuk tezligiga teng bo‘lishi ma’lum:

$$\vartheta_D = \vartheta_K = 4m/s$$

1 – blok uchun C nuqta tezliklar oniy markazini ifodalaydi. Shuning uchun

$$\vartheta_C = 0$$

Bunday holda 1 – blok burchak tezligi quyidagicha aniqlanadi:

ω_1 ning yo'nalishi $\vec{\vartheta}_D$ yo'nalishi orqali aniqalanadi.

1 – qo'zg'aluvchi blok B va E nuqtalarining tezliklari mazkur nuqatalarning tezliklar oniy markazi C nuqta atrofidagi aylanma harakat tezligi kabi topiladi:

$$V_B=\omega*CB$$

$V_E=\omega*CE$

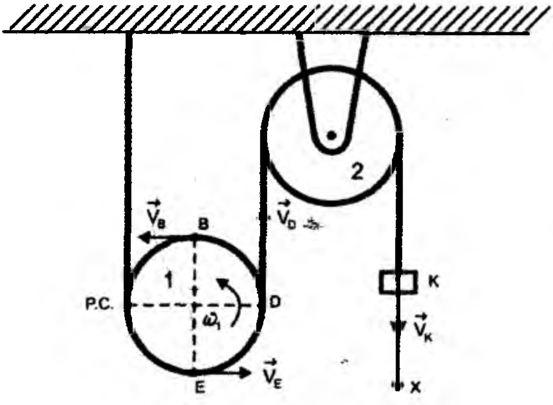
Bunda

$$CB = R_1\sqrt{2} = 0{,}2\cdot 1{,}41 = 0{,}28m,$$
$$CE = R_1\sqrt{2} = 0{,}2\cdot 1{,}41 = 0{,}28m,$$

ekanligini e'tiborga olsak,

$$V_B=10*0.28m/s$$
$$V_E=10*0.28=2.8m/s$$

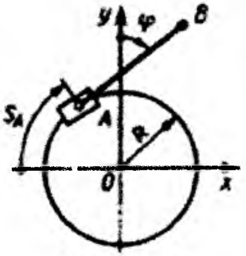
bo'ladi. $\vec{V}_B$ *va* $\vec{V}_E$ lar yo'nalishlari (4.29- rasm) da ko'rsatilgan.



4.29 – rasm

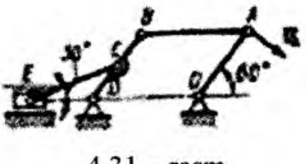
64-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. AB sterjenning A nuqtasi radiusi R=1m bo'lgan aylana bo'ylab $S_A=1,05t$ qonun bo'yicha harakat qiladi. Bir vaqtning o'zida sterjen φ=t qonun bilan aylanadi. Agar sterjenning uzunligi AB=1m bo'lsa, $t_1=1s$ paytda uning B nuqtasi tezligining Oy o'qiga proyeksiyasini aniqlang(4.30 –rasm).



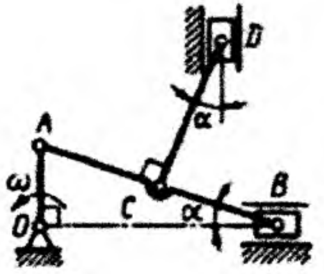
4.30 –rasm

2-muammo. Sharnirli parallelogram $OABD$ ga CE shatun biriktirilgan bo'lib, uning uchida E polzun harakatlanadi. Agar A nuqtaning tezligi 0,4m/s va parallelogrammning o'lchamlari $OA=BD$=20sm, $BC=BD/2$ bo'lsa, E polzunning tezligini toping (4.31 – rasm).



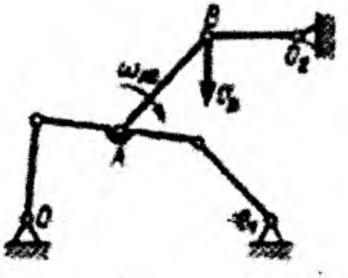
4.31 – rasm

3-muammo. Uzunligi 0,2m bo'lgan OA krivoship ω=8rad/s burchak tezlik bilan tekis aylanadi. AB shatunning C nuqtasiga CD shatun biriktirilgan. Berilgan holat, α=20° uchun D polzunning tezligini aniqlang (4.32- rasm).



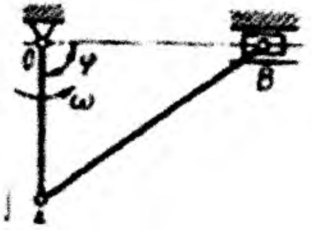
4.32 – rasm

4-muammo. Berilgan paytda B nuqtaning tezligi 20m/s va AB zvenoning burchak tezligi 10rad/s bo'lsa, B nuqtadan AB zvenoning tezliklar oniy markazigacha bo'lgan masofani aniqlang (4.33- rasm).



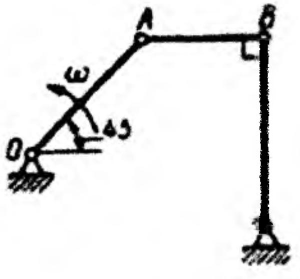
4.33- rasm

5-muammo. Mexanizmning OA krivoshipi tekis aylanma harakat qilib, OB yo'nalishga φ=90° burchak tashqil qilgan paytda B pozundan AB zveno tezliklari oniy markazigacha bo'lgan masofani toping (4.34- rasm).



4.34- rasm

6-muammo. Uzunligi AB=0,6m bo'lgan mexanizmning krivoshipi ω=10rasd/s burchak tezlik bilan aylansa, shakilda ko'rastilgan holat uchun A nuqtada AB sterjenning tezliklar oniy markazigacha bo'lgan masofani toping (4.35- rasm).



4.36 – rasm

65-§. Tekis shakl nuqtasining tezlanishi

Tekis shakl nuqtalarining tezlanishlari orasidagi bog'lanish quyidagi teorema yordamida aniqlanadi:

Teorema. *Tekis shakl ixtiyoriy nuqtasining tezlanishi qutbning tezlanishi bilan, mazkur nuqtaning qutb atrofida aylanishidagi tezlanishining geometrik yig'indisiga teng.*

Ma'lumki, A nuqtani qutb deb olsak, tekis shakl ixtiyoriy B nuqtasining tezligi (4.8) formula orqali aniqlanar edi:

$$\vec{\vartheta}_B = \vec{\vartheta}_A + \vec{\omega} \times \overline{AB}.$$

B nuqtaning tezlanishini aniqlash uchun (4.8) dan vaqt bo'yicha hosila olamiz:

$$\vec{a}_B = \frac{d\vec{\vartheta}_B}{dt} = \frac{d\vec{\vartheta}_A}{dt} + \frac{d\vec{\omega}}{dt} \times \overline{AB} + \vec{\omega} \times \frac{d\overline{AB}}{dt}. \quad (4.24)$$

Bu yerda,

$$\frac{d\vec{\vartheta}_A}{dt} = \vec{a}_A, \quad \frac{d\vec{\omega}}{dt} = \vec{\varepsilon}, \quad \frac{d\overline{AB}}{dt} = \vec{\vartheta}_{BA} = \vec{\omega} \times \overline{AB}.$$

Shuning uchun,

$$\vec{a}_B = \vec{a}_A + \vec{\varepsilon} \times \overline{AB} + \vec{\omega} \times \vec{\vartheta}_{BA}, \quad (4.25)$$

bunda $\vec{a}_A$ – *A* nuqtaning tezlanishi; $\varepsilon \times \overline{AB} = \vec{a}^{\tau}_{BA}$ – *B* nuqtaning A qutb atrofida aylanishidagi aylanma tezlanishi; $\vec{\omega} \times \vec{\vartheta}_{BA} = \vec{a}^{n}_{BA}$ B nuqtaning A qutb atrofida aylanishidagi markazga intilma tezlanishi.

Shuning uchun,

$$\vec{a}_B = \vec{a}_A + \vec{a}^{\tau}_{BA} + \vec{a}^{n}_{BA}.$$

Agar

$$\vec{a}^{\tau}_{BA} + \vec{a}^{n}_{BA} = \vec{a}_{BA}$$

ekanligini e'tiborga olsak,

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} \quad (4.26)$$

ifodaga ega bo'lamiz.

$\vec{a}_{BA}$ ning moduli quyidagicha aniqlanadi:

$$a^{\tau}_{BA} = AB \cdot \varepsilon; \qquad a^{n}_{BA} = AB \cdot \omega^2;$$

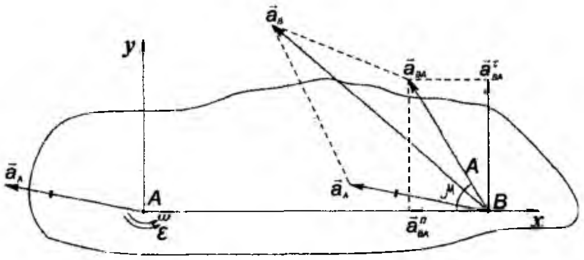
$$a_{BA} = \sqrt{(a^{\tau}_{BA})^2 + (a^{n}_{BA})^2} = AB\sqrt{\varepsilon^2 + \omega^4} \quad (4.27)$$

$\vec{a}_{BA}$ ning yo'nalishi esa quyidagicha aniqlanadi:

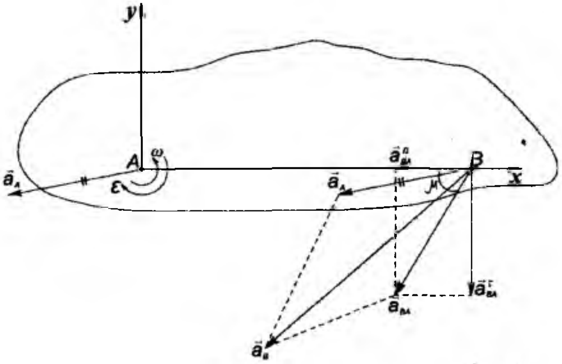
$$tg\mu = \frac{|\varepsilon|}{\omega^2}. \quad (4.28)$$

B nuqtaning A qutb atrofida aylanishi tezlanuvchan bo'lganda, B nuqtaning tezlanishi 4.37a-rasmda, sekinlanuvchan bo'lganda, 4.37b-rasmda ko'rsatilgan.

Masala yechishda, B nuqtaning tezlanishini proeksiyalash usulida aniqlash qulay bo'ladi. Buning uchun o'qlardan birini, masalan, x o'qni aylanish radiusi bo'ylab, ikkinchisini esa, unga perpendikulyar ravishda o'tkazish maqsadga muvofiq bo'ladi.



