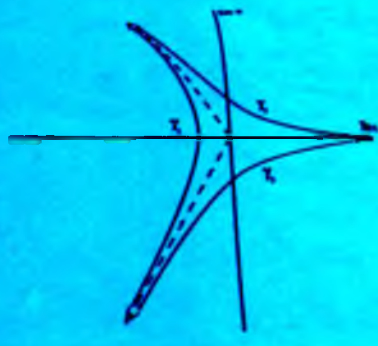
OLIY MATEMATIKADAN INDIVIDUAL TOPSHIRIQLAR TO'PLAMI

2-qism



Toshkent – 2018

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

OLIY MATEMATIKADAN INDIVIDUAL TOPSHIRIQLAR TO'PLAMI

To'plamning ikkinchi qismi

Fizika-matematika fanlari doktori,
professor A.P.Ryabushkoning
umumiy tahriri ostida

O'zbekiston Respublikasi Oliy va O'rta maxsus ta'lim vazirligining muvofiqlashtirish kengashi tomonidan o'quv qo'llanma sifatida tavsiya etilgan

«Sano-standart» nashriyoti
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Ushbu o'quv qo'llanma uch qismli «Oliy matematikadan individual topshiriqlar to'plami» nomli o'quv qo'llanmaning ikkinchi qismi bo'lib, talabalarning individual tohshiriqlarni, auditoriyalik amaliy mashg'ulotlarni va mustaqil ishlarni bajarishdagi aqliy kamoloti va faoliyatini oshirishga qaratilgan. Qo'llanmada nazariy ma'lumotlar, auditoriyalik va individual topshiriqlar masalalar to'plami quyidagi bo'limlarni o'z ichiga oladi: kompleks sonlar, aniqmas va aniq integrallar, bir necha o'zgaruvchili funksiyalar va differensial tenglamalar tartibida berilgan.

Qo'llanma ushbu nomdagi boshqa qo'llanmalarga nisbatan keltirilgan misollarning xilma- xilligi hamda texnikaning deyarli barcha sohalarini qamrab olishi bilan farqlanadi, ushbu sabablarga ko'ra tarjima mualliflari tomonidan ko'p yillardan buyon keng foydalanilmoqda.

Taqrizchilar:

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So'zboshi

Qo'lingizdagi ushbu kitob, "Oliy matematikadan individual topshiriqlar" nomli o'quv qo'llanmalar majmuasining ikkinchi qismi bo'lib, u oliy o'quv yurtlarining muhandis-texnik mutaxassislari uchun mo'ljallangan 380–450 soatlik dastur asosida yozilgan. Shuningdek, mazkur majmuadan, oliy matematika fanini o'qitish uchun ajratilgan soatlar anchagina kam bo'lgan boshqa yo'nalishdagi mutaxassislar tayyorlaydigan oliy o'quv yurtlarining talabalari ham foydalanishlari mumkin. (Buning uchun taqdim etilayotgan materiallardan keraklilarini tanlab olinishi lozim).

Tavsiya etilayotgan ushbu o'quv qo'llanma, auditoriyada amaliy mashg'ulotlar va mustaqil (nazorat) ishlarni o'tkazish uchun hamda oliy matematikaning barcha bo'limlari bo'yicha individual uy topshiriqlarini bajarish uchun mo'ljallangan.

O'quv majmuaning ikkinchi qismida kompleks sonlar, aniqmas va aniq integrallar, ko'p o'zgaruvchili funksiyalar va differensial tenglamalarga bag'ishlangan mavzular bo'yicha materiallar keltirilgan.

Kitobning ikkinchi qismi tuzilishi ham uning birinchi qismiga aynan o'xshash ko'rinishda yozilgan. Boblar, paragraflar va rasmlarning raqamlanishi birinchi qismga mos ravishda davom ettirilgan.

Kitobning yaxshilanishi borasidagi bebaho ko'rsatma va maslahatlarini ayamaganliklari uchun mualliflar jamoasi, mazkur majmuaning taqrizchilari bo'lgan Moskva energetika instituti, FA muxbir a'zosi, fizika-matematika fanlari doktori, professor S.I. Poxojayev rahbarligidagi "Oliy matematika" kafedrasining jamoasiga, Minsk radiotexnika institutining "Oliy matematika" kafedrasining mudiri, fizika-matematika fanlari doktori, professor L.A. Cherkasga hamda shu kafedraning dotsentlari, fizika-matematika fanlari nomzodlari L.A. Kuznetsov, P.A. Shmelyov, A.A. Karpuklarga, o'zlarining minnatdorchiliklarini bildiradilar.

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USLUBIY TAVSIYALAR

Tavsiya etilayotgan qo'llanmaning shakli, undan foydalanish uslubi, talabaning ko'nikma va bilimlarini baholash mezonlarini tavsiflab chiqamiz.

Oliy matematika kursi bo'yicha barcha ma'lumotlar boblarga taqsimlangan bo'lib, ularning har birida masala va misollarni yechish uchun zarur bo'ladigan nazariy bilimlar (asosiy ta'riflar, tushunchalar, teoremalar va formulalar) keltirilgan.

Ushbu ma'lumotlar yechilgan mashqlar yordamida mustahkamlanadi. (Misollar yechishning boshlanishi – ► va oxiri – ◄ belgilar yordamida berilgan.) So'ngra auditoriya mashg'ulot (AT) va o'tkazilayotgan mashg'ulotlarda 10–15 minutga mo'ljallangan mustaqil (kichik-nazoratli) ishlar uchun javoblari bilan birgalikda masala va misollar tanlab olingan. Va nihoyat 30 variantdan iborat haftalik individual uy topshiriqlari (IUT), namunaviy misollar yechimi bilan birgalikda berilgan. IUT ma'lum qismining javoblari ham keltirilgan. Har bobning nihoyasida amaliy ahamiyatga molik, darajasi yuqori qiyinchilikka ega bo'lgan qo'shimcha topshiriqlar joylashtirilgan.

Ilovada muhim mavzular bo'yicha bir va ikki soatga mo'ljallangan (har biri 30 variantlik) nazorat ishlari keltirilgan.

AT topshiriqlarining raqamlanishi uzluksiz bo'lgan ikki sondan iborat: birinchi-qismi bobni aniqlasa, ikkinchisi ushbu bobdagi AU tartib raqamini belgilaydi. Masalan AT 9.1 shifri ikkinchi bobga tegishli birinchi topshiriqni aniqlaydi. Qo'llanmaning ikkinchi qismida 26 AT va 12 IUT berilgan.

IUT uchun ham boblar bo'yicha raqamlash kiritilgan. Masalan IUT 9.2 belgisi beshinchi bobdagi ikkinchi IUT ekanligini ta'kidlaydi. Har bir IUT ning ichida esa quyidagicha raqamlash kiritilgan: birinchi son topshiriqdagi masalaning tartib raqamiga tegishli bo'lsa, ikkinchisi variantning tartib raqamini aniqlaydi. Shunday qilib, IUT 9.2:16 shifri talabaning IUT 5.2 dan 16 variantdagi topshiriqlarini bajarishini belgilab, ushbu variantda 1.16, 2.16, 3.16, 4.16 masalalar borligini ta'kidlaydi. IUT bo'yicha variantlarni tanlab olishda oldingi topshiriqdan keyingisiga o'tganida tasodifiy yoki boshqa usulda almashtirish usulini qo'llash mumkin. Bundan tashqari, ixtiyoriy talabaga IUT berilishida bir xil turdagi masalalarni har xil variantlardan olish mumkin. Masalan, IUT -3.1;1.2;2.4;3.6 shifri talaba IUT -3.1 dan birinchi masalani 1 – variantdan, ikkinchisini 4 – variantdan, uchinchisini 6 – variantdan yechishini ta'kidlaydi. Bu ko'rinishdagi kombinatsion usul 30 ta variantdan keng qamrovli ko'p variantlar hosil qilishni ta'minlaydi.

IUT larni ba'zi oliy texnika o'quv yurtlari (Belorussiya qishloq xo'jaligini mexanizatsiyalash instituti, Belorussiya politexnika instituti, Uzoq sharq politexnika instituti v.b.) ning o'quv jarayonida qo'llanilishi, IUT ni har bir haftalik auditoriya topshiriqlaridan keyin alohida har safar berishning o'rniga, ikki haftada bir marta, ikki haftalik auditoriya mashg'ulotlari mazmuniga mos ravishda berish maqsadga muvofiq ekanligini ko'rsatdi. Ushbu qo'llanmaga muvofiq talabalar bilan ishlashni tashkil etish bo'yicha umumiy tavsiyalarni beramiz.

1. Oliy o'quv yurtlarining 25 talik guruhlari uchun har haftada ikkita auditoriya mashg'ulotlari, talabalar erkin qatnashadigan maslahat darslari rejalashtiriladi va haftalik IUT beriladi. Ushbu tadbirlarni samarali tashkil etish maqsadida, talabalar bilimini, xato va kamchiliklarini aniqlash va tuzatish yo'llarini ko'rsatgan holda, tizimli baholash uchun kafedra tomonidan oldindan tayyorlangan professor-o'qituvchilarga IUT ning javoblar varaqasi va yechimlar majmuasi beriladi (talabalar mustasno). Javoblar varaqasi har bir topshiriqlar uchun tayyorlansa, yechimlar majmuasi faqat yechish usulini, amallar ketma-ketligi va hisoblashlardagi ko'nikmalarning to'g'riligini tekshirish uchun zarur bo'lgan muhim bo'lgan masala va variantlarga ishlab chiqiladi. Kafedra tomonidan yechimlar varaqasi qaysi IUT lar uchun zarurligini belgilanadi. Yechimlar varaqasi (bitta variant bitta varaqda joylashadi) talabalar tomonidan bajarilgan topshiriqlar bajarilishida o'z o'zini nazorat qilish uchun, talabalar o'rtasida o'zaro nazorat tashkil etishda ishlatiladi. Lekin ko'pchilik hollarda yechimlar varaqasi yordamida o'qituvchi usulning to'g'riligini tekshirsa, talabalar o'zining hisob-kitoblari to'g'riligini nazoratdan o'tkazishi

mumkin. Ushbu usullar 25 talabaning IUT larini 15–20 minut davomida tekshirib baholash imkonini beradi.

2. Oliy o'quv yurtlarining 15 talik guruhlarida esa har haftada ikkita auditoriya mashg'ulotlari, guruhlar dars jadvalida mustaqil tayyorlanish uchun, o'qituvchi nazorati ostida haftalik yuklamaga kiritilgan ikki soatlik maslahat darslari rejalashtiriladi. Dars jarayonini ushbu taxlitda tashkil etish (Belorussiya qishloq xo'jaligini mexanizatsiyalash instituti), talabalarning mustaqil va ijodiy ishlashlari hamda bilim sifatini o'qituvchilar tomonidan tezkor ravishda nazorat qilish darajasi sezilarli tarzda oshishi kuzatiladi. Yuqorida tavsiya etilgan usullar bu yerda ham o'zining samarasini beradi. Lekin, ushbu guruhlarda AT va IUT larni tekshirish tezlashadi va topshiriqlarni bajarishda nazariy bilimlarni nazorat qilish imkoni oshadi, o'zlashtirmovchi talabalardan mavjud qarzdorliklarni kamaytirish imkoniyati paydo bo'ladi. Shuningdek, yana IUT, mustaqil va nazorat ishlari bo'yicha baholar jamlamasi yordamida o'quv jarayonini boshqarish, nazorat qilish, talabalar olgan bilimlari sifatini baholash imkoni ham paydo bo'ladi.

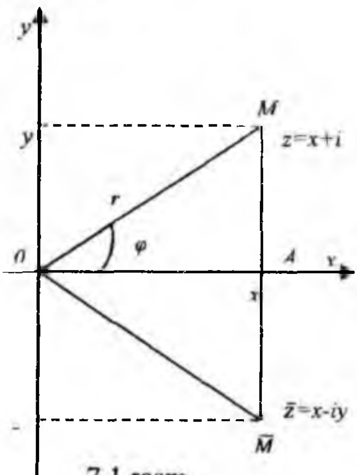
Yuqorida aytilgan tadbirlarni amalga oshirish natijasida semestr mobaynida o'rganilgan bilimlar bo'yicha an'anaviy semestr (yillik) imtihonlardan voz kechish, hamda talabalar ko'nikmalari va bilimlarini baholash bo'yicha blokli-siklik (modulli-siklik) deb ataluvchi usuldan foydalanish mumkin bo'ladi. Ushbu usulning mohiyati quyidagilardan iborat: Fanning semestrdagi (yillik) yuklamasi 3–5 ta blok (modul) larga bo'linadi va ularning har biri bo'yicha AT, IUT bajarilib, sikl yakunida esa ikki soatlik yozma nazorat o'tkazilib, bu yerda 2–3 ta nazariy savollar, 5–6 ta masala va misolllar beriladi. AT, IUT va yakuniy nazorat ballarining yig'indisi talabalarning har bir blok (modul) va semestr (o'quv yilida) hamma bloklar (modullar) bo'yicha olgan bilimlarini ham alohida obektiv baholash imkonini beradi. Shunga o'xshash usul Belorussiya qishloq xo'jaligini mexanizatsiyalash institutida tadbiq qilingan.

Fikrimiz yakunida, ushbu qo'llanma o'rtacha imkoniyatli talabalarga mo'ljallanganligini va bu yerdagi bilimlarni egallash oliy matematika fanidan qoniqarli va yaxshi ko'nikmalarga ega bo'lishlarini ta'minlashini ta'kidlashimiz mumkin. Iqtidorli va a'lo bahoga o'quvchi talabalar uchun esa, rag'batlantirishning chora-tadbirlarini e'tiborga olgan holda alohida murakkab topshiriqlar (ta'limda individual yondashuv) tayyorlanishi zarur. Masalan, bu talabalarga, o'z ichiga ushbu qo'llanmadagi yuqori murakkablikka ega masalalar va nazariy mashqlar (ushbu maqsad uchun, xususan, har bir bob oxiridagi qo'shimcha topshiriqlar mo'ljallangan) butun semestr uchun ishlab chiqilishi lozim. O'qituvchi ushbu topshiriqlarni semestr boshida berib, ularning bajarilish ketma-ketligini belgilab (o'zining shaxsiy nazoratida), talabalarga oliy matematikadan ma'ruza va amaliyot darslarida erkin qatnashishga ruxsat berishi mumkin va hamma topshiriqlar muvaffaqiyatli bajarilgandan so'ng sessiyada a'lo baho qo'yiladi

7. KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR

7.1. ASOSIY TUSHUNCHALAR. KOMPLEKS SONLAR USTIDA AMALLAR

Kompleks son deb, $z = x + iy$ turdagi songa aytiladi. Bu yerda, x va y lar haqiqiy sonlar $i = \sqrt{-1}$ esa, mavhum birlikdir, ya'ni, kvadrati -1 ga teng bo'lgan son yoki $z^2 + 1 = 0$ tenglamaning ildizidir. Odatda, x ni kompleks sonning haqiqiy qismi, y ni esa, uning mavhum qismi deb yuritiladi. Ular uchun quyidagi belgilashlar kiritilgan: $x = Rez$ va y$= Imz$. Agar $y = 0$ unga $z = x \in R$ agar $x = 0$ bo'lsa, $z = iy$ ni sof mavhum son deyiladi.



7.1 rasm

Geometrik nuqtai nazardan qaralganda, har qanday $z = x + iy$ kompleks songa tekislikning biror $M(x, y)$ nuqtasi (yoki $\overrightarrow{OM}$ vektor) mos keladi va aksincha, tekislikning har qanday $M(x, y)$ nuqtasiga $z = x + iy$ kompleks son mos keladi. Umuman, kompleks sonlar to'plami bilan Oxy tekislikdagi nuqtalar orasida o'zaro bir qiymatli moslik o'rnatilganki, Oxy tekislikni kompleks tekisligi deb yuritiladi va uni z kabi belgilanadi (7.1-rasm).

Barcha kompleks sonlar to'plamini C harfi bilan belgilanadi. Har doim, $R \subset C$ ekanligini ta'kidlaymiz. Barcha $z = x$ haqiqiy sonlarga mos keladigan nuqtalar Ox o'qida joylashadi, shu boisdan, Ox o'qini kompleks sonlar tekisligidagi haqiqiy o'q deb yuritiladi. Barcha $z = iy$ mavhum sonlarga mos nuqtalar Oy o'qida joylashadi va kompleks sonlar tekisligining mavhum o'qi deb ataladi.

Agar ikkita kompleks sonlarning haqiqiy va mavhum qismlari o'zaro teng bo'lsalar, ularni o'zaro teng kompleks sonlar deb yuritiladi.

$z = x + iy$ va $\bar{z} = x - iy$ turdagi sonlar o'zaro tutashgan (bog'langan) kompleks sonlar deb ataladi (7.1-rasm).

Agarda, $z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$ ikki kompleks sonlar bo'lsa, ular ustidagi arifmetik amallar quyidagicha bajariladi:

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2),$$
$$z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) = (x_1 - x_2) + i(y_1 - y_2),$$
$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + y_2 x_1),$$
$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{z_1 \bar{z_2}}{z_2 \bar{z_2}} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

(oxirgi amal $z_2 \neq 0$ bo'lsagina o'rinli bo'ladi). Yuqorida bajarilgan amallar natijasida, umuman yana kompleks sonlar hosil bo'ladi. Shuningdek, kompleks sonlar ustidagi mazkur amallar, haqiqiy sonlar ustidagi arifmetik amallarga o'xshash barcha xossalarga egadir, ya'ni, qo'shish va ko'paytirish amallari kommutativ va assotsiativdir, hamda ular distributivlik xossasiga ega bo'lib, ular uchun teskari amallar bo'lgan ayirish va bo'lish (nolga bo'lishdan tashqari) amallari ham mavjuddir.

1-misol. $z_1 = 2 + 3i$, $z_2 = 3 - 4i$ va $z_3 = 1 + i$ kompleks sonlar berilgan. $z = \frac{z_1 + z_1 z_2 + z_2^2}{z_1 + z_3}$ ni topilsin.

►Ketma-ket hisoblaymiz:

$$z_1 + z_3 = (2 + 3i) + (1 + i) = 3 + 4i,$$
$$z_1 z_2 = (2 + 3i)(3 - 4i) = (6 + 12) + i(9 - 8) = 18 + i,$$
$$z_2^2 = (3 - 4i)^2 = 9 - 24i - 16 = -7 - 24i,$$
$$z_1 + z_1 z_2 + z_2^2 = 2 + 3i + 18 + i - 7 - 24i = 13 - 20i.$$

U holda: $z = \frac{13-20i}{3+4i} = \frac{(13-20i)(3-4i)}{(3+4i)(3-4i)} = \frac{(39-80)+i(-60-52)}{25} = -\frac{41}{25} - i\frac{112}{25}$.◄

Berilgan $z = x + iy$ kompleks sonning moduli deb, $r = |\overline{OM}| = \sqrt{z\bar{z}}$ songa aytiladi. $\overline{OM}$ vektorning Ox o'qning musbat yo'nalishi bilan tashkil etgan φ burchagi ni kompleks sonning argumenti deb ataladi va $\varphi = Arg z$ kabi belgilanadi.

Har qanday kompleks son uchun quyidagilarni yozish mumkin (7.1 rasmga qaralsin):

$$x = r\cos\varphi, y = r\sin\varphi,$$
$$\cos\varphi = \frac{x}{r}, \sin\varphi = \frac{y}{r}, \qquad (7.1)$$
$$r = \sqrt{x^2 + y^2}$$

Bu yerda, kompleks son argumentining bosh qiymati $\varphi = argz$ uchun quyidagi shartlar o'rinli bo'ladi: $-\pi < argz \le \pi$ yoki $0 \le argz < 2\pi$.

Har qanday $z = x + iy$ kompleks sonning trigonometrik shakli deb,

$$z = r(cos\varphi + isin\varphi) \qquad (7.2)$$

ifodaga aytiladi. Agar *Eyler formulasi* deb ataluvchi $e^{i\varphi} = cos\varphi + isin\varphi$ ni inobatga olinsa, (7.2) dan kompleks sonning ko'rsatkichli shakli deb ataluvchi

$$z = re^{i\varphi} \qquad (7.3)$$

ni hosil bo'ladi.

Yuqorida keltirilgan (7.2) bilan (7.3) formulalarni kompleks sonlarni ko'paytirish va ularning darajasini oshirishda qo'llash maqsadga muvofiqdir.

Agar $z_1 = r_1(cos\varphi_1 + isin\varphi_1)$, $z_2 = r_2(cos\varphi_2 + isin\varphi_2)$ ifodalar berilgan bo'lsa, u holda quyidagilar o'rinli bo'ladi:

$$z_1z_2 = r_1r_2(cos(\varphi_1 + \varphi_2) + isin(\varphi_1+\varphi_2)) = r_1r_2e^{i(\varphi_1+\varphi_2)},$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos(\varphi_1 - \varphi_2) + isin(\varphi_1 - \varphi_2)\right) = \frac{r_1}{r_2}e^{i(\varphi_1-\varphi_2)}\ (z_2 \neq 0),$$

$$z^n = r^n(cosn\varphi + isinn\varphi) = r^ne^{in\varphi} \qquad (7.4)$$

(7.4) formulani Muavr formulasi deb ataladi.

Agar (7.2) kabi berilgan kompleks sondan n-darajali ($n>1$, $n \in Z$) ildiz chiqarish lozim bo'lsa, ushbu ildizning n ta qiymatini beruvchi quyidagi formuladan foydalaniladi:

$$z_k = \sqrt[n]{z} = \sqrt[n]{r}\left(cos\frac{\varphi+2\pi k}{n} + isin\frac{\varphi+2\pi k}{n}\right) = \sqrt[n]{r}e^{i(\varphi+2\pi k)/n}\ (k = \overline{0, n-1}) \qquad (7.5)$$

$\sqrt[n]{r}$- arifmetik ildiz deb tushuniladi.

2-misol. $(1 + i)^{12}$ hisoblansin.

► (7.1) formuladan foydalanib, $z = 1 + i$ ning trigonometrik yoki ko'rsatkichli shakllarni yozib olamiz: $r = \sqrt{1+1} = \sqrt{2}, cos\varphi = \frac{1}{\sqrt{2}}, sin\varphi = \frac{1}{\sqrt{2}}, \varphi = \frac{\pi}{4}$

$$z = \sqrt{2}\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right) = \sqrt{2}e^{\pi i/4}$$

Muavr formulasiga binoan,

$z^{12} = (\sqrt{2})^{12}\left(\cos\left(12\cdot\frac{\pi}{4}\right) + i\sin\left(12\cdot\frac{\pi}{4}\right)\right) = \sqrt{2^{12}}e^{3\pi i} =$
$64(cos3\pi + +isin3\pi) = -64.$◀

3-misol. $z^6 + 1 = 0$ tenglamaning ildizlari topilsin.

▶Berilgan tenglamani $z^6 = -1\, yoki\, z = \sqrt[6]{-1}$ kabi yozib olish mumkin. (7.1) formulaga binoan, -1 ning trigonometrik shakli $-1 = 1\cdot(cos\pi + isin\pi)$ kabi yoziladi. (7.3) formulaga ko‘ra, qaralayotgan tenglamaning ildizlarini

$z_k = \sqrt[6]{-1} = 1\left(cos\frac{\pi+2k\pi}{6} + isin\frac{\pi+2k\pi}{6}\right) = e^{i(\varphi+2\pi k)/n}$, bu yerda $k = \overline{0,5}$

dan foydalanib aniqlaymiz. k ga ketma-ket 0,1, …, 5, qiymatlarni berib $z^6 + 1 = 0$ tenglamaning barcha mumkin bo‘lgan 6 ta ildizlarini topamiz:

$$z_0 = cos\frac{\pi}{6} + isin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i = e^{\frac{\pi i}{6}},$$
$$z_1 = cos\frac{\pi}{2} + isin\frac{\pi}{2} = i = e^{\pi i/2},$$
$$z_2 = cos\frac{5}{6}\pi + isin\frac{5}{6}\pi = -\frac{\sqrt{3}}{2} + i\frac{1}{2} = e^{\frac{-5\pi i}{6}},$$
$$z_3 = cos\frac{7}{6}\pi + isin\frac{7}{6}\pi = -\frac{\sqrt{3}}{2} - \frac{1}{2}i = e^{\frac{7\pi i}{6}} = e^{\frac{-5\pi i}{6}},$$
$$z_4 = cos\frac{3}{2}\pi + isin\frac{3}{2}\pi = -i = e^{\frac{3\pi i}{2}} = e^{\frac{3\pi i}{2}},$$
$$z_5 = cos\frac{11}{6}\pi + isin\frac{11}{6}\pi = \frac{\sqrt{3}}{2} - \frac{1}{2}i = e^{\frac{11\pi}{6}i} = e^{\frac{-\pi i}{6}}.$$◀

4-misol. $z^3 - 1 + i\sqrt{3} = 0$ tenglamaning ildizlari topilsin.

▶$z^3 = 1 - i\sqrt{3} = 2(cos\frac{\pi}{3} - isin\frac{\pi}{3})$ bo‘lganligi uchun (7.5) formulaga binoan, $z_k = \sqrt[3]{z} = \sqrt[3]{2}(cos\frac{\frac{\pi}{3}+2\pi k}{3} - isin\frac{\frac{\pi}{3}+2\pi k}{3})$ ni yoza olamiz ($k = \overline{0,2}$).

Demak, berilgan tenglamaning ildizlari quyidagicha bo‘ladi:

$$z_0 = \sqrt[3]{2}(cos\frac{\pi}{9} - isin\frac{\pi}{9}),\ z_1 = \sqrt[3]{2}\left(cos\frac{7\pi}{9} - isin\frac{7\pi}{9}\right),$$
$$z_2 = \sqrt[3]{2}\left(cos\frac{13\pi}{9} - isin\frac{13\pi}{9}\right).$$◀

AT- 7.1

1. Agar $z_1 = 2 + 3i$, $z_2 = 3 + 2i$, $z_3 = 5 - 2i$ bo‘lsa, $(z_1 + 2z_2)z_3$ hisoblansin. (*Javob:* 54+19i).

2. $z_1 = 3 + 5i$, $z_2 = 3 - 4i$, $z_3 = 1 - 2i$ kompleks sonlar berilgan. $z = \frac{(z_1+z_3)z_2}{z_3}$ ni yoni topilsin. (*Javob:* $\frac{38}{5} + \frac{41}{5}i$).

3. $z_1 = 2 - 2i$, $z_2 = -1 + i$, $z_3 = -i$ va $z_4 = -4$ larni trigonometrik va ko‘rsatkichli shakllarda ifodalansin.

4. $z^8 - 1 = 0$ tenglamaning ildizlari topilsin. (*Javob:* $z_0 = 1$, $z_1 = \frac{\sqrt{2}}{2} + + \frac{\sqrt{2}}{2}i$, $z_2 = i$, $z_3 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $z_4 = -1$, $z_5 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$, $z_6 = -i$, $z_7 = \frac{\sqrt{2}}{2} - -\frac{\sqrt{2}}{2}i$).

Mustaqil ish

1. 1. Agar $z_1 = 4 + 5i$, $z_2 = 1 + i$, $z_3 = 7 - 9i$ bo‘lsa, $z = \frac{z_1(z_2+z_3)}{z_2}$ ifodaning qiymati topilsin. (*Javob:* 40−32i).

2. $z_1 = \sqrt{3} + i$, $z_2 = -1 + \sqrt{3}i$ va $z_3 = -\frac{1}{2}$ larning trigonometrik va ko‘rsatkichli shakllari keltirilgan.

2. 1. Agar $z_1 = 4 + 8i$, $z_2 = 1 - i$, $z_3 = 9 + 13i$ bo‘lsa, $\frac{(z_1+z_2z_3)}{z_2}$ ning qiymati topilsin. (*Javob:* 7+19i).

2. $z^2 - i = 0$ tenglama yechilsin. (*Javob:* $\pm\left(\frac{(1+i)}{\sqrt{2}}\right)$.

3. 1. Agar $z_1 = 2 - i$, $z_2 = -1 + 2i$, $z_3 = 8 + 12i$ bo‘lsa, $\frac{(z_1^2+z_2+z_3)}{z_2}$ topilsin. (*Javob:* 2+2i).

2. $z_1 = 2/(1 + i)$, $z_2 = -\sqrt{3} - i$ kompleks son trigonometrik va ko‘rsatkichli shakllarda ifodalansin.

7.2. 7- bobga qo‘shimcha mashqlar

1. Quyidagi kompleks sonlar ko‘rsatkichli shaklda ifodalansin

a) $z = -\sqrt{12} - 2i$, b) $z = -cos\frac{\pi}{7} + isin\frac{\pi}{7}$. (*Javob:* a) $4e^{7\pi i/6}$, b) $e^{6\pi i/7}$).

2. Isbotlansin $(1+cos\alpha+isin\alpha)^{2n}=\left(2cos\frac{\alpha}{2}\right)^{2n}e^{in\alpha}.(n\in N,\alpha\in R)$.

3. Yig'indi topilsin $\sum_{k=0}^{n}e^{ik\varphi}$. (*Javob*: $\frac{e^{i(n+1)\varphi}-1}{e^{i\varphi}-1}$.)

4. n ning qanday butun qiymatlarida quyidagi tenglik o'rinli bo'ladi?

$(1+i)^n=(1-i)^n$. (*Javob*: $n=4k,k\in Z$.)

5. Eyler formulasidan foydalanib

$cosx+cos2x+cos3x+\cdots+cosnx$ yig'indi hisoblansin.

(*Javob*: $\left(sin\frac{nx}{2}cos\frac{n+1}{2}x\right)sin\frac{x}{2}$.)

6. Ayniyat isbotlansin

$$x^5-1=(x-1)(x^2-2xcos72°+1)(x^2-2xcos144°+1).$$

$z=x+iy$ nuqtalarda ko'rsatilgan shartlarni qanoatlantiruvchi sohalarni (z) kompleks tekisligida topilsin va ular tasvirlansin.

7. $|z-z_1|<4$, bu yerda: $z_1=3-5i$. (*Javob*: markazi $z_1$nuqtada bo'lib, radiusi R=4 bo'lgan ochiq doira.)

8. $|z+z_1|>6$, bu yerda: $z_1=1-i$. (*Javob*: markazi $-z_1$ nuqtada bo'lib, radiusi R=6 bo'lgan doiraning tashqarisi.)

9. $1<|z-i|<3$. (*Javob*: markazi $z=i$ nuqtada bo'lib, radiuslari $r_1=1$ va $r_2=3$ bo'lgan aylanalar orasidagi halqa.)

10. $0<|z+i|<1$. (*Javob*: radiusi R=1 doiraning $z=-i$ nuqtadagi markazini chiqarib tashlangan ichki qismi.)

11. $0<Re(3iz)<2$. (*Javob*: $y=0,y=-\frac{2}{3}$ to'g'ri chiziqlar orasidagi gorizontal tasma.)

12. $Re\left(\frac{1}{z}\right)>a,a=const,a\in R$. (*Javob*: agar $a=0$ bo'lsa, u holda $x>0$, ya'ni, chegarasiz o'ng yarim tekislik; agar $a>0$ yoki $a<0$ bo'lsa, u holda, $\left(x-{}^1/_{2a}\right)^2+y^2={}^1/_{4a^2}$ aylananing ichki va tashqi qismlari nuqtalarini hosil qilamiz.)

13. $Re\frac{z-ai}{z+ai}=0, bu\ yerda\ a=const,a\in R$ (*Javob*: $z=ai$ nuqta.)

14. $Im(iz)<2$ (*Javob*: $x=2$ to'g'ri chiziqdan chapda joylashgan yarim tekislik.)

8. ANIQMAS INTEGRAL

8.1. BOSHLANG'ICH FUNKSIYA VA ANIQMAS INTEGRAL

Faraz qilaylik, $(a; b)$ oraliqda $f(x)$ funksiya berilgan bo'lsin. Agar shu oraliqning barcha nuqtalarida $F'(x) = f(x)$ kabi tenglik o'rinli bo'ladigan bo'lsa, u holda, $F(x)$ funksiyani $f(x)$ funksiyaning $(a; b)$ oraliqdagi *boshlang'ich funksiyasi* deb yuritiladi. Berilgan $f(x)$ funksiyaning har qanday ikkita boshlang'ich funksiyalari bir-biridan ixtiyoriy o'zgarmas son bilan farq qiladi.

Agar C ixtiyoriy o'zgarmas son bo'lganda, $(a; b)$ oraliqda berilgan $f(x)$ funksiyaning $F(x) + C$ kabi barcha boshlang'ich funksiyalari to'plamini $f(x)$ funksiyaning *aniqmas itegrali* deb ataladi va u quyidagicha yoziladi:

$$\int f(x)dx = F(x) + C.$$

Integrallashning asosiy qoidalarini keltiramiz:

1) $\int f'(x)dx = \int df(x) = f(x) + C$,

2) $d\int f(x)dx = d(F(x) + C) = f(x)dx$;

3) $\int [f(x) \pm \varphi(x)]dx = \int f(x)dx \pm \int \varphi(x)dx$;

4) $\int af(x)dx = a\int f(x)dx \ (a = const)$;

5) $\int f(x)dx = F(x) + C$ bo'lib, a va b ($a \neq 0$) lar o'zgarmas sonlar bo'lganda, har doim quyidagi munosabat o'rinli bo'ladi:

$$\int f(ax + b)dx = \frac{1}{a}F(ax + b) + C;$$

6) agar $\int f(x)dx = F(x) + C$ bo'lib, $u = \varphi(x)$, ixtiyoriy differensiallanuvchi funksiya bo'lsa, u holda:

$$\int f(u)du = F(u) + C.$$

Integrallash natijasining to'g'riligini tekshirish uchun, topilgan boshlang'ich funksiyaning hosilasi hisoblanadi, ya'ni: $(F(x) + C)' = f(x)$.

Aniqmas integralning ta'rifiga ko'ra, integrallashning asosiy qoidalari va asosiy elementar funksiyalar hosilalar jadvaliga asoslanib, asosiy aniqmas integrallarning jadvalini tuzish mumkin:

1) $\int u^a du = \frac{u^{a+1}}{a+1} + C \ (a \neq -1)$;

2) $\int \frac{du}{u} = ln|u| + C;$

3) $\int a^u du = \frac{a^u}{\ln a} + C;$

4) $\int e^u du = e^u + C;$

5) $\int \sin u du = -\cos u + C;$

6) $\int \cos u du = \sin u + C;$

7) $\int \frac{du}{a^2+u^2} = \frac{1}{a} arctg\frac{u}{a} + C = -\frac{1}{a} arcctg\frac{u}{a} + C$ ($a \neq 0$);

8) $\int \frac{du}{a^2-u^2} = \frac{1}{2a} ln\left|\frac{u+a}{u-a}\right| + C = -\frac{1}{2a} ln\left|\frac{u-a}{u+a}\right| + C;$

9) $\int \frac{du}{\sqrt{u^2 \pm a^2}} = ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$ ($a \neq 0$);

10) $\int \frac{du}{\sqrt{a^2-u^2}} = arcsin\frac{u}{a} + C = -arccos\frac{u}{a} + C$ ($a>0$);

11) $\int \frac{du}{cos^2 u} = tg\, u + C;$

12) $\int \frac{du}{sin^2 u} = -ctg\, u + C;$

13) $\int \frac{du}{\sin u} = ln\left|tg\frac{u}{2}\right| + C = ln\left|\frac{1}{\sin u} - ctg\, u\right| + C;$

14) $\int \frac{du}{\cos u} = ln\left|tg(\frac{u}{2} + \frac{\pi}{4})\right| = ln\left|\frac{1}{\cos u} + tg\, u\right| + C;$

15) $\int sh\, udu = ch\, u + C;$

16) $\int ch\, udu = sh\, u + C;$

17) $\int \frac{du}{ch^2 u} = th\, u + C;$

18) $\int \frac{du}{sh^2 u} = -cth\, u + C.$

Yuqorida keltirilgan munosabatlar *integrallar jadvali* deb ataladi.

Eslatib o‘tamizki, keltirilgan jadvaldagi u harfi, erkli o‘zgaruvchi ham bo‘lishi yoki uzluksiz differensiallanuvchi $u=\varphi(x)$ funksiya ham bo‘lishi mumkin.

Quyida, aniqmas integrallarni hisoblashga doir ayrim misollarni keltiramiz:

1-misol. $\int \left(4x^3 - 2\sqrt[3]{x^2} + \frac{2}{x^3} + 1\right) dx$ hisoblansin.

► $\int \left(4x^3 - 2\sqrt[3]{x^2} + \frac{2}{x^3} + 1\right) dx = 4\int x^3 dx - 2\int x^{\frac{2}{3}} dx + 2\int x^{-3} dx + + \int dx = 4\frac{x^4}{4} - 2\frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 2\frac{x^{-2}}{-2} + x + C = x^4 - \frac{6}{5}\sqrt[3]{x^5} - \frac{1}{x^2} + x + C.$ ◄

2-misol. $\int \frac{1+2x^2}{x^2(1+x^2)} dx$ hisoblansin.

► $\int \frac{1+2x^2}{x^2(1+x^2)} dx = \int \frac{(1+x^2)+x^2}{x^2(1+x^2)} dx = \int \frac{1+x^2}{x^2(1+x^2)} +$
$\int \frac{x^2}{x^2(1+x^2)} dx = \int \frac{dx}{x^2} + + \int \frac{dx}{1+x^2} = -\frac{1}{x} + arctgx + C.$◄

3-misol. $\int 3^x e^{2x} dx$ hisoblansin.

► $\int 3^x e^{2x} dx = \int (3e^2)^x dx = \frac{(3e^2)^x}{\ln(3e^2)} + C$.◄

4-misol. $\int (2x-7)^9 dx$ hisoblansin.

► $\int (2x-7)^9 dx = \frac{1}{2}\int (2x-7)^9 \cdot 2dx = \frac{1}{2}\frac{(2x-7)^{10}}{10} + C =$
$\frac{1}{20}(2x-7)^{10} + +C.$◄

5-misol. $\int \cos(7x-3)\, dx$ hisoblansin.

► $\int \cos(7x-3)\, dx = \frac{1}{7}\int \cos(7x-3)\, d(7x-3) =$
$\frac{1}{7}\sin(7x-3) + C.$ ◄

6-misol. $\int \frac{x - arctgx}{1+x^2} dx$ hisoblansin.

► $\int \frac{x-arctgx}{1+x^2} dx = \int \frac{x}{1+x^2} dx - \int \frac{arctgx}{1+x^2} dx = \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} -$
$-\int arctgx d(arctgx) = \frac{1}{2}\ln(1+x^2) - \frac{1}{2} arctg^2 x + C.$◄

7-misol. $\int ctg3xdx$ hisoblansin.

► $\int ctg3xdx = \int \frac{cos3x}{sin3x} dx = \frac{1}{3}\int \frac{cos3x \cdot 3dx}{sin3x} = \frac{1}{3}\int \frac{d(sin3x)}{sin3x} =$
$\frac{1}{3} ln|sin3x| + C.$◄

Yuqorida keltirilgan 4–7 misollardagi integrallarni hisoblash jarayonida 5-qoidani qo‘llash maqsadida integral belgisi ostida qatnashgan ayrim ko‘paytuvchilarni differensial belgisi ostiga kiritilib, undan keyin esa, kerakli jadval integralidan foydalanildi. Bu xildagi almashtirishlarni *differensial belgisi ostiga kiritish usuli* deb yuritiladi. Masalan, differensialluvchi bo‘lgan har qanday $f(x)$ funksiya uchun

$$\int \frac{f'(x)}{f(x)} dx = \int \frac{df(x)}{f(x)} = ln|f(x)| + C$$

deb yozish mumkin.

8-misol. $\int \frac{sin2x}{4+sin^2x} dx$ hisoblansin.

▶ $\int \frac{\sin 2x}{4+\sin^2 x}dx = \int \frac{2\sin x\cos x}{4+\sin^2 x}dx = \int \frac{2\sin x}{4+\sin^2 x}d(\sin x) =$
$\frac{(4+\sin^2 x)'}{4+\sin^2 x} = = \ln(4+\sin^2 x) + C.$ ◀

9-misol. $\int \frac{x+2}{x^2+4x+5}dx$ hisoblansin.

▶ $\int \frac{x+2}{x^2+4x+5}dx = \frac{1}{2}\int \frac{(x^2+4x+5)'}{x^2+4x+5}dx = \ln\sqrt{x^2+4x+5} + C.$ ◀

AT-8.1

Ko'rsatilgan integrallarni hisoblang va integrallash natijasini differensiallab tekshiring.

1. $\int(5x^7 - 3\sqrt[5]{x^3} + \frac{3}{x^4})dx.$
2. $\int \frac{\cos 2x}{\cos^2 x\sin^2 x}dx.$
3. $\int(3\sin x + 2^x 3^{2x} - \frac{1}{9+x^2})dx.$
4. $\int \sqrt[7]{(5x+3)^3}\,dx.$
5. $\int \frac{x^2}{\sqrt[3]{(x^3+7)^2}}dx.$
6. $\int(\sin 7x - e^{3-2x} + \frac{1}{\cos^2 4x})\,dx.$
7. $\int(e^{-3x} - \frac{1}{3x+2} + 3^{2x} - -\sin^3 x\cos x)dx.$
8. $\int tg3x dx.$
9. $\int \frac{\sqrt{\arcsin x} - x}{\sqrt{1-x^2}}dx.$
10. $\int \frac{x-x^3}{\sqrt{9-x^4}}dx.$
11. $\int \frac{3^x}{\sqrt{9-9^x}}dx.$
12. $\int \frac{x-3}{1-x^2}dx.$

Mustaqil yechish uchun topshiriqlar

Aniqmas integrallarni hisoblang va integrallash natijasini differensiallab tekshiring.

1. *a)* $\int(3x - \sqrt[7]{x^5} + 2\sin x - 3)dx$; *b)* $\int(\sin 3x + x\sqrt{1+x^2})dx$; *c)* $\int \frac{e^{2x}}{e^{2x}+3}dx.$
2. *a)* $\int(x^7 + \frac{1}{\sqrt[3]{x}} + 2^x)dx$; *b)* $\int\left(x^2\sqrt[3]{4-x^2} + \frac{1}{\sin^2 4x}\right)dx$; *c)* $\int \frac{x+1}{x^2+2x-3}dx.$
3. *a)* $\int(x^{-2} + 7x^6 - \frac{1}{2\sqrt{x}})dx$; *b)* $\int\left(\frac{x^3}{\sqrt{1+x^4}} - \cos^7 x\sin x\right)dx$; *c)* $\int ctg(3x-2)dx.$

8.2 FUNKSIYALARNI BEVOSITA INTEGRALLASH

Ko'plab funksiyalarning aniqmas integrallarini topish masalasida, ularni jadval integrallaridan biriga keltirish usulidan foydalaniladi. Uning uchun esa, integrallanuvchi funksiyalar ustida algebraik ayniy almashtirishlar bajariladi yoki ayrim ko'paytuvchilarni differensial belgisi ostiga kiritish yo'li tanlaniladi.

1-misol. $\int tg^3xdx$ hisoblansin.

►$\int tg^3xdx = \int\left(\frac{1}{cos^2x} - 1\right)tgxdx = \int\frac{1}{cos^2x}tgxdx -$ $\int tgxdx = \int tgxd(tgx) - -\int\frac{\sin x}{\cos x}dx = \frac{tg^2x}{2} + \int\frac{d(\cos x)}{\cos x} =$ $\frac{1}{2}tg^2x + ln|\cos x| + C.$◄

2-misol. $\int\frac{x+3}{x+5}dx$ hisoblasin.

►$\int\frac{x+3}{x+5}dx = \int\frac{x+5-2}{x+5}dx = \int dx - \int\frac{2}{x+5}dx = x -$ $2\int\frac{d(x+5)}{x+5} = x - -2ln|x+5| + C.$◄

3-misol. $\int\frac{dx}{x^2-4x+8}$ hisoblansin.

►$\int\frac{dx}{x^2-4x+8} = \int\frac{dx}{x^2-4x+4+4} = \int\frac{dx}{(x-2)^2+4} =$ $\int\frac{d(x-2)}{4+(x-2)^2} = \frac{1}{2}arctg\frac{x-2}{2} + C.$ ◄

$\int sinmxcosnxdx$, $\int sinmxsinnxdx$, $\int cosmxcosnxdx$ kabi integrallarni hisoblashda, mos ravishda quyidagi formulalardan foydalaniladi:

$$sinmxcosnx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x],$$

$$sinmxsinnx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x],$$

$$cosmxcosnx = \frac{1}{2}[\cos(m-n)x + \cos(m+n)x].$$

4-misol. $\int\cos(2x-1)\cos(3x+5)\,dx$ hisoblansin.

►$\int\cos(2x-1)\cos(3x+5)\,dx = \frac{1}{2}\int(\cos(x+6) +$ $\cos(5x+4))dx = =\frac{1}{2}\int\cos(x+6)\,d(x+6) + \frac{1}{10}\int\cos(5x+$ $4)d(5x+4) = \frac{1}{2}\sin(x+6) + +\frac{1}{10}\sin(5x+4) + C.$ ◄

$\int cos^m x sin^n x dx$ $(m, n \in Z)$ kabi ko'rinishdagi integrallarni hisoblashda quyidagi hollarni ko'rib o'tamiz:

1) m va n sonlaridan biri toq son bo'lsin, masalan, $m=2k+1$ bo'lsin. U holda:

$$\int cos^m x sin^n x dx = \int cos^{2k} x sin^n x cosx dx = \int (1 - sin^2 x)^k sin^n x d(sinx).$$

Bu esa, darajali funksiyalarning integrallaridir.

2) m va n sonlarining har ikkalasi ham juft sonlar bo'lsin. Bu holda, $sin^2 x = \frac{1-cos2x}{2}$ va $cos^2 x = \frac{1+cos2x}{2}$ kabi formulalar orqali, trigonometrik funksiyalarning darajalari pasaytiriladi.

5-misol. $\int cos^7 x sin^3 x dx$ hisoblansin.

► $\int cos^7 x sin^3 x dx = \int cos^7 x sin^2 x\, sinx dx = -\int cos^7 x\,(1 - cos^2 x) d(cosx) == -\int cos^7 x\, d(cosx) + \int cos^9 x\, d(cosx) = -\frac{1}{8} cos^8 x + \frac{1}{10} cos^{10} x + C.$ ◄

6-misol. $\int cos^2 3x dx$ hisoblansin.

► $\int cos^2 3x dx = \int \frac{1+cos6x}{2} dx =$

$= \frac{1}{2}\int dx + \frac{1}{2}\int cos6x dx = \frac{1}{2}x + \frac{1}{12}\int cos6x d(6x) = \frac{1}{2}x + \frac{1}{12} sin6x + C.$ ◄

7-misol. $\int \frac{dx}{5-4x-x^2}$ hisoblansin.

Mazkur integralni hisoblash uchun kasr maxrajidagi kvadrat uchhaddan to'la kvadrat ajratamiz. Natijada:

► $\int \frac{dx}{5-4x-x^2} = \int \frac{dx}{9-(x^2+4x+4)} =$

$= \int \frac{d(x+2)}{3^2-(x+2)^2} = \frac{1}{2\cdot 3} ln\left|\frac{x+2+3}{x+2-3}\right| + C = \frac{1}{6} ln\left|\frac{x+5}{x-1}\right| + C.$ ◄

8-misol. $\int \frac{x^5+1}{x^2+4} dx$ hisoblansin.

Ko'phadni ko'phadga bo'lish qoidasidan foydalanib, integral belgisi ostidagi funksiyaning suratini maxrajiga bo'lamiz. Natijada, integrallanuvchi funksiya butun darajali ko'phad bilan to'g'ri ratsional kasrning yig'indisi ko'rinishida ifodalanadi. Kerakli almashtirishlarni bajarib, quyidagini hosil qilamiz.

► $\int \frac{x^5+1}{x^2+4}dx = \int (x^3 - 4x + \frac{16x+1}{x^2+4})dx = \int (x^3 - 4x)dx + 8\int \frac{2xdx}{x^2+4} + + \int \frac{1}{x^2+4}dx = \frac{x^4}{4} - 2x^2 + 8\ln(x^2+4) + \frac{1}{2}arctg\frac{x}{2} + C.$ ◄

AT-8.2

Berilgan aniqmas integrallarni hisoblang.

1. $\int (e^{2x} + e^{-2x})dx.$

2. $\int \sqrt[6]{1-7x^3}x^2dx.$

3. $\int \frac{2x-3}{\sqrt{4+x^2}}dx.$

4. $\int \cos^3 2x \sin^4 2xdx.$

5. $\int \cos^2 3x \cdot \sin^2 3\,xdx.$

6. $\int ctg^3 2xdx.$

7. $\int \frac{x^2-9}{x^2+9}dx.$

8. $\int \sin 7x \cdot \sin 9xdx.$

9. $\int \frac{dx}{x^2+6x+13}.$

10. $\int \frac{dx}{x^2-6x+7}.$

11. $\int \frac{1}{ch^2 3x}dx.$

12. $\int \frac{x^2+x+1}{x+1}dx.$

Mustaqil yechish uchun topshiriqlar

Aniqmas integrallarni hisoblang.

1. *a)* $\int \frac{\sin^3 x}{\cos x}dx$; *b)* $\int \cos 2x \cdot \sin 10xdx$;
c) $\int tg^2 7xdx.$

2. *a)* $\int \frac{1}{x^2+2x+5}dx$; *b)* $\int \sin(7x-1)\sin 5xdx$;
c) $\int \frac{3x+2}{x^2+1}dx.$

3. *a)* $\int \frac{x^2-1}{x^2+1}dx$; *b)* $\int \sin^3(1-3x)dx$;
c) $\int \frac{x+3}{x+1}dx.$

8.3. KVADRAT UCHHAD QATNASHGAN FUNKSIYALARNI INTEGRALLASH

$\int \frac{Ax+B}{x^2+bx+c}dx$ (8.1)

kabi integralni hisoblash lozim bo'lsin.

Aytaylik, $A\neq0$ bo'lsin. Integral belgisi ostida ba'zi ayniy almashtirishlarni bajarib, quyidagini hosil qilamiz.

$$\int \frac{Ax+B}{x^2+bx+c}dx = \frac{A}{2}\int \frac{(2x+b)+(2\frac{B}{A}-b)}{x^2+bx+c}dx = \frac{A}{2}\int \frac{d(x^2+bx+c)}{x^2+bx+c} + (B - \frac{Ab}{2})\int \frac{dx}{x^2+bx+c} = \frac{A}{2}ln|x^2+bx+c| + +(B-\frac{Ab}{2})\int \frac{dx}{x^2+bx+c}$$

Oxirgi ifodadagi integralni hisoblash uchun x^2+bx+c dan to'la kvadrat ajratamiz, ya'ni: $x^2+bx+c = (x+\frac{b}{2})^2 + c - \frac{b^2}{4}$.

Natijada, $c-\frac{b^2}{4}$ ning ishorasiga qarab, $\int \frac{du}{u^2\pm a^2}$ kabi jadval integrallarining biriga kelamiz.

1-misol. $\int \frac{3x-2}{x^2+4x+13}dx$ hisoblansin.

► $\int \frac{3x-2}{x^2+4x+13}dx = \frac{3}{2}\int \frac{2x+4-4-\frac{4}{3}}{x^2+4x+13}dx = \frac{3}{2}\int \frac{2x+4}{x^2+4x+13}dx - 8\int \frac{dx}{(x+2)^2+9} =$

$= \frac{3}{2}ln|x^2+4x+13| - 8\cdot\frac{1}{3}arctg\frac{x+2}{3} + C.$◄

2-misol. $\int \frac{5x-7}{x^2-8x+7}dx$ hisoblansin.

► $\int \frac{5x-7}{x^2-8x+7}dx = \frac{5}{2}\int \frac{2x-8+8-\frac{14}{5}}{x^2-8x+7}dx =$

$\frac{5}{2}\int \frac{2x-8}{x^2-8x+7}dx + 13\int \frac{dx}{x^2-2\cdot4x+16-9} == \frac{5}{2}ln|x^2-8x+7| +$

$13\int \frac{dx}{(x-4)^2-9} = \frac{5}{2}ln|x^2-8x+7| + 13\frac{1}{2\cdot3}ln\left|\frac{x-4-3}{x-4+3}\right| + +C =$

$\frac{5}{2}ln|x^2-8x+7| + \frac{13}{6}ln\left|\frac{x-7}{x-1}\right| + C.$ ◄

Eslatma. Agar (8.1) integraldagi kvadrat uch hadning ko'rinishi ax^2+bx+c ($a\neq0$) kabi bo'ladigan bo'lsa, u holda u integralni hisoblashda a koeffitsientni qavsdan tashqariga chiqariladi, ya'ni: $x^2+bx+c = a(x^2++\frac{b}{a}x+\frac{c}{a})$.

3-misol. $\int \frac{4x-3}{-2x^2+12x-10}dx$ hisoblansin.

► $\int \frac{4x-3}{-2x^2+12x-10}dx = -\frac{1}{2}\int \frac{4x-3}{x^2-6x+5}dx =$

$$= -\int \frac{2x-6+6-\frac{3}{2}}{x^2-6x+5}dx = -\int \frac{2x-6}{x^2-6x+5}dx - \frac{9}{2}\int \frac{dx}{(x-3)^2-4} =$$

$$= -ln|x^2-6x+5| + \frac{9}{2}\frac{1}{4}ln\left|\frac{2+x-3}{2-x+3}\right| + C. ◀$$

$\int \frac{(Ax+B)dx}{\sqrt{ax^2+bx+c}}$ kabi integrallarni hisoblashda ham yuqorida bayon etilgan usuldan foydalaniladi, ammo bu yerda, yuqoridagidan boshqacharoq jadval integrallari hosil bo‘ladi. Agar $A\neq 0$ bo‘lsa,

$$\int \frac{(Ax+B)dx}{\sqrt{ax^2+bx+c}} = \frac{A}{2a}\int \frac{\left(2ax+b-b+\frac{2Ba}{A}\right)dx}{\sqrt{ax^2+bx+c}} = \frac{A}{2a}\int \frac{d(ax^2+bx+c)}{\sqrt{ax^2+bx+c}} +$$

$$+\left(B-\frac{bA}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{A}{a}\sqrt{ax^2+bx+c} + \left(B-\frac{bA}{2a}\right)\int \frac{dx}{\sqrt{a(x+\frac{b}{2a})^2+(c-\frac{b^2}{4a})}}$$

ni hosil qilamiz. U holda, oxirgi integral yoki,

$$\int \frac{du}{\sqrt{u^2 \pm q^2}} = ln\left|u+\sqrt{u^2 \pm q^2}\right| + C$$

yoki,

$$\int \frac{du}{\sqrt{q^2-u^2}} = arcsin\frac{u}{q} + C$$

integrallarning biriga keltiriladi.

4-misol. $\int \frac{3x-1}{\sqrt{x^2-4x+8}}dx$ hisoblansin.

$$▶\int \frac{3x-1}{\sqrt{x^2-4x+8}}dx = \frac{3}{2}\int \frac{(2x-4)+\left(4-\frac{2}{3}\right)}{\sqrt{x^2-4x+8}}dx =$$

$$= \frac{3}{2}\int \frac{2x-4}{\sqrt{x^2-4x+8}}dx - 5\int \frac{dx}{\sqrt{(x-2)^2+4}} =$$

$$= 3\sqrt{x^2-4x+8} - 5ln\left|x-2+\sqrt{(x-2)^2+4}\right| + C. ◀$$

5-misol. $\int \frac{4x-5}{\sqrt{-x^2+2x+3}}dx$ hisoblansin.

$$▶\int \frac{4x-5}{\sqrt{-x^2+2x+3}}dx =$$

$$-2\int \frac{-2x+2+\frac{5}{2}-2}{\sqrt{-x^2+2x+3}}dx = -2\int \frac{-2x+2}{\sqrt{-x^2+2x+3}}dx - -\int \frac{dx}{\sqrt{4-(x-1)^2}} =$$

$$-4\sqrt{-x^2+2x+3} - arcsin\frac{x-1}{2} + C. ◀$$

endi

$$\int \frac{Ax+B}{(x^2+px+q)^k} \quad (8.2)$$

ko‘rinishdagi integralni qaraymiz, bu yerda, $k>0$ butun son bo‘lib,$p^2-4q<0$ dir.

Agarda, $A\neq 0$ va $k\neq 1$ bo‘ladigan bo‘lsa, aynan (8.1) holga o‘xshash ayniy almashtirishlardan foydalanib,

$$\frac{A}{2}\int\frac{(2x+p)dx}{(x^2+px+q)^k}=\frac{A}{2}\frac{(x^2+px+q)^{-k+1}}{-k+1}+C$$

ni ajratib olamiz. Natijada, (8.2) kabi integralni integrallash, quyidagi integralni integrallashga keltiriladi:

$$\int\frac{dx}{(x^2+px+q)^k}=\int\frac{dx}{\left[(x+\frac{p}{2})^2+\frac{4q-p^2}{4}\right]^k}=\int\frac{du}{(u^2+a^2)^k}\ (8.3).$$

Bu yerda: $u=x+\frac{p}{2}, a^2=\frac{4q-p^2}{4}$ $(4q-p^2>0)$ deb belgilash kiritilgan.

O'z navbatida, (8.3) ko'rinishidagi integrallarni hisoblash uchun maxrajning darajasini pasaytirishga asoslangan quyidagi rekurrent formuladan foydalaniladi:

$$\int\frac{du}{(u^2+a^2)^k}=\frac{u}{2a^2(k-1)(u^2+a^2)^{k-1}}+\frac{2k-3}{2a^2(k-1)}\int\frac{du}{(u^2+a^2)^{k-1}}\ (8.4)$$

6-misol. $\int\frac{3x+5}{(x^2+2x+5)^2}dx$ hisoblansin.

► $\int\frac{3x+5}{(x^2+2x+5)^2}dx=\frac{3}{2}\int\frac{2x+2-2+10/3}{(x^2+2x+5)^2}dx=\frac{3}{2}\int\frac{d(x^2+2x+5)}{(x^2+2x+5)^2}+$
$2\int\frac{dx}{((x+1)^2+4)^2}=\underline{\underline{(8.4)}}=-\frac{3}{2}\frac{1}{x^2+2x+5}+2\left(\frac{x+1}{8((x+1)^2+4)}+\right.$
$\left.\frac{1}{8}\int\frac{dx}{4+(x+1)^2}\right)=-\frac{3}{2}\frac{1}{x^2+2x+5}+\frac{1}{4}\frac{x+1}{x^2+2x+5}++\frac{1}{8}arctg\frac{x+1}{2}+C.$ ◄

Bu yerdagi $\underline{\underline{(8.4)}}$ kabi yozuv, keyingi hisoblashlarga o'tishda (8.4) formuladan foydalanganlik belgisini anglatadi. (Bu xildagi qisqa va qulay yozuvdan bundan keyin ham foydalanamiz).

AT-8.3

1. $\int\frac{dx}{x^2+4x+20}$. (*Javob*: $\frac{1}{4}arctg\frac{x+2}{4}+C$)
2. $\int\frac{3x-7}{x^2+x+1}dx$ (*Javob*: $\frac{3}{2}ln|x^2+x+1|-\frac{17}{\sqrt{3}}arctg\frac{2x+1}{\sqrt{3}}+C$)
3. $\int\frac{x-2}{x^2-8x+7}dx.$ (*Javob*: $\frac{1}{2}ln|x^2-8x+7|+\frac{11}{6}ln\left|\frac{x-7}{x-1}\right|+C$))
4. $\int\frac{x^3+3x}{x^2+2x+2}dx$ (*Javob*: $\frac{(x^2-2)^2}{2}+\frac{5}{2}ln|x^2+2x+2|-9arctg(x-1)+C$)

5. $\int \frac{3x-1}{\sqrt{x^2-6x+18}}dx$. (*Javob*: $3\sqrt{x^2-6x+18}+$
$5ln|x-3++\sqrt{x^2-6x+18}|+C$)

6. $\int \frac{8x-11}{\sqrt{5+2x-x^2}}dx$. (*Javob*: $-8\sqrt{5+2x-x^2}-3arcsin\frac{x-1}{\sqrt{6}}+$
C)

7. $\int \frac{3x-1}{(x^2+2x+10)^2}dx$. (*Javob*:$-\frac{4x+13}{x^2+2x+10}+\frac{1}{54}arctg\frac{x+1}{3}+C$)

8. $\int \frac{2-3x}{\sqrt{4+x^2}}dx$. (*Javob*: $2ln|x+\sqrt{4+x^2}|-3\sqrt{4+x^2}+C$)

Mustaqil yechish uchun topshiriqlar

Aniqmas integrallarni hisoblang.

1. a) $\int \frac{3x+9}{x^2-6x+12}dx$; b) $\int \frac{x-3}{\sqrt{x^2+2x+2}}dx$.
2. a) $\int \frac{x-7}{x^2-10x+9}dx$; b) $\int \frac{7x-2}{\sqrt{5-4x-x^2}}dx$.
3. a) $\int \frac{7x+3}{2x^2+4x+9}dx$; b) $\int \frac{4x-5}{\sqrt{x^2+10x+29}}dx$.

8.4. O‘ZGARUVCHINI ALMASHTIRISH YOKI O‘RNIGA QO‘YISH USULIDA INTEGRALLASH

Agar $x=\varphi(t)$ funksiya uzluksiz differensiallanuvchi bo‘lsa, u holda, berilgan $\int f(x)dx$ integralda har doim yangi t o‘zgaruvchiga nisbatan integralga kelish mumkin bo‘ladi, ya’ni:

$$\int f(x)dx = \int f[\varphi(t)]\cdot\varphi'(t)dt \quad (8.5)$$

O‘ng tomondagi integralni hisoblab, natijada eski x o‘zgaruvchiga qaytsak, berilgan integral hisoblangan bo‘ladi.

Aniqmas integrallarni bu usulda integrallashga *o‘zgaruvchini almashtirish yoki o‘rniga qo‘yish usuli* deb yuritiladi.

Bu yerda, shuni ta’kidlash kerakki, $x=\varphi(t)$ almashtirish kiritilayotganda, $\varphi(t)$ bilan $f(x)$ funksiyalar orasida o‘zaro bir qiymatli moslik bo‘lishi va $\varphi(t)$ funksiya o‘zgaruvchi x ning barcha qiymatlarini qabul qilishi lozim.

1-misol. $\int x\sqrt{x-1}dx$ hisoblansin.

►$t=\sqrt{x-1}$ almashtirish kiritamiz, u holda, $x=t^2+1$ va $dx=2dt$

Natijada,

$$\int x\sqrt{x-1}dx = \int (t^2+1)\cdot t\cdot 2tdt = 2\int (t^4+t^2)\,dt =$$
$$\frac{2}{5}t^5 + \frac{2}{3}t^3 + C == \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C \blacktriangleleft$$

2-misol. $\int \frac{\sqrt{x^2+a^2}}{x^2}dx$ hisoblansin.

► Bu yerda, $x = a\cdot tgt$ $(-\frac{\pi}{2} < t < \frac{\pi}{2})$ almashtirishdan foydalanamiz. $dx = \frac{adt}{cos^2t}$ bo'lganligi uchun, (8.5) formulaga binoan quyidagini hosil qilamiz:

$$\int \frac{\sqrt{x^2+a^2}}{x^2}dx = \int \frac{\sqrt{a^2tg^2t+a^2}}{a^2tg^2t}\frac{adt}{cos^2t} = \int \frac{\sqrt{1+tg^2t}}{sin^2t}dt ==$$
$$\int \frac{1}{costsin^2t}dt = \int \frac{cos^2t+sin^2t}{costsin^2t}dt = \int \frac{cost}{sin^2t}dt + \int \frac{1}{cost}dt ==$$
$$-\frac{1}{sint} + ln\left|tgt + \frac{1}{cost}\right| + C = -\frac{\sqrt{1+tg^2t}}{tgt} + ln|tgt +$$
$$+\sqrt{1+tg^2t}| + C = -\frac{\sqrt{a^2+x^2}}{x} + ln|x+\sqrt{a^2+x^2}| + C. \blacktriangleleft$$

3-misol. $\int \sqrt{a^2-x^2}dx$ hisoblansin.

►Bu yerda, $x = asint$ almashtirishdan foydalanamiz. $dx = acostdt$ $(-\frac{\pi}{2} \le t \le \frac{\pi}{2}$ va $-a \le x \le a)$ ga ko'ra, quyidagini yoza olamiz:

$$\int \sqrt{a^2-x^2}dx = \int \sqrt{a^2-a^2sin^2t}\,acostdt =$$
$$a^2\int cos^2t\,dt == a^2\int \frac{1+cos2t}{2}dt = \frac{a^2}{2}t + \frac{a^2}{4}sin2t + C = \frac{a^2}{2}t +$$
$$\frac{a^2}{2}sintcost + C.$$

Agar $t = arcsin\frac{x}{a}$ va $cost = \sqrt{1-sin^2t} = \sqrt{1-\frac{x^2}{a^2}}$ ekanligini inobatga olsak, natijada,

$$\int \sqrt{a^2-x^2}dx = \frac{a^2}{2}arcsin\frac{x}{a} + \frac{a^2}{2}\frac{x}{a}\sqrt{1-\frac{x^2}{a^2}} + C =$$
$$\frac{a^2}{2}arcsin\frac{x}{a} + +\frac{x}{2}\sqrt{a^2-x^2} + C. \blacktriangleleft$$

hosil bo'ladi.

Ayrim funksiyalarni integrallashda, ko'pincha, $x = \varphi(t)$ almashtirish emas, balki, $t = \varphi(x)$ almashtirshdan foydalanish maqsadga muvofiq bo'ladi.

4-misol. $\int \sqrt[3]{1+sinx}\,cosxdx$ integrallansin.

►Yechish: $1 + sinx = t$ kabi almashtirish kiritamiz. U holda, $cosxdx == dt$ bo'lganligi uchun,

$$\int \sqrt[3]{1 + sinx}cosxdx = \int t^{\frac{1}{3}}dt = \frac{3t^{\frac{4}{3}}}{4} + C = \frac{3}{4}\sqrt[3]{(1 + sinx)^4} + C. \blacktriangleleft$$

5-misol. $\int e^{-x^3} \cdot x^2dx$ hisoblansin.

►Yechish: $-x^3 = t$ deb olsak, $-3x^2dx = dt$ yoki $x^2dx = -\frac{dt}{3}$ bo'ladi. Natijada,

$$\int e^{-x^3}x^2dx = \int e^t\left(-\frac{1}{3}\right)dt = -\frac{1}{3}e^t + C = -\frac{1}{3}e^{-x^2} + C$$

hosil bo'ladi. ◄

6-misol. $\int \frac{dx}{(x+1)\sqrt{x^2+2x+10}}$ hisoblansin.

►Bu yerda, $t = \frac{1}{x+1}$ almashtirish kiritish maqsadga muvofiq bo'ladi. U holda, $x = \frac{1}{t} - 1$ va $dx = -\frac{dt}{t^2}$ bo'ladi. Natijada esa, quyidagini yozamiz:

$$\int \frac{dx}{(x+1)\sqrt{x^2+2x+10}} = \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{(\frac{1}{t}-1)^2+2\left(\frac{1}{t}-1\right)+10}} = -\int \frac{dt}{t\sqrt{t^{-2}+9}} = -\int \frac{dt}{\sqrt{9t^2+1}} =$$

$$= -\frac{1}{3}ln\left|3t + \sqrt{9t^2 + 1}\right| + C = -\frac{1}{3}ln\left|\frac{3}{x+1} + \sqrt{\frac{9}{(x+1)^2} + 1}\right| + C. \blacktriangleleft$$

Eslatma. Aniqmas integrallarni o'rniga qo'yish (o'zgaruvchini almashtirish usuli) usulidan foydalanish jarayonida qo'yidagicha sxemani qo'llash tavsiya etiladi. Bu sxemaning qo'llanishini yuqoridagi 3-misolni yechish jarayoni uchun bayon etamiz:

$$\blacktriangleright \int \sqrt{a^2 - x^2}dx = \left|\begin{matrix} x = asint \\ dx = acostdt \end{matrix}\right| =$$

$$\int \sqrt{a^2 - a^2sin^2t}\, acostdt = a^2\int |cost|\, costdt =$$

$$= a^2\int cos^2tdt = a^2\int \frac{1+cos2t}{2}dt = \frac{a^2}{2}\int dt + \frac{a^2}{2}\int cos2tdt ==$$

$$\frac{a^2}{2}t + \frac{a^2}{4}sin2t + C = \frac{a^2}{2}t + \frac{a^2}{2}sintcost + C =$$

$$= \left|\begin{matrix} t = arcsin\frac{x}{a}, sint = x/a \\ cost = \sqrt{1 - sin^2t} = \sqrt{1 - x^2/a^2} \end{matrix}\right| = \frac{a^2}{2}arcsin\frac{x}{a} +$$

$$+\frac{a}{2}x\sqrt{1 - \frac{x^2}{a^2}} + C = \frac{a^2}{2}arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + C. \blacktriangleleft$$

Bundan keyin ham barcha oraliqdagi hisoblashlarni yozish uchun ularni vertikal chiziqlar orasiga joylashtiramiz.

AT-8.4

1. $\int \frac{dx}{1+\sqrt{x+3}}$. (*Javob:* $2(\sqrt{x+3} - ln|1+\sqrt{x+3}|) + C$.)

2. $\int x\sqrt[5]{(5x^2-3)^7}\,dx$. (*Javob:* $\frac{1}{24}\sqrt[5]{(5x^2-3)^{12}} + C$.)

3. $\int \frac{dx}{x^2\sqrt{x^2+a^2}}$. (*Javob:* $-\frac{\sqrt{x^2+a^2}}{a^2x}+C$.)

4. $\int \frac{\sqrt{1+ln\,x}}{x\,ln\,x}dx$. (*Javob:* $2\sqrt{1+ln\,x} - ln\,ln\,x +$ $2ln|\sqrt{1+ln\,x}-1| + C$.)

5. $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$. (*Javob:* $2\sqrt{x} - 4\sqrt[4]{x} + 4(1+\sqrt[4]{x}) + C$.)

6. $\int \frac{dx}{x\sqrt{x^2+x+1}}$. (*Javob:* $-ln\frac{x+2+2\sqrt{x^2+x+1}}{x} + C$.)

7. $\sqrt{144-x^2}dx$. (*Javob:* $72arcsin\frac{x}{12} + \frac{x}{2}\sqrt{144-x^2} +$ C.)

8. $\int \frac{dx}{x^2\sqrt{x^2+9}}$ (*Javob:* $C - \frac{\sqrt{x^2+9}}{9x}$.)

9. $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx$. (*Javob:* $\frac{2}{3}(e^x-2)\sqrt{e^x+1} + C$.)

10. $\int \frac{dx}{x\sqrt{1+x^2}}$. (*Javob:* $ln\left|\frac{x}{1+\sqrt{x^2+1}}\right| + C$.)

Mustaqil yechish uchun topshiriqlar

1. $a)$ $\int x^3\sqrt{4-3x^4}dx$;b)$\int \frac{1+x}{1+\sqrt{x}}dx$. (*Javob*: a) $-\frac{1}{8}\sqrt{(4-3x^4)^3} + C$; b) $\frac{2}{3}\sqrt{x^3} - x + 4\sqrt{x} - 4\ln(1+\sqrt{x})^8 + C$.)

2. a) $\int \frac{x^2}{\sqrt[3]{9-2x^3}}dx$; b)$\int \frac{dx}{x\sqrt{4-x^2}}$. (*Javob*: a) $-\frac{1}{4}\sqrt[3]{(9-2x^3)^2} + C$; b) $-\frac{1}{2}ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C$.)

3. a)$\int \sqrt[7]{1+\cos^2 x}\sin 2x\,dx$; b)$\int \frac{\sqrt{1-x^2}}{x^2}dx$. (*Javob*: a)$-\frac{7}{8}\sqrt{(1+\cos^2 x)^8} + C$; b) $C - \frac{\sqrt{1-x^2}}{x} - \arcsin x$.)

8.5. BO'LAKLAB INTEGRALLASH

Bo'laklab integrallash usuli deb ataluvchi usul, quyidagi

$$\int udv = uv - \int vdu \quad (8.6)$$

formulaga asoslangan. Bu yerdagi, $u(x)$ bilan $v(x)$ lar uzluksiz differensiallanuvchi funksiyalardir. Ushbu (8.6) formula, bo'laklab integrallash formulasi deb yuritiladi. (8.6) tenglikning o'ng tomonidagi integral, chap tomondagisiga nisbatan soddaroq integrallanadigan hollarda ushbu formulani qo'llash maqsadga muvofiqdir. Shuningdek, ayrim hollarda, (8.6) formulani bir necha marta qo'llash kerak bo'ladi.

Bo'laklab integrallash usulini, $x^k sin\alpha x, x^k cos\alpha x, x^k e^{\alpha x}$, $x^n ln^k x, x^k ch\alpha x$, $x^k sh\alpha x$, $a^{\beta x} cos\alpha x$, $a^{\beta x} sin\alpha x, arcsinx, arctgx$ (n, k lar butun musbat sonlar bo'lib, $\alpha, \beta \in R$) va boshqa xildagi funksiyalarni integrallash uchun qo'llash tavsiya qilinadi.

1-misol. $\int xe^{-2x}dx$ hisoblansin.

►Bo'laklab integrallash usulidan foydalanamiz. $u = x$ va $dv == e^{-2x}dx$ deb olsak, $du = dx$ va $v = \int e^{-2x}dx = -\frac{1}{2}e^{-2x} + S$ (har doim $C = 0$ deb hisoblash mumkin). U holda, (8.6) formulaga binoan quyidagini hosil qilamiz:

$$\int xe^{-2x}dx = x\left(-\frac{1}{2}e^{-2x}\right) - \int(-\frac{1}{2}e^{-2x})dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C. ◄$$

2-misol. $\int(x^2 + 2x)cos2xdx$ hisoblansin.

$$► \int(x^2 + 2x)cos2xdx = \left| \begin{matrix} u = x^2 + 2x, du = (2x + 2)dx, \\ dv = cos2xdx, \\ v = \int cos2xdx = \frac{1}{2}sin2x \end{matrix} \right| =$$

$$= \frac{1}{2}(x^2 + 2x)sin2x - \int(x + 1)sin2xdx = \left| \begin{matrix} u = x + 1, du = dx, \\ dv = sin2xdx, \\ v = -\frac{1}{2}cos2x \end{matrix} \right| =$$

$$= \frac{1}{2}(x^2 + 2x)sin2x + (x + 1)\frac{1}{2}cos2x - \int\frac{1}{2}cos2xdx =$$

$$= \frac{1}{2}(x^2 + 2x)sin2x + \frac{1}{2}(x + 1)cos2x + \frac{1}{4}sin2x + C. ◄$$

3-misol. $\int xarctgxdx$ hisoblansin

$$► \int xarctgxdx = \left| \begin{matrix} u = arctgx, du = \frac{dx}{1+x^2} \\ dv = xdx, v = \frac{x^2}{2} \end{matrix} \right| = \frac{x^2}{2}arctgx -$$

$$-\frac{1}{2}\int\frac{x^2dx}{1+x^2} = \frac{x^2}{2}arctgx - \frac{1}{2}\int\frac{x^2+1-1}{1+x^2}dx = \frac{x^2}{2}arctgx - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{dx}{1+x^2} =$$

$$= \frac{x^2}{2}arctgx - \frac{1}{2}x + \frac{1}{2}arctgx + C. ◄$$

4-misol. $\int e^{2x}sinxdx$ hisoblansin.

$$\blacktriangleright \int e^{2x}sinxdx = \left| \begin{matrix} u = sinx, du = cosxdx, \\ dv = e^{2x}dx, v = \frac{1}{2}e^{2x} \end{matrix} \right| = \frac{1}{2}e^{2x}sinx -$$

$$-\frac{1}{2}\int e^{2x}cosxdx = \left| \begin{matrix} u = cosx, du = -sinxdx, \\ dv = e^{2x}dx, v = \frac{1}{2}e^{2x} \end{matrix} \right| = \frac{1}{2}e^{2x}sinx -$$

$$-\frac{1}{2}\left(\frac{1}{2}e^{2x}cosx - \int \frac{1}{2}e^{2x}sinxdx\right) = \frac{1}{2}e^{2x}sinx - \frac{1}{4}e^{2x}cosx +$$

$$+\frac{1}{4}\int e^{2x}sinxdx.$$

Oxirgi integralni chap tarafga o'tkazsak

$$\frac{3}{4}\int e^{2x}sinxdx = \frac{1}{2}e^{2x}sinx - \frac{1}{4}e^{2x}cosx + \frac{3}{4}C.$$

Natijada,

$$\int e^{2x}sinxdx = \frac{2}{3}e^{2x}sinx - \frac{1}{3}e^{2x}cosx + C. \blacktriangleleft$$

5-misol. $\int x^2ln^2xdx$ hisoblansin.

$$\blacktriangleright \int x^2ln^2xdx = \left| \begin{matrix} u = ln^2x, du = 2lnx \cdot \frac{1}{x}dx, \\ dv = x^2dx, v = \frac{x^3}{3} \end{matrix} \right| == \frac{x^3}{3}ln^2x -$$

$$\frac{2}{3}\int x^3lnx \cdot \frac{1}{x}dx = \frac{x^3}{3}ln^2x - -\frac{2}{3}\int x^2lnxdx = = \left| \begin{matrix} u = lnx, du = \frac{dx}{x}, \\ dv = x^2dx, v = x^3/3 \end{matrix} \right| =$$

$$= \frac{x^3}{3}ln^2x - \frac{2}{3}\left(\frac{x^3}{3}lnx - \int \frac{x^3}{3}\frac{1}{x}dx\right) = \frac{x^3}{3}ln^2x - \frac{2}{9}x^3lnx +$$

$$+\frac{2}{9}\int x^2dx = \frac{1}{3}x^3ln^2x - \frac{2}{9}x^3lnx + \frac{2}{27}x^3 + C. \blacktriangleleft$$

AT-8.5

Aniqmas integrallarni hisoblang.

1. $\int x\cos 3xdx$. (*Javob*: $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C$.)
2. $\int \arccos xdx$. (*Javob*: $x\arccos x - \sqrt{1-x^2} + C$.)
3. $\int (x^2 - 2x + 5)e^{-x}dx$. (*Javob*: $-e^{-x}(x^2 + 5) + C$.)
4. $\int ln^2 xdx$. (*Javob*: $xln^2 x - 2x\,ln\,x + 2x + C$.)
5. $\int \frac{x\cos x}{\sin^2 x}dx$. (*Javob*: $-\frac{x}{\sin x} + ln\left|tg\frac{x}{2}\right| + C$.)
6. $\int x^3e^{-x^2}dx$. (*Javob*: $-\frac{1}{2}e^{-x^2}(x^2 + 1) + C$.)
7. $\int e^{\sqrt{x}}dx$. (*Javob*: $2e^{x^{\frac{1}{2}}}(\sqrt{x} - 1) + C$.)
8. $\int \sin(\ln x)dx$. (*Javob*: $\frac{x}{2}(\sin\ln x - \cos\ln x) + C$.)

Mustaqil yechish uchun topshiriqlar

Aniqmas integrallarni hisoblang.

1. a) $\int \frac{\ln x}{x} dx$; b) $\int x e^{-7} dx$; c) $\int x \arcsin x dx$.
2. a) $\int x e^{11x-1} dx$; b) $\int \ln(1 + x^2 dx)$; c) $\int x \cos\left(\frac{x}{2} + 1\right) dx$.
3. a) $\int \ln(x - 3) dx$; b) $\int x \cos(2x - 1) dx$; c) $\int x \cdot 2^{3x} dx$.

8.6. RATSIONAL FUNKSIYALARNI INTEGRALLASH

Ratsional funksiya deb, ikkita ko‘phadning nisbatini ifodalaydigan funksiyaga aytiladi, ya’ni:

$$R(x) = \frac{Q_m(x)}{P_n(x)} = \frac{b_0 x^m + b_1 x^{m-1} + \dots b_m}{a_0 x^n + a_1 x^{n-1} + \dots a_n} \quad (8.7)$$

bu yerda, *m* va *n* lar butun musbat sonlardir: $b_i, a_j \in R$ $(i = \overline{0, m}; j = \overline{0, n})$.

Agar *m*<*n* bo‘lsa, $R(x)$ ni to‘g‘ri kasr, aksincha, *m*≥*n* bo‘lganda esa, uni noto‘g‘ri kasr deb yuritiladi.

Har qanday noto‘g‘ri kasrning suratidagi ko‘phadni maxrajidagi ko‘phadga bo‘lib, uni biron bir ko‘phad bilan to‘g‘ri kasrning yig‘indisi shaklida ifodalash mumkin bo‘ladi, ya’ni:

$$\frac{Q_m(x)}{P_n(x)} = M_{n-m}(x) + \frac{Q_l(x)}{P_n(x)}$$

bu yerda, $M_{n-m}(x)$ va $Q_l(x)$ lar ko‘phadlardir; $\frac{Q_l(x)}{P_n(x)}$ esa, to‘g‘ri kasr (*l*<*n*).

Masalan, $\frac{x^4+4}{x^2+3x-1}$ ko‘rinishidagi ratsional funksiya, noto‘g‘ri kasrdir. Ko‘phadni ko‘phadga bo‘lish qoidasidan foydalanib, suratni maxrajga bo‘lsak, quyidagini hosil qilamiz:

$$\frac{x^4 + 4}{x^2 + 3x - 1} = x^2 - 3x + 10 + \frac{-33x + 14}{x^2 + 3x - 1}$$

Ma’lumki, ko‘phadni osongina integrallash mumkin, shu boisdan, har qanday ratsional funksiyani integrallash, to‘g‘ri kasrni integrallashga keltiriladi. Bundan buyon, *m*<*n* shartni qanoatlantiruvchi $R(x)$ funksiyalarni integrallash masalasini o‘rganamiz.

Eng sodda kasr funksiya deb, quyida keltirilgan 4 xildagi kasr funksiyalarning biriga aytiladi:

1) $\frac{A}{x-a}$; 2) $\frac{A}{(x-a)^k}$;

3) $\frac{Mx+N}{x^2+px+q}$; 4) $\frac{Mx+N}{(x^2+px+q)^k}$

bu yerdagi A, a, M, N, p, q lar ixtiyoriy sonlar bo‘lib, k esa ($k \geq 2$), butun son hamda $p^2-4q<0$.

Birinchi va ikkichi xildagi eng sodda kasrlarni integrallash bevosita integrallash yo‘li bilan amalga oshiriladi, ya’ni:

$$\int \frac{Adx}{x-a} = Aln|x-a| + C,$$

$$\int \frac{A}{(x-a)^k} dx = A\int (x-a)^{-k} d(x-a) = \frac{A}{(x-a)^{k-1}(1-k)} + C.$$

Shuningdek, uchinchi va to‘rtinchi xildagi eng sodda kasrlarni integrallash usullari esa, §8.4 da qaralgan edi.

Demak, har qanday eng sodda kasrni integrallash, elementar funksiyalar orqali amalga oshirilar ekan.

Ma’lumki, koeffitsientlari haqiqiy sonlardan iborat bo‘lgan har qanday $P_n(x)$ kabi ko‘phadni haqiqiy sonlar sohasida quyidagicha ifodalash mumkin bo‘lar edi:

$$P_n(x) = a_0(x-\alpha_1)^{k_1} \dots (x-\alpha_\beta)^{k_\beta} \cdot$$
$$\cdot (x^2+p_1x+q_1)^{t_1} \dots (x^2+p_sx+q_s)^{t_s} \quad (8.8)$$

bu yerda: $\alpha_1, \alpha_2, \dots, \alpha_\beta$ lar $P_n(x)$ ko‘pxadning mos ravishda k_1, $k_2, \dots, k_\beta$ karrali haqiqiy ildizlari bo‘lib, $p_\gamma{}^2 - 4q_\gamma < 0$ $(\gamma = \overline{1,s})$;

$$k_1 + k_2 + \dots + k_\beta + 2t_1 + 2t_2 + \dots + 2t_s = n,.$$

$(k_1, k_2, \dots, k_\beta, t_1, t_2, \dots, t_\beta$ lar manfiy bo‘lmagan butun sonlardir). U holda, quyidagi teorema o‘rinlidir:

Teorema (to‘g‘ri kasrni eng sodda kasrlar yig‘indisiga yoyish).