4.37a-rasm



4.37b-rasm

Davomi: Tekislik ixtiyoriy P nuqtasining tezlanishini, qutb sifatida O nuqtani tanlab olib 4.14 - rasmdagi quyidagi formula yordamida ham aniqlash mumkin:

$$\vec{a}_p = \vec{a}_o + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \quad (4.29)$$

Agar

$$\vec{a}_o = a_{o_x} + a_{o_y}\vec{j};$$

$$\vec{\varepsilon} = \varepsilon\vec{k}; \qquad \vec{\omega} = \omega\vec{k};$$

$$\vec{r} = x \cdot \vec{i} + y\vec{j}$$

Ekanligini e'tiborga olsak (4.29) ifoda quyidagi ko'rinishda yoziladi:

$$\vec{a}_p = \vec{i}\left(a_{o_x} - \varepsilon y - \omega^2 x\right) + \vec{j}\left(a_{o_y} + \varepsilon x - \omega^2\right).$$

66-§. Tezlanishlarning oniy markazi va undan foydalanib tekis shakl nuqtalarining tezlanishlarini aniqlash

Tekis shaklning berilgan ondagi tezlanishi nolga teng bo'lgan nuqtasi (yoki tekis shaklga bog'langan va u bilan birga harakatlanuvchi tekislikning nuqtasi) tezlanishlarning oniy markazi deyiladi.

Tezlanishlarning oniy markazini aniqlash uchun tekis shakl biror A nuqtasining tezlanishi $\vec{a}_A$ va tekis shaklning burchak tezligi ω hamda burchak tezlanishi ε berilgan bo'lishi kerak.

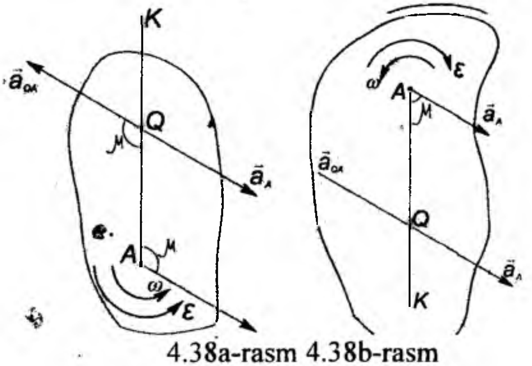
Dastlab,

$$tg\mu = \frac{|\varepsilon|}{\omega^2}$$

formula orqali μ burchak topiladi. So'ngra, tezlanuvchan aylanma harakatda $\vec{a}_A$ vektorga harakat yo'nalishida, sekinlanuvchan aylanma harakatda, harakat yo'nalishiga teskari yo'nalishda μ burchak ostida to'g'ri chiziq o'tkazib, A nuqtadan

$$AQ = \frac{a_A}{\sqrt{\varepsilon^2 + \omega^4}} \qquad (4.30)$$

masofa uzoqlikda yotuvchi Q nuqtani aniqlaymiz (4.12a,b-rasmlar). Bu nuqta tekis shakl nuqtalari tezlanishlarining oniy markazini ifodalaydi.



4.38a-rasm 4.38b-rasm

Haqiqatdan:

$$\vec{a}_Q = \vec{a}_A + \vec{a}_{QA}; \quad a_{QA} = QA\sqrt{\varepsilon^2 + \omega^4} = \vec{a}_A \,.$$

$\vec{a}_{QA}$ miqdor jihatdan $\vec{a}_A$ ga teng, yo'nalishi esa, $\vec{a}_A$ ga qarama – qarshi. Shu sababli,

$$\vec{a}_Q = \vec{a}_A + \vec{a}_{QA} = 0 \,.$$

Agar tekis shakl nuqtalari tezlanishlarining oniy markazini qutb deb olsak, tekis shakl ixtiyoriy B nuqtasining tezlanishi (4.26) va (4.27) formulalarga asosan:

$$\vec{a}_B = \vec{a}_{BQ} \tag{4.31}$$

va

$$a_B = BQ\sqrt{\varepsilon^2 + \omega^4} \tag{4.32}$$

bo'ladi.

(4.32) dan ko'rinib turibdiki, tekis shakl nuqtalarining berilgan ondagi tezlanishlari, mazkur nuqtalardan tezlanishlarning oniy markazigacha bo'lgan masofalarga mutanosib bo'lar ekan:

$$\frac{a_B}{BQ} = \frac{a_A}{AQ} = \frac{a_C}{CQ} = \dots = \sqrt{\varepsilon^2 + \omega^4} \,. \tag{4.33}$$

Takrorlash uchun savollar.

1. Tekis shakl nuqtalarining tezlanishlari orasidagi bog'lanish qanday ta'riflanadi?

2. Tekis shakl ixtiyoriy nuqtasining tezlanishini aniqlash uchun tekis shakl harakatini harakterlovchi qanday kinematik harakteristikalar ma'lum bo'lishi lozim?

3. Tezlanishlarning oniy markazi deb qanday nuqtaga aytiladi?

4. Tekis shakl nuqtalarining berilgan ondagi tezlanishlari va bu nuqtalardan tezlanishlar oniy markazigacha bo'lgan masofalar orasidagi qanday munosabat mavjud?

5. Tekis shakl nuqtalari tezlanishlarining oniy markazini aniqlash uchun qanday kattaliklar ma'lum bo'lishi lozim?

6. Tekis shakl nuqtalarining berilgan ondagi tezlanishlari va bu nuqtalardan tezlanishlarning oniy markazigacha bo'lgan masofalar orasida qanday bog'lanish mavjud?

67-§. Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Tekis shakl nuqtalari tezliklarining oniy markazini aniqlash usullaridan foydalanib, berilgan masalada, tekis shakl nuqtalari tezliklarining oniy markazi aniqlanadi.

2. Tekis shaklning burchak tezligini bilgan holda, tekis shakl ikkinchi nuqtasining birinchi nuqta atrofidagi aylanma harakatidagi markazga intilma tezlanishi topiladi.

3. Ikkinchi nuqtaga uning tezlanishini tashkil etuvchi tezlanishlar vektorlari qo'yiladi. Agar birinchi nuqta A, ikkinchi nuqta B bo'lsa:

$$\vec{a}_B = \vec{a} + \vec{a}_{BA} = \vec{a}_A^{\tau} + \vec{a}_A^{n} + \vec{a}_{BA}^{ayl} + \vec{a}_{BA}^{mi}.$$

4. Koordinata o'qlarini o'tkazib, yuqoridagi vektor tenglikning har ikki tomoni koordinata o'qlariga proeksiyalanadi.

5. Hosil bo'lgan proeksiyalar tenglamalaridan noma'lum $\vec{a}_{BA}^{ayl}$ va $\vec{a}_B$ lar aniqlanadi.

6. Proeksiyalar tenglamalaridan topilgan a_{BA}^{ayl} tezlanish modulini bilgan holda, tekis shakl burchak tezlanishi aniqlanadi:

$$a_{BA}^{ayl} = \varepsilon \cdot AB;$$

$$\varepsilon = \frac{a_{BA}^{ayl}}{AB}.$$

7. Tekis shakl burchak tezligi va burchak tezlanishini bilgan holda, tekis shakl nuqtalarining tezlanishlari haqidagi teorema yordamida, so'ralgan ixtiyoriy nuqtaning tezlanishi aniqlanadi.

Izoh: Tekis shaklda, $\vec{a}_B$ va $\vec{a}_{BA}^{ayl}$ larning modullarini, B nuqtada tanlangan masshtabda chizilgan, tomonlari tashkil etuvchi tezlanishlar, yopuvchi tomoni esa nuqtaning tezlanishi bo'lgan ko'p burchakdan, grafik usulda aniqlash mumkin.

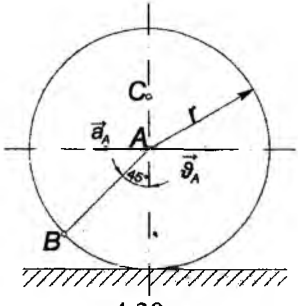
68-§. Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalar

1-masala. 4-masala. Radiusi $r_I = 30\ sm$. bo'lgan g'ildirak yo'l-ning to'g'ri chiziqli gorizontal uchastkasida sirg'anmay dumalaydi. Bu paytda g'ildirak markazining tezligi $\vartheta_A = 50\ m/s$, tezlanishi $a_A = 30\ m/s^2$, $AS = 10\ sm$.

G'ildirak *B* va *C* nuqtalarining tezligi va tezlanishi aniqlansin (4.39a-rasm).

***Yechimi:** 1. Nuqtalarning tezliklarini va g'ildirak burchak tezligini aniqlash.*

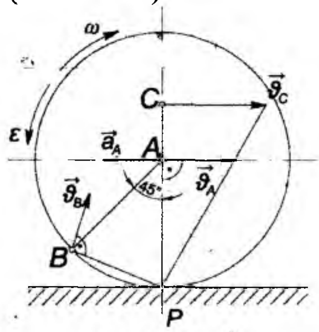
Masala shartida g'ildirak markazi *A* nuqtaning tezligi $\vec{\vartheta}_A$ berilgan.



4.39a-rasm

G'ildirakning qo'zg'almas chiziqqa tegib turgan nuqtasining tezligi nolga teng bo'lishi sababli, g'ildirak nuqtalari tezliklarining oniy markaz shu urinish nuqtasida yotadi (4.39b-rasm).

Berilgan onda g'ildirak nuqtalari tezliklarining oniy markazi *P* nuqtani qutb deb olsak, g'ildirak nuqtalarining shu ondagi tezliklarini, oniy markaz atrofida aylanma harakatdagi jism nuqtalarining tezliklari kabi topish mumkin bo'ladi:



4.39b-rasm

$$\vartheta_A = \omega * PA$$

$$\vartheta_B = \omega * PB$$

$$\vartheta_C = \omega * PC$$

yoki

$$\frac{\vartheta_A}{P_A}=\frac{\vartheta_B}{P_B}=\frac{\vartheta_C}{P_C}.$$

Masala shartiga ko‘ra:

$$PA=r=30\,sm;\quad PB=\sqrt{r^2+r^2-2r^2\cos 45^0}=22{,}8\,sm.$$

$$PC=r+AC=30+10=40\,sm.$$

Shuning uchun,

$$\vartheta_B=\frac{\vartheta_A\cdot PB}{PA}=38{,}1\,sm/s;$$

$$\vartheta_C=\frac{\vartheta_A\cdot PC}{PA}=66{,}7\,sm/s.$$

G‘ildirak nuqtalarining tezliklarini g‘ildirakning burchak tezligi orqali ham topish mumkin:

$\vartheta_A=\omega_F*PA$

Bundan,

$$\omega_1=\frac{\vartheta_A}{PA}=\frac{50}{30}=1{,}67\text{ rad/s}.$$

Burchak tezlikning yo‘nalishi $\vec{\vartheta}_A$ yo‘nalishi orqali aniqlanadi (4.39b-rasm).