Maxraji (8.8) kabi ko‘rinishda tasvirlanadigan har qanday (8.7) to‘g‘ri ratsional kasrni har doim yuqorida keltirilgan 1–4 xildagi eng sodda kasrlarning yig‘indisi shaklida yoyish mumkin bo‘ladi. Xususan, (8.8) ifodadagi har bir k_r karrali α_r ildiz $(r = \overline{1,\beta})$ $((x-\alpha_r)^{k_r}$ ko‘paytmaga) ga, yoyilmada quyidagicha ko‘rinishda bo‘lgan k_r kasrlar yig‘indisi mos keladi:

$$\frac{A_1}{x-\alpha_r} + \frac{A_2}{(x-\alpha_r)^2} + \dots + \frac{A_{k_r}}{(x-\alpha_r)^{k_r}} \quad (8.9)$$

$P_n(x)$ ko‘phadning har bir t_γ karrali juft o‘zaro qo‘shma bo‘lgan kompleks ildizlar

$((x^2+p_\gamma x+q_\gamma)^{t_\gamma}$ *ko'paytuvchilarga) ga esa, quyidagicha* t_γ *dona elementar kasrlarning yig'indisi mos keladi:*

$$\frac{M_1x+N_1}{x^2+p_\gamma x+q_\gamma}+\frac{M_2x+N_2}{(x^2+p_\gamma x+q_\gamma)^2}+\dots+\frac{M_{t_\gamma}x+N_{t_\gamma}}{(x^2+p_\gamma x+q_\gamma)^{t_\gamma}} \qquad (8.10)$$

Yuqorida keltirilgan yoyilmalardagi A, M, N larning qiymatlarini aniqlash uchun ko'pincha noma'lum koeffitsientlar usulidan foydalaniladi.

Mazkur usulning mohiyati quyidagichadir:

Berilgan to'g'ri ratsional kasr $R(x)$ ni (8.9) va (8.10) kabi eng sodda kasrlarning yig'indisi ko'rinishida yozib olamiz. U esa, o'z navbatida ayniyatdir. Shuning uchun, barcha kasrlarni umumiy maxrajga keltirsak, suratda (n–1) darajali $Q^*_{n-1}(x)$ kabi ko'phad hosil bo'ladiki, u esa, o'z navbatida (8.7) ning suratidagi $Q_m(x)$ ko'phadga aynan teng bo'ladi. Ushbu ko'phadlar oldidagi koeffitsientlarni x ning darajalariga nisbatan tenglashtirib, A, M, N (indekslari bilan) noma'lum koeffitsientlarga nisbatan n noma'lumli n ta algebraik tenglamalar sistemasini hosil qilamiz.

Hisoblash ishlarini soddalashtirish maqsadida ayrim hollarda quyidagi mulohazadan foydalanish ham mumkin, ya'ni, $Q_m(x)$ bilan $Q_{n-1}{}^*(x)$ ko'phadlar ayniy teng bo'lganliklari sababli, x ning har qanday sonli qiymatlarida ham ularning qiymatlari o'zaro teng bo'ladi. x ga muayyan qiymatlari berib, noma'lum koeffitsientlarni aniqlash uchun tenglamalar sistemasini hosil qilamiz. Mazkur usulni odatda, xususiy qiymatlar usuli deb yuritiladi. Agarda, x larning qiymatlari maxrajning haqiqiy ildizlari bilan bir xil bo'ladigan bo'lsa, bitta noma'lum koeffitsientga nisbatan tenglamaga ega bo'lamiz.

1-misol. $\int\frac{2x-3}{x(x-1)(x-2)}dx$ hisoblansin.

►(8.9) formulaga binoan, quyidagini yozamiz:

$$\int\frac{(2x-3)dx}{x(x-1)(x-2)}=\int\left(\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x-2}\right)dx \quad (1)$$

Agarda, mazkur yoyilmadagi kasrlarda umumiy maxrajga keltirilsa, u umumiy maxraj integrallanuvchi funksiyaning maxraji bilan bir xil bo'lib, (1) formulaning chap va o'ng tomonlaridagi integral ostidagi ifodalarning ham suratlari aynan bir xil bo'ladi, ya'ni:

$$2x-3=A(x-1)(x-2)+Bx(x-2)+Cx(x-1) \quad (2)$$

Mazkur ayniy tenglikning har ikkala tomonida x ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{array}{l|c} x^2 & 0 = A + B + C \\ x^1 & 2 = -3A - 2B - C \\ x^0 & -3 = 2A \end{array}$$

va uni yechib, A=-3/2, B=1, C=1/2 larni topamiz.

Endi, yoyilmadagi koeffitsientlarni xususiy qiymatlar usuli yordamida aniqlaymiz. Shu maqsadda, (2) ifodadagi x ning o'rniga maxrajning ildizlari bo'lgan $\alpha_1 = 0, \alpha_2 = 1, va\ \alpha_3 = 2$ xususiy qiymatlarni qo'yamiz. Natijada, -3=2A, -1=-B, 1=2C lar yoki ulardan A=-3/2, B=1, C=1/2 larni topamiz.

Ushbu topilgan qiymatlarni keltirib, (1) tenglikka qo'ysak,

$$\int \frac{2x-3}{x(x-1)(x-2)}dx = \int \left(\frac{-\frac{3}{2}}{x} + \frac{1}{x-1} + \frac{\frac{1}{2}}{x-2} \right) dx =$$

$$= -\frac{3}{2}ln|x| + ln|x-1| + \frac{1}{2}ln|x-2| + C^*$$

ni hosil qilamiz (bu yerda, C^*, integrallashdagi ixtiyoriy o'zgarmasdir).◄

2-misol. $\int \frac{xdx}{(x-1)(x+1)^2}$ hisoblansin.

►To'g'ri kasrni eng sodda kasrlarning yig'indisi ko'rinishida ifodalash haqidagi teoremaga ko'ra, quyidagini yozamiz:

$$\int \frac{xdx}{(x-1)(x+1)^2} = \int \left(\frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1} \right) dx$$

Umumiy maxrajga keltirilgandan so'ng,

$$x = A(x+1)^2 + B(x-1) + C(x^2-1) \quad (1)$$

ni hosil qilamiz.

Agar $x = 1$ va $x = -1$ deb olsak, $4A = 1$ va $-2B = -1$ ni va $A = 1/4$, $B = 1/2$ larni topamiz. Uchinchi noma'lum C koeffitsientni aniqlash maqsadida (1) tenglikdagi x^2 ning oldidagi koeffitsientlarni tenglashtirib, 0=A+C ni va undan, $C = -1/4$ ni topamiz. Natijada,

$$\int \frac{xdx}{(x-1)(x+1)^2} = \int \frac{1/4}{x-1}dx + \int \frac{1/2}{(x+1)^2}dx + \int \frac{-1/4}{x+1}dx = \frac{1}{4}ln|x-1| - \frac{1}{2}\frac{1}{x+1} - \frac{1}{4}ln|x+1| + C^* = \frac{1}{4}ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\frac{1}{x+1} + C^*$$

ni hosil qilamiz.◀

3-misol. $\int \frac{xdx}{(x-1)(x^2+1)}$ hisoblansin

▶Yuqorida keltirilgan (8.9) bilan (8.10) formulalarga binoan integral ishorasi ostidagi ratsional kasrni eng sodda kasrlarning yig'indisiga keltiramiz.

$$\int \frac{xdx}{(x-1)(x^2+1)} = \int \left(\frac{A}{x-1} + \frac{Mx+N}{x^2+1}\right)dx = \int \frac{A(x^2+1)+(Mx+N)(x-1)}{(x-1)(x^2+1)}dx.$$

Bundan esa, $x = A(x^2+1) + (Mx+N)(x-1)$ ni hosil qilamiz.

Ushbu tenglikda $x = 1$ deb olsak, $1=2A$ yoki $A=1/2$ ni topamiz. $\left.\begin{matrix} x^2 \\ x^0 \end{matrix}\right| \left.\begin{matrix} A+M=0 \\ A-N=0 \end{matrix}\right\}$ dan esa, $M=-1/2$ bilan $N=1/2$ larni topamiz. U holda:

$$\int \frac{xdx}{(x-1)(x^2+1)} = \int \left(\frac{\frac{1}{2}}{x} + \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}\right)dx = \frac{1}{2}ln|x| - \frac{1}{4}ln|x^2+1| + \frac{1}{2}arctgx + C$$ ◀

4-misol. $I(x) = \int \frac{x^4+3x^2-5}{x^3+2x^2+5x}dx$ hisoblansin.

▶Integrallanuvchi funksiya noto'g'ri ratsional kasr bo'lganligidan, suratni maxrajga bo'lib, uning butun qismi bilan to'g'ri kasr qismlarini ajratib olamiz:

$$\frac{x^4+3x^2-5}{x^3+2x^2+5x} = x - 2 + \frac{2x^2+10x-5}{x^3+2x^2+5x}.$$

Endi. (8.9) va (8.10) formulalarga binoan,

$$I(x)=\int(x-2)dx+\int\frac{2x^2+10x-5}{x(x^2+2x+5)}dx=\frac{(x-2)^2}{2}+\int\left(\frac{A}{x}+\frac{Mx+N}{x^2+2x+5}\right)dx$$

ni hosil qilamiz.

Oxirgi integral ostidagi ifodada umumiy maxraj topib, tenglikning chap va o'ng tomonlaridagi suratlarni tenglashtirsak, $2x^2+10x-5=A(x^2+2x+5)+Mx^2+Nx$ ni hosil qilamiz.

Bu yerda, x ning darajalariga qarab ular oldidagi koeffitsientlarni tenglashtirib, quyidagiga ega bo'lamiz:

$$\begin{array}{c|l} x^2 & 2=A+M \\ x & 10=2A+N \\ x^0 & -5=5A \end{array}$$

Mazkur sistemani yechib, $A=-1$, $M=3$, $N=12$ larni topamiz. U holda:

$$I(x)=\frac{(x-2)^2}{2}+\int\left(-\frac{1}{x}+\frac{3x+12}{x^2+2x+5}\right)dx=\frac{(x-2)^2}{2}-ln|x|+$$
$$+\frac{3}{2}\int\frac{2x+2+6}{x^2+2x+5}dx=\frac{(x-2)^2}{2}--ln|x|+\frac{3}{2}\int\frac{(2x+2)dx}{x^2+2x+5}+9\int\frac{dx}{(x+1)^2+4}=$$
$$\frac{(x-2)^2}{2}--ln|x|+\frac{3}{2}ln|x^2+2x+5|+\frac{9}{2}arctg\frac{x+1}{2}+C.$$ ◄

AT-8.6

1. $\int\frac{x-4}{x^2-5x+6}dx.$ (*Javob*: $ln\frac{(x-2)^2}{|x-3|}+C.$)
2. $\int\frac{x^5+x^4-8}{x^3-4x}dx.$ (*Javob*: $\frac{x^3}{3}+\frac{x^2}{2}+4+ln\left|\frac{x^2(x-2)^5}{(x+2)^3}\right|+C.$)
3. $\int\frac{x^3+1}{x^3-x^2}dx.$ (*Javob*: $x+\frac{1}{x}+ln\frac{(x-1)^2}{|x|}+C.$)
4. $\int\frac{x^2-2x+3}{(x-1)(x^3-4x^2+3x)}dx.$ (*Javob*: $\frac{1}{x-1}+ln\frac{\sqrt{(x-1)(x-3)}}{|x|}+C.$)
5. $\int\frac{(2x^2-3x-3)dx}{(x-1)(x^2-2x+5)}.$ (*Javob*: $ln\frac{\sqrt{(x^2-2x+5)^3}}{|x-1|}+\frac{1}{2}arctg\frac{x-1}{2}+C.$)
6. $\int\frac{x^2dx}{x^4-1}.$ (*Javob*: $\frac{1}{2}arctg\,x+\frac{1}{4}ln\left|\frac{1-x}{1+x}\right|+C.$)
7. $\int\frac{2xdx}{(x+1)(x^2+1)^2}.$ (*Javob*: $\frac{x-1}{2(x^2+1)}-\frac{1}{2}ln|x+1|+\frac{1}{4}ln(1+x^2)+C.$)

Mustaqil yechish uchun topshiriqlar

1. a) $\int \frac{dx}{(x-1)(x+2)(x+3)}$; b) $\int \frac{4dx}{x(x^2+4)}$.

(*Javob*: a) $\frac{1}{12} \ln \left|\frac{(x-1)(x+3)^3}{(x+2)^4}\right| + C$; b) $\ln \frac{\sqrt{x^2+4}}{|x|} + C$.)

2. a) $\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} dx$; b) $\int \frac{dx}{x(x+1)^2}$.

(*Javob*: a) $\ln \left|\frac{(x-1)^4(x-4)^5}{(x+3)^7}\right| + C$; b) $\frac{1}{x+1} + \ln\left|\frac{x}{x+1}\right| + C$.)

3. a) $\int \frac{dx}{x(x^2-1)}$; b) $\int \frac{13dx}{x(x^2+6x+13)}$.

(*Javob*: a) $\ln \frac{\sqrt{x^2-1}}{|x|} + C$; b) $\ln \frac{x}{\sqrt{x^2+6x+13}} + 5arctg \frac{x+3}{2} + C$.)

8.7. AYRIM IRRATSIONAL FUNKSIYALARNI INTEGRALLASH

Har qanday irratsional funksiya uchun elementar funksiyalar orqali ifodalanadigan boshlang'ich funksiyani topish har doim ham mumkin bo'lavermaydi.

Quyida, ayrim irratsional funksiyalarning integrallarini qarab o'tamizki, ular o'z navbatida biron bir o'rniga qo'yish usuli orqali yangi o'zgaruvchiga nisbatan ratsional funksiyalarni integrallashga keltiriladi.

Quyidagi $\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{r_1}{s_1}}, \ldots, \left(\frac{ax+b}{cx+d}\right)^{\frac{r_\gamma}{s_\gamma}}\right) dx$ kabi integralni qaraylik.

Bu yerda: *R*- ratsional funksiya bo'lib, *a*, *b*, *c*, *d* lar ixtiyoriy sonlar, r_i, s_i $(i = \overline{1,\gamma})$ lar esa, butun musbat sonlar.

Mazkur integralni $\frac{ax+b}{cx+d} = u^m$ almashtirish orqali yangi o'zgaruvchi *u* ga nisbatan ratsional funksiyani integrallashga keltiriladi. Bu yerda, m=EKUK $(s_1, s_2, \ldots, s_\gamma)$ dir.

Xususan, $\int R\left(x, x^{\frac{r_1}{s_1}}, \ldots, x^{\frac{r_\gamma}{s_\gamma}}\right) dx$ kabi integral $x=u^m$ almashtirish orqali ratsionallashtiriladi.

1-misol. $\int \frac{\sqrt{x}dx}{\sqrt[4]{x^3}+4}$ hisoblansin.

►Bu yerda, EKUK(2,4)=4 bo'lganligi uchun $x=u^4$ almashtirish kiritiladi, ya'ni:

$$\int \frac{\sqrt{x}dx}{\sqrt[4]{x^3}+4} = \int \frac{x^{\frac{1}{2}}dx}{x^{3/4}+4} = \begin{vmatrix} x = u^4 \\ dx = 4u^3du \end{vmatrix} =$$

$$= 4\int \frac{u^2}{u^3+4}u^3du = 4\int \left(u^2 - \frac{4u^2}{u^3+4}\right)du = \frac{4}{3}u^3$$

$$-\frac{16}{3}ln|u^3+4| + C =$$

$$= \frac{4}{3}\sqrt[4]{x^3} - \frac{16}{3}ln\left|\sqrt[4]{x^3}+4\right| + C.$$

(chunki, $u = \sqrt[4]{x}$ edi).◄

2-misol. $\int \frac{\sqrt[6]{x+1}dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$ hisoblansin.

►Bu yerda, EKUK (2,3,6)=6 bo'lganligi uchun

$$\int \frac{\sqrt[6]{x+1}dx}{\sqrt{x+1}+\sqrt[3]{x+1}} = \begin{vmatrix} x+1 = u^6 \\ dx = 6u^5du \end{vmatrix} = \int \frac{u\cdot 6u^5du}{u^3+u^2} = 6\int \frac{u^4du}{u+1} =$$

$$= 6\int \left(u^3 - u^2 + u - 1 + \frac{1}{u+1}\right)du = \frac{3}{2}u^4 - 2u^3 + 3u^2 -$$

$$6u + +6ln|u+1| + C = \frac{3}{2}\sqrt[3]{(x+1)^2} - 2\sqrt{x+1} + 3\sqrt[3]{x+1} -$$

$$6\sqrt[6]{x+1} + +6ln\left|\sqrt[6]{x+1}+1\right| + C.◄$$

Eslatma: $\sqrt{ax^2+bx+c}$ ga nisbatan ratsional bo'lgan ayrim irratsionalliklarni integrallash §8.3 va §8.4 larda bayon etilgan edi.

Quyidagi ko'rinishdagi integralni qaraylik $\int \frac{P_n(x)dx}{\sqrt{ax^2+bx+c}}$.

Bu integralni har doim quyidagicha yoyish mumkin bo'lar ekan:

$$\int \frac{P_n(x)dx}{\sqrt{ax^2+bx+c}} = Q_{n-1}(x)\cdot\sqrt{ax^2+bx+c} + \int \frac{dx}{\sqrt{ax^2+bx+c}} \quad (8.11)$$

Bu yerda, $\lambda \in R$ hamda $Q_{n-1}(x)$ esa, koeffitsientlari noma'lum bo'lgan (n-1) darajali ko'phad bo'lib, uning koeffitsientlarini aniqlash uchun (8.11) ni differetsiallab yuboriladi. Natijada hosil bo'lgan ayniyatdan, ham $Q_{n-1}(x)$ ning koeffitsientlari, ham λ ni aniqlanadi.

3-misol. $\int \frac{x^4+4x^2}{\sqrt{x^2+4}}dx$ hisoblansin.

►(8.11) formulaga binoan,

$$\int \frac{x^4+4x^2}{\sqrt{x^2+4}}dx = (Ax^3+Bx^2+Cx+D)\sqrt{x^2+4} + \lambda\int \frac{dx}{\sqrt{x^2+4}}$$

ni yozib olamiz.

Oxirgi tenglikni differensiallab, quyidagini hosil qilamiz

$$\frac{x^4+4x^2}{\sqrt{x^2+4}} = (3Ax^2 + 2Bx + C)\sqrt{x^2+4} +$$

$$+(Ax^3 + Bx^2 + Cx + D)\frac{x}{\sqrt{x^2+4}} + \frac{\lambda}{\sqrt{x^2+4}} \quad (1)$$

Tenglikning har ikkala tomonini $\sqrt{x^2+4}$ ga ko‘paytirsak, u holda:

$$x^4 + 4x^2 = (3Ax^2 + 2Bx + C)(x^2 + 4) + (Ax^3 + Bx^2 + Cx + D)\cdot x + \lambda$$

Noma’lum koeffitsientlar usulini qo‘llab, quyidagi tenglamalar sistemasini aniqlaymiz, ya’ni:

$$\begin{array}{l|c} x^4 & 1 = 3A + A \\ x^3 & 0 = 2B + B \\ x^2 & 4 = 12A + C + B \\ x^1 & 0 = 4B + D \\ x^0 & 0 = 4C + \lambda \end{array}\Bigg\}$$

Bu sistemani yechib, A=1/4, B=0, C=1/2, D=0 va λ=–2 larni topamiz.

Natijada,

$$\int \frac{x^4+4x^2}{\sqrt{x^2+4}}dx = \frac{x^3+2x}{4}\sqrt{x^2+4} - 2ln\left|x + \sqrt{x^2+4}\right| + C$$

ni hosil qilamiz.

Differensial binom deb ataluvchi ifodaning integrali, $\int x^m(a + +bx^n)^p dx$ ni (bu yerda, a va b lar noldan farqli bo‘lgan o‘zgarmas sonlar bo‘lib, m, n, p lar esa, ratsional sonlardir) integrallash uchun uni Chebishev almashtirishlari yordamida ratsional funksiyalarning integrallariga keltiriladi. Quyidagi uch holni qaraymiz:

1) agar p butun son bo‘ladigan bo‘lsa, yuqorida ko‘rib o‘tilgan eng sodda irratsional funksiyalarni integrallash holiga kelinadi;

2) agar $(m+1)/n$ butun son bo‘ladigan bo‘lsa, $a + bx^n = u^s, p = \frac{r}{s}, s > 0$ kabi almashtirish qo‘llaniladi;

3) agar $\frac{m+1}{n} + p$ butun son bo‘ladigan bo‘lsa, $a + bx^n = u^s x^n$ kabi almashtirish kiritiladi.

4-misol. $\int \frac{dx}{x^7\sqrt{1+x^4}}$ hisoblansin.

▶Bu yerda, $m=-7, n=4, p=-1/2$ bo'lganligi uchun, $\frac{m+1}{n}+p==-\frac{3}{2}--\frac{1}{2}=-2$ butun sondir. Shu boisdan, yuqorida qaralgan 3-holdan foydalanamiz:

$$\int\frac{dx}{x^7\sqrt{1+x^4}}=\left|\begin{array}{c}1+x^4=u^2x^4\\ x=(u^2-1)^{-\frac{1}{4}}\\ dx=-\frac{1}{2}(u^2-1)^{-5/4}udu\end{array}\right|=$$

$$=\int(u^2-1)^{\frac{7}{4}}\cdot u^{-1}\cdot(u^2-1)^{\frac{1}{2}}\left(-\frac{1}{2}\right)(u^2-1)^{-\frac{5}{4}}udu=$$

$$=-\frac{1}{2}\int(u^2-1)\,du=-\frac{1}{6}u^3+\frac{1}{2}u+C=\left|u=\frac{\sqrt{1+x^4}}{x^2}\right|=$$

$$=\left(-\frac{1}{6x^6}+\frac{1}{3x^2}\right)\sqrt{1+x^4}+C.$$ ◀

AT-8.7

Aniqmas integrallarni hisoblang.

1. $\int\frac{dx}{3x-4\sqrt{x}}$. (*Javob*: $\frac{2}{3}ln|\sqrt{3x}+4|+C.$)
2. $\int\frac{\sqrt{x}dx}{\sqrt[3]{x^2}-\sqrt[4]{x}}$. (*Javob*: $\frac{6}{5}\sqrt[6]{x^5}+\frac{12}{5}\sqrt[12]{x^5}+\frac{12}{5}ln|\sqrt[12]{x^5}-1|+C.$)
3. $\int\frac{dx}{\sqrt{3x+4}+2\sqrt[4]{3x+4}}$. (*Javob*: $\frac{4}{3}\Big(\frac{1}{2}\sqrt{3x+4}-2\sqrt[4]{3x+4}+$
$+4\,ln\left(\sqrt[4]{3x+4}+21\right)\Big)+C.$)
4. $\int\frac{dx}{\sqrt{x}-7\sqrt[4]{x}}$. (*Javob*: $4(\frac{1}{2}\sqrt{x}+7\sqrt[4]{x}+49ln|\sqrt[4]{x}-7|+C.$)
5. $\int\sqrt{\frac{1-x}{1+x}}\frac{dx}{x}$. (*Javob*: $ln\left|\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right|+2arctg\sqrt{\frac{1-x}{1+x}}+C.$)
6. $\int x^5\sqrt[3]{(1+x^3)^2}dx$. (*Javob*: $\frac{1}{8}\sqrt[3]{(1+x^3)^8}-\frac{1}{5}\sqrt[3]{(1+x^3)^5}+C.$)

Mustaqil yechish uchun topshiriqlar

Aniqmas integrallarni hisoblang.

1. a) $\int\frac{\sqrt{x}}{\sqrt[4]{x^3}+1}dx$; b) $\int\frac{x^3dx}{\sqrt{x^2+2}}$.
(*Javob*: a) $\frac{4}{3}\left(\sqrt[4]{x^3}-\ln\left(\sqrt[4]{x^3}+1\right)\right)+C$; b) $\frac{(x^2-4)\sqrt{x^2+2}}{3}+C.$)
2. a) $\int\frac{\sqrt{x^3}-\sqrt[3]{x}}{\sqrt[4]{x}}dx$; b) $\int\frac{4xdx}{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}$.
(*Javob*: a) $\frac{2}{9}\sqrt[4]{x^9}-\frac{12}{13}\sqrt[12]{x^{13}}+C$; b) $3\sqrt[3]{x+1}-4(x+1)+C.$)
3. a) $\int\frac{dx}{\sqrt[3]{x}+\sqrt{x}}$; b) $\int\frac{4xdx}{\sqrt[3]{(3x-8)^2}-2\sqrt[3]{3x-8}+4}$.

(*Javob*: a) $6\left(\frac{\sqrt{x}}{3} - \frac{\sqrt[3]{x}}{2} + \sqrt[6]{x} - ln(1 + \sqrt[6]{x}) + C.\right)$;
b)$\frac{1}{3}\sqrt[3]{(3x-8)^4} + \frac{8}{9}(3x - 8) + C$.)

8.8. TRIGONOMETRIK IFODALARNI INTEGRALLASH

$\int R(cosx; sinx)dx$ (8.12)

integralni qaraymiz. Bu yerda, R ratsional funksiya. Bu xildagi integralni integrallash, universal almashtirish deb ataluvchi $tg\frac{x}{2} = u$ kabi almashtirish yordamida yangi u o'zgaruvchiga nisbatan ratsional funksiyani integrallashga keltiriladi. Bu yerda,

$$cosx = \frac{1-u^2}{1+u^2}, sinx = \frac{2u}{1+u^2}, dx = \frac{2du}{1+u^2} \qquad (8.13)$$

belgilashlar inobatga olinadi (§8.6 ga qaralsin).

1-misol. $\int \frac{dx}{1+sinx+cosx}$ hisoblansin.

► $tg\frac{x}{2} = u$ deb olib, (8.13) lardan foydalansak, quyidagiga ega bo'lamiz:

$$\int \frac{dx}{1+sinx+cosx} = \int \frac{2du/(1+u^2)}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}} = \int \frac{du}{1+u} = ln|1+u| + C ==$$
$$ln\left|1 + tg\frac{x}{2}\right| + C.$$ ◄

Agarda, $R(-cosx, -sinx) = R(cosx, sinx)$ kabi ayniyat o'rinli bo'lsa, integral belgisi ostidagi funksiyani ratsional ko'rinishga keltirish uchun nisbatan soddaroq bo'lgan $tgx = u$ almashtirishni qo'llash mumkin. Bu yerda esa,

$$sinx = \frac{u}{\sqrt{1+u^2}}, cosx = \frac{1}{\sqrt{1+u^2}}, dx = \frac{du}{1+u^2} \qquad (8.14)$$

kabi ifodalar inobatga olinadi.

2-misol. $\int \frac{dx}{3+sin^2x}$ hisoblansin.

► $tgx = u$ deb olib, (8.14) lardan foydalanamiz:

$$\int \frac{dx}{3+sin^2x} = \int \frac{\frac{du}{1+u^2}}{3+\frac{u^2}{1+u^2}} = \int \frac{du}{3+4u^2} = \frac{1}{2\sqrt{3}} arctg\frac{2u}{\sqrt{3}} + C ==$$
$$\frac{1}{2\sqrt{3}} arctg\frac{2tgx}{\sqrt{3}} + C..$$ ◄

3-misol. $\int tg^5 2xdx$ ni hisoblang.

► Bu integralni hisoblash uchun $tg2x = u$ almashtirish kiritamiz. U holda, $x = \frac{1}{2} arctgu$ va $dx = \frac{1}{2}\frac{du}{1+u^2}$ larni inobatga olib, quyidagiga ega bo'lamiz:

$$\int tg^5 2xdx = \frac{1}{2}\int u^5 \frac{1}{1+u^2}du = \frac{1}{2}\int \left(u^3 - u + \frac{u}{1+u^2}\right)du = \frac{1}{8}u^4 - \frac{1}{4}u^2 + +\frac{1}{4}\ln(1+u^2) + C = \frac{1}{8}tg^4 2x - \frac{1}{4}tg^2 2x + \frac{1}{4}\ln(1+tg^2 2x) + C. ◄$$

$\int f(cosx)sinxdx$ va $\int f(sinx)cosxdx$ kabi integrallarni hisoblash uchun mos ravishda $cosx = t$ va $sinx = t$ almashtirishlardan foydalanish maqsadga muvofiqdir.

4-misol. $\int \frac{sin^3x}{cos^4x}dx$ hisoblansin.

► $cosx = t$ deb olamiz, u holda:

$$\int \frac{sin^3x}{cos^4x}dx = \int \frac{1-cos^2x}{cos^4x}sinxdx = \int \frac{1-t^2}{t^4}(-dt) = -\int \frac{dt}{t^4} + \int \frac{dt}{t^2} =$$
$$= \frac{1}{3}t^{-3} - \frac{1}{t} + C = \frac{1}{3}\frac{1}{cos^3x} - \frac{1}{cosx} + C. ◄$$

5-misol. I=$\int \frac{cos2xdx}{\sqrt[3]{(2+3sin2x)^2}}$ hisoblansin.

► $2 + 3sin2x = t^3$ deb olsak, $cos2x = \frac{1}{2}t^2dt$ bo'ladi. U holda:

$$I = \frac{1}{2}\int \frac{t^2dt}{\sqrt[3]{t^6}} = \frac{1}{2}\int dt = \frac{1}{2}t + C = \frac{1}{2}\sqrt[3]{(2+3sin2x)} + C. ◄$$

AT- 8.8

Berilgan aniqmas integrallar hisoblansin.

1. $\int \frac{dx}{3+5\cos x}$.(*Javob*: $\frac{1}{4}ln\left|\frac{2+tg\frac{x}{2}}{2-tg\frac{x}{2}}\right| + C$.)
2. $\int \frac{dx}{3\sin^2 x+5\cos^2 x}$. (*Javob*: $\frac{1}{\sqrt{15}}arctg\frac{\sqrt{3}tg\,x}{\sqrt{5}} + S$.)
3. $\int \frac{dx}{8-4\sin x+7\cos x}$. (*Javob*: $ln\left|\frac{tg\frac{x}{2}-5}{tg\frac{x}{2}-3}\right| + C$.)
4. $\int \cos^3 x\sin^{10} xdx$. (*Javob*: $\frac{\cos^{11}x}{11} - \frac{\cos^{13}x}{13} + C$.)
5. $\int \frac{dx}{\sin^2 x+3\sin x\cos x+\cos^2 x}$.(*Javob*: $\frac{1}{\sqrt{13}}ln\left|\frac{\frac{1}{2}tgx+3-\sqrt{3}}{2tgx+3+\sqrt{3}}\right| + C$.)
6. $\int \sin^4 3xdx$. (*Javob*: $\frac{3}{8}x - \frac{1}{2}\sin 6x + \frac{1}{96}\sin 12x + C$.)
7. $\int \frac{\cos^4 x+\sin^4 x}{\cos^2 x-\sin^2 x}dx$. (*Javob*: $\frac{1}{4}ln\left|\frac{1+tgx}{1-tgx}\right| + \frac{1}{2}\sin x\cos x + C$.)
8. $\int \frac{dx}{\cos x\sin^3 x}$. (*Javob*: $ln|tgx| - \frac{1}{2\sin^2 x} + C$.)

Mustaqil yechish uchun topshiriqlar

1. a) $\int \frac{\sin^3 x}{\sqrt[3]{\cos^4 x}}dx$; b) $\int \frac{dx}{4-5\sin x}$.

(*Javob*: a) $\frac{3}{5}\cos^{5/3} x - 3\cos^{-1/3} x + C$; b) $\frac{1}{3}\ln\left|\frac{tg\frac{x}{2}-2}{2tg\frac{x}{2}-1}\right| + C$.)

2. a) $\int \frac{\cos 2x}{\sqrt{3+4\sin 2x}}dx$; b) $\int \frac{\sin x dx}{\sin x+1}$.

(*Javob* : a) $\frac{1}{4}\sqrt{3+4\sin 2x} + C$; b) $\frac{2}{1+tg\frac{x}{2}} + x + C$.)

3. a) $\int \frac{\sin 3x dx}{\sqrt[3]{(3+2\cos 3x)^2}}$; b) $\int \frac{\sin^2 x dx}{1+\cos^2 x}$.

(*Javob*: a) $\frac{1}{2}\sqrt{3+2\cos 3x} + C$; b) $\sqrt{2}arctg\left(\frac{tgx}{\sqrt{2}}\right) - x + C$.)

8.9. 8- BOBGA DOIR INDIVIDUAL UY TOPSHIRIQLARI

IUT-8.1

Aniqmas integrallarni hisoblang (1–5 topshiriqlarda integrallash natijasini differensiallab tekshiring).

1.

1.1. $\int \frac{3+\sqrt[3]{x^2}-2x}{\sqrt{x}}dx.$

1.2. $\int \frac{2x^2+3\sqrt{x}-1}{2x}dx.$

1.3. $\int \frac{3\sqrt{x}+4x^2-5}{2x^2}dx.$

1.4. $\int \frac{2\sqrt{x}-x^2+3}{\sqrt[3]{x}}dx.$

1.5. $\int \frac{\sqrt[4]{x}-2x+5}{x^2}dx.$

1.6. $\int \frac{2x^3-\sqrt{x}+4}{\sqrt{x}}dx.$

1.7. $\int \left(\sqrt[3]{x} - \frac{2\sqrt[4]{x}}{x} + 3\right)dx.$

1.8. $\int \frac{2x^3-\sqrt{x^5+1}}{\sqrt{x}}dx.$

1.9. $\int \frac{3x^2-\sqrt[5]{x}+3}{x}dx.$

1.10. $\int \frac{2x^3-\sqrt{x}+4}{x^2}dx.$

1.11. $\int \frac{\sqrt[6]{x^5}-5x^2+3}{x}dx.$

1.12. $\int \left(x\sqrt{x} - \frac{1}{\sqrt{x^3}} + 1\right)dx.$

1.13. $\int (x^2 - \frac{\sqrt[6]{x}}{x} - 3)dx.$

1.14. $\int \frac{\sqrt[3]{x^2}-2x^5+3}{x}dx.$

1.15. $\int (\frac{\sqrt[3]{x}}{x} + 2x^3 - 4)dx.$

1.16. $\int \frac{\sqrt{x^3}-3x^4+2}{x}dx.$

1.17. $\int (2x^3 - 3\sqrt{x^5} + \frac{4}{x})dx.$

1.18. $\int \frac{2x^3-\sqrt[3]{x^2}+1}{x^2}dx.$

1.19. $\int \frac{3x^2-\sqrt{x^3}+7}{x^3}dx.$

1.20. $\int \frac{3x^4-\sqrt[3]{x^2}+1}{x^2}dx.$

1.21. $\int \left(\sqrt[5]{x^2} - \frac{2}{x^3} + 4\right)dx.$

1.22. $\int \frac{\sqrt{x}-2x^2+6}{x}dx.$

1.23. $\int \frac{\sqrt[5]{x}-2x^3+4}{x^2}dx.$

1.24. $\int \left(\sqrt{x} - \frac{3x^2}{\sqrt{x^3}} + 2\right) dx.$

1.25. $\int \left(\sqrt[5]{x} - \frac{4}{x^5} + 2\right) dx.$

1.26. $\int \frac{\sqrt[7]{x^5} - 2x^2 + 3}{x} dx.$

1.27. $\int (\frac{\sqrt[3]{x}}{x} - \frac{2}{x^3} + 1)\, dx.$

1.28. $\int \left(\frac{2x^2}{\sqrt{x}} - \frac{5}{x} + 6\right) dx.$

1.29. $\int \left(\frac{\sqrt[3]{x^2}}{x} - \frac{7}{x^3} + 5\right) dx.$

1.30. $\int \left(\frac{5x^2}{\sqrt{x}} - \sqrt[3]{x^2} + 2\right) dx.$

2.

2.1. $\int \sqrt{3+x}dx.$

2.2. $\int \sqrt[3]{1+x}dx.$

2.3. $\int \sqrt[3]{(1+x)^2}dx.$

2.4. $\int \frac{dx}{\sqrt{1+x}}.$

2.5. $\int \frac{dx}{\sqrt{(1-x)^3}}.$

2.6. $\int \frac{dx}{\sqrt[3]{2+x}}.$

2.7. $\int (1-4x^7)dx.$

2.8. $\int (1+4x)^5 dx.$

2.9. $\int (1-3x)^4 dx.$

2.10. $\int \sqrt{1+3x}\, dx.$

2.11. $\int \sqrt{5-4x}\, dx.$

2.12. $\int \frac{dx}{\sqrt[3]{5+3x}}.$

2.13. $\int \frac{dx}{\sqrt[3]{(1-4x)^5}}.$

2.14. $\int \frac{dx}{\sqrt[3]{(3-4x)^2}}.$

2.15. $\int \frac{dx}{\sqrt[3]{2-5x}}.$

2.16. $\int \sqrt[5]{3-2x}\, dx.$

2.17. $\int \sqrt[4]{1+3x}\, dx.$

2.18. $\int \sqrt[3]{1+3x}\, dx.$

2.19. $\int \frac{dx}{\sqrt{(3-x)^5}}.$

2.20. $\int \frac{dx}{\sqrt[3]{3+x}}.$

2.21. $\int \frac{dx}{(2+x)^3}.$

2.22. $\int \sqrt[3]{5-2x}\, dx.$

2.23. $\int \sqrt{5-4x}\, dx.$

2.24. $\int \sqrt[5]{(6-5x)^2}dx.$

2.25. $\int \sqrt[4]{2-5x}\, dx.$

2.26. $\int \sqrt[3]{4-2x}\, dx.$

2.27. $\int \sqrt{3-4x}\, dx.$

2.28. $\int \sqrt[5]{3+2x}\, dx.$

2.29. $\int \sqrt[4]{(3+5x)^3}dx.$

2.30. $\int \sqrt[3]{(2-x)^2}dx.$

3.

3.1. $\int \frac{dx}{3-x}.$

3.2. $\int \frac{dx}{3x+9}.$

3.3. $\int \frac{dx}{2-3x}.$

3.4. $\int \frac{dx}{1-4x}.$

3.5. $\int \frac{dx}{2+3x}.$

3.6. $\int \frac{dx}{2-5x}.$

3.7. $\int \frac{dx}{3x-2}.$

3.8. $\int \frac{dx}{2x+3}.$

3.9. $\int \frac{dx}{3x-4}.$

3.10. $\int \frac{dx}{4-3x}$.

3.11. $\int \frac{dx}{3x+4}$.

3.12. $\int \frac{dx}{4x-2}$.

3.13. $\int \frac{dx}{5-3x}$.

3.14. $\int \frac{dx}{4-7x}$.

3.15. $\int \frac{dx}{5x-3}$.

3.16. $\int \frac{dx}{3-2x}$.

3.17. $\int \frac{dx}{5+3x}$.

3.18. $\int \frac{dx}{3-5x}$.

3.19. $\int \frac{dx}{5+4x}$.

3.20. $\int \frac{dx}{6-3x}$.

3.21. $\int \frac{dx}{6+5x}$.

3.22. $\int \frac{dx}{1-7x}$.

3.23. $\int \frac{dx}{1+6x}$.

3.24. $\int \frac{dx}{2+7x}$.

3.25. $\int \frac{dx}{7-3x}$.

3.26. $\int \frac{dx}{5-2x}$.

3.27. $\int \frac{dx}{2x+7}$.

3.28. $\int \frac{dx}{2x+9}$.

3.29. $\int \frac{dx}{7x-3}$.

3.30. $\int \frac{dx}{6x+1}$.

4.

4.1. $\int \sin(2-3x)\,dx$.
4.2. $\int \sin(3-2x)\,dx$.
4.3. $\int \sin(5-3x)\,dx$.
4.4. $\int \cos(2+3x)\,dx$.
4.5. $\int \cos(3+2x)\,dx$.
4.6. $\int \sin(4-2x)\,dx$.
4.7. $\int \cos(5-2x)\,dx$.
4.8. $\int \cos(7x+3)\,dx$.
4.9. $\int \sin(8x-3)\,dx$.
4.10. $\int \sin(3+4x)\,dx$.
4.11. $\int \sin(3-4x)\,dx$.
4.12. $\int \cos(4x+3)\,dx$.
4.13. $\int \cos(3-4x)\,dx$.
4.14. $\int \cos(2+5x)\,dx$.
4.15. $\int \cos(3x+5)\,dx$.
4.16. $\int \sin(4x+3)\,dx$.
4.17. $\int \sin(5-3x)\,dx$.
4.18. $\int \sin(3x+6)\,dx$.
4.19. $\int \cos(5x-8)\,dx$.
4.20. $\int \cos(3x-7)\,dx$.
4.21. $\int \cos(5x-6)\,dx$.
4.22. $\int \sin(7x+1)\,dx$.
4.23. $\int \cos(7x+3)\,dx$.
4.24. $\int \sin(7-4x)\,dx$.
4.25. $\int \cos(3x-7)\,dx$.
4.26. $\int \sin(8x-5)\,dx$.
4.27. $\int \cos(8x-4)\,dx$.
4.28. $\int \sin(9x-1)\,dx$.
4.29. $\int \cos(10x-3)\,dx$.
4.30. $\int \sin(9x+7)\,dx$.

5.

5.1. $\int \frac{\sqrt{3}dx}{9x^2-3}$.

5.2. $\int \frac{dx}{\sqrt{9x^2+3}}$.

5.3. $\int \frac{dx}{9x^2+3}$.

5.4. $\int \frac{dx}{\sqrt{9x^2-3}}$.

5.5. $\int \frac{dx}{\sqrt{3-9x^2}}$.

5.6. $\int \frac{dx}{7x^2-4}$.

5.7. $\int \frac{3dx}{\sqrt{7x^2-4}}$.

5.8. $\int \frac{dx}{5x^2+3}$.

5.9. $\int \frac{dx}{5x^2-3}$.

5.10. $\int \frac{dx}{\sqrt{3-5x^2}}$.

5.11. $\int \frac{dx}{\sqrt{5x^2+3}}$.

5.12. $\int \frac{dx}{\sqrt{4-7x^2}}$.

5.13. $\int \frac{\sqrt{5}dx}{\sqrt{3-4x^2}}$.

5.14. $\int \frac{dx}{\sqrt{2x^2-9}}$.

5.15. $\int \frac{dx}{2x^2+7}$.

5.16. $\int \frac{dx}{\sqrt{3x^2+1}}$.

5.17. $\int \frac{dx}{3x^2+2}$.

5.18. $\int \frac{\sqrt{2}dx}{\sqrt{7-2x^2}}$.

5.19. $\int \frac{\sqrt{14}dx}{2x^2-7}$.

5.20. $\int \frac{dx}{8x^2+9}$.

5.21. $\int \frac{dx}{3x^2-2}$.

5.22. $\int \frac{dx}{4x^2+3}$.

5.23. $\int \frac{dx}{\sqrt{4x^2+3}}$.

5.24. $\int \frac{dx}{\sqrt{3-4x^2}}$.

5.25. $\int \frac{dx}{\sqrt{9-8x^2}}$.

5.26. $\int \frac{dx}{4x^2-3}$.

5.27. $\int \frac{dx}{8x^2-9}$.

5.28. $\int \frac{dx}{4x^2+7}$.

5.29. $\int \frac{2dx}{4+3x^2}$.

5.30. $\int \frac{2dx}{\sqrt{4x^2-3}}$.

6.

6.1. $\int \frac{2xdx}{\sqrt{5-4x^2}}$.

6.2. $\int \frac{xdx}{\sqrt{5-3x^2}}$.

6.3. $\int \frac{3xdx}{4x^2+1}$.

6.4. $\int \frac{4xdx}{\sqrt{3-4x^2}}$.

6.5. $\int \frac{2xdx}{\sqrt{8x^2-9}}$.

6.6. $\int \frac{4xdx}{\sqrt{4x^2+3}}$.

6.7. $\int \frac{xdx}{\sqrt{9-8x^2}}$.

6.8. $\int \frac{\sqrt{3}dx}{\sqrt{3x^2-2}}$.

6.9. $\int \frac{2xdx}{\sqrt{3x^2-2}}$.

6.10. $\int \frac{2xdx}{\sqrt{7-2x^2}}$.

6.11. $\int \frac{xdx}{2x^2-7}$.

6.12. $\int \frac{xdx}{3x^2+8}$.

6.13. $\int \frac{2xdx}{3x^2-7}$.

6.14. $\int \frac{2xdx}{\sqrt{2x^2+5}}$.

6.15. $\int \frac{xdx}{\sqrt{7-3x^2}}$.

6.16. $\int \frac{xdx}{2x^2+9}$.

6.17. $\int \frac{5xdx}{\sqrt{3-5x^2}}$.

6.18. $\int \frac{xdx}{\sqrt{3x^2+8}}$.

6.19. $\int \frac{5xdx}{\sqrt{5x^2+3}}$.

6.20. $\int \frac{xdx}{3x^2-6}$.

6.21. $\int \frac{xdx}{5x^2+1}$.

6.22. $\int \frac{5xdx}{5x^2-3}$.

6.23. $\int \frac{xdx}{2x^2-7}$.

6.24. $\int \frac{9xdx}{\sqrt{1-9x^2}}$.

6.25. $\int \frac{3xdx}{9x^2+2}$.

6.26. $\int \frac{5xdx}{\sqrt{7x^2-1}}$.

6.27. $\int \frac{3xdx}{\sqrt{9x^2+5}}$.

6.28. $\int \frac{2xdx}{5x^2-3}$.

6.29. $\int \frac{xdx}{3x^2-2}$.

6.30. $\int \frac{7xdx}{7x^2+1}$.

7.

7.1. $\int \frac{dx}{\sqrt{7x^2-3}}$.

7.2. $\int \frac{dx}{2x^2-5}$.

7.3. $\int \frac{dx}{\sqrt{2-5x^2}}$.

7.4. $\int \frac{dx}{5x^2+2}$.

7.5. $\int \frac{dx}{2x^2+3}$.

7.6. $\int \frac{dx}{\sqrt{5x^2+1}}$.

7.7. $\int \frac{dx}{2x^2+9}$.

7.8. $\int \frac{dx}{\sqrt{9-2x^2}}$.

7.9. $\int \frac{dx}{\sqrt{9x^2+2}}$.

7.10. $\int \frac{dx}{5x^2-4}$.

7.11. $\int \frac{dx}{3x^2-7}$.

7.12. $\int \frac{dx}{3x^2+7}$.

7.13. $\int \frac{dx}{6x^2-7}$.

7.14. $\int \frac{dx}{7x^2+6}$.

7.15. $\int \frac{dx}{\sqrt{7-3x^2}}$.

7.16. $\int \frac{dx}{6x^2+1}$.

7.17. $\int \frac{dx}{\sqrt{5x^2-1}}$.

7.18. $\int \frac{dx}{3x^2-5}$.

7.19. $\int \frac{dx}{\sqrt{2-3x^2}}$.

7.20. $\int \frac{dx}{\sqrt{8-3x^2}}$.

7.21. $\int \frac{dx}{\sqrt{3x^2+8}}$.

7.22. $\int \frac{dx}{\sqrt{3x^2+2}}$.

7.23. $\int \frac{dx}{2x^2+7}$.

7.24. $\int \frac{dx}{4x^2-3}$.

7.25. $\int \frac{dx}{3x^2+4}$.

7.26. $\int \frac{dx}{\sqrt{8x^2-9}}$.

7.27. $\int \frac{dx}{\sqrt{5-4x^2}}$.

7.28. $\int \frac{dx}{\sqrt{1-3x^2}}$.

7.29. $\int \frac{dx}{\sqrt{4x^2+5}}$.

7.30. $\int \frac{dx}{3x^2-2}$.

8.

8.1. $\int e^{2x-7}dx$.

8.2. $\int e^{3+5x}dx$.

8.3. $\int e^{2-3x}dx$.

8.4. $\int e^{2x+1}dx$.

8.5. $\int e^{7x-2}dx$.

8.6. $\int e^{5x-7}dx$.

8.7. $\int e^{5x+7}dx$.

8.8. $\int e^{7-2x}dx$.

8.9. $\int e^{3-4x}dx$.

8.10. $\int e^{10x+2}dx$.

8.11. $\int e^{2x-10}dx$.

8.12. $\int e^{4x+3}dx$.

8.13. $\int e^{4x+5}dx$.

8.14. $\int e^{6x-1}dx$.

8.15. $\int e^{5-2x}dx$.

8.16. $\int e^{4-3x}dx$.

8.17. $\int e^{3-5x}dx$.

8.18. $\int e^{1-4x}dx.$

8.19. $\int e^{2-5x}dx.$

8.20. $\int e^{6x-4}dx.$

8.21. $\int e^{8x+1}dx.$

8.22. $\int e^{2-6x}dx.$

8.23. $\int e^{2-4x}dx.$

8.24. $\int e^{3-6x}dx.$

8.25. $\int e^{4-5x}dx.$

8.26. $\int e^{5-x}dx.$

8.27. $\int e^{7+3x}dx.$

8.28. $\int e^{2x+3}dx.$

8.29. $\int e^{8x+1}dx.$

8.30. $\int e^{4-7x}dx.$

9.

9.1. $\int \frac{dx}{(2x+1)\sqrt[3]{ln^2(2x+1)}}.$

9.2. $\int \frac{\sqrt[3]{ln^2(1-x)}}{x-1}dx.$

9.3. $\int \frac{dx}{(1-x)\sqrt[3]{ln^2(1-x)}}.$

9.4. $\int \frac{dx}{(1-x)\sqrt{ln^3(1-x)}}.$

9.5. $\int \frac{ln^3(1-x)}{x-1}dx.$

9.6. $\int \frac{\sqrt{\ln(2x-1)}}{2x-1}dx.$

9.7. $\int \frac{\sqrt[3]{\ln(3x+1)}}{3x+1}dx.$

9.8. $\int \frac{dx}{(x+1)ln^2(x+1)}.$

9.9. $\int \frac{dx}{(x+1)\sqrt[3]{\ln(x+1)}}.$

9.10. $\int \frac{\sqrt[5]{ln^2(x+1)}}{x+1}dx.$

9.11. $\int \frac{\sqrt{ln^5(x+1)}}{x+1}dx.$

9.12. $\int \frac{\sqrt[7]{ln^2(x+1)}}{x+1}dx.$

9.13. $\int \frac{\sqrt{ln^3(x+1)}}{x+1}dx.$

9.14. $\int \frac{dx}{(x+1)\sqrt[5]{ln^2(x+1)}}.$

9.15. $\int \frac{\sqrt{ln^7(x+1)}}{x+1}dx.$

9.16. $\int \frac{dx}{(x+2)\sqrt{ln(x+2)}}.$

9.17. $\int \frac{ln^4(3x+1)}{3x+1}dx.$

9.18. $\int \frac{dx}{(x-3)ln^4(x-3)}.$

9.19. $\int \frac{dx}{(x+5)ln^3(x+5)}.$

9.20. $\int \frac{ln^3(x-5)}{x-5}dx.$

9.21. $\int \frac{\sqrt[3]{ln(x+4)}}{x+4}dx.$

9.22. $\int \frac{ln^5(x-7)}{x-7}dx.$

9.23. $\int \frac{\sqrt{ln^3(x+3)}}{x+3}dx.$

9.24. $\int \frac{\sqrt[3]{ln^4(x-5)}}{x-5}dx.$

9.25. $\int \frac{dx}{(x+3)ln^4(x+3)}.$

9.26. $\int \frac{ln^5(x-8)}{x-8}dx.$

9.27. $\int \frac{\sqrt{ln^3(x+6)}}{x+6}dx.$

9.28. $\int \frac{dx}{(x-4)ln^5(x-4)}.$

9.29. $\int \frac{ln^6(x+9)}{x+9}dx.$

9.30. $\int \frac{ln(3x+5)}{3x+5}dx.$

10.

10.1. $\int sin^4 2x cos2x dx.$

10.2. $\int \frac{cos2x}{sin^3 2x} dx.$

10.3. $\int \frac{sin3x}{cos^3 3x} dx.$

10.4. $\int \frac{sinx}{\sqrt[3]{cosx}} dx.$

10.5. $\int \frac{sinx}{cos^5 x} dx.$

10.6. $\int cos^7 2x sin2x dx.$

10.7. $\int \frac{cosx\, dx}{sinx+2}.$

10.8. $\int \frac{cosx\, dx}{3-sinx}.$

10.9. $\int \frac{sinxdx}{\sqrt{cosx+3}}.$

10.10. $\int \frac{sinxdx}{\sqrt[3]{cosx+1}}.$

10.11. $\int \frac{cosxdx}{\sqrt{(sinx-4)^3}}.$

10.12. $\int \frac{sin3x}{cos^2 3x} dx.$

10.13. $\int \frac{sin5x}{\sqrt{cos5x}} dx.$

10.14. $\int \frac{cos4x}{sin^3 4x} dx.$

10.15. $\int sin^3 5x cos5x dx.$

10.16. $\int \sqrt[3]{cos2x} sin2x dx.$

10.17. $\int \sqrt{cos^3 2x} sin2x dx.$

10.18. $\int \frac{sin4x}{\sqrt[3]{cos^2 4x}} dx.$

10.19. $\int sin^3 5x cos5x dx.$

10.20. $\int \frac{cos5x}{\sqrt{sin^3 5x}} dx.$

10.21. $\int \frac{sin5x}{cos^4 5x} dx.$

10.22. $\int \sqrt{cos7x} sin7x dx.$

10.23. $\int sin^6 3x cos3x dx.$

10.24. $\int \frac{cos6x}{sin^7 6x} dx.$

10.25. $\int \sqrt{cos^3 2x} sin2x dx.$

10.26. $\int sin^4 8x cos8x dx.$

10.27. $\int sin^5 4x cos4x dx.$

10.28. $\int \frac{sin4x}{\sqrt[3]{cos4x}} dx.$

10.29. $\int \frac{sin2x}{\sqrt[3]{cos^4 2x}} dx.$

10.30. $\int \frac{cos6x}{sin^4 6x} dx.$

11.

11.1. $\int \frac{\sqrt{tg^3 x}}{cos^2 x} dx.$

11.2. $\int \frac{dx}{cos^2 x \sqrt{tg^3 x}}.$

11.3. $\int \frac{dx}{sin^2 x ctg^4 x}.$

11.4. $\int \frac{ctg^5 2x dx}{sin^2 2x}.$

11.5. $\int \frac{tg^3 4x}{cos^2 4x} dx.$

11.6. $\int \frac{\sqrt[3]{tg5x}}{cos^2 5x} dx.$

11.7. $\int \frac{\sqrt[3]{ctg^2 x}}{sin^2 x} dx.$

11.8. $\int \frac{dx}{sin^2 x ctg^3 x}.$

11.9. $\int \frac{dx}{cos^2 3x tg^4 3x}.$

11.10. $\int \frac{\sqrt{ctg7x}}{sin^2 x} dx.$

11.11. $\int \frac{\sqrt[5]{ctg3x}}{sin^2 3x} dx.$

11.12. $\int \frac{tg^4 7x}{cos^2 7x} dx.$

11.13. $\int \frac{ctg^5 6x}{sin^2 6x} dx.$

11.14. $\int \frac{\sqrt[3]{tg^5 4x}}{cos^2 4x} dx.$

11.15. $\int \frac{ctg^4 3x}{sin^2 3x} dx.$

11.16. $\int \frac{dx}{cos^2 4x\sqrt{tg4x}}.$

11.17. $\int \frac{dx}{sin^2 3x ctg^3 3x}.$

11.18. $\int \frac{tg6x}{cos^2 6x} dx.$

11.19. $\int \frac{dx}{sin^2 x ctg^3 x}.$

11.20. $\int \frac{\sqrt{ctg4x}}{sin^2 4x} dx.$

11.21. $\int \frac{ctg^5 4x}{sin^2 4x} dx.$

11.22. $\int \frac{\sqrt[3]{tg7x}}{cos^2 7x} dx.$

11.23. $\int \frac{\sqrt[5]{tg^2 3x}}{cos^2 3x} dx.$

11.24. $\int \frac{\sqrt{ctg^3 5x}}{sin^2 5x} dx.$

11.25. $\int \frac{dx}{sin^2 x\sqrt[5]{ctg^4 x}}.$

11.26. $\int \frac{dx}{cos^2 x\sqrt[5]{tg^2 x}}.$

11.27. $\int \frac{tg^6 2x}{cos^2 2x} dx.$

11.28. $\int \frac{\sqrt{ctg^5 x}}{sin^2 x} dx.$

11.29. $\int \frac{\sqrt[5]{ctg^2 x}}{sin^2 x} dx.$

11.30. $\int \frac{tg^7 4x}{cos^2 3x} dx.$

12.

12.1. $\int \frac{\sqrt{arctg^6 3x}}{1+9x^2} dx.$

12.2. $\int \frac{\sqrt[3]{arcsinx}}{\sqrt{1-x^2}} dx.$

12.3. $\int \frac{arccos^2 3x}{\sqrt{1-9x^2}} dx.$

12.4. $\int \frac{arcctg^2 2x}{1+4x^2} dx.$

12.5. $\int \frac{\sqrt[3]{arccos^2 3x}}{\sqrt{1-9x^2}} dx.$

12.6. $\int \frac{dx}{(1+x^2)arctg^3 x}.$

12.7. $\int \frac{arccos^3 x}{\sqrt{1-9x^2}} dx.$

12.8. $\int \frac{\sqrt[3]{arctg^2 x}}{1+x^2} dx.$

12.9. $\int \frac{arcsin^5 2x}{\sqrt{1-4x^2}} dx.$

12.10. $\int \frac{dx}{\sqrt{1-x^2}arcsin^4 x}.$

12.11. $\int \frac{arccos^3 2x}{\sqrt{1-4x^2}} dx.$

12.12. $\int \frac{arcctg^7 3x}{1+9x^2} dx.$

12.13. $\int \frac{arccos4x}{\sqrt{1-16x^2}} dx.$

12.14. $\int \frac{arcsin^4 x}{\sqrt{1-x^2}} dx.$

12.15. $\int \frac{arcsin^3 2x}{\sqrt{1-4x^2}} dx.$

12.16. $\int \frac{dx}{(1+x^2)arctg^7 x}.$

12.17. $\int \frac{\sqrt[3]{arctg2x}}{1+4x^2} dx.$

12.18. $\int \frac{arccos^6 3x}{1+9x^2} dx.$

12.19. $\int \frac{\sqrt{arctg^3 x}}{1+x^2} dx.$

12.20. $\int \frac{dx}{(1+x^2)\sqrt{arctgx}}.$

12.21. $\int \frac{dx}{(1+x^2)arctg^5 x}.$

12.22. $\int \frac{arccos^7 x dx}{\sqrt{1-x^2}}.$

12.23. $\int \frac{\sqrt[3]{arccos2x}}{\sqrt{1-4x^2}}dx.$

12.24. $\int \frac{arcctg^4 5x}{1+25x^2}dx.$

12.25. $\int \frac{arcsin^2 5x}{\sqrt{1-25x^2}}dx.$

12.26.

$\int \frac{dx}{\sqrt{1-25x^2}arcsin5x}.$

12.27. $\int \frac{arctg^8 3x}{1+9x^2}dx.$

12.28. $\int \frac{arccos^2 7x}{\sqrt{1-49x^2}}dx.$

12.29. $\int \frac{\sqrt[5]{arctg^3 x}}{1+x^2}dx.$

12.30. $\int \frac{arcctg^4 8x}{1+64x^2}dx.$

13.

13.1. $\int \frac{xdx}{e^{3x^2+4}}.$

13.2. $\int \frac{xdx}{e^{x^2+3}}.$

13.3. $\int \frac{x^2dx}{e^{x^3+1}}.$

13.4. $\int e^{cosx}sinxdx.$

13.5. $\int e^{2x^3-1}x^2dx.$

13.6. $\int \frac{sinx}{e^{cosx}}dx.$

13.7. $\int e^{7x^2+2}xdx.$

13.8. $\int e^{3-x^2}xdx.$

13.9. $\int e^{4x^2+5}xdx.$

13.10. $\int \frac{dx}{\sqrt{1-x^2}e^{arcsinx}}.$

13.11. $\int e^{5x^2-3}xdx.$

13.12. $\int e^{1-4x^2}xdx.$

13.13. $\int e^{3x^2+4}xdx.$

13.14. $\int e^{sinx+1}cosxdx.$

13.15. $\int e^{4-x^2}xdx.$

13.16. $\int e^{tgx}\frac{1}{cos^2x}dx.$

13.17. $\int e^{3cosx+2}sinxdx.$

13.18. $\int e^{4sinx-1}cosxdx.$

13.19. $\int e^{5x^2-3}xdx.$

13.20. $\int e^{5-2x^2}xdx.$

13.21. $\int e^{4-3x^2}xdx.$

13.22. $\int e^{cos2x}sin2xdx.$

13.23. $\int e^{1-6x^2}xdx.$

13.24. $\int e^{x^3+1}x^2dx.$

13.25. $\int \frac{e^{arctgx}}{1+x^2}dx.$

13.26. $\int e^{3x^3}x^2dx.$

13.27. $\int \frac{x^4dx}{e^{x^5+1}}.$

13.28. $\int \frac{xdx}{e^{x^2-3}}.$

13.29. $\int \frac{xdx}{e^{2x^2+1}}.$

13.30. $\int e^{4-5x^2}xdx.$

14.

14.1. $\int \frac{x-1}{7x^2+4}dx.$

14.2. $\int \frac{1-2x}{5x^2-1}dx.$

14.3. $\int \frac{2x+1}{5x^2+1}dx.$

14.4. $\int \frac{x+3}{\sqrt{x^2+4}}dx.$

14.5. $\int \frac{3x-2}{2x^2+7}dx.$

14.6. $\int \frac{5-x}{3x^2+1}dx.$

14.7. $\int \frac{5+x}{3x^2+1} dx.$

14.8. $\int \frac{2x-5}{\sqrt{7x^2+3}} dx.$

14.9. $\int \frac{2x-3}{\sqrt{x^2+9}} dx.$

14.10. $\int \frac{3x-2}{3x^2+1} dx.$

14.11. $\int \frac{x-1}{5-2x^2} dx.$

14.12. $\int \frac{2x+3}{1-3x^2} dx.$

14.13. $\int \frac{2x+3}{5x^2+2} dx.$

14.14. $\int \frac{x-3}{4x^2+1} dx.$

14.15. $\int \frac{x-3}{1-4x^2} dx.$

14.16. $\int \frac{3x-1}{4-x^2} dx.$

14.17. $\int \frac{5x-2}{x^2+9} dx.$

14.18. $\int \frac{2x+5}{\sqrt{5x^2+1}} dx.$

14.19. $\int \frac{1-2x}{\sqrt{3x^2+2}} dx.$

14.20. $\int \frac{2x-4}{x^2+16} dx.$

14.21. $\int \frac{2x-3}{\sqrt{4-x^2}} dx.$

14.22. $\int \frac{2x-1}{\sqrt{5-3x^2}} dx.$

14.23. $\int \frac{3x+4}{5-2x^2} dx.$

14.24. $\int \frac{3x-3}{\sqrt{1-x^2}} dx.$

14.25. $\int \frac{5x+2}{\sqrt{x^2+9}} dx.$

14.26. $\int \frac{3-2x}{x^2-8} dx.$

14.27. $\int \frac{x-5}{8-4x^2} dx.$

14.28. $\int \frac{x+4}{7x^2+3} dx.$

14.29. $\int \frac{3x+2}{\sqrt{2x^2-1}} dx.$

14.30. $\int \frac{x-5}{\sqrt{4-9x^2}} dx.$

Namunaviy variant yechimi

Aniqmas integrallarni hisoblang (1–5 topshiriqlarda integrallash natijasini differensiallab tekshiring).

1. $\int \frac{3-2x^4+\sqrt[3]{x^2}}{\sqrt[4]{x}} dx.$

► Integral ostidagi funksiya suratini maxrajiga bo‘lamiz va integrallash usulining ikkinchi va uchinchi qoidalarini qo‘llaymiz. Aniqmas integrallar jadvalini qo‘llasak:

$$\int \frac{3-2x^4+\sqrt[3]{x^2}}{\sqrt[4]{x}} dx = 3\int x^{-\frac{1}{4}} dx - 2\int x^{\frac{15}{4}} dx + \int x^{\frac{5}{12}} dx =$$

$$= 4x^{3/4} - \frac{8}{19}x^{\frac{19}{4}} + \frac{12}{17}x^{\frac{17}{12}} + C = 4\sqrt[4]{x^3} - \frac{8}{19}\sqrt[4]{x^{19}} + \frac{12}{17}\sqrt[12]{x^{17}} + C.$$

Hosil bo‘lgan natijani tekshiramiz:

$$\left(4x^{\frac{3}{4}} - \frac{8}{19}x^{19/4} + \frac{12}{17}x^{17/12} + C\right)' = 4\cdot\frac{3}{4}x^{-1/4} - \frac{8}{19}\cdot\frac{19}{4}x^{\frac{15}{4}} +$$

$$+\frac{12}{17}\cdot\frac{17}{12}x^{5/12} = 3x^{-1/4} - 2x^{\frac{15}{4}} + x^{\frac{5}{12}}. ◄$$

2. $\int \frac{dx}{\sqrt[5]{(4-8x)^2}}.$

► $\int \frac{dx}{\sqrt[5]{(4-8x)^2}} = \int (4-8x)^{-2/5}dx = -\frac{5}{8\cdot 3}(4-8x)^{\frac{3}{5}} + C =$

$$= -\frac{5}{24}\sqrt[5]{(4-8x)^3} + C.$$

Hosil bo'lgan natijani tekshiramiz:

$\left(-\frac{5}{24}(4-8x)^{\frac{3}{5}} + C\right)' = -\frac{5}{24}\frac{3}{5}(4-8x)^{-\frac{2}{5}}(-8) = (4-8x)^{-2/5}$ ◄

3. $\int \frac{dx}{6-7x}$.

► $\int \frac{dx}{6-7x} = -\frac{1}{7}\ln|6-7x| + C.$

Hosil bo'lgan natijani tekshiramiz:

$\left(-\frac{1}{7}\ln|6-7x| + C\right)' = -\frac{1}{7}\frac{1}{6-7x}\cdot(-7) = \frac{1}{6-7x}$. ◄

4. $\int \cos(2-5x)\,dx$.

► $\int \cos(2-5x)\,dx = -\frac{1}{5}\sin(2-5x) + C.$

Hosil bo'lgan natijani tekshiramiz:

$(-\frac{1}{5}\sin(2-5x) + C)' = -\frac{1}{5}\cos(2-5x)\cdot(-5) = \cos(2-5x)$. ◄

5. $\int \frac{3dx}{\sqrt{4x^2-3}}$.

► $\int \frac{3dx}{\sqrt{4x^2-3}} = \frac{3}{2}\int \frac{2dx}{\sqrt{(2x)^2-(\sqrt{3})^2}} = \frac{3}{2}\ln\left|2x - \sqrt{4x^2-3}\right| + C$

Hosil bo'lgan natijani tekshiramiz:

$\left(\frac{3}{2}\ln\left|2x + \sqrt{4x^2-3}\right| + C\right)' = \frac{3}{2}\left(\frac{2+\frac{8x}{2\sqrt{4x^2-3}}}{2x+\sqrt{4x^2-3}}\right) =$

$\frac{3}{2}\frac{2(\sqrt{4x^2-3}+2x)}{(2x+\sqrt{4x^2-3})\sqrt{4x^2-3}} = \frac{3}{\sqrt{4x^2-3}}$. ◄

6. $\int \frac{7xdx}{3x^2+4}$.

► $\int \frac{7xdx}{3x^2+4} = \frac{7}{6}\int \frac{6xdx}{3x^2+4} = \frac{7}{6}\ln|3x^2+4| + C.$ ◄

7. $\int \frac{dx}{\sqrt{6-5x^2}}$.

► $\int \frac{dx}{\sqrt{6-5x^2}} = \frac{1}{\sqrt{5}}\int \frac{d(\sqrt{5}x)}{\sqrt{(\sqrt{6})^2-(\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}}\arcsin\frac{\sqrt{5}x}{\sqrt{6}} + C.$ ◄

8. $\int e^{5-4x}dx$.

► $\int e^{5-4x}dx = -\frac{1}{4}\int e^{5-4x}d(5-4x) = -\frac{1}{4}e^{5-4x} + C.$ ◄

9. $\int \frac{\sqrt[7]{\ln^3(x+2)}}{x+2}dx$.

► $\int \frac{\sqrt[7]{ln^3(x+2)}}{x+2}dx$

$$= \int ln^{\frac{3}{7}}(x+2)d(\ln(x+2)) = \frac{7}{10}ln^{\frac{10}{7}}(x+2) + C =$$

$= \frac{7}{10}\sqrt[7]{ln^{10}(x+2)} + C.$ ◄

10. $\int \frac{cos3xdx}{\sqrt[5]{sin3x-4}}$.

► $\int \frac{cos3xdx}{\sqrt[5]{sin3x-4}} = \frac{1}{3}\int (sin3x-4)^{-\frac{1}{5}} \cdot 3cos3xdx = \frac{1}{3}\int (sin3x -$

$-4)^{-\frac{1}{5}}d(sin3x-4) = \frac{1}{3}\cdot\frac{5}{4}(sin3x-4)^{\frac{4}{5}} + C = \frac{5}{12}\sqrt[5]{(sin3x-4)^4} + C.$ ◄

11. $\int \frac{dx}{sin^2 4x\sqrt[3]{ctg^2 4x}}$

► $\int \frac{dx}{sin^2 4x\sqrt[3]{ctg^2 4x}} = -\frac{1}{4}\int ctg^{-\frac{2}{3}}4x\left(-\frac{4}{sin^2 4x}dx\right) =$

$= -\frac{1}{4}\int ctg^{-\frac{2}{3}}4xd(ctg4x) = -\frac{3}{4}ctg^{\frac{1}{3}}4x + C = -\frac{3}{4}\sqrt[3]{ctg4x} + C.$ ◄

12. $\int \frac{\sqrt[3]{arcctg^5 2x}}{1+4x^2}dx.$

► $\int \frac{\sqrt[3]{arcctg^5 2x}}{1+4x^2}dx = -\frac{1}{2}\int arcctg^{\frac{5}{3}}2x\left(-\frac{2}{1+4x^2}\right)dx =$

$= -\frac{1}{2}\int arcctg^{\frac{5}{3}}2xd(arctg2x) = -\frac{1}{2}\cdot\frac{3}{8}arcctg^{\frac{8}{3}}2x + C =$

$= -\frac{3}{16}\sqrt[3]{arcctg^8 2x} + C.$ ◄

13. $\int e^{3cosx+2}sinxdx.$

► $\int e^{3cosx+2}sinxdx = -\frac{1}{3}\int e^{3cosx+2}d(3cosx+2) =$

$-\frac{1}{3}e^{3cosx+2} + +C.$ ◄

14. $\int \frac{3x+10}{6x^2-4}dx.$

► $\int \frac{3x+10}{6x^2-4}dx = \int \frac{3xdx}{6x^2-4} + 10\int \frac{dx}{6x^2-4} = \frac{1}{4}\int \frac{12xdx}{6x^2-4} +$

$\frac{10}{\sqrt{6}}\int \frac{dx}{(\sqrt{6}x)^2-2^2} =$

$= \frac{1}{4}ln|6x^2-4| + \frac{5}{2\sqrt{6}}ln\left|\frac{\sqrt{6}x-2}{\sqrt{6}x+2}\right| + C.$ ◄

IUT-8.2

Aniqmas integrallarni hisoblang

1

1.1. $\int \frac{2-3x}{x^2+2}dx.$ (*Javob:* $\sqrt{2}\,arctg\frac{x}{\sqrt{2}} - \frac{3}{2}\ln|x^2+2| + C.$)

1.2. $\int \frac{3-5x}{\sqrt{1-x^2}}dx$. (*Javob*: $3arcsinx + 5\sqrt{1-x^2} + C$.)

1.3. $\int \frac{8-13x}{\sqrt{x^2-1}}dx$. (*Javob*: $8\,ln\left|x + \sqrt{x^2-1}\right| - 13\sqrt{x^2-1} + C$.)

1.4. $\int \frac{6x+1}{2x^2-1}dx$. (*Javob*: $\frac{3}{2}ln|2x^2-1| + \frac{\sqrt{2}}{4}ln\left|\frac{\sqrt{2}x-1}{\sqrt{2}x+1}\right| + C$.)

1.5. $\int \frac{x-2}{\sqrt{2-x^2}}dx$. (*Javob*: $-\sqrt{2-x^2} - 2arcsin\frac{x}{\sqrt{2}} + C$.)

1.6. $\int \frac{3-7x}{\sqrt{1-4x^2}}dx$. (*Javob*: $\frac{3}{2}arcsin2x + \frac{7}{4}\sqrt{1-4x^2} + C$.)

1.7. $\int \frac{5-3x}{\sqrt{2x^2+1}}dx$. (*Javob*: $\frac{5}{\sqrt{2}}ln\left|\sqrt{2}x + \sqrt{2x^2+1})\right| - \frac{3}{2}\sqrt{2x^2+1} + C$.)

1.8. $\int \frac{1+x}{\sqrt{2-x^2}}dx$. (*Javob*: $arcsin\frac{x}{\sqrt{2}} - \sqrt{2-x^2} + C$.)

1.9. $\int \frac{3x+2}{2x^2+1}dx$. (*Javob*: $\frac{3}{4}ln|2x^2+1| + \sqrt{2}arctg\sqrt{2}x + C$.)

1.10. $\int \frac{1-5x}{1+25x^2}dx$. (*Javob*: $\frac{1}{5}\,arctg\,5x - \frac{1}{10}\,ln|1+25x^2| + C$.)

1.11. $\int \frac{4x-3}{3x^2-4}dx$. (*Javob*: $\frac{2}{3}ln|3x^2-4| - \frac{\sqrt{3}}{4}ln\left|\frac{\sqrt{3}x-2}{\sqrt{3}x+2}\right| + C$.)

1.12. $\int \frac{x-3}{9x^2+7}dx$. (*Javob*: $\frac{1}{18}ln|9x^2+7| - \frac{1}{\sqrt{7}}arctg\frac{3x}{\sqrt{7}} + C$.)

1.13. $\int \frac{5-3x}{\sqrt{4-3x^2}}dx$. (*Javob*: $\frac{5}{\sqrt{3}}arcsin\frac{\sqrt{3}x}{2} + \sqrt{4-3x^2} + C$.)

1.14. $\int \frac{4-2x}{\sqrt{1-4x^2}}dx$. (*Javob*: $2arcsin2x + \frac{1}{2}\sqrt{1-4x^2} + C$.)

1.15. $\int \frac{5-x}{2+x^2}dx$. (*Javob*: $\frac{5}{\sqrt{2}}\,arctg\frac{x}{\sqrt{2}} - \frac{1}{2}\,ln|2+x^2| + C$.)

1.16. $\int \frac{1+3x}{\sqrt{1+4x^2}}dx$. (*Javob*: $\frac{1}{2}ln\left|2x + \sqrt{1+4x^2}\right| + \frac{3}{4}\sqrt{1+4x^2} + C$.)

1.17. $\int \frac{5-4x}{\sqrt{1-x^2}}dx$. (*Javob*: $5arcsinx + 4\sqrt{1-x^2} + C$.)

1.18. $\int \frac{5x-1}{\sqrt{x^2-3}}dx$. (*Javob*: $5\sqrt{x^2-3} - ln\left|x + \sqrt{x^2-3}\right| + C$.)

1.19. $\int \frac{1-3x}{4x^2-1}dx$. (*Javob*: $\frac{1}{4}ln\left|\frac{2x-1}{2x+1}\right| - \frac{3}{8}\,ln|4x^2-1| + C$.)

1.20. $\int \frac{x-5}{3-2x^2}dx$. (*Javob*: $-\frac{1}{4}ln|3-2x^2| + \frac{5}{2\sqrt{6}}\,ln\left|\frac{\sqrt{2}x-\sqrt{3}}{\sqrt{2}x+\sqrt{3}}\right| + C$.)

1.21. $\int \frac{x+4}{\sqrt{9-x^2}}dx$. (*Javob*: $-\sqrt{9-x^2} + 4arcsin\frac{x}{3} + C$.)

1.22. $\int \frac{2x-7}{x^2-5}dx$. (*Javob*: $ln|x^2-5| - \frac{7}{2\sqrt{5}}\,ln\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + C$.)

1.23. $\int \frac{7x-2}{\sqrt{x^2-1}}dx$. (*Javob*: $7\sqrt{x^2-1} - 2\,ln\left|x + \sqrt{x^2-1}\right| + C$.)

1.24. $\int \frac{1+3x}{\sqrt{x^2+1}}dx$. (*Javob*: $ln\left|x + \sqrt{x^2+1}\right| + 3\sqrt{x^2+1} + C$.)

1.25. $\int \frac{x-5}{x^2+7}dx$. (*Javob:* $\frac{1}{2}ln|x^2+7| - \frac{5}{\sqrt{7}}arctg\frac{x}{\sqrt{7}} + C$.)

1.26. $\int \frac{3-7x}{1+x^2}dx$. (*Javob:* $3\,arctgx - \frac{7}{2}ln|1+x^2| + C$.)

1.27. $\int \frac{8-2x}{1+3x^2}dx$. (*Javob:* $\frac{8}{\sqrt{3}}arctg\sqrt{3}x - \frac{1}{3}ln|1+3x^2| + C$.)

1.28. $\int \frac{3x+7}{\sqrt{x^2+4}}dx$. (*Javob:* $3\sqrt{x^2+4} + 7\,ln|x+\sqrt{x^2+4}| + C$.)

1.29. $\int \frac{2x-1}{\sqrt{3x^2-4}}dx$. (*Javob:* $\frac{2}{3}\sqrt{3x^2-4} - \frac{1}{\sqrt{3}}ln|\sqrt{3}x + \sqrt{3x^2-4}| + C$)

1.30. $\int \frac{5x+1}{\sqrt{x^2-6}}dx$. (*Javob:* $5\sqrt{x^2-6} + ln|x+\sqrt{x^2-6}| + C$.)

2.

2.1. $\int \frac{\sin 2x}{1+3\cos 2x}dx$. (*Javob:* $-\frac{1}{6}ln|1+3cos2x| + C$.)

2.2. $\int \frac{3x^3}{1-x^4}dx$. (*Javob:* $-\frac{3}{4}ln|1-x^4| + C$.)

2.3. $\int \frac{\sin 3x}{3-\cos 3x}dx$. (*Javob:* $\frac{1}{3}ln|3-cos3x| + C$.)

2.4. $\int \frac{e^x dx}{2e^x+3}$. (*Javob:* $\frac{1}{2}ln|2e^x+3| + C$.)

2.5. $\int \frac{\sin 2x}{\cos^2 x-4}dx$. (*Javob:* $-ln|\cos^2 x - 4| + C$.)

2.6. $\int \frac{e^x dx}{4-3e^x}$. (*Javob:* $-\frac{1}{3}ln|4-3e^x| + C$.)

2.7. $\int \frac{x^2}{7-5x^3}dx$. (*Javob:* $-\frac{1}{15}ln|7-5x^3| + C$.)

2.8. $\int \frac{\sin 2x}{3\sin^2 x+4}dx$. (*Javob:* $\frac{1}{3}ln|3\sin^2 x+4| + C$.)

2.9. $\int \frac{e^{2x}dx}{5+e^{2x}}$. (*Javob:* $\frac{1}{2}ln|5+e^{2x}| + C$.)

2.10. $\int \frac{4x^3}{7+2x^4}dx$. (*Javob:* $\frac{1}{2}ln|7+2x^4| + C$.)

2.11. $\int \frac{4x-5}{2x^2-5x+17}dx$. (*Javob:* $ln|2x^2-5x+17| + C$.)

2.12. $\int \frac{7x^3}{2x^4-5}dx$. (*Javob:* $\frac{7}{8}ln|2x^4-5| + C$.)

2.13. $\int \frac{cos3x}{\sqrt{sin3x-2}}dx$. (*Javob:* $\frac{2}{3}\sqrt{sin3x-2} + C$.)

2.14. $\int \frac{\sin 2x}{\sqrt{1+\cos^2 x}}dx$. (*Javob:* $-2\sqrt{1+\cos^2 x} + C$.)

2.15. $\int \frac{\sin x}{1+3cosx}dx$. (*Javob:* $-\frac{1}{3}ln|1+3cosx| + C$.)

2.16. $\int \frac{\sin 2x}{4-\sin^2 x}dx$. (*Javob:* $-ln|4-\sin^2 x| + C$.)

2.17. $\int \frac{e^{3x}dx}{e^{3x}-5}$. (*Javob:* $\frac{1}{3}ln|e^{3x}-5| + C$.)

2.18. $\int \frac{x^2}{7+3x^3}dx$. (*Javob*: $\frac{1}{9}ln|7+3x^3|+C$.)

2.19. $\int \frac{3x+3}{x^2+2x}dx$. (*Javob*: $\frac{3}{2}ln|x^2+2x|+C$.)

2.20. $\int \frac{e^{2x}dx}{\sqrt{e^{2x}+3}}$. (*Javob*: $\sqrt{e^{2x}+3}+C$.)

2.21. $\int \frac{3x^2+1}{x^3+x-10}dx$. (*Javob*: $ln|x^3+x-10|+C$.)

2.22. $\int \frac{x^5}{3x^6-7}dx$. (*Javob*: $\frac{1}{18}ln|3x^6-7|+C$.)

2.23. $\int \frac{x^4}{\sqrt{x^5+3}}dx$. (*Javob*: $\frac{2}{5}\sqrt{x^5+3}+C$.)

2.24. $\int \frac{3x^2-2}{\sqrt{2x^3-4x}}dx$. (*Javob*: $\sqrt{2x^3-4x}+C$.)

2.25. $\int \frac{cos7x}{\sqrt{5-sin7x}}dx$. (*Javob*: $-\frac{2}{7}\sqrt{5-sin7x}+C$.)

2.26. $\int \frac{\sin 4x}{\sqrt{cos4x+3}}dx$. (*Javob*: $-\frac{1}{2}\sqrt{cos4x+3}+C$.)

2.27. $\int \frac{12x^2+5x^4}{4x^3+x^5}dx$. (*Javob*: $ln|4x^3+x^5|+C$.)

2.28. $\int \frac{4e^{2x}dx}{\sqrt{1-e^{2x}}}$. (*Javob*: $-4\sqrt{1-e^{2x}}+C$.)

2.29. $\int \frac{\sin 2x}{\sqrt{6-\cos^2 x}}dx$. (*Javob*: $2\sqrt{6-\cos^2 x}+C$.)

2.30. $\int \frac{7x}{\sqrt{5x^2-4}}dx$. (*Javob*: $\frac{7}{5}\sqrt{5x^2-4}+C$.)

3.

3.1. $\int \frac{1-2x-x^3}{1+x^2}dx$. (*Javob*: $-\frac{x^2}{2}-\frac{1}{2}ln|x^2+1|+arctgx+C$.)

3.2. $\int \frac{7-x^2}{1-x}dx$. (*Javob*: $\frac{x^2}{2}+x-6\,ln|1-x|+C$.)

3.3. $\int \frac{x^3+2}{x^2-1}dx$. (*Javob*: $\frac{x^2}{2}+\frac{1}{2}ln|x^2-1|+ln\left|\frac{x-1}{x+1}\right|+C$.)

3.4. $\int \frac{8x^3-1}{2x+1}dx$. (*Javob*: $\frac{4}{3}x^3-x^2+x-ln|2x+1|+C$.)

3.5. $\int \frac{x^5-2}{x^2-4}dx$. (*Javob*: $\frac{1}{4}x^4+2x^2+8\,ln|x^2-4|-\frac{1}{2}ln\left|\frac{x-2}{x+2}\right|+C$.)

3.6. $\int \frac{2x^4-3}{x^2+1}dx$. (*Javob*: $\frac{2}{3}x^3-2x-arctgx+C$.)

3.7. $\int \frac{x^3-1}{2x+1}dx$. (*Javob*: $\frac{1}{6}x^3-\frac{1}{8}x^2+\frac{1}{8}x-\frac{9}{16}ln|2x+1|+C$.)

3.8. $\int \frac{x^5}{1-x^3}dx$. (*Javob*: $-\frac{1}{3}x^3-\frac{1}{3}ln|1-x^3|+C$.)

3.9. $\int \frac{x^2}{x^2+3}dx$. (*Javob*: $x-\sqrt{3}arctg\frac{x}{\sqrt{3}}+C$.)

3.10. $\int \frac{6x^3+x^2-2x+1}{2x-1}dx$. (*Javob*: $x^3 + x^2 + \frac{1}{2}ln|2x-1| + C$.)