Bunday holda g‘ildirak *B* va *S* nuqtalarining tezligi quyidagilarga teng bo‘ladi:

$$\vartheta_B=\omega_g\cdot PB=1{,}67\cdot 22{,}8=38{,}1\,sm/s.$$

$$\vartheta_C=\omega_g\cdot PC=1{,}67\cdot 40=66{,}7\,sm/s.$$

II. *G‘ildirak nuqtalarning tezlanishlari va g‘ildirak burchak tezlanishini aniqlash.*

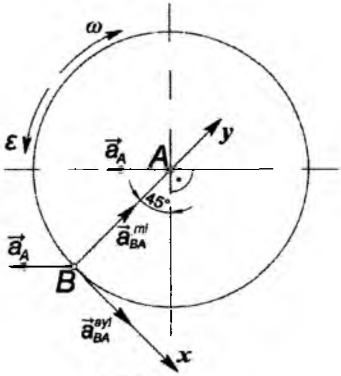
Masala shartida *A* nuqtaning tezlanishi a_A berilgan.

Tekis shakl nuqtalarining tezlanishlari haqidagi teoremaga asosan:

$$\vec{a}_B = \vec{a}_B + a_{BA}^{mi} + a_{BA}^{ayl}$$

Bunda *A* nuqta qutb sifatida qabul qilindi.

G‘ildirakning *A* qutb atrofida aylanma harakatida *B* nuqtasining markazga intilma tezlanishi:



4.39v-rasm

$$a_{BA}^{mi} = \omega_g^2 \cdot BA = 83{,}7\ sm/s^2 .$$

$\vec{a}_{BA}^{mi}$ vektor *B* nuqtadan *A* nuqtaga qarab yo‘naladi.

G‘ildirakning *A* qubt atrofida aylanma harakatida *B* nuqtasining aylanma tezlanishi:

$$a_{BA}^{ayl} = \varepsilon_g \cdot BA$$

G‘ildirakning burchak tezlanishini aniqlaymiz:

$$\varepsilon_g = \frac{d\omega g}{at} = \frac{d}{at}\left(\frac{v_A}{PA}\right) = \frac{1}{PA}\frac{dv_A}{at} = \frac{a_A}{PA} = 1\ rad/s^2 .$$

Shuning uchun,

$$a_{BA}^{ayl} = \varepsilon_g \cdot BA = 30\ sm/s^2 .$$

$\vec{a}_{BA}^{ayl}$ vektor g‘ildirakning *B* nuqtasiga ε_g yo‘nalishida o‘tkazilgan urinma bo‘ylab yo‘naladi (4.39v-rasm).

B nuqtaning tezlanishini proeksiyalash yo‘li bilan aniqlaymiz:

$$(a_B)_x = a_{BA}^{ayl} - a_A \cos 45^0 = 30 - 30 \cdot 0{,}71 = 8{,}7\ sm/s^2;$$

$$(a_B)_y = a_{BA}^{mi} - a_A \cos 45^0 = 83{,}7 - 30 \cdot 0{,}71 = 62{,}4\ sm/s;$$

$$a_B = \sqrt{(a_B^2)_x^{\ 2} + (a_B^2)_y^{\ 2}} = \sqrt{75{,}69 + 3893{,}76} = 63\ sm/s^2.$$

G‘ildirak *C* nuqtasining tezlanishini aniqlaymiz.

Tekis shakl nuqtalarining tezlanishlari haqidagi teoremaga asosan:

$$\vec{a}_c = \vec{a}_A + \vec{a}_{CA},$$

yoki

$$\vec{a}_c = \vec{a}_A + \vec{a}_{CA}^{mi} + \vec{a}_{CA}^{ayl}.$$

G'ildirakning A qutb atrofida aylanma harakatida C nuqtasining markazga intilma tezlanishi:

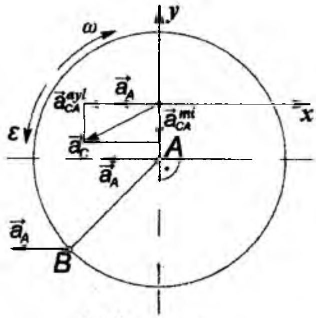
$$a_{CA}^{mi} = \omega_g^2 \cdot CA = 16{,}7\ sm/s^2$$

G'ildirakning A qutb atrofida aylanma harakatida S nuqtasining aylanma tezlanishi:

$$a_{CA}^{ayl} = \varepsilon_g \cdot CA = 10\ sm/s^2.$$

$\vec{a}_{CA}^{mi}$, $\vec{a}_{CA}^{ayl}$ vektorlar 4.39g-rasmda ko'rsatilgan.

C nuqtaning tezlanishini ham proeksiyalash yo'li bilan aniqlaymiz:



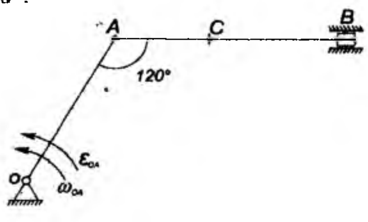
4.39g-rasm

$$(a_c)_x = -a_A - a_{CA}^{ayl} = -30 - 10 = -40\ sm/s^2,$$

$$(a_c)_y = -a_{CA}^{mi} = -16{,}7\ sm/s^2,$$

$$a_c = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 43{,}34 sm/s^2.$$

2-masala. Mexanizmning berilgan holati uchun A,B,C nuqtalarining tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zvenoning burchak tezligi va burchak tezlanishi topilsin (4.40-rasm)



4.40-rasm

Masalada: OA=40sm
AB=80sm,AC=30sm
ω_{OA}=2rad/s, ε_{OA}=6rad/s^2

Yechish.

1. Nuqtalarning tezliklarini va zvenoning burchak tezligini aniqlash.

Mexanizmning berilgan harakatida OA krivoship A panjasi tezligining modulini hisoblaymiz:

$$\vartheta_A=\omega_{OA}*OA=2*80sm/s \quad (1.1)$$

A nuqtaning tezligi $\vec{\vartheta}_A$ OA krivoshipga perpendikulyar holda ω_{OA} yo'nalishi bo'yicha yo'naladi.

B polzunning tezligi gorizontal holda B nuqtadan A nuqta tomon yo'nalgan. AB shatun nuqtalari tezliklarining oniy markazi P_{AB} A va B nuqtalardan, ularning tezliklariga o'tkazilgan perpendikulyarlarning kesishgan nuqtasida yotadi.

AB shatun burchak tezligi quyidagi formuladan topiladi

$$\vartheta_A=\omega_{AB}=BP_{AB},$$

bundan,

$$\omega_{AB}=\frac{\vartheta_A}{AP_{AB}}. \quad (1.2)$$

ω_{AB} ning yo'nalishi $\vec{\vartheta}_A$vektor yo'nalishi orqali aniqlanadi.

Shatun B va C nuqtalari tezliklarining modullari quyidagi formulalardan aniqlanadi:

$$\vartheta_B=\omega_{AB}*BP_{AB}, \vartheta_C=\omega_{AB}*CP_{AB} \quad (1.3)$$

$AP_{AB}, BP_{AB}, CP_{AB}$ masofalar chizmadagi ABP_{AB} va ABP_{AB} va ACP_{AB} uchburchaklardan topiladi (4.40a-rasm).

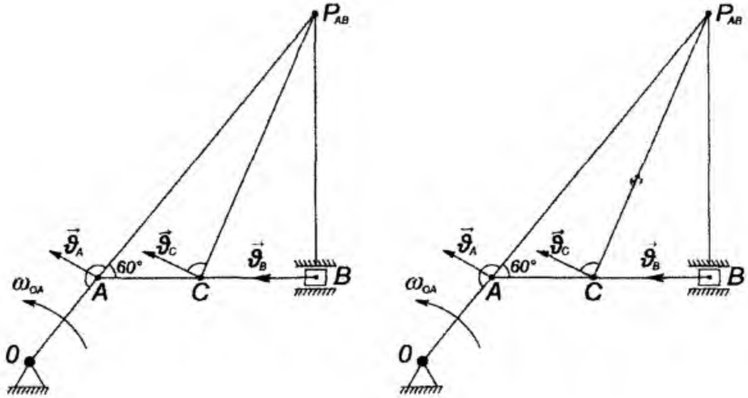
$$AP_{AB}=\frac{AB}{cos60^0}=160\,sm,\ BP_{AB}=AP_{AB}\cdot sin60^0=137{,}6\,sm;$$

$$CP_{AB}=\sqrt{(BC)^2+(BP_{AB})^2}=\sqrt{2500+18933{,}8}=146{,}4\,sm.$$

Yuqoridagilarni e'tiborga olsak:

$$\omega_{AB}=0.5rad/s; \vartheta_B=68.8sm/s, \vartheta_C=73.2sm/s$$

$\vec{\vartheta}_C$ vektor CP_{AB} kesmaga perpendikulyar holda, ω_{AB} yo'nalishi tomon yo'nalgan (4.40a-rasm).



4.40a-rasm

Bajarilgan hisoblashlarning to'g'riligiga ishonch hosil qilish uchun, B nuqtaning tezligini, tekis shakl ikki nuqtasi tezliklarining bu nuqtalardan o'tuvchi o'qdagi proeksiyalarining o'zaro tengligi haqidagi teoremadan foydalanib aniqlaymiz.

Y o'qini shatun bo'ylab B nuqtadan A nuqtaga qarab yo'naltiramiz. Teoremaga asosan:

$$\vartheta_A \cos(\vec{\vartheta}_A \widehat{\ } y) = \vartheta_B \cos(\vec{\vartheta}_B \widehat{\ } y) \quad (1.4)$$

4.40a-rasmdan:

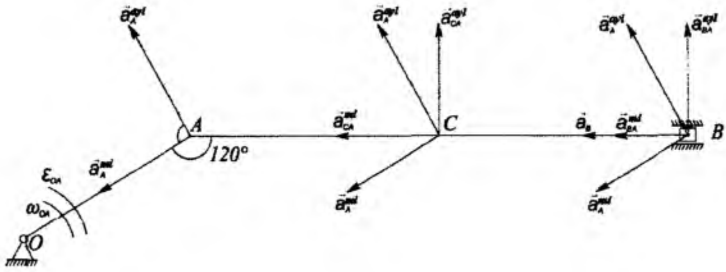
$$\vartheta_A \cos 30^\circ = \vartheta_B$$

Demak, ϑ_B=68.8 sm/s., hisoblashlar to'g'ri bajarilgan.