3.11. $\int \frac{x^4}{x^2-3}dx$. (*Javob*: $\frac{x^3}{3} + 3x + \frac{9}{2\sqrt{3}}ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + C$.)

3.12. $\int \frac{x^3+5x}{x^2+1}dx$. (*Javob*: $\frac{x^2}{2} + 2\,ln|x^2+1| + C$.)

3.13. $\int \frac{x^2-5x+6}{x^2+4}dx$. (*Javob*: $x - \frac{5}{2}ln|x^2-4| + arctg\frac{x}{2} + C$.)

3.14. $\int \frac{x^3-1}{x+3}dx$. (*Javob*: $\frac{x^3}{3} - \frac{3}{2}x^2 + 9x - 28ln\,|x+3| + C$.)

3.15. $\int \frac{x^3}{x^2-1}dx$. (*Javob*: $\frac{1}{2}x^2 + \frac{1}{2}ln|x^2-1| + C$.)

3.16. $\int \frac{x^4+1}{x^2+1}dx$. (*Javob*: $\frac{1}{3}x^3 - x + 2arctgx + C$.)

3.17. $\int \frac{x^4-2x^2-1}{x^2+1}dx$. (*Javob*: $\frac{x^3}{3} - 3x + 2arctgx + C$.)

3.18. $\int \frac{x^4+2}{x^2-4}dx$. (*Javob*: $\frac{x^3}{3} + 4x + \frac{9}{2}ln\left|\frac{x-2}{x+2}\right| + C$.)

3.19. $\int \frac{x^3-3}{x+5}dx$. (*Javob*: $\frac{x^3}{3} - \frac{5}{2}x^2 + 25x - 128ln|x+5| + C$.)

3.20. $\int \frac{x^3+1}{x^2+1}dx$. (*Javob*: $\frac{1}{2}x^2 - \frac{1}{2}ln|x^2+1| + arctgx + C$.)

3.21. $\int \frac{1-2x^4}{x^2+1}dx$. (*Javob*: $-\frac{2}{3}x^3 + 2x - arctgx + C$.)

3.22. $\int \frac{2x^3-3}{x-2}dx$. (*Javob*: $\frac{2}{3}x^3 + 2x^2 + 8x + 13ln|x-2| + C$.)

3.23. $\int \frac{2x^2+5}{x+1}dx$. (*Javob*: $2x + 3arctgx + C$.)

3.24. $\int \frac{x^3+3x+1}{x^2+2}dx$. (*Javob*: $\frac{x^2}{2} + \frac{1}{2}ln|x^2+2| + \frac{1}{\sqrt{2}}arctg\frac{x}{\sqrt{2}} + C$.)

3.25. $\int \frac{x^2+x}{2-x}dx$. (*Javob*: $-\frac{x^2}{2} - 3x - 6ln|x-2| + C$.)

3.26. $\int \frac{2x^2+5}{x-7}dx$. (*Javob*: $x^2 + 14x + 103ln|x-7| + C$.)

3.27. $\int \frac{2x^3+3}{x-1}dx$. (*Javob*: $\frac{2}{3}x^3 + x^2 + 2x + 5ln|x-1| + C$.)

3.28. $\int \frac{1-x^4}{x^2+4}dx$. (*Javob*: $-\frac{x^3}{3} + 4x - \frac{15}{2}arctg\frac{x}{2} + C$.)

3.29. $\int \frac{x^2+4}{x-3}dx$. (*Javob*: $\frac{x^2}{2} + 3x + 13ln|x-3| + C$.)

3.30. $\int \frac{2x^2+3}{2x^2-1}dx$. (*Javob*: $x + \sqrt{2}ln\left|\frac{\sqrt{2}x-1}{\sqrt{2}x+1}\right| + C$.)

4.

4.1. $\int \sin^2(1-x)\,dx$. (*Javob*: $\frac{1}{2}x + \frac{1}{4}\sin2(1-x) + C$.)

4.2. $\int \sin^3(1-x)\,dx$. (*Javob*: $\cos(1-x) - \frac{1}{3}\cos^3(1-x) + C$.)

4.3. $\int(1-2\sin\frac{x}{5})^2dx$. (*Javob*: $3x+20\cos\frac{x}{5}-5\sin\frac{2x}{5}+C$.)

4.4. $\int\cos^3 5x\sin 5x\,dx$. (*Javob*: $-\frac{1}{20}\cos^4 5x+C$.)

4.5. $\int\cos^3(1-x)\,dx$. (*Javob*: $-\sin(1-x)+\frac{1}{3}\sin^3(1-x)+C$.)

4.6. $\int(3-\sin 2x)^2dx$. (*Javob*: $\frac{19}{2}x+3\cos 2x-\frac{1}{8}\sin 4x+C$.)

4.7. $\int\sin^2\frac{3x}{2}dx$. (*Javob*: $\frac{1}{2}x-\frac{1}{6}\sin 3x+C$.)

4.8. $\int(\cos x+3)^2dx$. (*Javob*: $\frac{19}{2}x+6\sin x+\frac{1}{4}\sin 2x+C$.)

4.9. $\int\cos^3(x+3)dx$. (*Javob*: $\sin(x+3)-\frac{1}{3}\sin^3(x+3)+C$.)

4.10. $\int\sin^3\frac{4x}{5}dx$. (*Javob*: $-\frac{5}{4}\cos\frac{4x}{5}+\frac{5}{12}\cos^3\frac{4x}{5}+C$.)

4.11. $\int(1-\cos x)^2dx$. (*Javob*: $\frac{3}{2}x-2\sin x+\frac{1}{4}\sin 2x+C$.)

4.12. $\int\sin^2(2x-1)\,dx$. (*Javob*: $\frac{x}{2}-\frac{1}{8}\sin(4x-2)+C$.)

4.13. $\int\sin^3 6x\,dx$. (*Javob*: $-\frac{1}{6}\cos 6x+\frac{1}{18}\cos^3 6x+C$.)

4.14. $\int\sin^2 0{,}5x\,dx$. (*Javob*: $\frac{x}{2}-\frac{1}{2}\sin x+C$.)

4.15. $\int\sin^2\left(\frac{x}{2}+1\right)dx$. (*Javob*: $\frac{x}{2}-\frac{1}{2}\sin(x+2)+C$.)

4.16. $\int\cos^2 2x dx$. (*Javob*: $\frac{x}{2}+\frac{1}{8}\sin 4x+C$.)

4.17. $\int(1+2\cos\frac{x}{2})^2dx$. (*Javob*: $3x+8\sin\frac{x}{2}+2\sin x+C$.)

4.18. $\int\cos^2 3x dx$. (*Javob*: $\frac{x}{2}+\frac{1}{12}\sin 6x+C$.)

4.19. $\int\sin^4 2x\,dx$. (*Javob*: $\frac{3}{8}x-\frac{1}{8}\sin 4x+\frac{1}{64}\sin 8x+C$.)

4.20. $\int\sin^2 3x\,dx$. (*Javob*: $\frac{x}{2}-\frac{1}{12}\sin 6x+C$.)

4.21. $\int(1-\cos 3x)^2dx$. (*Javob*: $\frac{3}{2}x-\frac{2}{3}\sin 3x+\frac{1}{12}\sin 6x+C$.)

4.22. $\int\cos^2\frac{2x}{5}dx$. (*Javob*: $\frac{x}{2}+\frac{5}{8}\sin\frac{4x}{5}+C$.)

4.23. $\int\sin^3 5x\,dx$. (*Javob*: $-\frac{1}{5}\cos 5x+\frac{1}{15}\cos^3 5x+C$.)

4.24. $\int\sin^4 x\,dx$. (*Javob*: $\frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$.)

4.25. $\int\cos^4 x dx$. (*Javob*: $\frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$.)

4.26. $\int\cos^3 4x dx$. (*Javob*: $\frac{1}{4}\sin 4x-\frac{1}{12}\sin^3 4x+C$.)

4.27. $\int\cos^2 7x dx$. (*Javob*: $\frac{x}{2}+\frac{1}{28}\sin 14x+C$.)

4.28. $\int(\sin x-5)^2dx$. (*Javob*: $\frac{51}{2}x-\frac{1}{4}\sin 2x+10\cos x+C$.)

4.29. $\int \sin^3 4x\,dx$. (*Javob*: $-\frac{1}{4}\cos 4x + \frac{1}{12}\cos^3 4x + C$.)

4.30. $\int \sin^2 \frac{3x}{4}\,dx$. (*Javob*: $\frac{x}{2} - \frac{1}{3}\sin\frac{3x}{2} + C$.)

5

5.1. $\int tg^2 x dx$. (*Javob*: $tgx - x + C$.)

5.2. $\int ctg^3(x-6)\,dx$. (*Javob*: $-\frac{1}{2}ctg^2(x-6) - ln|\sin(x-6)| + C$.)

5.3. $\int tg^4 3x dx$. (*Javob*: $\frac{1}{9}tg^3 3x - \frac{1}{3}tg3x + x + C$.)

5.4. $\int tg^2 7x dx$. (*Javob*: $\frac{1}{7}tg7x - x + C$.)

5.5. $\int tg^5 x dx$. (*Javob*: $\frac{1}{4}tg^4 x - \frac{1}{2}tg^2 x - ln|cosx| + C$.)

5.6. $\int x tg^2 x^2 dx$. (*Javob*: $\frac{1}{2}tgx^2 - \frac{1}{2}x^2 + C$.)

5.7. $\int ctg^3 x\,dx$. (*Javob*: $-\frac{1}{2}ctg^2 x - ln|\sin x| + C$.)

5.8. $\int tg^2 \frac{x}{2} dx$. (*Javob*: $2tg\frac{x}{2} - x + C$.)

5.9. $\int tg^3 \frac{x}{2} dx$. (*Javob*: $tg^2\frac{x}{2} + 2ln\left|cos\frac{x}{2}\right| + C$.)

5.10. $\int tg^2 4x dx$. (*Javob*: $\frac{1}{4}tg4x - x + C$.)

5.11. $\int ctg^3 x\,dx$. (*Javob*: $-\frac{1}{2}ctg^2 x - ln|\sin x| + C$.)

5.12. $\int ctg^2 5x\,dx$. (*Javob*: $-\frac{1}{5}ctg5x - x + C$.)

5.13. $\int tg^3 \frac{x}{3} dx$. (*Javob*: $\frac{3}{2}tg^2\frac{x}{3} + 3ln\left|cos\frac{x}{3}\right| + C$.)

5.14. $\int (1 - tg2x)^2\,dx$. (*Javob*: $ln|cos2x| + \frac{1}{2}tg2x + C$.)

5.15. $\int tg^5 2x dx$. (*Javob*: $\frac{1}{8}tg^4 2x - \frac{1}{4}tg^2 2x - \frac{1}{2}ln|cosx| + C$.)

5.16. $\int (2x + tg^2 7x)\,dx$. (*Javob*: $x^2 + \frac{1}{7}tg7x - x + C$.)

5.17. $\int tg^4 \frac{2x}{3} dx$. (*Javob*: $\frac{1}{2}tg^3\frac{2x}{3} - \frac{3}{2}tg\frac{2x}{3} + x + C$.)

5.18. $\int (tg2x + ctg2x)^2\,dx$. (*Javob*: $\frac{1}{2}tg2x - \frac{1}{2}ctg2x + C$.)

5.19. $\int (1 - ctgx)^2\,dx$. (*Javob*: $-2ln|sinx| - ctgx + C$.)

5.20. $\int ctg^3 3x\,dx$. (*Javob*: $-\frac{1}{6}ctg^2 3x - \frac{1}{3}ln|sin3x| + C$.)

5.21. $\int ctg^4 x dx$. (*Javob*: $-\frac{1}{3}ctg^3 x + ctgx + x + C$.)

5.22. $\int tg^2 \frac{x}{2} dx$. (*Javob*: $6tg\frac{x}{6} - x + C$.)

5.23. $\int tg^4(x-6) dx$. (*Javob*: $\frac{1}{3}tg^3(x-6) - tg(x-6) + x + C$.)

5.24. $\int tg^3 4x\,dx$. (*Javob*: $\frac{1}{8}tg^2 4x + \frac{1}{4}ln|cos4x| + C$.)

5.25. $\int tg^4 \frac{x}{4} dx$. (*Javob*: $\frac{4}{3}tg^3\frac{x}{4} - 4tg\frac{x}{4} + x + C$.)

5.26. $\int tg^4(x+5)dx$. (*Javob*: $\frac{1}{3}tg^3(x+5) - tg(x+5) + x + C$.)

5.27. $\int tg^3(x-3)\,dx$. (*Javob*: $\frac{1}{2}tg^2(x-3) + ln|cos(x-3)| + C$.)

5.28. $\int tg^2(5x+1)\,dx$. (*Javob*: $\frac{1}{5}tg(5x+1) - x + C$.)

5.29. $\int tg^2\frac{7x}{4}dx$. (*Javob*: $\frac{4}{7}tg\frac{7x}{4} - x + C$.)

5.30. $\int tg^5 4x dx$. (*Javob*: $\frac{1}{16}tg^4 4x - \frac{1}{8}tg^2 4x + \frac{1}{4}ln|1 + tg^2 4x| + C$.)

6

6.1. $\int sin3x\,cosx\,dx$. (*Javob*: $-\frac{1}{8}cos4x - \frac{1}{4}cos2x + C$.)

6.2. $\int sin^5 2x\,cos2x\,dx$. (*Javob*: $\frac{1}{12}sin^6 2x + C$.)

6.3. $\int sin^2 3x\,cos3x\,dx$. (*Javob*: $\frac{1}{9}sin^3 3x + C$.)

6.4. $\int cos^3 5x\,sin5x\,dx$. (*Javob*: $-\frac{1}{20}cos^4 5x + C$.)

6.5. $\int sin\frac{x}{2}\,cos\frac{x}{4}\,dx$. (*Javob*: $-\frac{2}{3}cos\frac{3x}{4} - 2cos\frac{x}{4} + C$.)

6.6. $\int cosx\,sin9x\,dx$. (*Javob*: $-\frac{1}{20}cos10x - \frac{1}{16}cos8x + C$.)

6.7. $\int sin^4 2x\,cos2x\,dx$. (*Javob*: $\frac{1}{10}sin^5 2x + C$.)

6.8. $\int sin\frac{x}{2}\,cos\frac{3x}{2}\,dx$. (*Javob*: $-\frac{1}{4}cos2x + \frac{1}{2}cosx + C$.)

6.9. $\int cos^5 x\,sinx\,dx$. (*Javob*: $-\frac{1}{6}cos^6 x + C$.)

6.10. $\int \cos 2x \cos 3x\,dx$. (*Javob*: $\frac{1}{10}sin\,5x + \frac{1}{2}sin\,x + C$.)

6.11. $\int sin\,5x\;sin\,7x\;dx$. (*Javob*: $\frac{1}{4}sin\,2x - \frac{1}{24}sin\,12x + C$.)

6.12. $\int sin\,4x \cos 2x\,dx$. (*Javob*: $-\frac{1}{12}cos6x - \frac{1}{4}cos2x + C$.)

6.13. $\int cos^3 4x\,sin\,4x\;dx$. (*Javob*: $-\frac{1}{16}cos^4 4x + C$.)

6.14. $\int cos^{-3} 2x\,sin\,2x\;dx$. (*Javob*: $\frac{1}{4}cos^{-2} 2x + C$.)

6.15. $\int \cos x\,sin\,9x\,dx$. (*Javob*: $-\frac{1}{20}\cos 10x - \frac{1}{16}\cos 8x + C$.)

6.16. $\int sin\,4x \cos 2x\,dx$. (*Javob*: $-\frac{1}{2}cos6x - \frac{1}{4}cos2x + C$.)

6.17. $\int sin\,3x \cos 2x\,dx$. (*Javob*: $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$.)

6.18. $\int sin^3 7x\,cos7x\,dx$. (*Javob*: $\frac{1}{28}sin^4 7x + C$.)

6.19. $\int \frac{\sin x}{\cos^3 x} dx$. (*Javob*: $\frac{1}{2}\cos^{-2} x + C$.)

6.20. $\int \frac{\cos 2x}{\sin^4 2x} dx$. (*Javob*: $-\frac{1}{6\sin^3 2x} + C$.)

6.21. $\int \cos 2x \cos 5x\, dx$. (*Javob*: $\frac{1}{6}\sin 3x + \frac{1}{14}\sin 7x + C$.)

6.22. $\int \sin^2 2x \cos x\, dx$. (*Javob*: $\frac{4}{3}\sin^3 x - \frac{4}{5}\sin^5 x + C$.)

6.23. $\int \frac{\cos x}{\sin^4 x} dx$. (*Javob*: $-\frac{1}{3\sin^3 x} + C$.)

6.24. $\int \sin 2x \sin 3x\, dx$. (*Javob*: $\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$.)

6.25. $\int \sin x \cos^3 x\, dx$. (*Javob*: $-\frac{\cos^4 x}{4} + C$.)

6.26. $\int \sin 5x \cos x\, dx$. (*Javob*: $-\frac{1}{12}\cos 6x - \frac{1}{8}\cos 4x + C$.)

6.27. $\int \sin x \cos 4x\, dx$. (*Javob*: $-\frac{1}{10}\cos 5x + \frac{1}{6}\cos 3x + C$.)

6.28. $\int \cos 3x \cos x\, dx$. (*Javob*: $\frac{1}{4}\sin 2x + \frac{1}{8}\sin 4x + C$.)

6.29. $\int \cos^4 2x \sin 2x\, dx$. (*Javob*: $-\frac{1}{10}\cos^5 2x + C$.)

6.30. $\int \cos 7x \cos 5x\, dx$. (*Javob*: $\frac{1}{4}\sin 2x + \frac{1}{24}\sin 12x + C$.)

7

7.1. $\int \frac{dx}{4x^2-5x+4}$. (*Javob*: $\frac{2}{\sqrt{39}} arctg \frac{8x-5}{\sqrt{39}} + C$.)

7.2. $\int \frac{dx}{x^2-4x+10}$. (*Javob*: $\frac{1}{\sqrt{6}} arctg \frac{x+2}{\sqrt{6}} + C$.)

7.3. $\int \frac{dx}{2x^2-7x+1}$. (*Javob*: $\frac{1}{\sqrt{41}} \ln\left|\frac{4x-7-\sqrt{41}}{4x-7+\sqrt{41}}\right| + C$.)

7.4. $\int \frac{dx}{2x^2+x-6}$. (*Javob*: $\frac{1}{7} \ln\left|\frac{2x-3}{2x+4}\right| + C$.)

7.5. $\int \frac{dx}{5x^2+2x+7}$. (*Javob*: $\frac{1}{\sqrt{34}} arctg \frac{5x+1}{\sqrt{34}} + C$.)

7.6. $\int \frac{dx}{2x^2-2x+1}$. (*Javob*: $arctg(2x-1) + C$.)

7.7. $\int \frac{dx}{2x^2-11x+2}$. (*Javob*: $\frac{1}{\sqrt{105}} \ln\left|\frac{4x-11-\sqrt{105}}{4x-11+\sqrt{105}}\right| + C$.)

7.8. $\int \frac{dx}{2x^2+x+2}$. (*Javob*: $\frac{2}{\sqrt{15}} arctg \frac{4x+1}{\sqrt{15}} + C$.)

7.9. $\int \frac{dx}{3x^2-12x+3}$. (*Javob*: $\frac{1}{6\sqrt{3}} \ln\left|\frac{x-2-\sqrt{3}}{x-2+\sqrt{3}}\right| + C$.)

7.10. $\int \frac{dx}{2x^2+3x}$. (*Javob*: $\frac{1}{3} \ln\left|\frac{x}{x+3/2}\right| + C$.)

7.11. $\int \frac{dx}{x^2-5x+6}$. (*Javob*: $\ln\left|\frac{x-3}{x-2}\right| + C$.)

7.12. $\int \frac{dx}{2x-3-4x^2}$. (*Javob:* $-\frac{1}{\sqrt{11}} arctg \frac{4x-1}{\sqrt{11}} + C.$)

7.13. $\int \frac{dx}{3x^2-8x-3}$. (*Javob:* $\frac{1}{10} ln \left|\frac{3x-9}{3x+1}\right| + C.$)

7.14. $\int \frac{dx}{8-2x-x^2}$. (*Javob:* $-\frac{1}{6} ln \left|\frac{x-2}{x+4}\right| + C.$)

7.15. $\int \frac{dx}{5x-x^2-6}$. (*Javob:* $- ln \left|\frac{x-3}{x-2}\right| + C.$)

7.16. $\int \frac{dx}{x^2+4x+25}$. (*Javob:* $\frac{1}{\sqrt{21}} arctg \frac{x+2}{\sqrt{21}} + C.$)

7.17. $\int \frac{dx}{2x^2-8x+30}$. (*Javob:* $\frac{1}{2\sqrt{11}} arctg \frac{x-2}{\sqrt{11}} + C.$)

7.18. $\int \frac{dx}{3x^2-9x+6}$. (*Javob:* $\frac{1}{3} ln \left|\frac{x-2}{x-1}\right| + C.$)

7.19. $\int \frac{dx}{2x^2-2x+5}$. (*Javob:* $\frac{1}{3} arctg \frac{2x-1}{3} + C.$)

7.20. $\int \frac{dx}{2x^2-3x-2}$. (*Javob:* $\frac{1}{5} ln \left|\frac{2x-4}{2x+1}\right| + C.$)

7.21. $\int \frac{dx}{2x^2-6x+1}$. (*Javob:* $\frac{1}{2\sqrt{7}} ln \left|\frac{2x-3-\sqrt{7}}{2x-3+\sqrt{7}}\right| + C.$)

7.22. $\int \frac{dx}{2x^2-3x+2}$. (*Javob:* $\frac{2}{\sqrt{7}} arctg \frac{4x-3}{\sqrt{7}} + C.$)

7.23. $\int \frac{dx}{x^2+7x+11}$. (*Javob:* $\frac{1}{\sqrt{5}} ln \left|\frac{2x+7-\sqrt{5}}{2x+7+\sqrt{5}}\right| + C.$)

7.24. $\int \frac{dx}{2x^2-3x+1}$. (*Javob:* $ln \left|\frac{2x-2}{2x-1}\right| + C.$)

7.25. $\int \frac{dx}{5x^2-10x+25}$. (*Javob:* $\frac{1}{10} arctg \frac{x-1}{2} + C.$)

7.26. $\int \frac{dx}{2x^2+6x+3}$. (*Javob:* $\frac{1}{2\sqrt{3}} ln \left|\frac{2x+3-\sqrt{3}}{2x+3+\sqrt{3}}\right| + C.$)

7.27. $\int \frac{dx}{x^2-6x+8}$. (*Javob:* $\frac{1}{2} ln \left|\frac{x-4}{x-2}\right| + C.$)

7.28. $\int \frac{dx}{1-2x-3x^2}$. (*Javob:* $-\frac{1}{4} ln \left|\frac{3x-1}{3x+3}\right| + C.$)

7.29. $\int \frac{dx}{2x^2+3x+6}$. (*Javob:* $\frac{2}{\sqrt{39}} arctg \frac{4x+3}{\sqrt{39}} + C.$)

7.30. $\int \frac{dx}{3x^2+5x+1}$. (*Javob:* $\frac{1}{\sqrt{13}} ln \left|\frac{6x+5-\sqrt{13}}{6x+5+\sqrt{13}}\right| + C.$)

8

8.1. $\int \frac{dx}{\sqrt{4+8x-x^2}}$. (*Javob:* $arcsin \frac{x-4}{\sqrt{20}} + C.$)

8.2. $\int \frac{dx}{\sqrt{3x^2-4x+1}}$. (*Javob:* $\frac{1}{\sqrt{3}} ln \left|x - \frac{2}{3} + \sqrt{x^2 - \frac{4}{3}x + \frac{1}{3}}\right| + C.$)

8.3. $\int \frac{dx}{\sqrt{2-3x-2x^2}}$. (*Javob:* $\frac{1}{\sqrt{2}} arcsin \frac{4x+3}{5} + C.$)

8.4. $\int \frac{dx}{\sqrt{x^2+6x+8}}$. (*Javob:* $ln\left|x+3+\sqrt{x^2+6x+8}\right|+C$.)

8.5. $\int \frac{dx}{\sqrt{2+8x-2x^2}}$. (*Javob:* $\frac{1}{\sqrt{2}}arcsin\frac{x-2}{\sqrt{5}}+C$.)

8.6. $\int \frac{dx}{\sqrt{3+2x-2x^2}}$. (*Javob:* $\frac{1}{\sqrt{2}}arcsin\frac{2x-1}{\sqrt{7}}+C$.)

8.7. $\int \frac{dx}{\sqrt{2-2x-3x^2}}$. (*Javob:* $\frac{1}{\sqrt{3}}arcsin\frac{3x+1}{\sqrt{7}}+C$.)

8.8. $\int \frac{dx}{\sqrt{1+x-x^2}}$. (*Javob:* $arcsin\frac{2x-1}{\sqrt{5}}+C$.)

8.9. $\int \frac{dx}{\sqrt{5x^2-10x+4}}$. (*Javob:* $\frac{1}{\sqrt{5}}ln\left|x-1+\sqrt{x^2-2x+\frac{4}{5}}\right|+C$.)

8.10. $\int \frac{dx}{\sqrt{2x+3-x^2}}$. (*Javob:* $arcsin\frac{x-1}{2}+C$.)

8.11. $\int \frac{dx}{\sqrt{4x^2-8x+3}}$. (*Javob:* $\frac{1}{2}ln\left|x-1+\sqrt{x^2-2x+\frac{3}{4}}\right|+C$.)

8.12. $\int \frac{dx}{\sqrt{1+2x-x^2}}$. (*Javob:* $arcsin\frac{x-1}{\sqrt{2}}+C$.)

8.13. $\int \frac{dx}{\sqrt{4x^2-x+4}}$. (*Javob:* $\frac{1}{2}ln\left|x-\frac{1}{8}+\sqrt{x^2-\frac{1}{4}x+1}\right|+C$.)

8.14: $\int \frac{dx}{\sqrt{2+4x-3x^2}}$. (*Javob:* $\frac{1}{\sqrt{3}}arcsin\frac{3x-2}{\sqrt{10}}+C$.)

8.15. $\int \frac{dx}{\sqrt{4x^2+2x+4}}$. (*Javob:* $\frac{1}{2}ln\left|x+\frac{1}{4}+\sqrt{x^2+\frac{1}{2}x+1}\right|+C$.)

8.16. $\int \frac{dx}{\sqrt{3x+2-2x^2}}$. (*Javob:* $\frac{1}{\sqrt{2}}arcsin\frac{4x-3}{5}+C$.)

8.17. $\int \frac{dx}{\sqrt{2x^2-8x+1}}$. (*Javob:* $\frac{1}{\sqrt{2}}ln\left|x-2+\sqrt{x^2-4x+\frac{1}{2}}\right|+C$.)

8.18. $\int \frac{dx}{\sqrt{x^2-5x+6}}$. (*Javob:* $ln\left|x-\frac{5}{2}+\sqrt{x^2-5x+6}\right|+C$.)

8.19. $\int \frac{dx}{\sqrt{3x-2x^2}}$. (*Javob:* $\frac{1}{\sqrt{2}}arcsin\frac{4x-3}{3}+C$.)

8.20. $\int \frac{dx}{\sqrt{2x^2-x+3}}$. (*Javob:* $\frac{1}{\sqrt{2}}ln\left|x-\frac{1}{4}+\sqrt{x^2-\frac{1}{2}x+\frac{3}{2}}\right|+C$.)

8.21. $\int \frac{dx}{\sqrt{2-x-2x^2}}$. (*Javob:* $\frac{1}{\sqrt{2}}arcsin\frac{4x+1}{\sqrt{17}}+C$.)

8.22. $\int \frac{dx}{\sqrt{x^2+3x-1}}$. (*Javob:* $ln\left|x+\frac{3}{2}+\sqrt{x^2+3x-1}\right|+C$.)

8.23. $\int \frac{dx}{\sqrt{5-7x-3x^2}}$. (*Javob:* $\frac{1}{\sqrt{3}}arcsin\frac{6x+7}{\sqrt{109}}+C$.)

8.24. $\int \frac{dx}{\sqrt{3x^2-x+5}}$. (*Javob:* $\frac{1}{\sqrt{3}}ln\left|x-\frac{1}{4}+\sqrt{x^2-\frac{1}{3}x+\frac{5}{3}}\right|+C$.)

8.25. $\int \frac{dx}{\sqrt{1-x-x^2}}$. (*Javob:* $arcsin\frac{2x+1}{\sqrt{5}} + C$.)

8.26. $\int \frac{dx}{\sqrt{1-2x-x^2}}$. (*Javob:* $arcsin\frac{x+1}{\sqrt{2}} + C$.)

8.27. $\int \frac{dx}{\sqrt{4-3x-x^2}}$. (*Javob:* $arcsin\frac{2x+3}{5} + C$.)

8.28. $\int \frac{dx}{\sqrt{x^2+5x+1}}$. (*Javob:* $ln\left|x+\frac{5}{2}+\sqrt{x^2+5x+1}\right| + C$.)

8.29. $\int \frac{dx}{\sqrt{3-x-x^2}}$. (*Javob:* $arcsin\frac{2x+1}{\sqrt{13}} + C$.)

8.30. $\int \frac{dx}{\sqrt{x^2+4x+1}}$. (*Javob:* $ln\left|x+2+\sqrt{x^2+4x+1}\right| + C$.)

9

9.1. $\int \frac{x+1}{2x^2+3x-4}dx$. (*Javob:* $\frac{1}{4}ln|2x^2+3x-4| + \frac{1}{4\sqrt{41}}ln\left|\frac{4x+3-\sqrt{41}}{4x+3+\sqrt{41}}\right| + C$.)

9.2. $\int \frac{x+6}{3x^2+x+1}dx$. (*Javob:* $\frac{1}{6}ln|3x^2+x+1| + \frac{35}{3\sqrt{11}}arctg\frac{6x+1}{\sqrt{11}} + C$.)

9.3. $\int \frac{2x-1}{3x^2-2x+6}dx$. (*Javob:* $\frac{1}{3}ln|3x^2-2x+6| - \frac{1}{3\sqrt{17}}arctg\frac{3x-1}{\sqrt{17}} + C$.)

9.4. $\int \frac{xdx}{2x^2+x+5}$. (*Javob:* $\frac{1}{4}ln|2x^2+x+5| - \frac{1}{2\sqrt{39}}arctg\frac{4x+1}{\sqrt{39}} + C$.)

9.5. $\int \frac{x+5}{x^2+x-2}dx$. (*Javob:* $\frac{1}{2}ln|x^2+x-2| + \frac{3}{2}ln\left|\frac{x-1}{x+2}\right| + C$.)

9.6. $\int \frac{3x-2}{5x^2-3x+2}dx$. (*Javob:* $\frac{3}{10}ln|5x^2-3x+2| - \frac{11}{5\sqrt{31}}arctg\frac{10x-3}{\sqrt{31}} + C$.)

9.7. $\int \frac{x+4}{2x^2-6x-8}dx$. (*Javob:* $\frac{1}{4}ln|2x^2-6x-8| + \frac{11}{20}ln\left|\frac{x-4}{x+1}\right| + C$.)

9.8. $\int \frac{x+4}{2x^2-7x+1}dx$. (*Javob:* $\frac{1}{4}ln|2x^2-7x+1| + \frac{23}{4\sqrt{41}}ln\left|\frac{4x-7-\sqrt{41}}{4x-7+\sqrt{41}}\right| + C$.)

9.9. $\int \frac{5x-2}{2x^2-5x+2}dx$. (*Javob:* $\frac{5}{4}ln|2x^2-5x+2| + \frac{17}{12}ln\left|\frac{2x-4}{2x-1}\right| + C$.)

9.10. $\int \frac{4x-1}{4x^2-4x+5}dx$. (*Javob:* $\frac{1}{2}ln|4x^2-4x+5| + \frac{1}{4}arctg\frac{2x-1}{2} + C$.)

9.11. $\int \frac{x+1}{2x^2+x+1}dx$. (*Javob:* $\frac{1}{4}ln|2x^2+x+1| + \frac{3}{2\sqrt{7}}arctg\frac{4x+1}{\sqrt{7}} + C$.)

9.12. $\int \frac{x+1}{3x^2-2x-3}dx$. Javob: $\frac{1}{6}ln|3x^2-2x-3| + \frac{2}{3\sqrt{10}}ln\left|\frac{3x-1-\sqrt{10}}{3x-1+\sqrt{10}}\right| + C$.)

9.13. $\int \frac{4x+8}{4x^2+6x-13}dx$. (*Javob:* $\frac{1}{2}ln|4x^2+6x-13| + \frac{5}{2\sqrt{61}}ln\left|\frac{4x+3-\sqrt{61}}{4x+3+\sqrt{61}}\right| + C$.)

9.14. $\int \frac{5x+1}{x^2-4x+1}dx$. (*Javob:* $\frac{5}{2}ln|x^2-4x+1| + \frac{11}{2\sqrt{3}}ln\left|\frac{x-2-\sqrt{3}}{x-2+\sqrt{3}}\right| + C$.)

9.15. $\int \frac{xdx}{2x^2+2x+5}$. (*Javob:* $\frac{1}{4}ln|2x^2+2x+5| - \frac{1}{6}arctg\frac{2x+1}{3} + C$.)

9.16. $\int \frac{x-3}{x^2-5x+4}dx$. (*Javob:* $\frac{1}{2}ln|x^2-5x+4| - \frac{1}{6}ln\left|\frac{x-4}{x-1}\right| + C$.)

9.17. $\int \frac{2x-1}{2x^2+8x-6}dx$. (*Javob:* $\frac{1}{2}ln|2x^2+8x-6| - \frac{5}{4\sqrt{7}}ln\left|\frac{x+2-\sqrt{7}}{x+2+\sqrt{7}}\right| + C$.)

9.18. $\int \frac{2-x}{4x^2+16x-12}dx$. (*Javob:* $-\frac{1}{8}ln|4x^2+16x-12| + \frac{1}{2\sqrt{7}}ln\left|\frac{x+2-\sqrt{7}}{x+2+\sqrt{7}}\right| + C$.)

9.19. $\int \frac{2x-1}{3x^2-6x-9}dx$. (*Javob:* $\frac{1}{3}ln|3x^2-6x-9| + \frac{1}{12}ln\left|\frac{x-3}{x+1}\right| + C$.)

9.20. $\int \frac{2x-1}{3+x-2x^2}dx$. (*Javob:* $-\frac{1}{2}ln|2x^2-x-3|+\frac{1}{10}ln\left|\frac{2x-3}{2x+2}\right|+C$.)

9.21. $\int \frac{x-4}{3x^2+x-1}dx$. (*Javob:* $\frac{1}{6}ln|3x^2+x-1|-\frac{25}{6\sqrt{13}}ln\left|\frac{6x+1-\sqrt{13}}{6x+1+\sqrt{13}}\right|+C$.)

9.22. $\int \frac{3x+1}{x^2-4x-2}dx$. (*Javob:* $\frac{3}{2}ln|x^2-4x-2|+\frac{7}{2\sqrt{6}}ln\left|\frac{x-2-\sqrt{6}}{x-2+\sqrt{6}}\right|+C$.)

9.23. $\int \frac{x-5}{2x^2+x-4}dx$. (*Javob:* $\frac{1}{4}ln|2x^2+x-4|+\frac{21}{4\sqrt{33}}ln\left|\frac{4x+1-\sqrt{33}}{4x+1+\sqrt{33}}\right|+C$.)

9.24. $\int \frac{2x+3}{3x^2+2x-7}dx$. (*Javob:* $\frac{1}{3}ln|3x^2+2x-7|+\frac{7}{6\sqrt{22}}ln\left|\frac{3x+1-\sqrt{22}}{3x+1+\sqrt{22}}\right|+C$.)

9.25. $\int \frac{x-3}{4x^2+2x-3}dx$. (*Javob:* $\frac{1}{8}ln|4x^2+2x-3|-\frac{\sqrt{13}}{8}ln\left|\frac{4x+1-\sqrt{13}}{4x+1+\sqrt{13}}\right|+C$.)

9.26. $\int \frac{x+2}{3x^2-x+5}dx$. (*Javob:* $\frac{1}{6}ln|3x^2-x+5|+\frac{13}{3\sqrt{59}}arctg\frac{6x-1}{\sqrt{59}}+C$.)

9.27. $\int \frac{3x-2}{x^2+5x-1}dx$. (*Javob:* $\frac{3}{2}ln|x^2+5x-1|-\frac{19}{2\sqrt{29}}ln\left|\frac{2x+5-\sqrt{29}}{2x+5+\sqrt{29}}\right|+C$.)

9.28. $\int \frac{x-7}{4x^2+3x-1}dx$. (*Javob:* $\frac{1}{8}ln|4x^2+3x-1|-\frac{59}{40}ln\left|\frac{4x-1}{4x+4}\right|+C$.)

9.29. $\int \frac{2x+1}{5x^2+2x+10}dx$. (*Javob:* $\frac{1}{5}ln|5x^2+2x-10|+\frac{3}{5\sqrt{49}}arctg\frac{5x+1}{\sqrt{49}}+C$.)

9.30. $\int \frac{x-4}{5x^2-x+7}dx$. (*Javob:* $\frac{1}{10}ln|5x^2-x+7|-\frac{39}{5\sqrt{139}}arctg\frac{10x-1}{\sqrt{139}}+C$.)

10

10.1. $\int \frac{2x-13}{\sqrt{3x^2-3x-16}}dx$ (*Javob:* $\frac{2}{3}\sqrt{3x^2-3x-16}-$
$-4\sqrt{3}\,ln\left|x-\frac{1}{2}+\sqrt{x^2-x-\frac{16}{3}}\right|+C$.)

10.2. $\int \frac{x-3}{\sqrt{2x^2-4x-1}}dx$. (*Javob:* $\frac{1}{2}\sqrt{2x^2-4x-1}-$
$-\sqrt{2}\,ln\left|x-1+\sqrt{x^2-2x-\frac{1}{2}}\right|+C$.)

10.3. $\int \frac{x-1}{\sqrt{3x^2-x+5}}dx$ (*Javob:* $\frac{1}{3}\sqrt{3x^2-x+5}-$
$-\frac{5}{6\sqrt{3}}ln\left|x-\frac{1}{6}+\sqrt{x^2-\frac{x}{3}+\frac{5}{3}}\right|+C$.)

10.4. $\int \frac{2x+1}{\sqrt{1+x-3x^2}}dx$. (*Javob:* $\frac{2}{3}\sqrt{1+x-3x^2}+\frac{4}{3\sqrt{3}}arcsin\frac{6x-1}{\sqrt{3}}+C$.)

10.5. $\int \frac{2x+5}{\sqrt{4x^2+8x+9}}dx$. (*Javob:* $\frac{1}{2}\sqrt{4x^2+8x+9}+$
$+\frac{3}{2}ln\left|x+1+\sqrt{x^2+2x+\frac{9}{4}}\right|+C$.)

10.6. $\int \frac{2x-10}{\sqrt{1+x-x^2}}dx$. (*Javob:* $-2\sqrt{1+x-x^2}-9arcsin\frac{2x-1}{\sqrt{5}}+C$.)

10.7. $\int \frac{2x-8}{\sqrt{1-x+x^2}}dx$. (*Javob:* $2\sqrt{1-x+x^2}-$
$-7\,ln\left|x-\frac{1}{2}+\sqrt{x^2-x+1}\right|+C$.)

10.8. $\int \frac{3x+4}{\sqrt{x^2+6x+13}}dx$. (*Javob:* $3\sqrt{x^2+6x+13}-$
$-5\,ln\left|x+3+\sqrt{x^2+6x+13}\right|+C$.)