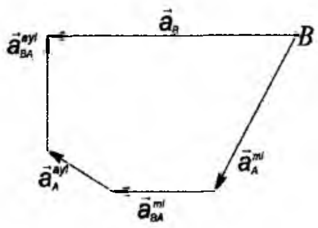
C nuqtaning avval topilgan tezligi ϑ_C ham shu teorema yordamida tekshirilishi mumkin.

2. Nuqtalarning tezlanishlari va zvenoning burchak tezlanishini aniqlash.

A *nuqta* O *nuqta atrofida* aylana bo'ylab harakatlanishi tufayli uning tezlanishi aylanma va markazga intilma tezlanishlardan tashkil topadi (4.40b-rasm).



4.40b-rasm



4.40v-rasm

$$\vec{a}_A = \vec{a}_A^{ayl} + \vec{a}_A^{mi} \qquad (1.5)$$

Bunda:

$a_A^{ayl} = \varepsilon_{OA} \cdot OA = 240$ sm/s^2, $a_A^{mi} = \omega_{OA}^2 \cdot OA = 160$ sm/s^2.

$\vec{a}_A^{ayl}$ vektor OA krivoshipga perpendikulyar holda ε_{OA} yo‘nalishi bo‘yicha yo‘naladi.

$\vec{a}_A^{mi}$ vektor A nuqtadan O nuqta tomon yo‘naladi.

Tekis shakl nuqtalarining tezlanishlari haqidagi teoremaga asosan:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA},$$

yoki

$$\vec{a}_B = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{BA}^{ayl} + \vec{a}_{BA}^{mi} \qquad (1.6)$$

Bunda tezlanishi $\vec{a}_A$ ma’lum bo‘lgan A nuqta qutb deb olindi.

AB shatunning A qutb atrofidagi aylanma harakatida B nuqtaning markazga intilma tezlanishi quyidagiga teng bo'ladi:

$$a_{BA}^{mi} = \omega_{AB}^2 \cdot AB = 20 \text{ m/s}^2 \qquad (1.7)$$

$\vec{a}_{BA}^{mi}$ vektor B nuqtadan A nuqta tomon yo'naladi.

B nuqtaning tezlanishi $\vec{a}_B$ va B nuqtaning A qutb atrofidagi aylanma harakatidagi aylanma tezlanishi $\vec{a}_{BA}^{ayl}$ larning faqat yo'nalish chiziqlari ma'lum: $\vec{a}_B$ gorizontal, $\vec{a}_{BA}^{ayl}$ esa AB shatunga perpendikulyar yo'nalgan. Ularning ko'rsatilgan yo'nalish chiziqlari bo'ylab qaysi tomonlarga yo'nalishlarini ixtiyoriy tanlab olamiz (4.40b-rasm).

Bu tezlanishlarning modullarini (1.6) vektor tenglikning koordinata o'qlariga proeksiyalari tenglamalaridan aniqlaymiz. Javobning ishorasiga qarab, vektorning haqiqiy yo'nalishini, hisoblashda qabul qilinganiga mos kelishi yoki kelmasligi aniqlanadi. x va y o'qlarining yo'nalishlarini 4.40b-rasmda ko'rsatilgandek o'tkazib, quyidagilarni hosil qilamiz:

$$0 = -a_A^{mi} cos30^0 + a_A^{ayl} cos60^0 + a_{BA}^{ayl}; \qquad (1.8)$$

$$a_B = a_{BA}^{mi} + a_A^{mi} cos60^0 + a_A^{ayl} cos30^0. \qquad (1.9)$$

(1.8) dan:

$$a_{BA}^{ayl} = a_A^{mi} cos30^0 - a_A^{ayl} cos60^0 = 17{,}6 \text{ sm/s}^2.$$

(1.9) dan

$$a_B = 306.4 \text{sm/s}^2$$

Javoblarning ishoralari musbat. Shuning uchun $\vec{a}_{BA}^{ayl}$ va $\vec{a}_B$ vektorlarning haqiqiy yo'nalishlari, hisoblashda qabul qilingan yo'nalishlarga mos kelar ekan.

AB shatunning burchak tezlanishini quyidagi formuladan topamiz:

$$a_{BA}^{ayl} = \varepsilon_{AB} \cdot AB.$$

Bundan,

$$\varepsilon_{AB} = \frac{a_{BA}^{ayl}}{BA} = 0{,}22 \text{ rad/s}^2. \qquad (1.10)$$

$\vec{a}_B$ va $\vec{a}_{BA}^{ayl}$ larni grafik usulda B nuqtada tezlanishlar ko'p burchagini chizish orqali ham aniqlash mumkin. Buning uchun (1.6) ga asosan B nuqtadan boshlab, tanlangan masshtabda ketma – ket $\vec{a}_A^{ayl}$, $\vec{a}_A^{mi}$ va $\vec{a}_{BA}^{mi}$ vektorlarni qo'yamiz (4.40v-rasm). $\vec{a}_A^{ayl}$ vektorning oxiri orqali AB shatunga perpendikulyar holda o'tkazilgan to'g'ri chiziqni, $\vec{a}_B$ tezlanishning yo'nalish chizig'i bilan kesishguncha davom ettiramiz.

Mazkur to'g'ri chiziq uzunligi tanlangan masshtabda $\vec{a}_{BA}^{ayl}$ ning modulini ifodalaydi. $\vec{a}_B$ vektori tezlanishlar ko'p burchagining yopuvchi tomoni kabi aniqlanadi. Shuning uchun ko'p-burchakning yopuvchi tomonining uzunligi tanlangan masshtabda $\vec{a}_B$ modulini ifodalaydi (4.40v-rasm).

C nuqtaning tezlanishini aniqlaymiz:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA} = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{CA}^{ayl} + \vec{a}_{CA}^{mi} \qquad (1.11)$$

AB shatunning A nuqta atrofidagi aylanma harakatida C nuqtaning aylanma va markazga intilma tezlanishlari quyidagilarga teng bo'ladi:

$$a_{CA}^{ayl} = \varepsilon_{AB} \cdot AC = 6{,}6\ \text{sm/s}^2,$$

$$a_{CA}^{mi} = \omega_{AB}^2 \cdot AC = 7{,}5\ \text{sm/s}^2.$$

$\vec{a}_{CA}^{mi}$ vektor C nuqtadan A nuqta tomon yo'naladi. $\vec{a}_{CA}^{ayl}$ vektor esa, $\vec{a}_{CA}^{mi}$ vektorga perpendikulyar holda, ε_{AB} burchak tezlanishining yo'nalishi tomon yo'naladi.

$\vec{a}_C$ ning modulini proeksiyalash usuli bilan aniqlaymiz.

Buning uchun (4.38) ni x va y o'qlarga proeksiyalaymiz (4.40b- rasm):

$$(a_C)_x = +a_A^{ayl}\cos 60^0 - a_A^{mi}\cos 30^0 + a_{CA}^{ayl} = -11\text{sm/s}^2;$$
$$(a_C)_y = a_A^{ayl}\cos 30^0 + a_A^{mi}\cos 60^0 + a_{CA}^{mi} = 293{,}9\text{sm/s}^2.$$

Natijada

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 294{,}1\ \text{sm/s}^2.$$

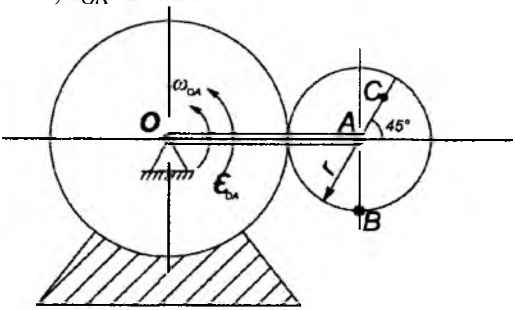
3-masala. Radius r=20 sm bo'lgan tishli g'ildirak radiusi R=40 sm bo'lgan qo'zg'almas tishli g'ildirakning O o'qi atrofida aylanuvchi OA krivoship bilan harakatga keltiriladi; krivoship shu paytda ω=2rad/s burchak tezligiga ega bo'lib, ε=2 rad/s^2 burchak tezlanish bilan aylanadi.

Krivoship A nuqtasining va qo‘zg‘aluvchi g‘ildirakning B va C nuqtalarining tezliklari va tezlanishlari hamda qo‘zg‘aluvchi g‘ildirakning burchak tezligi va burchak tezlanishi aniqlansin (4.41-rasm).

Masalada:

R=40 sm, r=20 sm, AC=10 sm.,

ω_{OA}=2rad/s , ε_{OA}=2 rad/s



4.41-rasm

Yechish.

1. Nuqtalarning tezliklarini va ko‘zg‘aluvchi g‘ildirak burchak tezligini aniqlash.

Mexanizmning berilgan holatida OA krivoship A panjasi tezligining modulini aniqlaymiz:

$$\vartheta_A=\omega_{OA}*OA120 \text{ sm/s}. \quad (1.1)$$

A nuqtaning tezligi $\vec{\vartheta}_A$ ***OA*** krivoshipga perpendikulyar holda ω_{OA} yo‘nalishi tomon yo‘naldi.

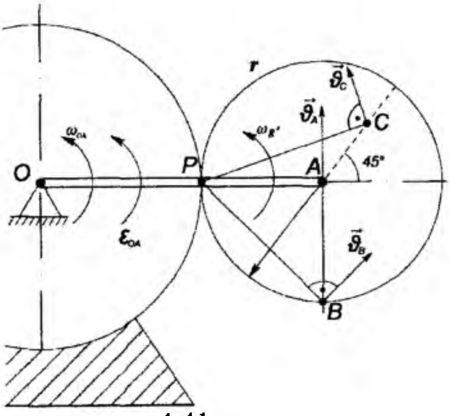
Qo‘zg‘aluvchi g‘ildirak nuqtalari tezliklarining oniy markazi P nuqta bo‘lganligi uchun (4.41a-rasmga qarang).

$$\vartheta_A=\omega_{g'}*r.$$

Bundan,

$$\omega_{g'} = \frac{v_A}{r} = 6 \text{ rad/s} \quad (1.2)$$

Qo‘zg‘aluvchi g‘ildirak burchak tezligi $\omega_{g'}$ ning yo‘nalishi $\vec{\vartheta}_A$ vektor yo‘nalishi orqali aniqlanadi (4.41a-rasm).