10.9. $\int \frac{3x-1}{\sqrt{2x^2-5x+1}}dx$. (*Javob:* $\frac{3}{2}\sqrt{2x^2-5x+1}+$

$\frac{11}{4\sqrt{2}}\ln\left|x-\frac{5}{4}+\sqrt{x^2-\frac{5}{2}x+\frac{1}{2}}\right|+C.)$

10.10. $\int\frac{5x+2}{\sqrt{x^2+3x-4}}dx.$ (*Javob:* $5\sqrt{x^2+3x-4}-$
$-\frac{11}{2}\ln\left|x+\frac{3}{2}+\sqrt{x^2+3x-4}\right|+C.)$

10.11. $\int\frac{x-4}{\sqrt{2x^2-x+7}}dx.$ (*Javob:* $\frac{1}{2}\sqrt{2x^2-x+7}-$
$\frac{15}{4\sqrt{2}}\ln\left|x-\frac{1}{4}+\sqrt{x^2-\frac{x}{2}-\frac{7}{2}}\right|+C.)$

10.12. $\int\frac{2x-1}{\sqrt{x^2-3x+4}}dx.$ (*Javob:* $2\sqrt{x^2-3x+4}+$
$+2\ln\left|x-\frac{3}{2}+\sqrt{x^2-3x+4}\right|+C.)$

10.13. $\int\frac{4x+1}{\sqrt{2+x-x^2}}dx.$ (*Javob:* $-4\sqrt{2+x-x^2}+3\arcsin\frac{2x-1}{3}+C.)$

10.14. $\int\frac{5x-3}{\sqrt{2x^2+4x-5}}dx$ (*Javob:* $\frac{5}{2}\sqrt{2x^2+4x-5}-$
$-4\sqrt{2}\ln\left|x+1+\sqrt{x^2+2x-\frac{5}{2}}\right|++C.)$

10.15. $\int\frac{3x+2}{\sqrt{4+2x-x^2}}dx.$ (*Javob:* $-3\sqrt{4+2x-x^2}+5\arcsin\frac{x-1}{\sqrt{5}}+C.)$

10.16. $\int\frac{x-7}{\sqrt{3x^2-2x+1}}dx$ (*Javob:* $\frac{1}{3}\sqrt{3x^2-2x+1}-$
$-\frac{20}{3\sqrt{3}}\ln\left|x-\frac{1}{3}+\sqrt{x^2-\frac{2}{3}x+\frac{1}{3}}\right|+C.)$

10.17. $\int\frac{x+5}{\sqrt{3-6x-x^2}}dx.$ (*Javob:* $-\sqrt{3-6x-x^2}+2\arcsin\frac{x+3}{\sqrt{12}}+C.)$

10.18. $\int\frac{2x+4}{\sqrt{3x^2+x-5}}dx.$ (*Javob:* $\frac{2}{3}\sqrt{3x^2+x-5}+$
$+\frac{11}{3\sqrt{3}}\ln\left|x+\frac{1}{6}+\sqrt{x^2+\frac{x}{3}-\frac{5}{3}}\right|+C.)$

10.19. $\int\frac{7x-2}{\sqrt{x^2-5x+1}}dx.$ (*Javob:* $7\sqrt{x^2-5x+1}+$
$+\frac{31}{2}\ln\left|x-\frac{5}{2}+\sqrt{x^2-5x+1}\right|+C.)$

10.20. $\int\frac{x-8}{\sqrt{4x^2+x-5}}dx.$ (*Javob:* $\frac{1}{4}\sqrt{4x^2+x-5}-$
$-\frac{65}{16}\ln\left|x+\frac{1}{8}+\sqrt{x^2+\frac{1}{4}x-\frac{5}{4}}\right|+C.)$

10.21. $\int\frac{3x+4}{\sqrt{2+3x-x^2}}dx.$ (*Javob:* $-3\sqrt{2+3x-x^2}+\frac{17}{2}\arcsin\frac{2x-3}{\sqrt{17}}+C.)$

10.22. $\int\frac{x-6}{\sqrt{3-2x-x^2}}dx.$ (*Javob:* $-\sqrt{3-2x-x^2}-7\arcsin\frac{x+1}{2}+C.)$

10.23. $\int\frac{2x+3}{\sqrt{2x^2-x+6}}dx.$ (*Javob:* $\sqrt{2x^2-x+6}+$
$+\frac{7}{2\sqrt{2}}\ln\left|x-\frac{1}{4}+\sqrt{x^2-\frac{x}{2}+3}\right|+C.)$

10.24. $\int\frac{x-9}{\sqrt{4+2x-x^2}}dx.$ (*Javob:* $-\sqrt{4+2x-x^2}-8\arcsin\frac{x-1}{\sqrt{5}}+C.)$

10.25. $\int\frac{2x+7}{\sqrt{x^2+5x-4}}dx.$ (*Javob:* $2\sqrt{x^2+5x-4}+$
$+2\ln\left|x+\frac{5}{2}+\sqrt{x^2+5x-4}\right|+C.)$

10.26. $\int \frac{3x-4}{\sqrt{2x^2-6x+1}}dx$. (*Javob:* $\frac{3}{2}\sqrt{2x^2-6x+1}+$
$+\frac{1}{2\sqrt{2}}ln\left|x-\frac{3}{2}+\sqrt{2x^2-6x+1}\right|+C.$)

10.27. $\int \frac{2x+5}{\sqrt{3x^2+9x-4}}dx$. (*Javob:* $\frac{2}{3}\sqrt{3x^2+9x-4}+$
$+\frac{2}{\sqrt{3}}ln\left|x+\frac{3}{2}+\sqrt{x^2+3x-\frac{4}{3}}\right|+C.$)

10.28. $\int \frac{4x+3}{\sqrt{2x^2-x+5}}dx$. (*Javob:* $2\sqrt{2x^2-x+5}+$
$+2\sqrt{2}\,ln\left|x-\frac{1}{4}+\sqrt{x^2-\frac{x}{2}+\frac{5}{2}}\right|+C.$)

10.29. $\int \frac{3x-7}{\sqrt{x^2-5x+1}}dx$. (*Javob:* $3\sqrt{x^2-5x+1}+$
$+\frac{1}{2}ln\left|x-\frac{5}{2}+\sqrt{x^2-5x+1}\right|+C.$)

10.30. $\int \frac{7x-1}{\sqrt{2-3x-x^2}}dx$. (*Javob:* $-7\sqrt{2-3x-x^2}-\frac{23}{2}arcsin\frac{2x+3}{\sqrt{17}}+C.$)

Namunaviy variant yechimi

Aniqmas integrallarni hisoblang.

1. $\int \frac{3-7x}{4x^2+5}dx$.

$$\blacktriangleright \int \frac{3-7x}{4x^2+5}dx = 3\int \frac{dx}{(2x)^2+\left(\sqrt{5}\right)^2}-7\int \frac{xdx}{4x^2+5}=$$

$=\frac{3}{2}\int \frac{d(2x)}{(2x)^2+\left(\sqrt{5}\right)^2}-\frac{7}{8}\int \frac{8xdx}{4x^2+5}=\frac{3}{2}\cdot\frac{1}{\sqrt{5}}arctg\frac{2x}{\sqrt{5}}-\frac{7}{8}ln|4x^2+5|+C.$ ◄

2. $\int \frac{dx}{e^{3x}(2-e^{-3x})}$.

► $u=2-e^{-3x}$ almashtirishdan foydalansak, u holda $du=3e^{-3x}dx$ va

$\int \frac{dx}{e^{3x}(2-e^{-3x})}=\frac{1}{3}\int \frac{3e^{-3x}dx}{2-e^{-3x}}=\frac{1}{3}ln|2-e^{-3x}|+C.$◄

3. $\int \frac{3x^5-4x}{x^2+1}dx$.

► Integral ostida turgan funksiyaning suratini maxrajiga bo‘lib noto‘g‘ri kasrning butun qismini ajratib olamiz. Natijada algebraik yig‘indini integrallashga kelamiz:

$\int \frac{3x^5-4x}{x^2+1}dx=\left(\int 3x^3-3x-\frac{x}{x^2+1}\right)dx=\frac{3}{4}x^4-\frac{3}{2}x^2-\frac{1}{2}ln|x^2+1|+C.$◄

4. $\int cos^3(7x+2)dx$.

► $cos^2(7x+2)=1-sin^2(7x+2)$ trigonometrik ayniyatdan foydalansak,

$\int cos^3(7x+2)dx=\int cos^2(7x+2)cos(7x+2)dx=$
$=\int\left(1-sin^2(7x+2)\right)cos(7x+2)dx=\int cos(7x+2)dx-$
$-\int sin^2(7x+2)\,cos(7x+2)dx=\frac{1}{7}sin(7x+2)-$
$-\frac{1}{7}\int sin^2(7x+2)\,d\left(sin(7x+2)\right)=\frac{1}{7}sin(7x+2)-\frac{1}{21}sin^3(7x+2)+C.$
◄

5. $\int ctg^4 5x dx$.

► $ctg^2 5x = \frac{1}{sin^2 5x} - 1$ ekanligidan foydalanib integralni almashtiramiz

$$\int ctg^4 5x dx = \int ctg^2 5x \left(\frac{1}{sin^2 5x} - 1\right) dx =$$

$$= \int ctg^2 5x \frac{1}{sin^2 5x} dx - \int ctg^2 5x dx = -\frac{1}{5}\int ctg^2 5x \left(-\frac{5}{sin^2 5x}\right) dx -$$

$$- \int \left(\frac{1}{sin^2 5x} - 1\right) dx = -\frac{1}{15} ctg^3 5x + \frac{1}{5} ctg 5x + x + C. ◄$$

6. $\int sin\frac{7}{2}x sin\frac{3}{2}x dx$

► $\int sin\frac{7}{2}x sin\frac{3}{2}x dx = \frac{1}{2}\int (cos2x - cos5x) dx = \frac{1}{4}sin2x - \frac{1}{10}sin5x + C.$◄

7. $\int \frac{dx}{6x^2-3x+2}$.

► Integral ostidagi funksiya maxrajida to'la kvadrat ajratamiz, u holda

$$\int \frac{dx}{6x^2-3x+2} = \frac{1}{6}\int \frac{dx}{x^2-\frac{1}{2}x+1/3} = \frac{1}{6}\int \frac{dx}{\left(x-\frac{1}{4}\right)^2+\left(\frac{\sqrt{13}}{4\sqrt{3}}\right)^2} = \frac{4\sqrt{3}}{6\sqrt{13}} arctg \frac{x-1/4}{\sqrt{13}/(4\sqrt{3})} + C =$$

$$= \frac{2\sqrt{3}}{3\sqrt{13}} arctg \frac{(4x-1)\sqrt{3}}{\sqrt{13}} + C. ◄$$

8. $\int \frac{3x-6}{2-5x-x^2} dx$

► Integral ostidagi funksiyaning suratida maxrajdagi funksiya hosilasiga teng qo'shiluvchi ajratib integralni ikkiga ajratamiz

$$\int \frac{3x-6}{2-5x-x^2} dx = -\frac{3}{2}\int \frac{-2x+4-5+5}{2-5x-x^2} dx = -\frac{3}{2}\int \frac{-2x-5}{2-5x-x^2} dx - \frac{3}{2}\cdot 9\int \frac{dx}{2-5x-x^2} =$$

$$= -\frac{3}{2} ln|2-5x-x^2| + \frac{27}{2}\int \frac{dx}{(x-5/2)^2-2-25/4} =$$

$$= -\frac{3}{2} ln|2-5x-x^2| + \frac{27}{2}\int \frac{dx}{(x-5/2)^2-(\sqrt{33}/2)^2} =$$

$$= -\frac{3}{2} ln|2-5x-x^2| + \frac{27}{2\sqrt{33}} ln\left|\frac{x-5/2-\sqrt{33}/2}{x-5/2+\sqrt{33}/2}\right| + C =$$

$$= -\frac{3}{2} ln|2-5x-x^2| + \frac{9\sqrt{3}}{2\sqrt{11}} ln\left|\frac{2x-5-\sqrt{33}}{2x-5+\sqrt{33}}\right| + C.◄$$

9. $\int \frac{dx}{\sqrt{5x^2+2x-7}}$.

► Integral ostidagi funksiya maxrajida to'la kvadrat ajratamiz, u holda

$$\int \frac{dx}{\sqrt{5x^2+2x-7}} = \frac{1}{\sqrt{5}}\int \frac{dx}{\sqrt{x^2+\frac{2}{5}x-7/5}} = \frac{1}{\sqrt{5}}\int \frac{d(x+1/5)}{\sqrt{(x+1/5)^2-7/5-1/25}} =$$

$$= \frac{1}{\sqrt{5}} ln\left|x + 1/5 + \sqrt{x^2 + \frac{2}{5}x - 7/5}\right| + C .◄$$

10. $\int \frac{2x-7}{\sqrt{1-4x-3x^2}} dx$.

► Integral ostidagi funksiyaning suratida maxrajdagi ildiz tagida turgan funksiya hosilasiga teng qo'shiluvchi ajratib berilgan integralni ikkita integral yig‘indisi ko‘rinishida ifodalaymiz:

$$\int \frac{2x-7}{\sqrt{1-4x-3x^2}}dx = -\frac{1}{3}\int \frac{-6x+21-4+4}{\sqrt{1-4x-3x^2}}dx =$$

$$= -\frac{1}{3}\int \frac{-6x-4}{\sqrt{1-4x-3x^2}}dx - \frac{25}{3\sqrt{3}}\int \frac{dx}{\sqrt{\frac{1}{3}-\frac{4}{3}x-x^2}} =$$

$$= -\frac{2}{3}\sqrt{1-4x-3x^2} - \frac{25}{3\sqrt{3}}\int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{3}\right)^2-\left(x+\frac{2}{3}\right)^2}} =$$

$$= -\frac{2}{3}\sqrt{1-4x-3x^2} - \frac{25}{3\sqrt{3}}arcsin\frac{x+2/3}{\sqrt{7}/3} + C =$$

$$= -\frac{2}{3}\sqrt{1-4x-3x^2} - \frac{25}{3\sqrt{3}}arcsin\frac{3x+2}{\sqrt{7}} + C. \blacktriangleleft$$

IUT-8.3

Aniqmas integrallarni hisoblang.

1

1.1. $\int \frac{\sqrt{1-x^2}}{x}dx$. $\left(Javob: \frac{1}{2}ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + \sqrt{1-x^2} + C.\right)$

1.2. $\int \frac{\sqrt{x^2-1}}{x}dx$. $\left(Javob: \sqrt{x^2-1} - arccos\frac{1}{x} + C.\right)$

1.3. $\int \frac{\sqrt{x^2+4}}{x}dx$. $\left(Javob: \sqrt{4+x^2} + ln\left|\frac{2-\sqrt{4+x^2}}{2+\sqrt{4+x^2}}\right| + C.\right)$

1.4. $\int \frac{\sqrt{1-x^2}}{x^4}dx$. $\left(Javob: C - \frac{1}{3}\frac{\sqrt{(1-x^2)^3}}{x^3}.\right)$

1.5. $\int \sqrt{4-x^2}\,dx$. $\left(Javob: 2arcsin\frac{x}{2} + \frac{x}{2}\sqrt{4-x^2} + C.\right)$

1.6. $\int \frac{\sqrt{x^2+9}}{x}dx$. $\left(Javob: \sqrt{x^2+9} + \frac{3}{2}ln\left|\frac{3-\sqrt{x^2+9}}{3+\sqrt{x^2+9}}\right| + C.\right)$

1.7. $\int \frac{\sqrt{x^2+4}}{x^2}dx$. $\left(Javob: ln\left|\frac{x+\sqrt{4+x^2}}{x-\sqrt{4+x^2}}\right| - \frac{\sqrt{4-x^2}}{x} + C.\right)$

1.8. $\int \frac{\sqrt{4-x^2}}{x^4}dx$. $\left(Javob: C - \frac{1}{12}\frac{\sqrt{(4-x^2)^3}}{x^3}.\right)$

1.9. $\int \frac{dx}{\sqrt{(1+x^2)^3}}$. $\left(Javob: \frac{x}{\sqrt{1+x^2}} + C.\right)$

1.10. $\int \frac{\sqrt{x^2+4}}{x^4}dx$. $\left(Javob: C - \frac{1}{12}\frac{\sqrt{(4+x^2)^3}}{x^3}.\right)$

1.11. $\int \frac{\sqrt{(4-x^2)^3}}{x^6}dx$. $\left(Javob: C - \frac{1}{20}\frac{\sqrt{(4-x^2)^5}}{x^5}.\right)$

1.12. $\int \frac{dx}{\sqrt{(1+x^2)^5}}$ $\left(Javob: \frac{x}{\sqrt{1+x^2}} - \frac{1}{3}\frac{x^3}{\sqrt{(1+x^2)^3}} + C\,.\right)$

1.13. $\int \frac{\sqrt{x^2-9}}{x}dx$. $\left(Javob: \sqrt{x^2-9} - 3arccos\frac{3}{x} + C\,.\right)$

1.14. $\int \frac{dx}{\sqrt{(x^2-1)^3}}$. $\left(Javob: C - \frac{x}{\sqrt{x^2-1}}.\right)$

1.15. $\int x^3\sqrt{9-x^2}dx$. $\left(Javob: \frac{1}{5}\sqrt{(9-x^2)^5} - 3\sqrt{(9-x^2)^3} + C.\right)$

1.16. $\int \frac{dx}{x^2\sqrt{(x^2-1)^3}}$. $\left(Javob: C - \frac{x}{\sqrt{x^2-1}} - \frac{\sqrt{x^2-1}}{x}.\right)$

1.17. $\int \frac{dx}{x^2\sqrt{x^2-1}}$. $\left(Javob: \frac{\sqrt{x^2-1}}{x} + C\,.\right)$

1.18. $\int \frac{\sqrt{x^2-9}}{x^2}dx$. $\left(Javob: \frac{1}{2}ln\left|\frac{\sqrt{x^2-9}+x}{\sqrt{x^2-9}-x}\right| - \frac{\sqrt{x^2-9}}{x} + C\,.\right)$

1.19. $\int \frac{dx}{x^3\sqrt{x^2-1}}$. $\left(Javob: \frac{1}{2}arccos\frac{1}{x} + \frac{\sqrt{x^2-1}}{2x^2} + C\,.\right)$

1.20. $\int \frac{\sqrt{9-x^2}}{x^4}dx$. $\left(Javob: C - \frac{1}{27}\frac{\sqrt{(9-x^2)^3}}{x^3}\,.\right)$

1.21. $\int \frac{dx}{x^2\sqrt{x^2+9}}$. $\left(Javob: C - \frac{\sqrt{x^2+9}}{9x}\,.\right)$

1.22. $\int x^2\sqrt{1-x^2}dx$. $\left(Javob: \frac{1}{8}arcsinx - \frac{1}{8}x\sqrt{1-x^2}(1-2x^2) + C\,.\right)$

1.23. $\int x^3\sqrt{1-x^2}dx$. $\left(Javob: \frac{1}{5}\sqrt{(1-x^2)^5} - \frac{1}{3}\sqrt{(1-x^2)^3} + C\,.\right)$

1.24. $\int \frac{\sqrt{(4-x^2)^3}dx}{x^4}$. $\left(Javob: arcsin\frac{x}{2} + \frac{\sqrt{4-x^2}}{x} - \frac{1}{3}\frac{\sqrt{(4-x^2)^3}}{x^3} + C\,.\right)$

1.25. $\int \frac{dx}{\sqrt{(4+x^2)^3}}$. $\left(Javob: \frac{x}{4\sqrt{4+x^2}} + C\,.\right)$

1.26. $\int \frac{\sqrt{x^2+9}}{x^4}dx$. $\left(Javob: C - \frac{1}{27}\frac{\sqrt{(9+x^2)^3}}{x^3}\,.\right)$

1.27. $\int \frac{dx}{\sqrt{(9+x^2)^3}}$. $\left(Javob: \frac{x}{9\sqrt{9+x^2}} + C\,.\right)$

1.28. $\int \frac{x^2dx}{\sqrt{9-x^2}}$. $\left(Javob: \frac{9}{2}arcsinx\frac{x}{3} - \frac{1}{2}x\sqrt{9-x^2} + C\,.\right)$

1.29. $\int \frac{\sqrt{16-x^2}}{x^4}dx$. $\left(Javob: C - \frac{1}{48}\frac{x^3}{\sqrt{16-x^2}}\,.\right)$

1.30. $\int \frac{\sqrt{16-x^2}}{x^2}dx$. $\left(Javob: C - arcsin\frac{x}{4} - \frac{x}{\sqrt{16-x^2}} + C\,.\right)$

2

2.1. $\int \frac{dx}{(x+1)\sqrt{1+x^2}}$. $\left(Javob: C - \frac{1}{\sqrt{2}} ln\left|\frac{1}{x+1} - \frac{1}{2}\frac{\sqrt{1+x^2}}{\sqrt{2}(x+1)}\right|.\right)$

2.2. $\int \frac{dx}{(x+1)\sqrt{x^2-1}}$. $\left(Javob: \sqrt{\frac{x-1}{x+1}} + C.\right)$

2.3. $\int \frac{dx}{(x+1)\sqrt{x^2-1}}$. $\left(Javob: \sqrt{\frac{x-1}{x+1}} + C.\right)$

2.4. $\int \frac{dx}{x\sqrt{1-x^2}}$. $\left(Javob: C - ln\left|\frac{1+\sqrt{1-x^2}}{x}\right|.\right)$

2.5. $\int \frac{dx}{x\sqrt{1+x^2}}$. $\left(Javob: C - ln\left|\frac{1+\sqrt{1+x^2}}{x}\right|.\right)$

2.6. $\int \frac{dx}{x\sqrt{x^2-1}}$. $\left(Javob: C - arcsin\frac{1}{x}.\right)$

2.7. $\int \frac{dx}{x\sqrt{x^2+x+1}}$. $\left(Javob: C - ln\left|\frac{1}{x} + \frac{1}{2} + \frac{\sqrt{x^2+x+1}}{x}\right|.\right)$

2.8. $\int \frac{dx}{x\sqrt{x^2-x+1}}$. $\left(Javob: C - ln\left|\frac{1+\sqrt{x^2-x+1}}{x} - \frac{1}{2}\right|.\right)$

2.9. $\int \frac{dx}{x\sqrt{x^2+x-1}}$. $\left(Javob: C - arcsin\frac{2-x}{\sqrt{5}x}.\right)$

2.10. $\int \frac{dx}{x\sqrt{x^2-x-1}}$. $\left(Javob: C - arcsin\frac{x+2}{\sqrt{5}x}.\right)$

2.11. $\int \frac{dx}{x\sqrt{1+x-x^2}}$. $\left(Javob: C - ln\left|\frac{1}{x} + \frac{1}{2} + \frac{\sqrt{1+x-x^2}}{x}\right|.\right)$

2.12. $\int \frac{dx}{x\sqrt{x^2+x-2}}$. $\left(Javob: C - \frac{1}{\sqrt{2}} arcsin\frac{4-x}{3x}.\right)$

2.13. $\int \frac{dx}{(x+1)\sqrt{x^2-x+1}}$. $\left(Javob: C - \frac{1}{\sqrt{3}} ln\left|\frac{1}{x+1} - \frac{1}{2} + \frac{\sqrt{x^2-x+1}}{\sqrt{3}(x+1)}\right|.\right)$

2.14. $\int \frac{dx}{(x+1)\sqrt{x^2-x-1}}$. $\left(Javob: C - \frac{1}{\sqrt{3}} ln\left|\frac{1}{x+1} - \frac{3}{2} + \frac{\sqrt{x^2-x-1}}{x+1}\right|.\right)$

2.15. $\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$. $\left(Javob: C - ln\left|\frac{1}{x+1} - \frac{1}{2} + \frac{\sqrt{x^2+x+1}}{x+1}\right|.\right)$

2.16. $\int \frac{dx}{(x+1)\sqrt{x^2+x-1}}$. $\left(Javob: C - arcsin\frac{x+3}{\sqrt{5}(x+1)}.\right)$

2.17. $\int \frac{dx}{(x+1)\sqrt{1+x-x^2}}$. $\left(Javob: arcsin\frac{3x+1}{\sqrt{5}(x+1)} + C.\right)$

2.18. $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$. $\left(Javob: C - \frac{1}{\sqrt{3}} ln\left|\frac{1}{x-1} + \frac{1}{2} + \frac{\sqrt{x^2+x+1}}{\sqrt{3}(x-1)}\right|.\right)$

2.19. $\int \frac{dx}{(x-1)\sqrt{x^2-x+1}}$. $\left(Javob: C - ln\left|\frac{1}{x-1} + \frac{1}{2} + \frac{\sqrt{x^2-x+1}}{x-1}\right|.\right)$

2.20. $\int \frac{dx}{(x-1)\sqrt{x^2+x-1}}$. $\left(Javob: C - ln\left|\frac{1}{x-1} + \frac{3}{2} + \frac{\sqrt{x^2+x-1}}{x-1}\right|.\right)$

2.21. $\int \frac{dx}{(x-1)\sqrt{x^2-x-1}}$. $\left(Javob: C - arcsin\frac{3-x}{\sqrt{5}(x-1)}.\right)$

2.22. $\int \frac{dx}{(x-1)\sqrt{1+x-x^2}}$. $\left(Javob: C - ln\left|\frac{1}{x-1} - \frac{1}{2} + \frac{\sqrt{1+x-x^2}}{x-1}\right|.\right)$

2.23. $\int \frac{dx}{(x+1)\sqrt{1-x-x^2}}$. $\left(Javob: C - ln\left|\frac{1}{x+1} + \frac{1}{2} + \frac{\sqrt{1-x-x^2}}{x+1}\right|.\right)$

2.24. $\int \frac{dx}{(x-1)\sqrt{1-x-x^2}}$. $\left(Javob: C - arcsin\frac{3x-1}{\sqrt{5}(x-1)}.\right)$

2.25. $\int \frac{dx}{x\sqrt{1-x-x^2}}$. $\left(Javob: C - ln\left|\frac{1}{x} - \frac{1}{2} + \frac{\sqrt{1-x-x^2}}{x}\right|.\right)$

2.26. $\int \frac{dx}{x\sqrt{x^2+x-3}}$. $\left(Javob: C - \frac{1}{3}arcsin\frac{6-x}{x\sqrt{3}}\ .\right)$

2.27. $\int \frac{dx}{(x+1)\sqrt{x^2+x-2}}$. $\left(Javob: C - \frac{1}{\sqrt{2}}arcsin\frac{x+5}{3(x+1)}\ .\right)$

2.28. $\int \frac{dx}{x\sqrt{x^2-3x+2}}$. $\left(Javob: C - \frac{1}{\sqrt{2}}ln\left|\frac{1}{x} - \frac{3}{4} + \frac{\sqrt{x^2-3x+2}}{2x}\right|\ .\right)$

2.29. $\int \frac{dx}{(x+1)\sqrt{2-x-x^2}}$. $\left(Javob: C - \frac{1}{\sqrt{2}}ln\left|\frac{1}{x+1} + \frac{1}{4} + \frac{\sqrt{2-x-x^2}}{x+1}\right|\ .\right)$

2.30. $\int \frac{dx}{x\sqrt{1-3x-2x^2}}$. $\left(Javob: C - ln\left|\frac{1}{x} - \frac{3}{2} + \frac{\sqrt{1-3x-2x^2}}{x}\right|\ .\right)$

3

3.1. $\int \frac{ln(cos\,x)}{cos^2x}dx$. $(Javob: tgx\,ln|cos\,x| + tgx - x + C\ .)$

3.2. $\int cos(lnx)dx$. $\left(Javob: \frac{x}{2}(sin(ln\,x) + cos(ln\,x)) + C\ .\right)$

3.3. $\int \frac{ln\,x}{x^2}dx$. $\left(Javob: C - \frac{ln\,x+1}{x}\ .\right)$

3.4. $\int ln(x+2)dx$. $(Javob: x\,ln(x+2) - x + 2\,ln(x+2) + C\ .)$

3.5. $\int \frac{ln(cos\,x)}{sin^2x}dx$. $(Javob: C - ctgx\,ln|cos\,x| - x\ .)$

3.6. $\int \frac{ln(ln\,x)}{x}dx$. $(Javob: ln\,x\,ln(ln\,x) - ln\,x + C\ .)$

3.7. $\int ln^2xdx$. $(Javob: xln^2x - 2x\,ln\,x + 2x + C\ .)$

3.8. $\int \frac{ln\,x}{\sqrt{x}}dx$. $(Javob: 2\sqrt{x}\,ln\,x - 4\sqrt{x} + C\ .)$

3.9. $\int x\,ln\frac{1-x}{1+x}dx$. $\left(Javob: \frac{x^2}{2}ln\frac{1-x}{1+x} - x - \frac{1}{2}ln\frac{1-x}{1+x} + C\ .\right)$

3.10. $\int ln\left(x + \sqrt{1+x^2}\right)dx$. $\left(Javob: x\,ln\left(x + \sqrt{1+x^2}\right) - \sqrt{1+x^2} + C.\right)$

3.11. $\int ln(x+4)dx$. $(Javob: x\,ln(x+4) - x + 4\,ln(x+4) + C.)$

3.12. $\int \frac{x\,ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$. $\left(Javob: \sqrt{1+x^2}\,ln\left(x + \sqrt{1+x^2}\right) - x + C.\right)$

3.13. $\int \frac{ln(sin\,x)}{sin^2x}dx$. $(Javob: C - x - ctgx - ctgx\,ln(sin\,x)\ .)$

3.14. $\int x^2\,ln(x+1)dx$. $\left(Javob: \frac{x^3}{3}ln(x+1) - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + \frac{1}{3}ln(x+1) + C\ .\right)$

3.15. $\int \frac{ln\,x\,ln(ln\,x)}{x}dx$. $\left(Javob: \frac{1}{2}ln^2x\,ln(ln\,x) - \frac{1}{4}ln^2x + C\ .\right)$

3.16. $\int ln(x^2+1)dx$. $(Javob: x\,ln(x^2+1) - 2x + 2arctgx + C\ .)$

3.17. $\int \frac{ln\,x}{x^3}dx$. $\left(Javob: C - \frac{ln\,x}{2x^2} - \frac{1}{4x^2}\ .\right)$

3.18. $\int \sqrt{x}ln^2xdx$. $\left(Javob: \frac{2}{3}\sqrt{x^3}ln^2x - \frac{8}{9}\sqrt{x^3}\,ln\,x + \frac{16}{27}\sqrt{x^3} + C.\right)$

3.19. $\int ln\frac{1-x}{1+x}dx$. $\left(Javob: x\,ln\frac{1-x}{1+x} - ln(x^2-1) + C\ .\right)$

3.20. $\int(x^2 - x + 1)\,ln\,x\,dx$. $\left(Javob: \left(\frac{x^3}{3} - \frac{x^2}{2} + x\right)ln\,x - \frac{x^3}{9} + \frac{x^2}{4} - x + C.\right)$

3.21. $\int \sqrt{x}\,ln\,x\,dx$. $\left(Javob: \frac{2}{3}\sqrt{x^3}\,ln\,x - \frac{4}{9}\sqrt{x^3} + C.\right)$

3.22. $\int \frac{ln(sin\,x)}{cos^2x}dx$. $(Javob: tgx\,ln(\sin x) - x + C.)$

3.23. $\int x\,ln(x^2+1)dx$. $\left(Javob: \frac{x^2}{2}ln(x^2+1) - \frac{x^2}{2} + \frac{1}{2}ln(x^2+1) + C.\right)$

3.24. $\int x\,ln^2xdx$. $\left(Javob: \frac{x^2}{2}ln^2x - \frac{x^2}{2}ln\,x + \frac{x^2}{4} + C.\right)$

3.25. $\int x^2 \ln x\, dx$. $\left(Javob: \frac{x^3}{3}\ln x - \frac{x^3}{9} + C.\right)$

3.26. $\int x \ln(x+1)\, dx$. $\left(Javob: \frac{x^2}{2}\ln(x+1) - \frac{x^2}{4} + \frac{x}{2} - \frac{1}{2}\ln(x+1) + C.\right)$

3.27. $\int \sin(\ln x)dx$. $\left(Javob: \frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + C.\right)$

3.28. $\int (x^2 - 4)\sin 5x\, dx$. $\left(Javob: \frac{2}{25}x \sin 5x - \frac{x^2-21}{5}\cos 5x + C.\right)$

3.29. $\int x \ln(x+5)\, dx$. (Javob: $x\ln(x+5) - x + 5\ln(x+5) + C.$)

3.30. $\int \ln\frac{2-x}{2+x}dx$. $\left(Javob: x\ln\frac{2-x}{2+x} - 2\ln|4-x^2| + C.\right)$

4

4.1. $\int \sqrt{1-x}\arccos\sqrt{x}dx$. $\left(Javob: \frac{2}{9}\sqrt{x^3} - \frac{2}{3}\sqrt{x} - \frac{2}{3}\sqrt{(1-x)^3}\arccos\sqrt{x} + C.\right)$

4.2. $\int \sqrt{1-x}\arcsin\sqrt{x}dx$. $\left(Javob: \frac{2}{3}\sqrt{x} - \frac{2}{9}\sqrt{x^3} - \frac{2}{3}\sqrt{(1-x)^3}\arcsin\sqrt{x} + C.\right)$

4.3. $\int x\, arctg2xdx$. $\left(Javob: \frac{x^2}{2}arctg2x - \frac{x}{4} + \frac{1}{8}arctg2x + C.\right)$

4.4. $\int \frac{\arcsin x}{\sqrt{x+1}}dx$. (Javob: $2\sqrt{x+1}\arcsin x + 4\sqrt{1-x} + C.$)

4.5. $\int \frac{\arcsin x}{\sqrt{1-x}}dx$. (Javob: $4\sqrt{1-x} - 2\sqrt{1-x}\arcsin x + C.$)

4.6. $\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx$. (Javob: $2\sqrt{x} - 2\sqrt{1-x}\arcsin\sqrt{x} + C.$)

4.7. $\int \frac{x\,arctgx}{\sqrt{1+x^2}}dx$. (Javob: $\sqrt{1+x^2}arctgx - \ln\left|x + \sqrt{1+x^2}\right| + C$.)

4.8. $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx$. (Javob: $x - \sqrt{1-x^2}\arcsin x + C.$)

4.9. $\int x\, arctgxdx$. $\left(Javob: \frac{x^2}{2}arctgx - \frac{x}{2} + \frac{1}{2}arctgx + C.\right)$

4.10. $\int x\, arcctgxdx$. $\left(Javob: \frac{x^2}{2}arcctgx + \frac{x}{2} + \frac{1}{2}arcctgx + C.\right)$

4.11. $\int \frac{x\arccos 2x}{\sqrt{1-4x^2}}dx$. $\left(Javob: C - \frac{x}{2} - \frac{1}{4}\sqrt{1-4x^2}\arccos 2x.\right)$

4.12. $\int \arccos 2xdx$. $\left(Javob: \arccos 2x - \frac{1}{2}\sqrt{1-4x^2} + C .\right)$

4.13. $\int arctgxdx$. $\left(Javob: x\,arctgx - \frac{1}{2}\ln(1+x^2) + C .\right)$

4.14. $\int \frac{\arccos\sqrt{x}}{\sqrt{1-x}}dx$. (Javob: $C - 2\sqrt{x} - 2\sqrt{1-x}\arccos\sqrt{x}.$)

4.15. $\int \frac{x\arccos x}{\sqrt{1-x^2}}dx$. (Javob: $C - x - \sqrt{1-x^2}\arccos x.$)

4.16. $\int \frac{\arccos x}{\sqrt{1-x}}dx$. (Javob: $C - 4\sqrt{1+x} - 2\sqrt{1-x}\arccos x.$)

4.17. $\int arcctg2xdx$. $\left(Javob: x\, arcctg2x + \frac{1}{4}\ln(1+4x^2) + C .\right)$

4.18. $\int \frac{x\, arcctgx}{\sqrt{1+x^2}}dx$. (Javob: $\sqrt{1+x^2}arcctgx + \ln\left|x + \sqrt{1+x^2}\right| + C$.)

4.19. $\int \arcsin 2xdx$. $\left(Javob: x \arcsin 2x + \frac{1}{2}\sqrt{1-4x^2} + C .\right)$

4.20. $\int \frac{x\arcsin 2x}{\sqrt{1-4x^2}}dx$. $\left(Javob: \frac{1}{2}x - \frac{1}{4}\sqrt{1-4x^2}\arcsin 2x + C .\right)$

4.21. $\int \frac{\arccos x}{\sqrt{1+x}}dx$. (Javob: $2\sqrt{1+x}\arccos x - 4\sqrt{1-x} + C$.)

4.22. $\int x^2 arctgxdx$. $\left(Javob: \frac{x^3}{3}arctgx - \frac{1}{6}x^2 + \frac{1}{6}\ln(x^2+1) + C .\right)$

4.23. $\int x\, arctg2xdx$. $\left(Javob: \frac{x^2}{2}arctg2x + \frac{x}{4} + \frac{1}{8}arctg\, 2x + C .\right)$

4.24. $\int arctg(x+5)dx$. $\left(Javob: x\, arctg(x+5) - \frac{1}{2}ln|x^2+10x+26| + \right.$
$\left. +5arctg(x+5) + C\,.\right)$

4.25. $\int x^2 arcctgx dx$. $\left(Javob: \frac{x^3}{3} arcctgx + \frac{x^2}{6} - \frac{1}{6}ln(x^2+1) + C\,.\right)$

4.26. $\int x\, arcctg^2 x dx$. $\left(Javob: \frac{x^2}{2} arctg^2 x + \frac{1}{2} arctg^2 x - x\, arctgx + \right.$
$\left. + \frac{1}{2}ln(x^2+1) + C\,.\right)$

4.27. $\int x^2 \cos\frac{x}{3} dx$. $\left(Javob: 3x^2 \sin\frac{x}{3} + 18x\cos\frac{x}{3} - 54\sin\frac{x}{3} + C\,.\right)$

4.28. $\int x\, arcctg^2 x dx$. $\left(Javob: \frac{x^2}{2} arcctg^2 x + \frac{1}{2} arcctg^2 x + x\, arcctgx + \right.$
$\left. + \frac{1}{2}ln(x^2+1) + C\,.\right)$

4.29. $\int x^2 \sin 2x\, dx$. $\left(Javob: \frac{x}{2}\sin 2x - \frac{x^2}{2}\cos 2x + \frac{1}{4}\cos 2x + C\,.\right)$

4.30. $\int (x^2+4)e^{2x} dx$. $\left(Javob: \frac{1}{2}(x^2+4)e^{2x} + \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C\,.\right)$

5

5.1. $\int x^2 \cos 2x\, dx$. $\left(Javob: \frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + C\,.\right)$

5.2. $\int x \sin^2 x dx$. $\left(Javob: \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + C\,.\right)$

5.3. $\int x\, \sin x\, \cos x\, dx$. $\left(Javob: \frac{1}{8}\sin 2x - \frac{x}{4}\cos 2x + C\,.\right)$

5.4. $\int x^2(\sin 2x - 3)dx$. $\left(Javob: \frac{x}{2}\sin 2x - \frac{x^2}{2}\cos 2x + \frac{1}{4}\cos 2x - x^3 + C\,.\right)$

5.5. $\int x^2(\sin x + 1)dx$. $\left(Javob: 2x\sin x - x^2\cos x + 2\cos x + \frac{x^3}{3} + C\,.\right)$

5.6. $\int (x^2+x)e^{-x}dx$. $(Javob: C - (x^2+3x+3)e^{-x}\,.)$

5.7. $\int (x^2+x)e^{x}dx$. $(Javob: (x^2-x+1)e^{x} + C\,.)$

5.8. $\int (x^2-x+1)e^{-x}dx$. $(Javob: C - (x^2+x+2)e^{-x}\,.)$

5.9. $\int (x^2-x+1)e^{x}dx$. $(Javob: (x^2-3x+4)e^{x} + C\,.)$

5.10. $\int x\, ctg^2 x dx$. $\left(Javob: ln|\sin x| - x\, ctgx - \frac{x^2}{2} + C\,.\right)$

5.11. $\int x^2 e^{-x} dx$. $(Javob: C - (x^2+2x+2)e^{-x}\,.)$

5.12. $\int \frac{xdx}{\sin^2 x}$. $(Javob: ln|\sin x| - x\, ctgx + C\,.)$

5.13. $\int \frac{xdx}{\cos^2 x}$. $(Javob: x\, tgx + ln|\cos x| + C\,.)$

5.14. $\int x\, tg^2 x dx$. $\left(Javob: x\, tgx + ln|\cos x| - \frac{x^2}{2} + C\,.\right)$

5.15. $\int (x^2+2)e^{-x}dx$. $(Javob: C - (x^2+2x+4)e^{-x}\,.)$

5.16. $\int x^2 \sin^2 x dx$. $\left(Javob: \frac{x^3}{6} - \frac{x^2}{4}\sin 2x + \frac{x}{4}\cos 2x + \frac{1}{8}\sin 2x + C\,.\right)$

5.17. $\int x^2(\cos 2x + 3)\, dx$. $\left(Javob: x^3 + \frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x + C\,.\right)$

5.18. $\int (x^2+2)e^{-x}dx$. $(Javob: (x^2-2x+4)e^{x} + C\,.)$

5.19. $\int (x^3+3)\sin x\, dx$. $(Javob: 2x\sin x - (x^2+1)\cos x + C\,.)$

5.20. $\int (x^2-3)\cos x\, dx$. $(Javob: (x^2-4)\sin x + 2x\cos x + C\,.)$

5.21. $\int (x^2+1)e^{-x}dx$. $(Javob: C - (x^2+2x+3)e^{-x}\,.)$

5.22. $\int (x^2-1)e^{x}dx$. $(Javob: (x-1)^2 e^{x} + C\,.)$

5.23. $\int x^2 \cos^2 x dx$. $\left(Javob: \frac{x^3}{6} + \frac{x^2}{4}\sin 2x + \frac{x}{4}\cos 2x - \frac{1}{8}\sin 2x + C .\right)$

5.24. $\int (x^2 + x) \sin x\, dx$. $(Javob: (2x + 1)\sin x - (x^2 + x - 2)\cos x + C .)$

5.25. $\int (x^2 + x) \cos x\, dx$. $(Javob: (x^2 + x - 1)\sin x + (2x + 1)\cos x + C .)$

5.26. $\int (x^2 + 1)e^x dx$. $(Javob: (x^2 - 2x + 3)e^x + C .)$

5.27. $\int (x^2 - 1)e^{-x} dx$. $(Javob: C - (x + 1)^2 e^{-x} .)$

5.28. $\int x \sin^2 x dx$. $\left(Javob: \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + C .\right)$

5.29. $\int \arcsin 9x\, dx$. $\left(Javob: x \arcsin 9x + \frac{1}{9}\sqrt{1 - 81x^2} + C .\right)$

5.30. $\int x\ arctg\, 2x\, dx$. $\left(Javob: \frac{x^2}{2} arctg 2x - \frac{x}{4} + \frac{1}{2} arctg\, 2x + C .\right)$

6

6.1. $\int (x + 1)e^{2x}\, dx$.

6.2. $\int (x - 2)e^x\, dx$.

6.3. $\int (x - 7) \cos 2x\ dx$.

6.4. $\int (x - 1) \cos 5x\ dx$.

6.5. $\int (x + 2) \cos 3x\ dx$.

6.6. $\int (x - 2) \cos 4x\ dx$.

6.7. $\int (x - 4) \sin 2x\ dx$.

6.8. $\int (x - 3) \cos x\ dx$.

6.9. $\int (x + 4) \sin 2x\ dx$.

6.10. $\int x \sin 3x\ dx$.

6.11. $\int (x + 5) \sin x\ dx$.

6.12. $\int (x - 5) \cos x\ dx$.

6.13. $\int (x + 9) \sin x\ dx$.

6.14. $\int (x + 7) \sin 2x\ dx$.

6.15. $\int (x + 4) \sin 3x\ dx$.

6.16. $\int (x + 3) \sin 5x\ dx$.

6.17. $\int (x - 4) \cos 2x\ dx$.

6.18. $\int (x - 8) \sin x\ dx$.

6.19. $\int (x + 4) \cos 3x\ dx$.

6.20. $\int (x + 8) \sin 3x\ dx$.

6.21. $\int (x + 6) \cos 4x\ dx$.

6.22. $\int (x - 6) \sin \frac{x}{2}\ dx$.

6.23. $\int (x + 1) \cos 7x\ dx$.

6.24. $\int (x + 2) \sin \frac{x}{2}\ dx$.

6.25. $\int x \sin \frac{x}{5}\ dx$.

6.26. $\int (x + 4) \cos \frac{x}{2}\ dx$.

6.27. $\int (x + 1) \sin \frac{x}{3}\ dx$.

6.28. $\int (x + 2) \cos \frac{x}{4}\ dx$.

6.29. $\int (x + 3) \sin \frac{x}{4}\ dx$.