4.41a-rasm

Qo'zg'aluvchi g'ildirak B va C nuqtalari tezliklarining modullari quyidagi formulalardan aniqlanadi:

$$\vartheta_B = \vartheta_B * BP \qquad (1.3)$$

$$\vartheta_C = \omega_{g'} * CP \qquad (1.4)$$

Chizmadan

$$BP = r\sqrt{2} = 28{,}2\ sm,$$

$$CP = \sqrt{r^2 + (AC)^2 + 2 \cdot r \cdot AC\cos 45^0} = 28\ sm$$

Yuqoridagilarni e'tiborga olsak:

$$\vartheta_B = 169.2 \text{sm/s},\ \vartheta_C = 168 \text{sm/s}$$

$\vec{\vartheta}_B$ vektor BP kesmaga, $\vec{\vartheta}_C$ vektor CP kesmaga perpendikulyar holda ω_g yo'nalishi tomon yo'naladi (4.41a-rasm).

2. Nuqtalarning tezlanishlari va qo'zg'aluvchi g'ildirak burchak tezlanishini aniqlash.

A nuqta O nuqta atrofida aylanma harakatda bo'lishi tufayli, uning tezlanishi aylanma va markazga intilma tezlanishlardan tashkil topadi (4.41b-rasm):

$$\vec{a}_A = \vec{a}_A^{ayl} + \vec{a}_A^{mi}. \qquad (1.5)$$

Bunda:

$$a_A^{ayl} = \varepsilon_{OA} \cdot OA = 120 \text{ sm/s}^2,\ a_A^{mi} = \omega_{OA}^2 \cdot OA = 240 \text{ sm/s}^2.$$

A nuqta tezlanishining moduli quyidagiga teng bo'ladi:

$$a_A = \sqrt{(a_A^{ayl})^2 + (a_A^{mi})^2} = 268{,}3 \text{ sm/s}^2.$$

$\vec{a}_A^{mi}$ vektor A nuqtadan O nuqta tomon yo‘naladi. $\vec{a}_A^{ayl}$ vektor $\vec{a}_A^{mi}$ vektorga perpendikulyar holda, ε_{OA} yo‘nalishi tomon yo‘naladi (4.41b-rasm).

Qo‘zg‘aluvchi g‘ildirak B va C nuqtalarining tezlanishlarini, tekis shakl nuqtalarining tezlanishlari haqidagi teoremadan foydalanib aniqlaymiz:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

yoki

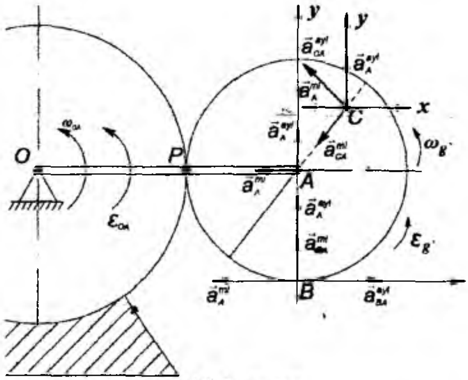
$$\vec{a}_B = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{BA}^{ayl} + \vec{a}_{BA}^{mi} \quad (1.6)$$

Bunda tezlanishi $\vec{a}_A$ ma’lum bo‘lgan A nuqta ***qutb*** deb olinadi.

$\vec{a}_{BA}^{mi}$ vektor B nuqtadan A nuqta tomon yo‘naladi, uning moduli quyidagi formuladan topiladi:

$$a_{BA}^{mi} = \omega_g^2 \cdot BA = 720 \text{ sm/s}^2. \quad (1.7)$$

$\vec{a}_{BA}^{ayl}$ vektorni aniqlash uchun qo‘zg‘aluvchi g‘ildirak burchak tezlanishini aniqlash lozim:



4.41.b-rasm

$$\varepsilon_g = \frac{d\omega_g}{dt} = \frac{1}{r}\frac{dv_A}{dt} = \frac{a_A^{ayl}}{r} = 6 \text{ rad/s}^2. \quad (1.8)$$

ε_g ishorasi ω_g ishorasi bilan bir xil bo‘lganligi uchun, ular bir xil yo‘nalishga ega bo‘ladilar.

$\vec{a}_{BA}^{ayl}$ vektor $\vec{a}_{BA}^{mi}$ vektorga perpendikulyar holda ε_g yo'nalishi tomon yo'naladi, uning moduli quyidagi formuladan topiladi:

$$a_{BA}^{ayl} = \varepsilon_g \cdot AB = 360 \text{ sm/s}^2. \qquad (1.9)$$

$\vec{a}_B$ vektor modulini aniqlash uchun $\vec{a}_A^{ayl}$, $\vec{a}_A^{mi}$, $\vec{a}_{BA}^{ayl}$, $\vec{a}_{BA}^{mi}$ vektorlarni B nuqtaga qo'yamiz va proeksiyalash usulidan foydalanamiz. x o'qini B nuqtadan gorizontal, y o'qini esa vertikal yo'naltiramiz. 1.6 ni xva y o'qlariga proeksiyalasak:

$$(a_B)_x = -a_A^{mi} + a_{BA}^{ayl} = 120 \text{sm/s}^2;$$

$$(a_B)_y = a_A^{ayl} + a_{BA}^{mi} = 840 \text{sm/s}^2;$$

Natijada

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = 848{,}5 \text{ sm/s}^2.$$

C nuqtaning tezlanishi B nuqtaning tezlanishi kabi topiladi:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA}, \qquad (1.10)$$

yoki

$$\vec{a}_C = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{CA}^{ayl} + \vec{a}_{CA}^{mi}. \qquad (1.11)$$

Bunda,

$$a_{CA}^{ayl} = \varepsilon_g \cdot AC = 60 \text{ sm/s}^2,$$

$$a_{CA}^{mi} = \omega_g^2 \cdot AC = 360 \text{ sm/s}^2.$$

$\vec{a}_C$ vektor modulini ham proeksiyalash usulidan foydalanib aniqlaymiz. Buning uchun C nuqtaga $\vec{a}_A^{ayl}$, $\vec{a}_A^{mi}$, $\vec{a}_{CA}^{ayl}$, $\vec{a}_{CA}^{mi}$ vektorlarni qo'yamiz. x o'qini C nuqtadan gorizontal, y o'qini esa vertikal yo'naltiramiz.

(1.11) ni x va y o'qlariga proeksiyalasak:

$$(a_C)_x^2 = -a_A^{mi} - a_{CA}^{mi}\cos 45^0 - a_{CA}^{ayl}\cos 45^0,$$

$$(a_C)_y^2 = a_A^{ayl} - a_{CA}^{mi}\cos 45^0 + a_{CA}^{ayl}\cos 45^0.$$

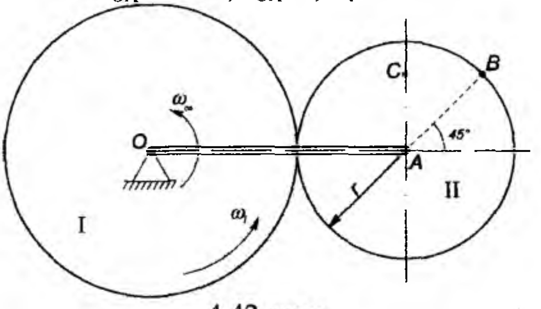
Natijada

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 546{,}2 \text{ sm/s}^2. \qquad (1.12)$$

4-masala. Uzunligi 60 sm bo'lgan OA krivoship chizma tekisligiga perpendikulyar bo'lgan qo'zg'almas Ox o'q atrofida

ω_{OA}=4rad/s burchak tezlik bilan aylanadi. Xuddi shu Ox o'qqa I – g'ildirak o'tqazilgan, krivoship A nuqtasiga esa radiusi r=20 sm bo'lgan, I g'ildirakka tashqari tomonidan ilashgan, II – g'ildirak o'rnatilgan. I-g'ildirakning burchak tezligi ω_I=15 rad/s. AC=10sm (4.42-rasm). Ikkinchi g'ildirak A,B,C – nuqtalarining tezliklari va tezlanishlari aniqlansin, hamda ikkinchi g'ildirakning burchak tezligi topilsin.

Masalada: OA =60sm, r=20sm, AC=10sm sm, ω_{OA}=4rad/s, ε_{OA}=0, ω_t=6rad/s.



4.42-rasm

Yechimi.

A) II-g'ildirak burchak tezligi va A,B,C – nuqtalarining tezliklarini aniqlash.

I- va II – g'ildiraklar tashqari tomonidan ilashganligi uchun, II – g'ildirakning burchak tezligi ω_{II} ni Villis formulasidan aniqlaymiz (4.42a-rasm):

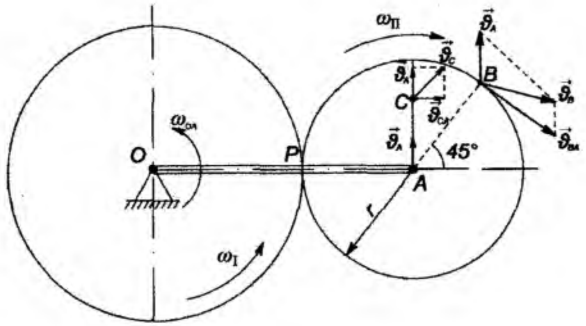
$$\frac{\omega_I-\omega_{OA}}{\omega_{II}-\omega_{OA}}=-\frac{r}{OA-r}, \qquad (1.1)$$

bundan ω_{II} =8 rad/s.

ω_I va ω_{II} larning yo'nalishlari chizmada ko'rsatilgan (4.19a-rasm).

Mexanizmning berilgan holatida krivoship A panjasi tezligining modulini aniqlaymiz:

$$\vartheta_A=\omega_{OA}*OA=4*60=240 \text{ sm/s}. \qquad (1.2)$$



4.42a-rasm

A nuqtaning tezligi $\vec{\vartheta}_A$ OA krivoshipga perpendikulyar holda, ω_{OA} yo‘nalishi tomon yo‘naladi.

II-g‘ildirak B va C nuqtalarining tezliklarini aniqlash uchun, tekis shakl nuqtalarining tezliklari haqidagi teoremadan foydalanamiz. Buning uchun, qutb sifatida tezligi $\vec{\vartheta}_A$ ma’lum bo‘lgan A nuqtani tanlaymiz. Teoremaga asosan, B nuqtaning tezligi quyidagicha aniqlanadi:

$$\vec{\vartheta}_B = \vec{\vartheta}_A + \vec{\vartheta}_{BA}.$$

Bunda:

$$\vartheta_{BA} = \omega_{II} * BA = 160 sm/s,$$

Shuning uchun,

$$\vartheta_B = \sqrt{\vartheta_A^2 + \vartheta_{BA}^2 + 2\vartheta_A\vartheta_{BA}\cos 45} = 288 sm/s. \quad (1.3)$$

II-g‘ildirak C nuqtasining tezligi ham B nuqtaning tezligi kabi topiladi:

$$\vec{\vartheta}_C = \vec{\vartheta}_A + \vec{\vartheta}_{CA},\ \vartheta_{CA} = \omega_{II} \cdot CA = 80 sm/s.$$

$$\vartheta_C = \sqrt{\vartheta_A^2 + \vartheta_{CA}^2} = 252,8 sm/s \quad (1.4)$$

$\vec{\vartheta}_B$ va $\vec{\vartheta}_C$ vektorlar $\vec{\vartheta}_A$ va $\vec{\vartheta}_{BA}$, hamda $\vec{\vartheta}_A$ va $\vec{\vartheta}_{CA}$ vektorlardan qurilgan parallelogrammlar diagonallari orqali ifodalanadi (4.42a-rasm).

B) *II – g‘ildirak A, B, C nuqtalarining tezlanishlarini aniqlash.*

A nuqta O nuqta atrofida aylana bo‘ylab harakatlanishi tufayli, uning tezlanishi aylanma va markazga intilma tezlanishlardan tashkil topadi (4.42b-rasm):

$$\vec{a}_A = \vec{a}_A^{ayl} + \vec{a}_A^{mi} \qquad (1.5)$$

Bunda:

$$a_A^{ayl} = \varepsilon_{OA} \cdot OA = 0, \text{ chunki } \varepsilon_{OA} = \frac{dw_{OA}}{at} = 0.$$

$$a_A^{mi} = \omega_{OA}^2 \cdot OA = 960 \text{sm/s}^2.$$

$$a_A = a_A^{mi} = 960 \text{ sm/s}^2. \qquad (1.6)$$

A nuqtaning tezlanishi $\vec{a}_A$ A nuqtaning O nuqta atrofidagi aylanma harakati markazga intilma tezlanishiga teng bo‘ladi.

II – g‘ildirak B va C nuqtalarining tezlanishlarini aniqlash uchun, tekis shakl nuqtalarining tezlanishlari haqidagi teoremadan foydalanamiz. Qutb sifatida tezlanishi $\vec{a}_A$ ma’lum bo‘lgan A nuqtani tanlaymiz. Teoremaga ko‘ra, B nuqtaning tezlanishi quyidagicha ifodalanadi:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA},$$

yoki

$$\vec{a}_B = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{BA}^{ayl} + \vec{a}_{BA}^{mi}. \qquad (1.7)$$

Bunda,

$a_{BA}^{ayl} = 0$, chunki $\varepsilon_{II} = \frac{dw_{II}}{at} = 0$.

$\vec{a}_{BA}^{mi}$ vektor B nuqtadan A nuqta tomon yo‘naladi. Uning moduli:

$$a_{BA}^{mi} = \omega_{II}^2 \cdot BA = 1280 \text{ sm/s}^2.$$

$\vec{a}_B$ vektor $\vec{a}_A^{mi}$ va $\vec{a}_{BA}^{mi}$ tezlanishlardan qurilgan parallelogrammning dioganali orqali ifodalanadi (4.42b-rasm). Uning moduli:

$$a_B = \sqrt{(a_A^{mi})^2 + (a_{BA}^{mi})^2 + 2a_A^{mi}a_{BA}^{mi}\cos 45} = 2074 \text{ sm/s}^2. \qquad (1.8)$$

C nuqtaning tezlanishi B nuqta tezlanishi kabi topiladi:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA} \qquad (1.9)$$

yoki

$$\vec{a}_C = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{CA}^{ayl} + \vec{a}_{CA}^{mi} \qquad (1.10)$$

Bunda:

$$a_{CA}^{ayl} = 0,$$

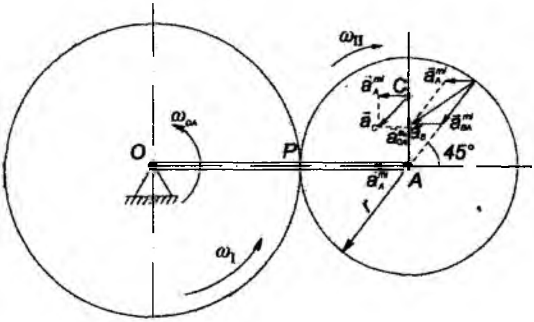
$$a_{CA}^{mi} = \omega_{II}^2 \cdot CA = 640 \text{ sm/s}^2.$$

$\vec{a}_{CA}^{mi}$ vektor C nuqtadan A nuqta tomon yo'naladi.

$\vec{a}_C$ vektor $\vec{a}_A^{mi}$ va $\vec{a}_{CA}^{mi}$ vektorlardan qurilgan parallelogramm dioganali orqali ifodalanadi (4.42b-rasm).

Uning moduli:

$$a_C = \sqrt{(a_A^{mi})^2 + (a_{CA}^{mi})^2} = 1153 \text{ sm/s}^2 \quad (1.11)$$

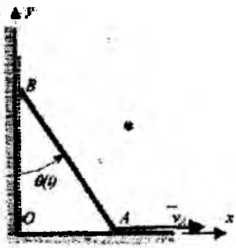


4.42b-rasm

5-masala. Uzunligi l_{AB}=50sm bo'lgan sterjen A nuqtada gorizontal polga, B nuqtada esa vertikal devorga tayanadi. Sterjenning shakl tekisligidagi harakatida A nuqta gorizontal polga $\vartheta_A = \vartheta_0$=0.8m/s tezlik harakatlanadi, ϑ_0 – A nuqtaning boshlang'ich tezligi. sterjenning B nuqtasi vertikal devor bo'ylab harakatlanadi (4.43- rasm).

a) Sterjen nuqtalari tezliklarining $t_1$0.2s vaqt onidagi taqsimoti aniqlansin.

b) Sterjen A va B nuqtalarining vaqt onidagi tezlanishi topilsin.



4.43 - rasm

Yechilishi: a) Masala shartiga ko‘ra AB sterjen tekislikka parallel harakat sodir etadi. Sterjenning shakl tekisligidagi harakatini o‘rganamiz. Sterjenning t_1=0.2s vaqt onidagi holatini aniqlaymiz. Masala shartidan:

$$\dot{x}_A = \vartheta_A \quad (1)$$

$$\dot{y}_A = 0 \quad (2)$$

Binobarin $y_A=0$

(1) tenglmani vaqt bo‘yicha integrallaymiz:

$$x_A(t) = \vartheta_A t + C_1 \quad (3)$$

Bu ifodada C_1 – integrallash doimiysi, *y* harakatning boshlang‘ich shartidan aniqlanadi. Agar reykaning boshlang‘ich holatida A nuqta –koordinata boshi bilan ustma-ust tushishini e’tiborga olsak, $C_1=0$ bo‘ladi. Natijada A nuqtaning t_1=0.2s vaqt onidagi holati quyidagicha aniqlanadi:

$$X_A(t_1)=0.8*0.2=0.16m$$

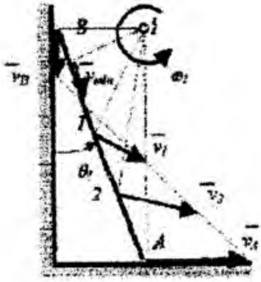
A nuqtaning t_1=0.2s vaqt onidagi holati va $\angle AOB=90^\circ$ ekanligini e’tiborga olsak AB sterjenning t_1=0.2s vaqt onidagi holatini aniqlashga imkon beruvchi θ_1 burchak qiymati aniqlanadi

$$\sin\theta_1 = \frac{OA}{AB} = \frac{0,16}{0,5} = 0,32.$$

Demak

$$\theta_1=\arcsin 0.32=18.66^\circ$$

AB sterjen nuqtalari tezliklarining t_1=0.2s vaqt onidagi taqsimotini aniqlash uchun AB sterjen nuqtalari tezliklarining oniy markazini aniqlaymiz. Mazkur nuqta $\vec{\vartheta}_A$ *va* $\vec{\vartheta}_B$ vektorlarga o‘tkazilgan perpendikulyar chizmalarining kesishish nuqtasi J da joylashadi (4.44– rasm).



4.44 – rasm

Rasmdan:

$$YA = AB\cos\theta.$$

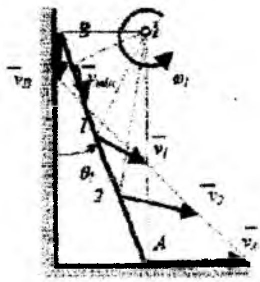
AB stejen oniy aylanish burchak tezligi quyidagi formula yordamida topiladi

$$\omega_1 = \frac{\vartheta_A}{Y_A} = \frac{0,8}{0,5 \cdot 0,947} = 1,688\frac{rad}{s}.$$

U paytda B nuqtaning tezligi quyidagi formula orqali aniqlanadi:

$V_B = \omega_1 * YB = \omega_1 * AB\sin\theta = 1.688*0.5*0.32 = 0.27 m/s$

B nuqtaning tezligi $\vec{\vartheta}_B$ YB kesmaga perpendikulyar holda ω_1 tomon yo'naladi. AB sterjen nuqtalari tezliklarining $t_1 = 0.2s$ vaqt onidagi taqsimotini aniqlash uchun $\vec{\vartheta}_B$ va $\vec{\vartheta}_A$ vektorlar uchlarini birlashtiramiz. AB sterjen nuqtalari tezliklari vektorlari mazkur nuqtalardan tezliklar oniy markaziga o'tkazilgan kesmalarga perpendikulyar holda yo'naladi, ularning uchlari $\vec{\vartheta}_A$ va $\vec{\vartheta}_B$ vektorlar uchlarini birlashtiruvchi chiziqda yotadi (4.45- rasm).



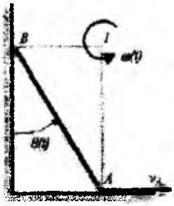
4.45- rasm

b) sterjen A nuqtasining t_1=0.2s vaqt onidagi tezlanishini aniqlashda, masal shartiga ko‘ra, A nuqta o‘zgarmas ϑ_A=0.8m/s tezlik bilan harakatlanishini e’tiborga olish lozim. Binobarin,

$$a_A = \frac{d\vartheta_A}{dt} = 0.$$

A nuqta tezlanishi qaralayotgan vaqt onida nolga teng ekanligi sababli mazkur nuqta AB sterjen nuqtalari tezlanishlarining oniy markazini ifodalaydi. Shuning sterjen boshqa nuqtalarini tezlanishlari mazkur nuqtalarning tezlanishlar oniy markazi atrofidgi aylanma harakat tezlanishi kabi aniqlanadi (4.46- rasm).