6.30. $\int (x - 9) \sin \frac{x}{2}\ dx$.

7

7.1. $\int \ln(x - 5)\, dx$.

7.2. $\int arctg 2x\, dx$.

7.3. $\int x^2 e^{-x}\, dx$.

7.4. $\int (x + 1)e^{-4x}\, dx$.

7.5. $\int x^2 e^{-2x}\, dx$.

7.6. $\int arctg 3x\, dx$.

7.7. $\int x \cos 8x\ dx$.

7.8. $\int arctg 4x\, dx$.

7.9. $\int \arcsin 5x\, dx$.

7.10. $\int (x + 1)e^{-x}\, dx$.

7.11. $\int x\, arctg x\, dx$.

7.12. $\int x^2 e^{3x}\, dx$.

7.13. $\int x\ \cos(x + 4)\ dx$.

7.14. $\int x\ \cos(x - 2)\, dx$.

7.15. $\int x\ \cos(x + 3)\ dx$.

7.16. $\int x e^{x+2}\, dx$.

7.17. $\int x e^{-7x}\, dx$.

7.18. $\int \arcsin 2x\, dx$.

7.19. $\int x\ \sin(x + 7)\ dx$.

7.20. $\int x \cos(x - 4)\ dx$.

7.21. $\int x\ \sin(x + 4)\ dx$.

7.22. $\int x \cos(x + 9)\ dx$.

7.23. $\int (x+3)e^{-x}\,dx$.
7.24. $\int \arccos x\,dx$.
7.25. $\int (x^2-3)e^{x}\,dx$.
7.26. $\int xe^{-4x}\,dx$.
7.27. $\int x\cos(x+7)\,dx$.
7.28. $\int xe^{-5x}\,dx$.
7.29. $\int xe^{x+3}\,dx$.
7.30. $\int x\cos(2-x)\,dx$.

8

8.1. $\int arctg2x\,dx$.
8.2. $\int x\cos 6x\,dx$.
8.3. $\int arsin3x\,dx$.
8.4. $\int arccos2x\,dx$.
8.5. $\int arctg8x\,dx$.
8.6. $\int x\sin(x-2)\,dx$.
8.7. $\int arsin8x\,dx$.
8.8. $\int x\sin(x+3)\,dx$.
8.9. $\int x\cos(x+4)\,dx$.
8.10. $\int arccos7x\,dx$.
8.11. $\int x\cos(x-7)\,dx$.
8.12. $\int x\sin(x-5)\,dx$.
8.13. $\int (x-4)e^{x}dx$.
8.14. $\int xe^{-6x}\,dx$.
8.15. $\int arctg7x\,dx$.
8.16. $\int arsin5x\,dx$.
8.17. $\int \ln(x-7)\,dx$.
8.18. $\int x\cos(x+6)\,dx$.
8.19. $\int arctg\frac{x}{2}\,dx$.
8.20. $\int \ln(x+8)\,dx$.
8.21. $\int arctg\frac{x}{5}\,dx$.
8.22. $\int \ln(x+12)\,dx$.
8.23. $\int \arcsin\frac{x}{5}\,dx$.
8.24. $\int \ln(2x-1)\,dx$.
8.25. $\int \ln(2x+3)\,dx$.
8.26. $\int \arccos\frac{x}{5}\,dx$.
8.27. $\int arctg\frac{x}{4}\,dx$.
8.28. $\int \arcsin\frac{x}{7}\,dx$.
8.29. $\int arctg6x\,dx$.
8.30. $\int \arccos\frac{x}{3}\,dx$.

Namunaviy variant yechimi

Aniqmas integrallarni hisoblang.

1. $\int x^2\sqrt{16-x^2}dx$.

$$\blacktriangleright \int x^2\sqrt{16-x^2}dx = \left|\begin{matrix} x = 4\sin t, dx = 4\cos tdt, \\ \sin t = x/4, t = \arcsin x/4\end{matrix}\right| =$$

$$= \int 16\sin^2 t\sqrt{16-16\sin^2 t}4\cos tdt = 256\int \sin^2 t\cos^2 tdt =$$

$$= 64\int \sin^2 2tdt = 32\int (1-\cos 4t)dt = 32t - 8\sin 4t + C =$$

$$= 32\arcsin\frac{x}{4} - \frac{x}{4}(8-x^2)\sqrt{16-x^2+C}. \blacktriangleleft$$

2. $\int \frac{dx}{x\sqrt{x^2+5x+1}}$

$$\blacktriangleright \int \frac{dx}{x\sqrt{x^2+5x+1}} = \left|\begin{matrix} x = \frac{1}{t}, t = \frac{1}{x}, \\ dx = -\frac{1}{t^2}dt\end{matrix}\right| = -\int \frac{dt}{t^2\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{5}{t}+1}} = -\int \frac{dt}{\sqrt{t^2+5t+1}} =$$

$$-\int \frac{dt}{\sqrt{\left(t+\frac{5}{2}\right)^2-\frac{21}{4}}} = -\ln\left|t+\frac{5}{2}+\sqrt{t^2+5t+1}\right| + C =$$

$$= -\ln\left|\frac{1}{x}+\frac{5}{2}+\sqrt{\frac{1}{x^2}+\frac{5}{x}+1}\right|+C. ◀$$

3. $\int(x-7)\sin 5x dx.$

▶ $\int(x-7)\sin 5xdx = \left|\begin{matrix} u = x-7, du = dx, \\ dv = \sin 5xdx, v = -\frac{1}{5}\cos 5x \end{matrix}\right| = -\frac{1}{5}(x-7)\cos 5x + + \frac{1}{5}\int \cos 5x\,dx = -\frac{1}{5}(x-7)\cos 5x + \frac{1}{25}\sin 5x + C.$ ◀

4. $\int \arccos 4xdx.$

▶ $\int \arccos 4xdx = \left|\begin{matrix} u = \arccos 4x, du = -\frac{4dx}{\sqrt{1-16x^2}}, \\ dv = dx, v = x \end{matrix}\right| =$

$= x\arccos 4x + 4\int\frac{xdx}{\sqrt{1-16x^2}} = x\arccos 4x - \frac{1}{4}\sqrt{1-16x^2}+C.$ ◀

5. $\int xe^{x-7}dx.$

▶ $\int xe^{x-7}dx = \left|\begin{matrix} u = x, du = dx \\ dv = e^{x-7}, v = e^{x-7} \end{matrix}\right| = xe^{x-7} - \int e^{x-7}dx =$

$= xe^{x-7} - e^{x-7} + C.$ ◀

6. $\int\frac{x\,\mathrm{arctg}x}{\sqrt{1+x^2}}dx.$

▶ $\int\frac{x\,\mathrm{arctg}x}{\sqrt{1+x^2}}dx = \left|\begin{matrix} u = \mathrm{arctg}x, du = \frac{dx}{1+x^2} \\ dv = \frac{xdx}{\sqrt{1+x^2}}, v = \sqrt{1+x^2} \end{matrix}\right| = \sqrt{1+x^2}\mathrm{arctg}x -$

$-\int\frac{dx}{\sqrt{1+x^2}} = \sqrt{1+x^2}\mathrm{arctg}x - \ln\left|x+\sqrt{1+x^2}\right| + C.$ ◀

7. $\int(x^2-4x+3)e^{-2x}dx.$

▶ $\int(x^2-4x+3)e^{-2x}dx = \left|\begin{matrix} u = x^2-4x+3, du = (2x-4)dx, \\ dv = e^{-2x}dx, v = -\frac{1}{2}e^{-2x} \end{matrix}\right| =$

$= -\frac{1}{2}\left((x^2-4x+3)e^{-2x}\right) + \int(x-2)e^{-2x}dx =$

$= \left|\begin{matrix} u = x-2, du = dx, \\ dv = e^{-2x}dx, v = -\frac{1}{2}e^{-2x} \end{matrix}\right| = -\frac{1}{2}(x^2-4x+3)e^{-2x} -$

$-\frac{1}{2}(x-2)e^{-2x} - \frac{1}{4}e^{-2x} + C.$ ◀

8. $\int\frac{\ln(\ln(x+1))\ln(x+1)}{x+1}dx.$

▶ $\int\frac{\ln(\ln(x+1))\ln(x+1)}{x+1}dx = \left|\begin{matrix} u = \ln(\ln(x+1)), du = \frac{dx}{(x+1)\ln(x+1)} \\ dv = \frac{\ln(x+1)}{x+1}dx, v = \frac{1}{2}\ln^2(x+1) \end{matrix}\right| =$

$= \frac{\ln^2(x+1)}{2}\ln(\ln(x+1)) - \frac{1}{2}\int\frac{\ln(x+1)}{x+1}dx =$

$= \frac{\ln^2(x+1)}{2}\ln(\ln(x+1)) - \frac{1}{4}\ln^2(x+1) + C.$ ◀

IUT-8.4

Aniqmas integrallarni hisoblang

1.

1.1. $\int \frac{3x^2+20x+9}{(x^2+4x+3)(x+5)}dx.$ $(Javob: 6\,ln|x+3| - ln|x+1| +$ $2\,ln|x+5| + +C.)$

1.2. $\int \frac{12}{(x-2)(x^2-2x+3)}dx.$ $(Javob: 3\,ln|x-3| - 4\,ln|x-2| +$ $+ln|x+1| + C.$

1.3. $\int \frac{43x-67}{(x-1)(x^2-x-12)}dx.$ $(Javob: 2\,ln|x-1| + 5\,ln|x-4| -$ $-7ln|x+3| + C.)$

1.4. $\int \frac{2x^4+8x^3+9x^2-7}{(x^2+x-2)(x+3)}dx.$ $(Javob: x^2 + 5ln|x+3| + ln|x+2| +$ $+ln|x-1| + C.)$

1.5. $\int \frac{8x}{(x^2+6x+5)(x+3)}dx.$ $(Javob: -5\,ln|x+5| + 6\,ln|x+3| -$ $-ln|x+1| + C.)$

1.6. $\int \frac{2x^4-7x^3+7x^2-8x}{(x^2-5x+6)(x+1)}dx.$ $(Javob: x^2 + x + 2\,ln|x+1| +$ $4\,ln|x-2| + +3\,ln\,|x-3| + C\,.)$

1.7. $\int \frac{2x^4+8x^3-45x-61}{(x-1)(x^2+5x+6)}dx.$ $(Javob: x^2 - 8\,ln|x-1| + 5\,ln|x+3| +$ $+ln\,|x+2| + C.)$

1.8. $\int \frac{2x^4+17x^3+32x^2-7x}{(x^2+4x+3)(x+5)}dx.$ $(Javob: x^2 - x - 5\,ln|x+5| +$ $3\,ln|x+ +1| - 3\,ln\,|x+3| + C.)$

1.9. $\int \frac{6x^2+6x-6}{(x+1)(x^2+x-2)}dx.$ $(Javob: 3ln|x+1| + ln|x-1| +$ $+2\,ln\,|x+2| + C.)$

1.10. $\int \frac{37x-85}{(x-4)(x^2+2x-3)}dx.$ $(Javob: 4\,ln|x-1| - 7\,ln|x+3| +$ $+3\,ln\,|x-4| + C\,.)$

1.11. $\int \frac{3x^2+3x-24}{(x-3)(x^2-x-2)}dx.$ $(Javob: 2ln|x-2| + 3\,ln|x-3| -$ $-2\,ln\,|x+1| + C.)$

1.12. $\int \frac{2x^4-7x^3+3x+20}{(x-2)(x^2-2x-3)}dx.$ $(Javob: x^2 + x - 4\,ln|x-2| +$ $3\,ln|x-3| + +3\,ln\,|x+1| + C.)$

1.13. $\int \frac{3x^2-15}{(x-1)(x^2+5x+6)}dx.$ $(Javob: ln|x+2| - ln|x-1| +$ $+3\,ln\,|x+3| + C.)$

1.14. $\int \frac{x^2-19x+6}{(x-1)(x^2+5x+6)}dx.$ $(Javob: 18ln|x+3| - ln|x-1| -$ $-16\,ln|x+2| + C.)$

1.15. $\int \frac{6x}{x^3+2x^2-x-2}dx$. (*Javob*: $ln|x-1| + 3\,ln|x+1| -$
$-4\,ln|x+2| + C$.)

1.16. $\int \frac{4x^2+32x+52}{(x^2+6x+5)(x+3)}dx$. (*Javob*: $3ln|x+1| + 2\,ln|x+3| -$
$-ln\,|x+5| + C$.)

1.17. $\int \frac{2x^2+41x-91}{(x^2+2x-3)(x-4)}dx$. (*Javob*: $4ln|x-1| - 7\,ln|x+3| +$
$+5\,ln\,|x-4| + C$.)

1.18. $\int \frac{2x^4+8x^3-17x-5}{(x^2+2x-3)(x+2)}dx$. (*Javob*: $x^2 - ln|x-1| + ln|x+2| -$
$-2\,ln|x+3| + C$.)

1.19. $\int \frac{2x^4+17x^3+40x^2+37x+36}{(x^2+8x+15)(x+1)}dx$. (*Javob*: $x^2 - x + 3ln|x+1| +$
$+3\,ln|x+3| - 3\,ln\,|x+5| + C$.)

1.20. $\int \frac{6x^2}{(x^2+3x+2)(x-1)}dx$. (*Javob*: $ln|x-1| - 3\,ln|x+1| +$
$+8\,ln\,|x+2| + C$.)

1.21. $\int \frac{6x^4}{(x^2-1)(x+2)}dx$. (*Javob*: $3x^2 - 12x + ln|x-1| -$
$-3\,ln|x+1| + 32\,ln\,|x+2| + C$.)

1.22. $\int \frac{2x^2-26}{(x^2+4x+3)(x+5)}dx$. (*Javob*: $2ln|x+3| - 3\,ln|x+1| +$
$+3\,ln\,|x+5| + C$.)

1.23. $\int \frac{2x^2+12x-6}{(x^2+8x+15)(x+1)}dx$. (*Javob*: $6ln|x+3| - 2\,ln|x+1| -$
$-2\,ln\,|x+5| + C$.)

1.24. $\int \frac{2x^4-5x^3-15x^2+40x-70}{(x^2+2x-3)(x-4)}dx$. (*Javob*: $x^2 - x + 4\,ln|x-1| -$
$-ln|x+3| + 2\,ln\,|x-4| + C$.)

1.25. $\int \frac{2x^4-7x^3+2x^2+13}{(x^2-5x+6)(x+1)}dx$. (*Javob*: $x^2 + x + 2\,ln|x+1| +$
$+ln|x-2| + ln\,|x-3| + C$.)

1.26. $\int \frac{6x^4-21x^2+3x+24}{(x^2+x-2)(x+1)}dx$. (*Javob*: $3x^2 - 12x + 2\,ln|x-1| -$
$-3\,ln|x+1| + 10\,ln\,|x+2| + C$.)

1.27. $\int \frac{2x^4-3x^3-21x^2-26}{(x^2-5x+4)(x+3)}dx$. (*Javob*: $x^2 + x + 4\,ln|x-1| +$
$+ln|x+3| - 2\,ln\,|x-4| + C$.)

1.28. $\int \frac{7x^2-17x}{(x^2-2x-3)(x-2)}dx$. (*Javob*: $2ln|x-2| + 3\,ln|x-3| +$
$+2\,ln\,|x+1| + C$.)

1.29. $\int \frac{6x^4-30x^2+30}{(x^2-1)(x+2)}dx$. (*Javob*: $3x^2 - 12x + ln|x-1| -$
$-3\,ln|x+1| + 2\,ln\,|x+2| + C$.)

1.30. $\int \frac{3x^2-17x+2}{(x^2+5x+6)(x-1)}dx$. (*Javob*: $20\ln|x+3| - \ln|x-1| - -16\ln|x+2| + C$.)

2

2.1. $\int \frac{x^3+1}{x^3-x^2}dx$. (*Javob*: $x + \frac{1}{x} - \ln|x| + 2\ln|x-1| + C$.)

2.2. $\int \frac{x^3-2x^2-2x+1}{x^3-x^2}dx$. (*Javob*: $x + \ln|x| + \frac{1}{x} - 2\ln|x-1| + C$.)

2.3. $\int \frac{3x^2+1}{(x-1)(x^2-1)}dx$. (*Javob*: $2\ln|x-1| - \frac{2}{x-1} + \ln|x+1| + C$.)

2.4. $\int \frac{x+2}{x^3-x^2}dx$. (*Javob*: $\frac{2}{x} - 3\ln|x| + 3\ln|x-1| + C$.)

2.5. $\int \frac{4x^4+8x^3-3x-3}{x^3+2x^2+x}dx$. (*Javob*: $2x^2 - 3\ln|x| - \ln|x+1| - \frac{4}{x+1} + C$.)

2.6. $\int \frac{x+2}{x^3+x^2}dx$. (*Javob*: $\ln|x+1| - \ln|x| - \frac{2}{x} + C$.)

2.7. $\int \frac{4x^2}{(x^2-2x+1)(x+1)}dx$. (*Javob*: $3\ln|x-1| - \frac{2}{x-1} + \ln|x+1| + C$.)

2.8. $\int \frac{2x^2-2x-1}{x^2-x^3}dx$. (*Javob*: $\frac{1}{x} - 3\ln|x| + \ln|x-1| + C$.)

2.9. $\int \frac{2x^2-5x+1}{x^3-2x^2+x}dx$. (*Javob*: $\ln|x| + \ln|x-1| + \frac{2}{x-1} + C$.)

2.10. $\int \frac{4x^4+8x^3-x-2}{x(x+1)^2}dx$. (*Javob*: $2x^2 - 2\ln|x| - 2\ln|x+1| - \frac{5}{x+1} + C$.)

2.11. $\int \frac{2x^4-4x^3+2x^2-4x+1}{x(x-1)^2}dx$. (*Javob*: $x^2 + \ln|x| - \ln|x-1| + \frac{3}{x-1} + C$.)

2.12. $\int \frac{3x-x^2-2}{x(x+1)^2}dx$. (*Javob*: $\ln|x+1| - 2\ln|x| - \frac{6}{x+1} + C$.)

2.13. $\int \frac{2x^3+1}{x^2(x+1)}dx$. (*Javob*: $2x - \ln|x| - \frac{1}{x} - \ln|x+1| + C$.)

2.14. $\int \frac{x^3-3}{(x-1)(x^2-1)}dx$. (*Javob*: $x + \frac{1}{x-1} + 2\ln|x-1| - \ln|x+1| + C$.)

2.15. $\int \frac{x^2-3x+2}{x^3+2x^2+x}dx$. (*Javob*: $2\ln|x| + \frac{6}{x+1} - \ln|x+1| + C$.)

2.16. $\int \frac{x+2}{x^3-2x^2+x}dx$. (*Javob*: $2\ln|x| - 2\ln|x-1| - \frac{3}{x-1} + C$.)

2.17. $\int \frac{4x^4+8x^3-1}{(x^2+x)(x+1)}dx$. (*Javob*: $2x^2 - \ln|x| - 3\ln|x+1| - \frac{5}{x+1} + C$.)

2.18. $\int \frac{4x}{(x^2-1)(x+1)}dx$. (*Javob*: $\ln|x-1| - \ln|x+1| - \frac{2}{x+1} + C$.)

2.19. $\int \frac{dx}{x^3+x^2}$. (*Javob*: $\ln|x+1| - \ln|x| - \frac{1}{x} + C$.)

2.20. $\int \frac{x^3-4x^2+2x-1}{x^3-x^2}dx$. (*Javob*: $x - \ln|x| - \frac{1}{x} - 2\ln|x-1| + C$.)

2.21. $\int \frac{6x-2x^2-1}{x^3-2x^2+x}dx$. (*Javob*: $-\ln|x| - \ln|x-1| - \frac{3}{x-1} + C$.)

2.22. $\int \frac{2x^3+2x^2+4x+3}{x^3+x^2}dx$. (*Javob*: $2x + \ln|x| - \frac{3}{x} - \ln|x+1| + C$.)

2.23. $\int \frac{x^3-4x+5}{(x^2-1)(x-1)}dx$. (*Javob*: $x - \ln|x-1| - \frac{1}{x-1} + 2\ln|x+1| + C$.)

2.24. $\int \frac{3x^2+2}{x(x+1)^2}dx$. (*Javob*: $2\ln|x| + \ln|x+1| + \frac{5}{x+1} + C$.)

2.25. $\int \frac{x+5}{x^3-x^2-x+1}dx$. (*Javob*: $\ln|x+1| - \ln|x-1| - \frac{3}{x-1} + C$.)

2.26. $\int \frac{3x^2-7x+2}{(x^2-x)(x-1)}dx$. (*Javob*: $2\ln|x| + \ln|x-1| + \frac{2}{x-1} + C$.)

2.27. $\int \frac{x^2+x+2}{x^3+x^2}dx$. (*Javob*: $2\ln|x+1| - \ln|x| - \frac{2}{x} + C$.)

2.28. $\int \frac{dx}{x^3-x^2}$. (*Javob*: $\frac{1}{x} - \ln|x| + \ln|x-1| + C$.)

2.29. $\int \frac{2x^2+1}{x^3-2x^2+x}dx$. (*Javob*: $\ln|x| + \ln|x-1| - \frac{3}{x-1} + C$.)

2.30. $\int \frac{2x^3+5x^2-1}{x^3+x^2}dx$. (*Javob*: $2x + \ln|x| + \frac{1}{x} + 2\ln|x+1| + C$.)

3

3.1. $\int \frac{3x+13}{(x-1)(x^2+2x+5)}dx$. (*Javob*: $2\ln|x-1| -$
$-\ln|x^2+2x+5| \frac{1}{2} arctg \frac{x+1}{2} + C$.)

3.2. $\int \frac{x^2-6x+8}{x^3+8}dx$. (*Javob*: $2\ln|x+2| - \frac{1}{2}\ln|x^2-2x+4| -$
$-\frac{1}{\sqrt{3}} arctg \frac{x-1}{\sqrt{3}} + C$.)

3.3. $\int \frac{12-6x}{(x+1)(x^2-4x+13)}dx$. (*Javob*: $\ln|x+1| - \frac{1}{2}\ln|x^2-4x+13| -$
$-arctg\frac{x-2}{3} + C$.)

3.4. $\int \frac{2x^2+2x+20}{(x-1)(x^2+2x+5)}dx$. (*Javob*: $3\ln|x-1| - \frac{1}{2}\ln|x^2+2x+5| -$
$-2\,arctg\frac{x+1}{2} + C$.)

3.5. $\int \frac{x^2+3x-6}{(x+1)(x^2+6x+13)}dx$. (*Javob*: $\ln|x^2+6x+13| - \ln|x+1| +$
$+\frac{1}{2} arctg\frac{x+3}{2} + C$.)

3.6.
$\int \frac{x^2+3x+2}{x^3-1}dx$. (*Javob*: $2\ln|x-1| - -\frac{1}{2}\ln|x^2+x+1| + \frac{1}{\sqrt{3}} arctg\frac{2x+1}{\sqrt{3}} +$
C.)

3.7. $\int \frac{36}{(x+2)(x^2-2x+10)}dx$. (*Javob*: $2\ln|x+2| - -\ln|x^2-2x+$
$10| + 2\,arctg\frac{x-1}{3} + C$.)

3.8. $\int \frac{9x-9}{(x+1)(x^2-4x+13)}dx$. (*Javob*: $\frac{1}{2}\ln|x^2-4x+13| -$
$-\ln|x+1| + 2\,arctg\frac{x-2}{3} + C$.)

3.9. $\int \frac{7x-10}{x^3+8}dx$. (*Javob*: $\ln|x^2-2x+4| - 2\ln|x+2| +$
$+\frac{1}{\sqrt{3}} arctg\frac{x-1}{\sqrt{3}} + C$.)

3.10. $\int \frac{4x^2+3x+17}{(x-1)(x^2+2x+5)}dx.$ (*Javob*: $3\ln|x-1| +$
$+\frac{1}{2}\ln|x^2+2x+5| - \frac{3}{2}arctg\frac{x+1}{2} + C.$)

3.11. $\int \frac{4x+2}{x^4+4x^2}dx.$ (*Javob*: $\ln|x| - \frac{1}{2x} -$
$-\frac{1}{2}\ln|x^2++4| - \frac{1}{4}arctg\frac{x}{2} + C.$)

3.12. $\int \frac{x^2-5x+40}{(x+2)(x^2-2x+10)}dx.$ (*Javob*: $3\ln|x+2| - \ln|x^2-2x+$
$+10| + arctg\frac{x-1}{3} + C.$)

3.13. $\int \frac{4x-x^2-12}{x^3+8}dx.$ (*Javob*: $\frac{1}{2}\ln|x^2-2x+4| - 2\ln|x+$
$+2| - \frac{1}{\sqrt{3}}arctg\frac{x-1}{\sqrt{3}} + C.$)

3.14. $\int \frac{x^2-13x+40}{(x+1)(x^2-4x+13)}dx.$ (*Javob*: $3\ln|x+1| - \ln|x^2-4x+13| -$
$-arctg\frac{x-2}{3} + C.$)

3.15. $\int \frac{3-9x}{x^3-1}dx.$ (*Javob*: $\ln|x^2+x+1| - 2\ln|x-1| -$
$-4\sqrt{3}\,arctg\frac{2x+1}{\sqrt{3}} + C.$)

3.16.
$\int \frac{6-9x}{x^3+8}dx.$ (*Javob*: $2\ln|x+2| - \ln|x^2-2x+\ +4| - \sqrt{3}\,arctg\frac{x-1}{\sqrt{3}} + C.$)

3.17. $\int \frac{4x-10}{(x+2)(x^2-2x+10)}dx.$ (*Javob*: $\frac{1}{2}\ln|x^2-2x+10| - \ln|x+2| +$
$+\frac{1}{3}arctg\frac{x-1}{3} + C.$)

3.18. $\int \frac{x^2+23}{(x+1)(x^2+6x+13)}dx.$ (*Javob*: $3\ln|x+1| - \ln|x^2+6x+$
$+13| - 5\,arctg\frac{x+3}{2} + C.$)

3.19. $\int \frac{2x^2+7x+7}{(x-1)(x^2+2x+5)}dx.$ (*Javob*: $2\ln|x-1| + \frac{3}{2}arctg\frac{x+1}{2} + C.$)

3.20. $\int \frac{19x-x^2-34}{(x+1)(x^2-4x+13)}dx.$ (*Javob*: $\ln|x^2-4x+13| - 3\ln|x+$
$+1| + 3\,arctg\frac{x-2}{3} + C.$)

3.21. $\int \frac{4x^2+38}{(x+2)(x^2-2x+10)}dx.$ (*Javob*: $3\ln|x+2| + \frac{1}{2}\ln|x^2-2x+$
$+10| + \frac{5}{3}arctg\frac{x-1}{3} + C.$)

3.22. $\int \frac{8}{(x+1)(x^2+6x+13)}dx.$ (*Javob*: $\ln|x+1| - \frac{1}{2}\ln|x^2+6x+$
$+13| - arctg\frac{x+3}{2} + C.$)

3.23. $\int \frac{2x^2+4x+20}{(x+1)(x^2-4x+13)}dx.$ (*Javob*: $\ln|x+1| + \frac{1}{2}\ln|x^2-4x+$
$+13| + 3\,arctg\frac{x-2}{3} + C.$)

3.24. $\int\frac{5x+13}{(x+1)(x^2+6x+13)}dx.$ $(Javob: ln|x+1|-\frac{1}{2}ln|x^2+6x+$
$13|+\frac{3}{2}arctg\frac{x+3}{2}+C.)$

3.25. $\int\frac{4x^2+x+10}{x^3+8}dx.$ $(Javob: 2\,ln|x+2|+ln|x^2-2x+4|+$
$+\sqrt{3}\,arctg\frac{x-1}{\sqrt{3}}+C.)$

3.26. $\int\frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx.$ $(Javob: 2\,ln|x-1|+ln|x^2+2x+5|+$
$+\frac{3}{2}arctg\frac{x+1}{2}+C.)$

3.27. $\int\frac{3x^2+2x+1}{x^3-1}dx.$ $(Javob: 2\,ln|x-1|+\frac{1}{2}ln|x^2+x+$
$+1|+\frac{1}{\sqrt{3}}arctg\frac{2x+1}{\sqrt{3}}+C.)$

3.28.
$\int\frac{6x}{x^3-1}dx.$ $(Javob: 2\,ln|x-1|-ln|x^2+x+\;+1|+2\sqrt{3}\,arctg\frac{2x+1}{\sqrt{3}}+$
$C.)$

3.29. $\int\frac{5x^2+17x+36}{(x+1)(x^2+6x+13)}dx.$ $(Javob: 3\,ln|x+1|+ln|x^2+6x+13|-$
$-\frac{9}{2}arctg\frac{x+3}{2}+C.)$

3.30. $\int\frac{2x+22}{(x+2)(x^2-2x+10)}dx.$ $(Javob: ln|x+2|-\frac{1}{2}ln|x^2-2x+$
$10|+\frac{5}{3}arctg\frac{x-1}{3}+C.)$

4.

4.1. $\int\frac{5x\,dx}{x^4+3x^2-4}$ (*Javob*: $\frac{1}{2}ln|x-1|+\frac{1}{2}ln|x+1|-\frac{1}{2}ln|x^2+4|+$
$+C.)$

4.2. $\int\frac{2x^5-2x+1}{1-x^4}dx$ (*Javob*: $\frac{1}{4}ln|x+1|-x^2-\frac{1}{4}ln|x-1|+$
$\frac{1}{2}arctgx++C.)$

4.3. $\int\frac{x^4+x^3+2x^2+x+2}{x^4+5x^2+4}dx$ (*Javob*: $x+\frac{1}{3}arctgx+\frac{1}{2}ln|x^2+4|-$
$\frac{5}{3}arctg\frac{x}{2}++C.)$

4.4. $\int\frac{5dx}{x^4+3x^2-4}$ (*Javob*: $\frac{1}{2}ln|x-1|-\frac{1}{2}ln|x+1|-\frac{1}{2}arctg\frac{x}{2}+C.)$

4.5. $\int\frac{x^3+8x-2}{x^4+4x^2}dx$ (*Javob*: $2ln|x|+\frac{1}{2x}-\frac{1}{2}ln|x^2+4|-\frac{1}{4}arctg\frac{x}{2}+C.)$

4.6. $\int\frac{2x^3-2x^2+5}{(x-1)^2(x^2+4)}dx$ (*Javob*: $ln|x^2+4|-\frac{1}{x-1}+\frac{1}{2}arctg\frac{x}{2}+C.)$

4.7. $\int\frac{x^3+x^2-x-3}{x^4-x^2}dx$ (*Javob*: $ln|x|-\frac{3}{x}-ln|x-1|+ln|x+1|+C.)$

4.8. $\int\frac{x^3-x-5}{x^4+3x^2-4}dx$ (*Javob*: $\frac{1}{2}ln|x+1|-\frac{1}{2}ln|x-1|+\frac{1}{2}ln|x^2+4|+$
$+\frac{1}{2}arctg\frac{x}{2}+C.)$

4.9. $\int \frac{x^3-x-1}{x^4-x^2}dx$ (*Javob*: $ln|x| - \frac{1}{x} - \frac{1}{2}ln|x-1| + \frac{1}{2}ln|x+1| + +C.$)

4.10. $\int \frac{2x^2-7x+10}{(x-1)(x^3-x^2+4x-4)}dx$ (*Javob*: $\frac{1}{2}ln|x^2+4| - ln|x-1| - \frac{1}{x-1} + +arctg\frac{x}{2} + C.$)

4.11. $\int \frac{4x+2}{x^4+4x^2}dx$ (*Javob*: $ln|x| - \frac{1}{2x} - \frac{1}{2}ln|x^2+4| - \frac{1}{4}arctg\frac{x}{2} + +C.$)

4.12. $\int \frac{x^3-x+2}{x^4+x^2}dx$ (*Javob*: $ln|x| + \frac{2}{x} + ln|x-1| - ln|x+1| + C.$)

4.13. $\int \frac{x^2+2x+4}{x^4+5x^2+4}dx$ (*Javob*: $\frac{1}{3}ln|x^2+1| + arctgx - \frac{1}{3}ln|x^2+4| + +C.$)

4.14. $\int \frac{2x^5-2x^3+x^2}{1-x^4}dx$ (*Javob*: $\frac{1}{4}ln|x+1| - \frac{1}{4}ln|x-1| - x^2 - -\frac{1}{2}arctgx + C.$)

4.15. $\int \frac{x^4}{x^4+5x^2+4}dx$ $\left(Javob: x + \frac{1}{3}arctgx - \frac{8}{3}arctg\frac{x}{2} + C.\right)$

4.16. $\int \frac{x^3-2x+5}{x^4-1}dx$ (*Javob*: $ln|x-1| - \frac{3}{2}ln|x+1| + +\frac{3}{4}ln|x^2+1| - -\frac{5}{2}arctgx + C.$)

4.17. $\int \frac{x^3+4x-3}{x^4+4x^2}dx$ (*Javob*: $ln|x| + \frac{3}{4x} + \frac{3}{8}arctg\frac{x}{2} + C.$)

4.18. $\int \frac{7x-2}{(x-1)(x^2+4)}dx$ $\left(Javob: ln|x-1| - \frac{1}{x-1} - \frac{1}{2}ln|x^2+4| - arctg\frac{x}{2} + +C.\right)$

4.19. $\int \frac{x^3+2x^2+4x-2}{x^4+3x^2-4}dx$ (*Javob*: $\frac{1}{2}ln|x-1| + \frac{1}{2}ln|x+1| + arctg\frac{x}{2} + C.$)

4.20. $\int \frac{4x^2-2}{x^4-x^2}dx$ $\left(Javob: ln|x-1| - \frac{2}{x} - ln|x+1| + C.\right)$

4.21. $\int \frac{2x^3-2x-5}{x^4+3x^2-4}dx$ (*Javob*: $\frac{1}{2}ln|x+1| - \frac{1}{2}ln|x-1| + ln|x^2+ +4| + \frac{1}{2}arctg\frac{x}{2} + C.$)

4.22. $\int \frac{3x-8}{(x-1)^2(x^2+4)}dx$ (*Javob*: $ln|x-1| + \frac{1}{x-1} - \frac{1}{2}ln|x^2+4| + C.$)

4.23. $\int \frac{x^2dx}{x^4+5x^2+4}dx$ (*Javob*: $\frac{2}{3}arctg\frac{x}{2} - \frac{1}{3}arctgx + C.$)

4.24. $\int \frac{2-8x}{x^4+4x^2}dx$ (*Javob*: $ln|x^2+4| - 2\,ln|x| - \frac{1}{2x} - \frac{1}{4}arctg\frac{x}{2} + +C.$)

4.25. $\int \frac{x^3-x^2+4x}{x^4-1}dx$ $\left(Javob: ln|x-1| + \frac{3}{2}ln|x+1| - \frac{3}{4}ln|x^2+1| - -\frac{1}{2}arctgx + C.\right)$

4.26. $\int \frac{2x^3+8x-3x^2-27}{x^4+13x^2+36}dx$ (*Javob*: $ln|x^2+9| - \frac{3}{2}arctg\frac{x}{2} + C.$)

4.27. $\int \frac{5x^3-x^2+21x-9}{x^4+10x^2+9}dx$ (*Javob:* $\frac{3}{2}ln|x^2+9|+ln|x^2+1|-$ $arctgx++C.$)

4.28. $\int \frac{2x^5-2x^3-x^2}{1-x^4}dx$ (*Javob:* $\frac{1}{4}ln|x-1|-\frac{1}{4}ln|x+1|-x^2+$ $+ln|x^2+1|+\frac{1}{2}arctgx+C.$)

4.29. $\int \frac{x^3+x^2+x-1}{x^4+5x^2+4}dx$ (*Javob:* $\frac{1}{2}ln|x^2+4|+\frac{5}{6}arctg\frac{x}{2}-\frac{2}{3}arctgx+$ $+C.$)

4.30. $\int \frac{(2x+3)dx}{(x-1)(x^3-x^2+4x-4)}$ (*Javob:* $-\frac{1}{x-1}-\frac{1}{2}arctg\frac{x}{2}+C.$)

5.

5.1. $\int \frac{dx}{2+\sqrt{x+3}}$. (*Javob:* $2\sqrt{x+3}-4ln|\sqrt{x+3}+2|+C.$)

5.2. $\int \frac{xdx}{\sqrt{x+3}}$. (*Javob:* $\frac{2}{3}\sqrt{(x+3)^3}-6\sqrt{x+3}+C.$)

5.3. $\int \frac{x^2dx}{\sqrt{x-3}}$ (*Javob* : $\frac{2}{5}\sqrt{(x-3)^5}-4\sqrt{(x-3)^3}+18\sqrt{x-3}++C.$)

5.4. $\int \frac{xdx}{2+\sqrt{x+4}}$. (*Javob:* $\frac{2}{3}\sqrt{(x+4)^3}-2(x+4)+2\sqrt{x+4}-$ $-4ln|\sqrt{x+4}+2|+C.$)

5.5. $\int \frac{x^3dx}{\sqrt{x+1}}$. (*Javob:* $\frac{2}{7}\sqrt{(x+1)^7}-\frac{18}{5}\sqrt{(x+1)^5}++9\sqrt{(x+1)^3}-$ $54\sqrt{x+1}+C.$)

5.6. $\int \frac{x+1}{x\sqrt{x+2}}dx$. (*Javob:* $2\sqrt{x+2}+\frac{1}{\sqrt{2}}ln\left|\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right|+C.$)

5.7. $\int \frac{dx}{(x+1)\sqrt{x+4}}$. (*Javob:* $\frac{1}{\sqrt{3}}ln\left|\frac{\sqrt{x+4}-\sqrt{3}}{\sqrt{x+4}+\sqrt{3}}\right|+C.$)

5.8. $\int \frac{\sqrt{x+2}}{x-3}dx$. (*Javob:* $2\sqrt{x+2}+\sqrt{5}ln\left|\frac{\sqrt{x+2}-\sqrt{5}}{\sqrt{x+2}+\sqrt{5}}\right|+C.$)

5.9. $\int \frac{dx}{\sqrt{x}+3}$. (*Javob:* $2\sqrt{x}-6ln|\sqrt{x}+3|+C.$)

5.10. $\int \frac{dx}{\sqrt{x}(x+3)}$. (*Javob:* $\frac{2}{\sqrt{3}}arctg\sqrt{\frac{x}{3}}+C.$)

5.11. $\int \frac{1+x}{x+\sqrt{x}}dx$. (*Javob:* $\frac{2}{3}\sqrt{x^3}-x+4\sqrt{x}-4ln|\sqrt{x}+1|+C.$)

5.12. $\int \frac{xdx}{\sqrt{x-1}}$. (*Javob:* $\frac{2}{3}\sqrt{(x-1)^3}+2\sqrt{x-1}+C.$)

5.13. $\int \frac{\sqrt{x}dx}{x-1}$. (*Javob:* $2\sqrt{x}+ln\left|\frac{\sqrt{x}-1}{\sqrt{x}+1}\right|+C.$)

5.14. $\int \frac{dx}{3+\sqrt{x+5}}$. (*Javob:* $2\sqrt{x+5}-6ln|\sqrt{x+5}+3|+C.$)

5.15. $\int \frac{dx}{1+\sqrt{x-1}}$. (*Javob:* $2\sqrt{x-1}-2ln|1+\sqrt{x-1}|+C.$)

5.16. $\int \frac{dx}{x\sqrt{x-7}}$. (*Javob:* $2arctg\sqrt{x-7}+C.$)

5.17. $\frac{x+1}{x\sqrt{x-1}}dx$. (*Javob:* $2\sqrt{x-1}+2arctg\sqrt{x-1}+C.$)

5.18. $\int \frac{x^3dx}{\sqrt{x-7}}$ (*Javob:* $\frac{2}{7}\sqrt{(x-7)^7} + \frac{6}{5}\sqrt{(x-7)^5} + +2\sqrt{(x-7)^3} + 2\sqrt{x-7} + C.$)

5.19. $\int \frac{x^2dx}{\sqrt{x-4}}$ (*Javob:* $\frac{2}{5}\sqrt{(x-4)^5} + \frac{4}{3}\sqrt{(x-4)^3} + 2\sqrt{x-4} + +C.$)

5.20. $\int \frac{\sqrt{x+4}}{x}dx$. (*Javob:* $2\sqrt{x+4} - 2arctg\sqrt{x+4} + C.$)

5.21. $\int \frac{x^3dx}{\sqrt{x+2}}$ (*Javob:* $\frac{2}{7}\sqrt{(x+2)^7} - \frac{6}{5}\sqrt{(x+2)^5} + +2\sqrt{(x+2)^3} - 2\sqrt{x+2} + C.$)

5.22. $\int \frac{\sqrt{x}dx}{x+10}$ (*Javob:* $2\sqrt{x} - 2\sqrt{10}arctg\sqrt{\frac{x}{10}} + C.$)

5.23. $\int \frac{dx}{\sqrt{x}(x-1)}$ (*Javob:* $ln\left|\frac{\sqrt{x}-1}{\sqrt{x}+1}\right| + C.$)

5.24. $\int \frac{dx}{1+\sqrt{x-2}}$ (*Javob:* $2\sqrt{x-2} - 2ln|1+\sqrt{x-2}| + C.$)

5.25. $\int \frac{dx}{x\sqrt{x-2}}$ (*Javob:* $\sqrt{2}arctg\sqrt{\frac{x-2}{2}} + C.$)

5.26. $\int \frac{x^2dx}{\sqrt{x-2}}$ (*Javob:* $\frac{2}{5}\sqrt{(x-2)^5} + \frac{8}{3}\sqrt{(x-2)^3} + 8\sqrt{x-2} + +C.$)

5.27. $\int \frac{x-1}{x\sqrt{x-2}}dx$. (*Javob:* $2\sqrt{x-2} - \sqrt{2}arctg\sqrt{\frac{x-2}{2}} + C.$)

5.28. $\int \frac{x^3dx}{\sqrt{x+6}}$ (*Javob:* $\frac{2}{7}\sqrt{(x+6)^7} + \frac{12}{5}\sqrt{(x+6)^5} + +8\sqrt{(x+6)^3} + 16\sqrt{x+6} + C.$)

5.29. $\int \frac{dx}{3+\sqrt{x-6}}$ (*Javob* : $2\sqrt{x-6} - 6ln|\sqrt{x-6}+3| + C.$)

5.30. $\int \frac{dx}{2+\sqrt{x-8}}$ (*Javob:* $2\sqrt{x-8} - 4ln|\sqrt{x-8}+2| + C.$)

6.