Tezlanishlar oniy markazi Q orqali belgilanadi A=Q.



4.46- rasm

Ma’lumki

$$\vartheta_A = \omega(t) * YA$$

Shuning uchun

$$\omega(t) = \frac{\vartheta_A}{YA} = \frac{V_A}{l\cos\theta(t)}$$

Sterjin burchak tezlanishi uning burchak tezligidan vaqt bo‘yicha hisoblangan birinchi tartibli hosilaga teng (4.47- rasm).

$$\varepsilon_1 = \varepsilon(t_1) = \frac{d\omega(t)}{at} = \frac{0,8 \cdot \theta(t_1) \cdot \sin\theta_1}{0,5\cos^2\theta_1} = \frac{0,8 \cdot \omega_1 \cdot \sin\theta_1}{0,5\cos^2\theta_1} = 0,96 rad/s^2$$

4.47- rasm

Burchak tezlanish yo'nalishi burchak tezlik yo'nalishi bilan ustma-ust tushadi, chunki ularning ishoralari bir xil.

B nuqtaning tezlanishini aniqlaymiz.

Birinchi yo'l.

Tekis shakl nuqtalarini tezlanishlari haqidagi teoremaga ko'ra, qutb sifatida AB sterjen nuqtalari tezlanishlaring oniy markazi θ nuqta olinsa

$$a_B = QB\sqrt{\varepsilon_1^2 + \omega_1^4} = 0{,}5 \cdot 3 = 1{,}5\frac{m}{s^2}.$$

$\vec{a}_B$ ning yo'nalishi φ_1 burchak orqali aniqlanadi:

$$\tan\varphi_1 = \frac{\varepsilon_1}{\omega_1^2} = 0{,}337; \qquad \varphi_1 = 18{,}66^0$$

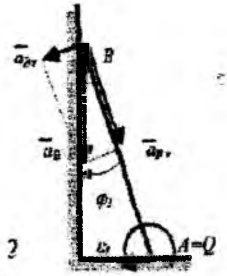
Ikkinchi yo'l:

$$\vec{a}_B = \vec{a}_B^\tau + \vec{a}_B^n.$$

Bunda:

$$a_B^\tau = AB \cdot \varepsilon_1 = 0{,}5 \cdot 0{,}96 = 0{,}48\ m/s^2$$
$$a_B^n = AB \cdot \omega_1^2 = 0{,}5 \cdot (1{,}688)^2 = 1{,}424\ m/s^2.$$

Mazkur tezlanishlarning yo'nalishlari 4.46- rasmda ko'rsatilgan.



4.46- rasm

Natijada B nuqta tezlanishi quyidagiga teng bo'ladi:

$$a_B = \sqrt{(a_B^\tau)^2 + (a_B^n)^2} = \sqrt{(0{,}48)^2 + (1{,}424)^2} = 1{,}5\,m/s^2.$$

$\vec{a}_B$ ning yo'nalishini aniqlaymiz:

$$tg\varphi_1 = \frac{a_B^\tau}{a_B^n} = \frac{0{,}48}{1{,}424} = 0{,}337;$$

$$\varphi_1 = 18.66^\circ$$

Ko'rinib turibdik, har ikkala yo'lda ham B nuqtaning tezlanishi bir xil miqdor va yo'nalishga ega bo'ldi.

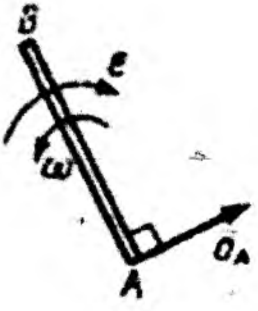
69-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Uzunligi AB=1,5m bo'lgan sterjen shakl tekisligida harakatlanadi.

Sterjen A nuqtasining tezligi gorizontal holda yo'nalgan bo'lib ϑ_A=0.5m/s ga teng ($\vec{\vartheta}_A = const$).

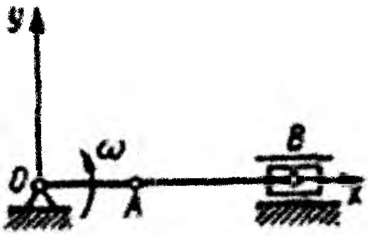
t_1= vaqt oni uchun sterjen B nuqtasining tezligi va tezlanishi aniqlansin AC=1m.

2-muammo. Uzunligi AB=1m bo'lgan sterjen tekislik bo'ylab harakatlanadi. Agar uning burchak tezligi ω=2rad/s, burchak tezlanishi ε=2rad/s^2 va A nuqtasining tezlanishi a_A=1m/s^2 bo'lsa, B nuqtasining tezlanishini hisoblag (4.47 –rasm).



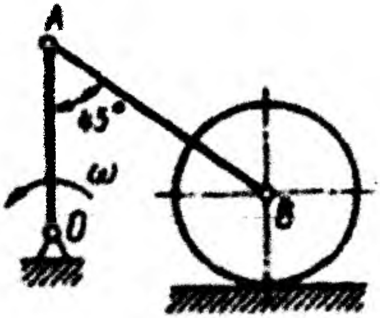
4.47- rasm

3-muammo. Krivoship –polzunli mexanizmning OA krivoshipi o'zgarmas ω=10rad/s burchak tezlik bilan aylanadi. Shaklda ko'rsatilgan holat uchun AB shatunning burchak tezlanishini toping (4.48-rasm).



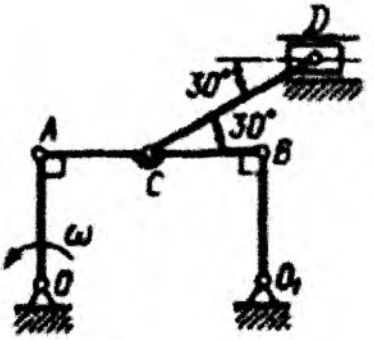
4.48– rasm

4-muammo. Krivoship –shatunli mexanizmning o‘lchamlari OA=0,3m va AB=0,45m bo‘lib, OA krivoship o‘zgarmas burchak tezlik ω=10rad/s bilan aylanadi. AB shatunning burchak tezlanishini hisoblang (4.49- rasm).



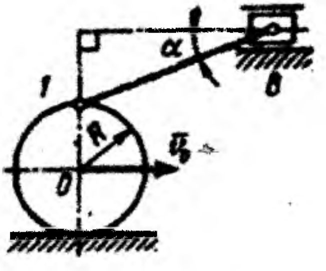
4.49 –rasm

5-muammo. Sharnirli parallelogrammning OA krivoshipi o‘zgarmas burchak tezlik ω=0.4rad/s bilan aylamadi. Agar mexanizmning o‘lchamlari OA=20sm, CD=30sm bo‘lsa, ko‘rsatilgan holat uchun CD shatunning burchak tezlanishini toping (4.50-rasm).



4.50- rasm

6-muammo. Mexanizmning polzuni B radius R=50sm li 1 g'ildirakka sharnir yordamida bog'langan bo'lib, uning mrkazi o'zgarmas ϑ_0=5m/s tezlik bilan harakatlansa, *B* polzunning tezlanishini aniqlang. Bunda $\boldsymbol{\alpha} = \mathbf{30^0}$ (4.51 – rasm).

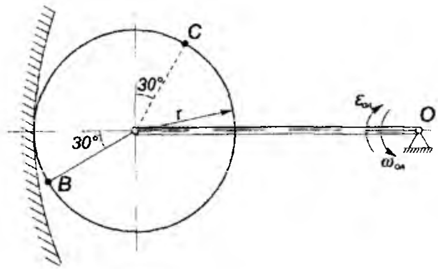
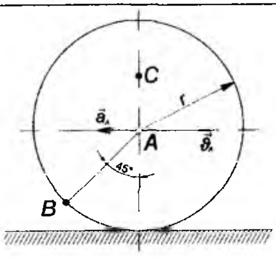
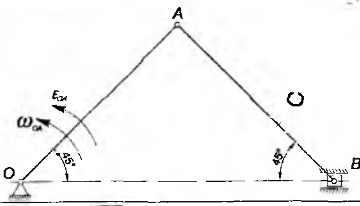
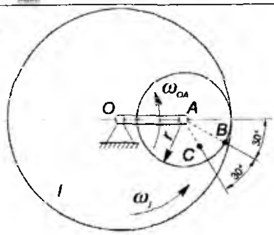


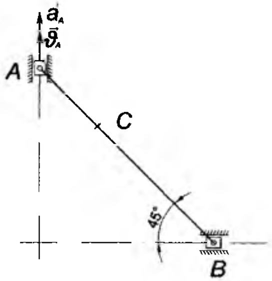
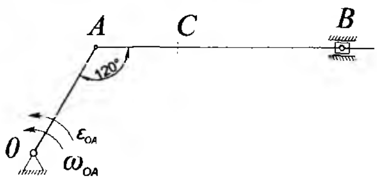
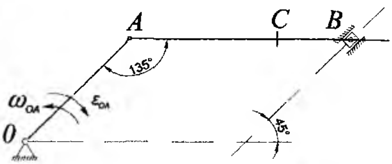
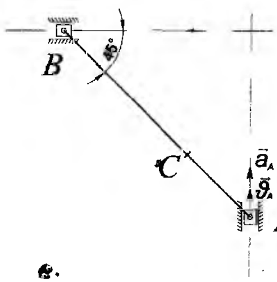
4.51 –rasm

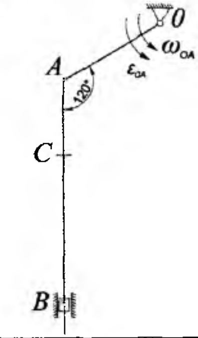
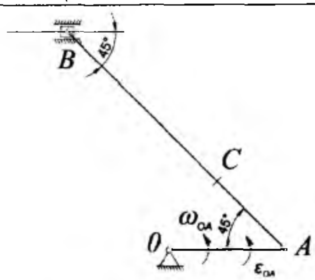
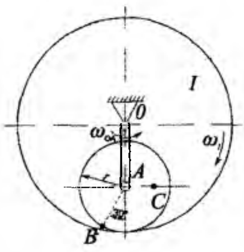
70-§. Mustaqil yechish uchun talabalarga tavsiya etiladigan masalalar

Mexanizmning berilgan holati uchun *B* va *C* nuqtalarning tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zvenoning burchak tezligi va burchak tezlanishi topilsin.

Mexanizmlarning sxemalari va hisoblash uchun kerakli ma'lumotlar quyidagi jadvalda keltirilgan.

Variant raqamlari	Mexanizmlarning sxemalari	Hisoblash uchun kerak ma'lumotlar
1.		OA=60 sm r=20 sm ω_{OA}=2 rad/s ε_{OA}=4 rad/s^2
2.		r=45 sm AS=15 sm $\vec{\vartheta}_A$=100 sm/s a_A=50sm/s^2
3		OA=20 sm AB=20 sm AC=10 sm ω_{OA}=2 rad/s ε_{OA}=6 rad/s^2
4		OA=30 sm r=20 sm AC=15 sm ω_{OA}=1rad/s ω_I=2,5rad/s ε_{OA}=0 rad/s^2

5		AB=30 sm AC=10 sm $\vec{\vartheta}_A$=10sm/s a_A=15sm/s^2
6		OA=30 sm AB=60 sm AC=20 sm ω_{OA}=2 rad/s ε_{OA}=6 rad/s^2
7		OA=40 sm AB=60 sm AC=40 sm ω_{OA}=3rad/s ε_{OA}=8 rad/s
8		AB=60 sm AC=20 sm $\vec{\vartheta}_A$=5sm/s a_A=10sm/s^2

<table>
<tr><td>9.</td><td>
</td><td>OA=30 sm
AB=40 sm
AC=15 sm
ω_{OA}=3rad/s
ε_{OA}=3 rad/s^2</td></tr>
<tr><td>10</td><td>
</td><td>OA=30 sm
AB=80 sm
AC=25 sm
ω_{OA}=1rad/s
ε_{OA}=2 rad/s^2</td></tr>
<tr><td>11</td><td>
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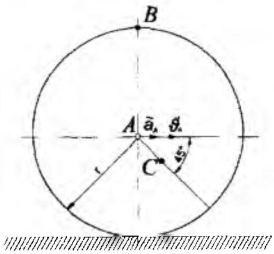
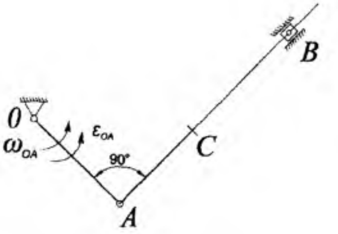
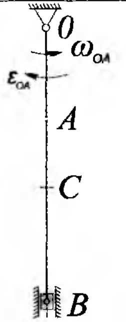
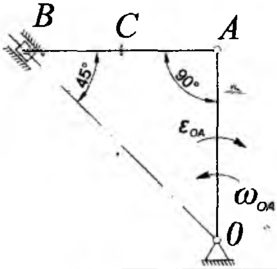
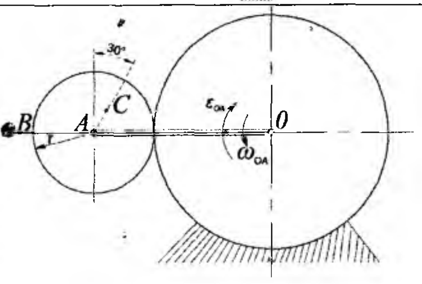
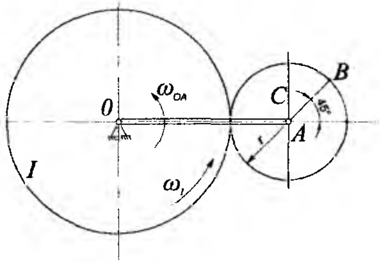
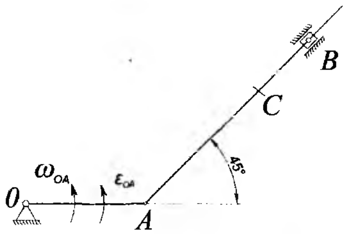
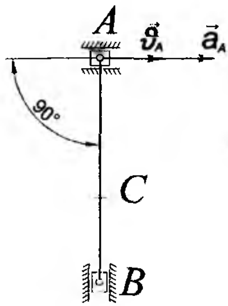
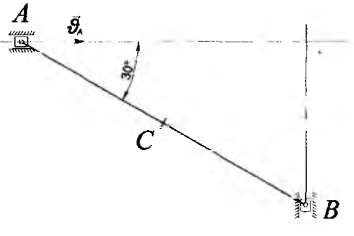
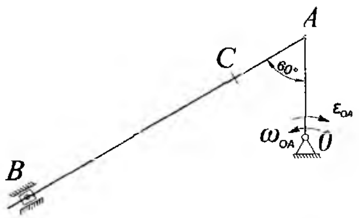
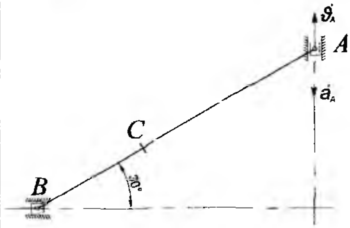
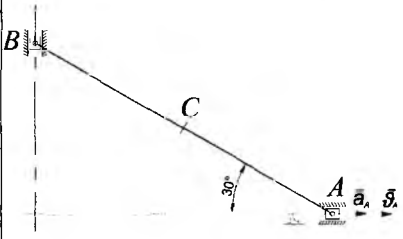
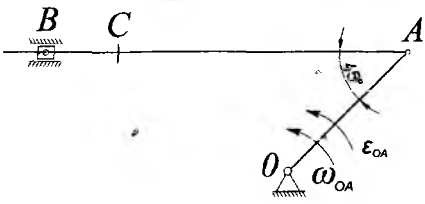
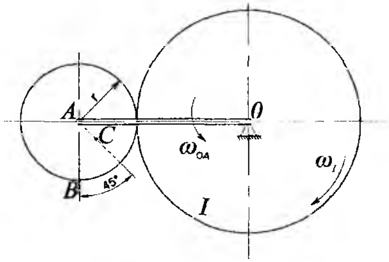
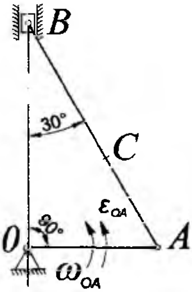
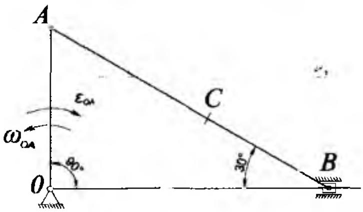
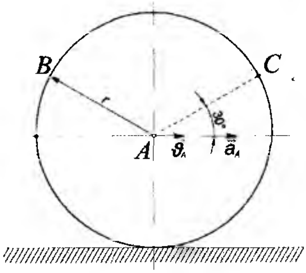
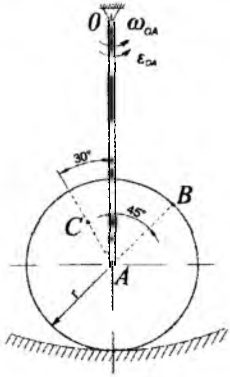
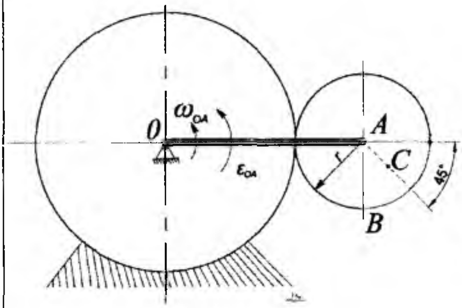
r=15 sm
AC=10 sm
ω_{OA}=7,0rad/s
ω_I=1,2rad/s
ε_{OA}=0</td></tr>
<tr><td>12</td><td>
</td><td>r=20 sm
AC=10 sm
$\vec{\vartheta}_A$=60 sm/s
a_A=30 sm/s^2</td></tr>
</table>

13		OA=30 sm AB=60 sm AC=25 sm ω_{OA}=1rad/s ε_{OA}=1 rad/s^2
14		OA=20 sm AB=40 sm AC=15 sm ω_{OA}=4rad/s ε_{OA}=6 rad/s^2
15.		OA=40 sm AC=20 sm ω_{OA}=4rad/s ε_{OA}=8 rad/s^2
16		OA=50 sm r=20 sm AC=10 sm ω_{OA}=1rad/s ε_{OA}=8 rad/s^2

17		OA=60 sm r=25 sm ω_{OA}=3rad/s AC=10 sm ω_I=12rad/s
18		OA=30 sm AB=60 sm AC=40 sm ω_{OA}=2rad/s ε_{OA}=4 rad/s^2
19		AB=40 sm AC=25 sm $\vec{\vartheta}_A$=20sm/s a_A=20sm/s^2
20		AB=40 sm AC=20 sm $\vec{\vartheta}_A$=10 sm/s a_A=0

21		OA=25 sm AB=80 sm AC=20 sm ω_{OA}=2rad/s ε_{OA}=2 rad/s^2
22		AB=50 sm AC=30 sm $\vec{\vartheta}_A$=20sm/s $\vec{a}_A$=10 sm /s^2
23		AB=30 sm AC=15 sm $\vec{\vartheta}_A$=40sm/s $\vec{a}_A$=20 sm/s^2
24		OA=35 sm AB=75 m AC=60 sm ω_{OA}=4rad/s ε_{OA}=10 rad/s^2

25		OA=60 sm r=15 sm AC=6 sm ω_{OA}=1rad/s ω_1=1rad/s ε_{OA}=0
26		OA=25 sm AC=20 sm ω_{OA}=1 rad/s^2 ε_{OA}=1 rad/s^2
27		OA=40 sm AC=50 sm ω_{OA}=4 rad/s ε_{OA}=8 rad/s^2
28		r=50 sm $\vec{\vartheta}_A$=50 sm/s a_A=100 sm/s^2

29		OA=40 sm r=20 sm AC=10 sm ω_{OA}=3rad/s ε_{OA}=2 rad/s^2
30		OA=40 sm r=15 sm AC=8 sm ω_{OA}=1rad/s ε_{OA}=1 rad/s^2

Eslatma. ω_{OA}, ε_{OA}-OA krivoship mexanizmning berilgan vaziyatidagi burchak tezligi va burchak tezlanishi; ω_1-1 g'ildirakning burchak tezligi (doimiy); v_A-va a_A – A nuqtaning tezligi va tezlanishi. G'ildiraklar sirpanishsiz aylanadi.

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K. KENJAYEV

NAZARIY MEXANIKA MISOL VA MASALALARDA

II-qism

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