6.1. $\int \frac{1-\sqrt{x+1}}{(1+\sqrt[3]{x+1})\sqrt{x+1}}dx$. (*Javob:* $3\sqrt[3]{x+1} - \frac{3}{2}\sqrt[3]{(x+1)^2} + 6\sqrt[6]{x+1} - -3ln|\sqrt[3]{x+1}+1| - 6arctg\sqrt[6]{x+1} + C.$)

6.2. $\int \frac{\sqrt[4]{x}+\sqrt{x}}{\sqrt{x}+1}dx$. (*Javob:* $x + \frac{4}{3}\sqrt[4]{x^3} - 2\sqrt{x} - 4\sqrt[4]{x} + +2ln|\sqrt{x}+1| + 4arctg\sqrt[4]{x} + C.$)

6.3. $\int \frac{\sqrt[3]{(x+1)^2}+\sqrt[6]{x+1}}{\sqrt{x+1}+\sqrt[3]{x+1}}dx$. (*Javob:* $\frac{6}{7}\sqrt[6]{(x+1)^7} - (x+1) + \frac{6}{5}\sqrt[6]{(x+1)^5} + C.$)

6.4. $\int \frac{(\sqrt[3]{x}+1)(\sqrt{x}+1)}{\sqrt[6]{x^5}}dx$. (*Javob:* $x + \frac{3}{2}\sqrt[3]{x^2} + 2\sqrt{x} + 6\sqrt[6]{x} + C.$)

6.5. $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx$. (*Javob:* $\frac{3}{2}\sqrt[3]{x^2} + 6arctg\sqrt[6]{x} + C.$)

6.6. $\int \frac{\sqrt{2x+1}+\sqrt[3]{2x+1}}{\sqrt{2x+1}}dx$. (*Javob:* $\frac{1}{2}(2x+1)+\frac{3}{5}\sqrt[6]{(2x+1)^5}+C$.)

6.7. $\int \frac{\sqrt{x-1}dx}{\sqrt[3]{x-1}+\sqrt[6]{x-1}}$ (*Javob:* $\frac{6}{7}\sqrt[6]{(x-1)^7}-(x-1)+\frac{6}{5}\sqrt[6]{(x-1)^5}-$
$-\frac{3}{2}\sqrt[3]{(x-1)^2}+2\sqrt{x-1}-3\sqrt[3]{x-1}+6\sqrt[6]{x-1}-6ln|\sqrt[6]{x-1}+1|+C$.)

6.8. $\int \frac{\sqrt{x-1}-2\sqrt[3]{x-1}}{2\sqrt[3]{x-1}+\sqrt{x-1}}dx$. (*Javob:* $x-1-\frac{24}{5}\sqrt[6]{(x-1)^5}+12\sqrt[3]{(x-1)^2}+$
$+96\sqrt[3]{x-1}-384\sqrt[6]{x-1}+768ln|\sqrt[6]{x-1}+2|+C$.)

6.9. $\int \frac{\sqrt{x+3}dx}{\sqrt[3]{x+3}+\sqrt[6]{x+3}}$ (*Javob:* $\frac{6}{7}\sqrt[6]{(x+3)^7}-(x+3)+\frac{6}{5}\sqrt[6]{(x+3)^5}-$
$-\frac{3}{2}\sqrt[3]{(x+3)^2}+2\sqrt{x+3}-3\sqrt[3]{x+3}+6\sqrt[6]{x+3}-6ln|\sqrt[6]{x+3}+1|+C$.)

6.10. $\int \frac{\sqrt[6]{x-1}dx}{\sqrt[3]{x-1}+\sqrt{x-1}}$ (*Javob:* $\frac{2}{3}\sqrt[3]{(x-1)^2}-2\sqrt{x-1}+3\sqrt[3]{x-1}-$
$-6\sqrt[6]{x-1}+6ln|\sqrt[6]{x-1}+1|+C$.)

6.11. $\int \frac{\sqrt{x+3}dx}{1+\sqrt[3]{x+3}}$ (*Javob:* $\frac{6}{7}\sqrt[6]{(x+3)^7}-\frac{6}{5}\sqrt[6]{(x+3)^5}+2\sqrt{x+3}-$
$-6\sqrt[6]{x+3}-arctg\sqrt[6]{x+3}+C$.)

6.12. $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}+\sqrt[6]{x}}dx$. (*Javob:* $x+\frac{6}{5}\sqrt[6]{x^5}-\frac{3}{2}\sqrt[3]{x^2}-2\sqrt{x}+3\sqrt[3]{x}+$
$+6\sqrt[6]{x}-\frac{1}{2}ln|\sqrt[3]{x}+1|-6arctg\sqrt[6]{x}+C$.)

6.13. $\int \frac{\sqrt[6]{x+3}dx}{\sqrt[3]{x+3}+\sqrt{x+3}}$ (*Javob:* $\frac{3}{2}\sqrt[3]{(x+3)^2}-2\sqrt{x+3}+3\sqrt[3]{x+3}-$
$6\sqrt[6]{x+3}++6ln|\sqrt[6]{x+3}+1|+C$.)

6.14. $\int \frac{x+1+\sqrt[3]{(x+1)^2}+\sqrt[6]{x+1}}{(x+1)\left(1+\sqrt[3]{x+1}\right)}dx$. (*Javob:*
$\frac{3}{2}\sqrt[3]{(x+2)^2}+6arctg\sqrt[6]{x+1}+C$.)

6.15. $\int \frac{\sqrt{x}-1}{(\sqrt[3]{x}+1)\sqrt{x}}dx$. (*Javob:* $\frac{3}{2}\sqrt[3]{x^2}-3\sqrt[3]{x}-6\sqrt[6]{x}+3ln|\sqrt[3]{x}+1|+$
$+6arctg\sqrt[6]{x}+C$.)

6.16. $\int \frac{\sqrt{3x+1}+2}{\sqrt{3x+1}+2\sqrt[3]{3x+1}}dx$. (*Javob:* $\frac{1}{3}(3x+1)-\frac{4}{5}\sqrt[6]{(3x+1)^5}+$
$2\sqrt[3]{(3x+1)^2}--4\sqrt{3x+1}+12\sqrt[3]{3x+1}-48\sqrt[6]{3x+1}+$
$96ln|\sqrt[6]{3x+1}+2|+C$.)

6.17. $\int \frac{dx}{\sqrt[3]{(2x+1)^2}-\sqrt{2x+1}}$ (*Javob* : $\frac{3}{2}\sqrt[3]{2x+1}+3\sqrt[6]{2x+1}+$
$+3ln|\sqrt[6]{2x+1}-1|+C$.)

6.18. $\int \frac{\sqrt{x}-\sqrt[3]{x}}{\sqrt[3]{x}-\sqrt[6]{x}-1}dx$. (*Javob:* $\frac{6}{7}\sqrt[6]{x^7}+\frac{6}{5}\sqrt[6]{x^5}+\frac{3}{2}\sqrt[3]{x^2}+4\sqrt{x}+$
$9\sqrt[3]{x}++30\sqrt[6]{x}+\frac{54}{\sqrt{5}}ln\left|\frac{2\sqrt[6]{x}-1-\sqrt{5}}{2\sqrt[6]{x}-1+\sqrt{5}}\right|+24ln|\sqrt[3]{x}-\sqrt[6]{x}-1|+C$.)

6.19. $\int \frac{\sqrt{x}dx}{1-\sqrt[4]{x}}$ (*Javob:* $-\frac{4}{5}\sqrt[4]{x^5}-x-\frac{4}{3}\sqrt[4]{x^3}-2\sqrt{x}-4\sqrt[4]{x}-4ln|1-\sqrt[4]{x}|+ +C.$)

6.20. $\int \frac{\sqrt[6]{3x+1}+1}{\sqrt{3x+1}-\sqrt[3]{3x+1}}dx$. (*Javob:* $\frac{1}{2}\sqrt[3]{(3x+1)^2}+\frac{4}{3}\sqrt{3x+1}+2\sqrt[3]{3x+1}+ +4\sqrt[6]{3x+1}+4ln|\sqrt[6]{3x+1}-1|+C.$)

6.21. $\int \frac{\sqrt{x}dx}{x-4\sqrt[3]{x^2}}$ (*Javob:* $2\sqrt{x}+24\sqrt[6]{x}+24ln\left|\frac{\sqrt[6]{x}-2}{\sqrt[6]{x}+2}\right|+C.$)

6.22. $\int \frac{x+\sqrt{x}+\sqrt[3]{x^2}}{x(1+\sqrt[3]{x})}dx$. (*Javob:* $\frac{3}{2}x^{\frac{2}{3}}+6x^{\frac{1}{6}}-6arctg\sqrt[6]{x}+C.$)

6.23. $\int \frac{\sqrt{x}dx}{x-\sqrt[3]{x^2}}$ (*Javob:* $2\sqrt{x}+6\sqrt[6]{x}+3ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right|+C.$)

6.24. $\int \frac{\sqrt{x}dx}{3x+\sqrt[3]{x^2}}$ (*Javob:* $\frac{2}{3}\sqrt{x}-\frac{2}{3}\sqrt[6]{x}+\frac{2\sqrt{3}}{9}arctg\sqrt[6]{9x}+C.$)

6.25. $\int \frac{\sqrt{x}dx}{1-\sqrt[3]{x}}$ (*Javob:* $3ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right|-\frac{6}{7}\sqrt[6]{x^7}-\frac{6}{5}\sqrt[6]{x^5}-2\sqrt{x}-6\sqrt[6]{x}+ +C.$)

6.26. $\int \frac{x-\sqrt[3]{x^2}}{x(1+\sqrt[6]{x})}dx$. (*Javob:* $\frac{1}{4}\sqrt[3]{x^2}-\sqrt[3]{x}+ln|\sqrt[3]{x}+1|+C.$)

6.27. $\int \frac{\sqrt{x}dx}{1+\sqrt[4]{x}}$ (*Javob:* $\frac{4}{5}\sqrt[4]{x^5}-x+\frac{4}{3}\sqrt[4]{x^3}-2\sqrt{x}+4\sqrt[6]{x}- -4ln|\sqrt[4]{x}+1|+C.$)

6.28. $\int \frac{\sqrt{3x+1}-1}{\sqrt[3]{3x+1}+\sqrt{3x+1}}dx$. (*Javob:* $\frac{1}{3}(3x+1)-\frac{2}{5}\sqrt[6]{(3x+1)^5}+\frac{1}{2}\sqrt[3]{(3x+1)^2}- -\frac{4}{3}\sqrt{3x+1}+2\sqrt[3]{3x+1}-4\sqrt[6]{3x+1}+4ln|\sqrt[6]{3x+1}+1|+C.$)

6.29. $\int \frac{\sqrt{x}dx}{4x-\sqrt[3]{x^2}}$ (*Javob:* $\frac{1}{2}\sqrt{x}+\frac{3}{8}\sqrt[6]{x}+\frac{3}{32}ln\left|\frac{2\sqrt[6]{x}-1}{2\sqrt[6]{x}+1}\right|+C.$)

6.30. $\int \frac{\sqrt{x+1}-1}{(\sqrt[3]{x+1}+1)\sqrt{x+1}}dx$. (*Javob:* $\frac{3}{2}\sqrt[3]{(x+1)^2}-3\sqrt[3]{x+1}-6\sqrt[6]{x+1}+ +3ln|\sqrt[3]{x+1}+1|+6arctg\sqrt[6]{x+1}+C.$)

7.

7.1. $\int \frac{dx}{5+2sinx+3cosx}$ (*Javob:* $\frac{1}{\sqrt{3}}arctg\left(\frac{tg\frac{x}{2}+1}{\sqrt{3}}\right)+C.$)

7.2. $\int \frac{dx}{5-4sinx+2cosx}$ (*Javob:* $\frac{2}{\sqrt{5}}arctg\left(\frac{3tg\frac{x}{2}-4}{\sqrt{5}}\right)+C.$)

7.3. $\int \frac{3sinx-2cosx}{1+cosx}dx$. (*Javob:* $2tg\frac{x}{2}+3ln\left|tg^2\frac{x}{2}+1\right|-4arctg\frac{x}{2}+C.$)

7.4. $\int \frac{dx}{5+3cosx-5sinx}$ (*Javob:* $\frac{1}{3}ln\left|\frac{tg\frac{x}{2}-4}{tg\frac{x}{2}-1}\right|+C.$)

7.5. $\int \frac{dx}{5\cos x+10\sin x}$. (*Javob:* $\left(-\frac{1}{5\sqrt{5}}\right) \ln \left|\frac{tg\frac{x}{2}-2-\sqrt{5}}{tg\frac{x}{2}-2+\sqrt{5}}\right| + C$.)

7.6. $\int \frac{dx}{3+2\cos x-\sin x}$. (*Javob:* $arctg\left(\frac{tg\frac{x}{2}-1}{2}\right) + C$.)

7.7. $\int \frac{dx}{5-3\cos x}$. (*Javob:* $\frac{1}{2} arctg\left(2tg\frac{x}{2}\right) + S$.)

7.8. $\int \frac{dx}{8-4\sin x+7\cos x}$. (*Javob:* $\ln\left|\frac{tg\frac{x}{2}-5}{tg\frac{x}{2}-3}\right| + C$.)

7.9. $\int \frac{dx}{3+5\cos x}$ (*Javob:* $-\frac{1}{4}\ln\left|\frac{tg\frac{x}{2}-2}{tg\frac{x}{2}+2}\right| + C$.)

7.10. $\int \frac{dx}{2\sin x+3\cos x+3}$. (*Javob:* $\frac{1}{2}\ln\left|2tg\frac{x}{2}+3\right| + C$.)

7.11. $\int \frac{dx}{5+4\sin x}$. (*Javob:* $\frac{2}{3} arctg\left(\frac{5tg\frac{x}{2}+4}{3}\right) + C$.)

7.12. $\int \frac{dx}{8+4\cos x}$. (*Javob:* $\frac{1}{2\sqrt{3}} arctg\left(\frac{tg\frac{x}{2}}{\sqrt{3}}\right) + C$.)

7.13. $\int \frac{dx}{3\sin x-4\cos x}$. (*Javob:* $\frac{1}{5}\ln\left|\frac{tg\frac{x}{2}-\frac{1}{2}}{tg\frac{x}{2}+2}\right| + C$.)

7.14. $\int \frac{dx}{7\sin x-3\cos x}$. (*Javob:* $\frac{1}{\sqrt{58}}\ln\left|\frac{3tg\frac{x}{2}+7-\sqrt{58}}{3tg\frac{x}{2}+7+\sqrt{58}}\right| + C$.)

7.15. $\int \frac{dx}{2+4\sin x+3\cos x}$. (*Javob:* $-\frac{1}{\sqrt{21}}\ln\left|\frac{tg\frac{x}{2}-4-\sqrt{21}}{tg\frac{x}{2}-4+\sqrt{21}}\right| + C$.)

7.16. $\int \frac{dx}{4\cos x+3\sin x}$. (*Javob:* $-\frac{1}{5}\ln\left|\frac{tg\frac{x}{2}-2}{tg\frac{x}{2}-\frac{1}{2}}\right| + C$.)

7.17. $\int \frac{2-\sin x+3\cos x}{1+\cos x}dx$. (*Javob:* $3x - tg\frac{x}{2} - \ln\left|tg^2\frac{x}{2}+1\right| + C$.)

7.18. $\int \frac{dx}{5+\sin x+3\cos x}$. (*Javob:* $\frac{2}{\sqrt{15}} arctg\left(\frac{2tg\frac{x}{2}+1}{\sqrt{15}}\right) + C$.)

7.19. $\int \frac{dx}{4\sin x+3\cos x+5}$. (*Javob:* $C - \frac{1}{tg\frac{x}{2}+2}$.)

7.20. $\int \frac{7+6\sin x-5\cos x}{1+\cos x}dx$. (*Javob:* $12tg\frac{x}{2} + 6\ln\left|tg^2\frac{x}{2}+1\right| - 5x + C$.)

7.21. $\int \frac{dx}{3+\cos x+\sin x}$. (*Javob:* $\frac{2}{\sqrt{7}} arctg\left(\frac{2tg\frac{x}{2}+1}{\sqrt{7}}\right) + C$.)

7.22. $\int \frac{6\sin x+\cos x}{1+\cos x}dx$. (*Javob:* $6\ln\left|tg^2\frac{x}{2}+1\right| - tg\frac{x}{2} + x + C$.)

7.23. $\int \frac{dx}{3\cos x-4\sin x}$. (*Javob:* $C - \frac{1}{5}\ln\left|\frac{tg\frac{x}{2}-\frac{1}{2}}{tg\frac{x}{2}+3}\right|$.)

7.24. $\int \frac{dx}{5+3\cos x}$. (*Javob:* $\frac{1}{2} arctg\left(\frac{tg\frac{x}{2}}{2}\right) + C$.)

7.25. $\int \frac{dx}{4\sin x-6\cos x}$. (*Javob:* $\frac{1}{2\sqrt{13}}\ln\left|\frac{3tg\frac{x}{2}+2-\sqrt{13}}{3tg\frac{x}{2}+2+\sqrt{13}}\right| + C$.)

7.26. $\int \frac{dx}{3+5sinx+3cosx}$. (*Javob:* $\frac{1}{5}ln\left|5tg\frac{x}{2}+3\right| + C.$)

7.27. $\int \frac{dx}{cosx-3sinx}$. (*Javob:* $\frac{1}{3}arctg\left(\frac{tg\frac{x}{2}}{\sqrt{3}}\right) + C.$)

7.28. $\int \frac{dx}{4-4sinx+3cosx}$. (*Javob:* $\frac{1}{3}ln\left|\frac{tg\frac{x}{2}-7}{tg\frac{x}{2}-1}\right| + C.$)

7.29. $\int \frac{dx}{3sinx-cosx}$. (*Javob:* $\frac{1}{\sqrt{10}}ln\left|\frac{tg\frac{x}{2}+3-\sqrt{10}}{tg\frac{x}{2}+3+\sqrt{10}}\right| + C.$)

7.30. $\int \frac{dx}{2-3cosx+sinx}$. (*Javob:* $\frac{1}{\sqrt{6}}ln\left|\frac{5tg\frac{x}{2}+1-\sqrt{6}}{5tg\frac{x}{2}+1+\sqrt{6}}\right| + C.$)

88.1. $\int \frac{dx}{8sin^2x-16sinxcosx}$. (*Javob:* $\frac{1}{16}ln\left|\frac{tgx-2}{tgx}\right| + C.$)

8.2. $\int \frac{dx}{16sin^2x-8sinxcosx}$. (*Javob:* $\frac{1}{8}ln\left|\frac{2tgx-1}{2tgx}\right| + C.$)

8.3. $\int \frac{dx}{1+3cos^2x}$. (*Javob* $\frac{1}{2}arctg\frac{tgx}{2} + C.$)

8.4. $\int \frac{2tgx+3}{sin^2x+2cos^2x}dx$. (*Javob:* $ln|tg^2x+2| + \frac{3}{\sqrt{2}}arctg\left(\frac{tgx}{\sqrt{2}}\right) + C.$)

8.5. $\int \frac{dx}{3cos^2x+4sin^2x}$. (*Javob:* $\frac{1}{2\sqrt{3}}arctg\left(\frac{2tgx}{\sqrt{3}}\right) + C.$)

8.6. $\int \frac{tgx}{1-ctg^2x}dx$. (*Javob:* $\frac{1}{4}ln|tg^4x-1| + C.$)

8.7. $\int \frac{dx}{4sin^2x-5cos^2x}$. (*Javob:* $\frac{1}{4\sqrt{5}}ln\left|\frac{2tgx-\sqrt{5}}{2tgx+\sqrt{5}}\right| + C.$)

8.8. $\int \frac{dx}{7cos^2x+2sin^2x}$. (*Javob:* $\frac{1}{\sqrt{14}}arctg\left(\frac{\sqrt{2}tgx}{\sqrt{7}}\right) + C.$)

8.9. $\int \frac{sin(2x)}{sin^4x+cos^4x}dx$. (*Javob:* $arctg(tg^2x) + C.$)

8.10. $\int \frac{dx}{cosxsin^3x}$. (*Javob:* $\frac{1}{2tg^2x} + ln|tgx| + C.$)

8.11. $\int \frac{dx}{1+sin^2x}$. (*Javob:* $\frac{1}{\sqrt{2}}arctg(\sqrt{2}tgx) + C.$)

8.12. $\int \frac{dx}{4sin^2x+8sinxcosx}$. (*Javob:* $\frac{1}{8}ln\left|\frac{tgx}{tgx+2}\right| + C.$)

8.13. $\int \frac{sin2x}{4sin^4x+cos^4x}dx$. (*Javob:* $\frac{1}{2}arctg(2tg^2x) + C.$)

8.14. $\int \frac{dx}{sin^2x-4sinxcosx+5cos^2x}$. (*Javob:* $arctg(tgx-2) + C.$)

8.15. $\int \frac{dx}{4cos^2x+3sin^2x}$. (*Javob:* $\frac{1}{2\sqrt{3}}arctg\left(\frac{\sqrt{3}tgx}{2}\right) + C.$)

8.16. $\int \frac{dx}{3cos^2x-2}$. (*Javob:* $\frac{1}{2\sqrt{2}}ln\left|\frac{1+\sqrt{2}tgx}{1-\sqrt{2}tgx}\right| + C.$)

8.17. $\int \frac{dx}{sin^2x+sin2x+3cos^2x}$. (*Javob:* $\frac{1}{\sqrt{2}}arctg\left(\frac{tgx+1}{\sqrt{2}}\right) + C.$)

8.18. $\int \frac{dx}{5sin^2x-3cos^2x}$. (*Javob.* $\frac{1}{2\sqrt{15}}ln\left|\frac{\sqrt{5}tgx-\sqrt{3}}{\sqrt{5}tgx+\sqrt{3}}\right| + C.$)

8.19. $\int \frac{dx}{sin^2x+3sinxcosx-cos^2x}$. (*Javob:* $\frac{1}{\sqrt{13}}ln\left|\frac{2tgx+3-\sqrt{13}}{2tgx+3+\sqrt{13}}\right| + C.$)

8.20. $\int \frac{\sin 2x}{\sin^4 x+4\cos^4 x}dx$. (Javob: $\frac{1}{2}arctg\left(\frac{tg^2x}{2}\right)+C$.)

8.21. $\int \frac{dx}{7\cos^2 x+16\sin^2 x}$. (Javob: $\frac{1}{4\sqrt{7}}arctg\left(\frac{4tgx}{\sqrt{7}}\right)+C$.)

8.22. $\int \frac{dx}{2\cos^2 x+3}$. (Javob: $\frac{1}{\sqrt{5}}arctg\left(\frac{\sqrt{3}tgx}{\sqrt{5}}\right)+C$.)

8.23. $\int \frac{dx}{3-2\sin^2 x}$. (Javob: $\frac{1}{\sqrt{3}}arctg\left(\frac{tgx}{\sqrt{3}}\right)+C$.)

8.24. $\int \frac{3tgx-1}{\sin^2 x+4\cos^2 x}dx$. (Javob. $\frac{3}{2}\ln(tg^2x+4)-\frac{1}{2}arctg\left(\frac{tgx}{2}\right)+C$.)

8.25. $\int \frac{dx}{5+3\sin^2 x}$. (Javob: $\frac{1}{2\sqrt{10}}arctg\left(\frac{2\sqrt{2}tgx}{\sqrt{5}}\right)+C$.)

8.26. $\int \frac{\cos^2 x}{1-\sin^4 x}dx$. (Javob: $\frac{1}{\sqrt{2}}arctg\left(\sqrt{2}tgx\right)+C$.)

8.27. $\int \frac{dx}{2\sin^2 x-\sin 2x+\cos^2 x}$. (Javob: $arctg(2tgx-1)+C$.)

8.28. $\int \frac{dx}{6-3\cos^2 x}$. (Javob: $\frac{1}{6}arctg(2tgx)+C$.)

8.29. $\int \frac{tgx}{\sin^2 x+3\cos^2 x}dx$. (Javob: $\frac{1}{2}\ln|tg^2x+3|+C$.)

8.30. $\int \frac{\sin^2 x}{3\sin^2 x-\cos^2 x}dx$. (Javob: $\frac{1}{3}tgx+\frac{\sqrt{3}}{9}arctg\left(\sqrt{3}tgx\right)+C$.)

9.1. $\int \cos^4 3x\sin^2 3x dx$. (Javob: $\frac{1}{16}x-\frac{1}{192}\sin 12x+\frac{1}{144}\sin^3 6x+C$.)

9.2. $\int \sqrt[5]{\sin^4 x}\cos^3 x dx$. (Javob: $\frac{5}{9}\sqrt[5]{\sin^9 x}-\frac{5}{19}\sqrt[5]{\sin^{19} x}+C$.)

9.3. $\int \cos^3 x\sin^8 x dx$. (Javob: $\frac{1}{9}\sin^9 x-\frac{1}{11}\sin^{11} x+C$.)

9.4. $\int \cos^4 x\sin^3 x dx$. (Javob: $\frac{1}{7}\cos^7 x-\frac{1}{5}\cos^5 x+C$.)

9.5. $\int \frac{\cos^3 x}{\sqrt[3]{\sin^4 x}}dx$. (Javob: $C-\frac{3}{\sqrt[3]{\sin x}}-\frac{3}{5}\sqrt[3]{\sin^5 x}$.)

9.6. $\int \sqrt[5]{\sin^3 2x}\cos^3 2x dx$. (Javob: $\frac{5}{16}\sqrt[5]{\sin^8 2x}-\frac{5}{36}\sqrt[5]{\sin^{18} 2x}+C$.)

9.7. $\int \frac{\cos^3 x}{\sqrt[3]{\sin^2 x}}dx$. (Javob: $3\sqrt[3]{\sin x}-\frac{3}{7}\sqrt[3]{\sin^7 x}+C$.)

9.8. $\int \frac{\sin^3 x}{\sqrt[3]{\cos^4 x}}dx$. (Javob: $3\left(\frac{1}{\sqrt[3]{\cos x}}\right)+\frac{3}{5}\sqrt[3]{\cos^5 x}+C$.)

9.9. $\int \frac{3\sin^3 x}{\cos^4 x}dx$. (Javob: $\frac{1}{\cos^3 x}-\frac{3}{\cos x}+C$.)

9.10. $\int \sin^5 x\cos^4 x dx$. (Javob: $\frac{2}{7}\cos^7 x-\frac{1}{5}\cos^5 x-\frac{1}{9}\cos^9 x+C$.)

9.11. $\int \frac{\sin^3 x}{\sqrt[5]{\cos^3 x}}dx$. (Javob: $\frac{5}{12}\sqrt[5]{\cos^{12} x}-\frac{5}{2}\sqrt[5]{\cos^2 x}+C$.)

9.12. $\int \sqrt[3]{\cos^2 x}\sin^3 x dx$. (Javob: $\frac{3}{11}\sqrt[3]{\cos^{11} x}-\frac{3}{5}\sqrt[3]{\cos^5 x}+C$.)

9.13. $\int \sqrt[3]{\sin^2 x}\cos^3 x dx$. (Javob: $\frac{3}{5}\sqrt[3]{\sin^5 x}-\frac{3}{11}\sqrt[3]{\sin^{11} x}+C$.)

9.14. $\int \sqrt[5]{\cos^3 2x}\sin^3 2x dx$. (Javob: $\frac{5}{36}\sqrt[5]{\cos^{18} 2x}-\frac{5}{16}\sqrt[5]{\cos^8 2x}+C$.)

9.15. $\int \frac{\cos^3 x dx}{\sqrt[5]{\sin^3 x}}$. (Javob: $\frac{5}{2}\sqrt[5]{\sin^2 x}-\frac{5}{12}\sqrt[5]{\sin^{12} x}+C$.)

9.16. $\int sin^2 2x cos^4 2x dx$. (*Javob:* $\frac{1}{16}x - \frac{1}{128}sin8x + \frac{1}{96}sin^3 4x + C$.)

9.17. $\int \frac{sin^3 x}{\sqrt[3]{cos^2 x}}dx$. (*Javob* : $\frac{3}{7}\sqrt[3]{cos^7 x} - 3\sqrt[3]{cosx} + C$.)

9.18. $\int \sqrt[5]{cos^4 x}sin^3 x dx$. (*Javob:* $\frac{5}{19}\sqrt[5]{cos^{19} x} - \frac{5}{9}\sqrt[5]{cos^9 x} + C$.)

9.19. $\int sin^4 2x cos^2 2x dx$. (*Javob:* $\frac{1}{16}x - \frac{1}{128}sin8x - \frac{1}{96}sin^3 4x + C$.)

9.20. $\int \frac{cos^3 2x}{\sqrt[3]{sin^2 2x}}dx$. (*Javob:* $\frac{3}{2}\sqrt[3]{sin2x} - \frac{3}{14}\sqrt[3]{sin^7 2x} + C$.)

9.21. $\int \frac{sin^3 2x}{\sqrt[3]{cos^2 2x}}dx$. (*Javob:* $\frac{3}{14}\sqrt[3]{cos^7 2x} - \frac{3}{2}\sqrt[3]{cos2x} + C$.)

9.22. $\int sin^4 x cos^3 x dx$. (*Javob:* $\frac{1}{5}sin^5 x - \frac{1}{7}sin^7 x + C$.)

9.23. $\int sin^2 x cos^4 x dx$. (*Javob:* $\frac{1}{16}x - \frac{1}{64}sin4x + \frac{1}{48}sin^3 2x + C$.)

9.24. $\int sin^4 x cos^2 x dx$. (*Javob:* $\frac{1}{16}x - \frac{1}{64}sin4x - \frac{1}{48}sin^3 2x + C$.)

9.25. $\int sin^3 x cos^8 x dx$. (*Javob:* $\frac{1}{11}cos^{11} x - \frac{1}{9}cos^9 x + C$.)

9.26. $\int \frac{3cos^3 x}{sin^4 x}dx$. (*Javob:* $\frac{3}{sinx} - \frac{1}{sin^3 x} + C$.)

9.27. $\int sin^5 x \sqrt[5]{cos^3 x}dx$. (*Javob:* $\frac{5}{9}\sqrt[5]{cos^{18} x} - \frac{5}{8}\sqrt[5]{cos^8 x} - \frac{5}{28}\sqrt[5]{cos^{28} x} + C$.)

9.28. $\int sin^4 x cos^5 x dx$. (*Javob:* $\frac{1}{5}sin^5 x - \frac{2}{7}sin^7 x + \frac{1}{9}sin^9 x + C$.)

9.29. $\int sin^4 3x cos^2 3x dx$. (*Javob:* $\frac{1}{16}x - \frac{1}{192}sin12x - \frac{1}{144}sin^3 6x + C$.)

9.30. $\int \frac{sin^3 x}{\sqrt[3]{cos^4 x}}dx$. (*Javob:* $\frac{3}{\sqrt[3]{cosx}} + \frac{3}{5}\sqrt[3]{cos^5 x} + C$.).

Namunaviy variant yechimi

Aniqmas integrallarni hisoblang.

1. $\int \frac{7x-x^2-4}{(x+1)(x^2-5x+6)}dx$.

► Integral ostidagi funksiya ratsional kasrdan iborat. Uning maxrajini ko‘paytuvchilarga ajratamiz: $(x+1)(x-2)(x-3)$. (8.9) formulaga asosan maxrajdagi har bir $(x-a)$ ko‘paytuvchiga bitta $\frac{A}{x-a}$ qo‘shiluvchi mos keladi. Shuning uchun bizning holimizda

$$\frac{7x-x^2-4}{(x+1)(x^2-5x+6)} = \frac{7x-x^2-4}{(x+1)(x-2)(x-3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-3}.$$

Oxirgi tenglikning o‘ng tomonini umumiy maxrajga keltirib va kasrlarning suratlarini tenglashtirib topsak,

$$7x - x^2 - 4 \equiv A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)$$

ayniyatni hosil qilamiz. A, V, S koffitsientlarni xususiy qiymatlar usuli bilan topamiz (§8.6 ni qarang):

$$\left.\begin{array}{l|r} x=-1 & -12=12A, \\ x=2 & 6=-3V, \\ x=3 & 8=4S, \end{array}\right|.$$

Bu yerdan $A=-1$, $V=-2$, $S=2$. Topilgan koeffitsientlarni integral ostidagi funksiyaning eng sodda kasrlarga yoyilmasiga qo'yib integrallasak

$\int \frac{7x-x^2-4}{(x+1)(x^2-5x+6)}dx=\int \left(-\frac{1}{x+1}-\frac{2}{x-2}+\frac{2}{x-3}\right)dx = -ln|x+1| + 2ln|x-3| - -2ln|x-2| + C^* = ln\frac{(x-3)^2}{|x+1|(x-2)^2} + C^*$ ni hosil qilamiz.

Bu yerda C^* – integrallash doimiysi. ◄

2. $\int \frac{15x-x^2-11}{(x-1)(x^2+x-2)}dx.$

► $\int \frac{15x-x^2-11}{(x-1)(x^2+x-2)}dx = \int \frac{15x-x^2-11}{(x-1)^2(x+2)}dx \overset{(8.9)}{=} \int \left(\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}\right)dx \overset{§8.6}{=}$

$$15x - x^2 - 11 \equiv A(x-1)(x+2) + B(x+2) + C(x-1)^2,$$
$$x=1 \quad 3=3V, \qquad V=1,$$
$$= x=-2 \; -45=9S, \; S=-5, =$$
$$x^2 \; -1=A+S, \; A=4$$
$$=\int \left(\frac{4}{x-1}+\frac{1}{(x-1)^2}-\frac{5}{x+2}\right)dx = -5ln|x+2| + 4ln|x-1| - \frac{1}{x-1} + C^*.$$

Shuni ta'kidlash lozimki, koeffitsientlarni topish uchun xususiy qiymatlar usuli va noma'lum koeffitsientlar usuli qo'llanildi. (§ 8.6 ni qarang). ◄

3. $I(x) = \int \frac{x^4-8x^3+23x^2-43x+27}{(x-2)(x^2-2x+5)}dx.$

► Integral ostidagi funksiya noto'g'ri kasr bo'lganligi uchun kasrning suratini maxrajiga bo'lib, uni butun ko'phad va to'g'ri kasrlar yig'indisi shaklida ifodalash mumkin:

$$I(x)= \int \left(x - 4 + \frac{-2x^2+3x-13}{(x-2)(x^2-2x+5)}\right)dx \overset{§ 8.6}{=} \frac{x^2}{2} - 4x + \int \left(\frac{A}{x-2} + \frac{Bx+C}{x^2-2x+5}\right)dx =$$

$$-2x^2+3x-13 \equiv A(x^2-2x+5)+(Bx+C)(x-2),$$
$= x=2\ -15=5A,\ A=-3, =$

$x^2\ -2=A+V,\ V=1.$

$x^0\ -13=5A-2S,\ S=-1$

$$=\frac{x^2}{2}-4x+\int\left(\frac{-3}{x-2}+\frac{x-1}{x^2-2x+5}\right)dx=\frac{x^2}{2}-4x-3ln|x-2|+\frac{1}{2}ln|x^2-2x+5|+C^*. ◄$$

4. $\int \frac{2x^3-5x^2+8x-22}{x^4+9x^2+20}dx.$

$$► \int \frac{2x^3-5x^2+8x-22}{x^4+9x^2+20}dx=\int \frac{2x^3-5x^2+8x-22}{(x^2+4)(x^2+5)}dx=\int\left(\frac{Ax+B}{x^2+4}+\frac{Cx+D}{x^2+5}\right)dx=$$
$$2x^3-5x^2+8x-22 \equiv (Ax+B)(x^2+5)+(Cx+D)(x^2+4),$$
$x^3\ 2=A+S,\ A=0$

$= x^2\ -5=V+D,\ V=-2, =$

$x\ 8=5A+4S,\ S=2,$

$x^0\ -22=5V+4D,\ D=-3$

$$=\int\left(\frac{-2}{x^2+4}+\frac{2x-3}{x^2+5}\right)dx=-arctg\left(\frac{x}{2}\right)+ln(|x^2+5|)-\frac{3}{\sqrt{5}}arctg\left(\frac{x}{\sqrt{5}}\right)+C^*. ◄$$

5. $\int \frac{x+1}{3-\sqrt{x-2}}dx.$

$$► \int \frac{x+1}{3-\sqrt{x-2}}dx= \begin{matrix}\sqrt{x-2}=t & x-2=t^2\\ x=t^2+2 & dx=2tdt\end{matrix} =$$
$$=-2\int\frac{(t^2+3)tdt}{t-3}=-2\int\left(t^2+3t+12+\frac{36}{t-3}\right)dt=$$
$$=-2\left(\frac{1}{3}t^3+\frac{3}{2}t^2+12t+36ln(|t-3|)\right)+C=$$
$$=-\frac{2}{3}\sqrt{(x-2)^3}-3(x-2)-24\sqrt{x-2}-72ln\left|\sqrt{x-2}-3\right|+C. ◄$$

6. $\int \frac{4\sqrt{x-2}+\sqrt[6]{x-2}}{\sqrt{x-2}+2\sqrt[3]{x-2}}dx.$

$$► \int \frac{4\sqrt{x-2}+\sqrt[6]{x-2}}{\sqrt{x-2}+2\sqrt[3]{x-2}}dx = \begin{matrix} m=EKUK(2,3,6)=6, & x-2=t^6,\\ x=t^6+2, & dx=6t^5dt\end{matrix} =$$
$$=\int\frac{(4t^3-t)6t^5dt}{t^3+2t^2}=6\int\frac{4t^6-t^4}{t+2}dt=$$
$$=6\int\left(4t^5-8t^4+15t^3-30t^2+60t-120+\frac{240}{t+2}\right)dt=$$
$$=6\left(240ln(|t+2|)+\frac{2t^6}{3}-\frac{8t^5}{5}+\frac{15t^4}{4}-10t^3+30t^2-120t\right)+C=$$
$$=4(x-2)-\frac{48}{5}\sqrt[6]{(x-2)^5}+\frac{45}{2}\sqrt[3]{(x-2)^2}-60\sqrt{x-2}-720\sqrt[6]{x-2}+$$
$$+180\sqrt[3]{x-2}+1440ln\left|\sqrt[6]{x-2}+2\right|+C. ◄$$

7. $\int \frac{dx}{3sinx-2cosx+1}.$

$$\blacktriangleright \int \frac{dx}{3sinx-2cosx+1} = \begin{matrix} t = tg\frac{x}{2}, sinx = \frac{2t}{1+t^2}, cosx = \frac{1-t^2}{1+t^2}, \\ dx = \frac{2dt}{1+t^2}, x = 2arctgt \end{matrix} =$$

$$= 2\int \frac{dt}{6t-2+2t^2+1+t^2} = 2\int \frac{dt}{3t^2+6t-1} = \frac{2}{3}\int \frac{dt}{t^2+2t-\frac{1}{3}} = \frac{2}{3}\int \frac{dt}{(t+1)^2-\frac{4}{3}} =$$

$$= \frac{2}{3}\frac{\sqrt{3}}{4} ln\left|\frac{t+1-2/\sqrt{3}}{t+1+2/\sqrt{3}}\right| + C = \frac{1}{2\sqrt{3}} ln\left|\frac{\sqrt{3}tg\frac{x}{2}+\sqrt{3}-2}{\sqrt{3}tg\frac{x}{2}+\sqrt{3}+2}\right| + C. \blacktriangleleft$$

8. $\int \frac{dx}{2sin^2x-sin2x+3cos^2x}$.

$$\blacktriangleright \int \frac{dx}{2sin^2x-sin2x+3cos^2x} \overset{(8.14)}{=}$$

$$= \begin{matrix} t = tgx, sin^2x = \frac{t^2}{1+t^2}, cos^2x = \frac{1}{1+t^2}, \\ sin\,x\,cos\,x = \frac{t}{1+t^2}, dx = \frac{dt}{1+t^2} \end{matrix} =$$

$$= \int \frac{dt}{2t^2-2t+3} = \frac{1}{2}\int \frac{dt}{t^2-t+\frac{3}{2}} = \frac{1}{2}\int \frac{dt}{\left(t-\frac{1}{2}\right)^2+\frac{5}{4}} = \frac{1}{2}\left(\frac{2}{\sqrt{5}}\right) arctg\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right) + C =$$

$$= \frac{1}{\sqrt{5}} arctg\left(\frac{2tgx-1}{\sqrt{5}}\right) + C. \blacktriangleleft$$

9. $\int \frac{cos^3 4x}{\sqrt[5]{sin4x}} dx$.

$$\blacktriangleright \int \frac{cos^3 4x}{\sqrt[5]{sin4x}} dx = \begin{matrix} sin4x = t, \\ dt = 4cos4xdx \end{matrix} = \frac{1}{4}\int \frac{(1-t^2)dt}{\sqrt[5]{t}} = \frac{1}{4}\int (t^{-\frac{1}{5}} - t^{\frac{9}{5}})dt =$$

$$= \frac{1}{4}\left(\frac{5}{4}t^{\frac{4}{5}} - \frac{5}{14}t^{\frac{14}{5}}\right) + C = \frac{5}{16}\sqrt[5]{sin^4 4x} - \frac{5}{56}\sqrt[5]{sin^{14} 4x} + C. \blacktriangleleft$$

8.10 8- bo'limga doir qo'shimcha topshiriqlar

Aniqmas integrallarni hisoblang.

1. $\int x^2\sqrt{4-x^2}dx$. (*Javob:* $\frac{x}{4}(x^2-2)\sqrt{4-x^2} + 2arcsin\frac{x}{2} + C$.)
2. $\int \frac{dx}{(x^2+4)\sqrt{4x^2+1}}$. (*Javob:* $\frac{1}{4\sqrt{15}} ln\left|\frac{x\sqrt{15}+2\sqrt{4x^2+1}}{x\sqrt{15}-2\sqrt{4x^2+1}}\right| + C$.)
3. $\int (x+1)\sqrt{x^2+2x}dx$. (*Javob:* $\frac{1}{3}\sqrt{(x^2+3x)^3} + C$.)
4. $\int ln(x+\sqrt{1+x^2})dx$. (*Javob:* $xln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + C$.)
5. $\int arccos\sqrt{\frac{x}{x+1}}dx$. (*Javob:* $xarccos\sqrt{\frac{x}{x+1}} + \sqrt{x} - arctg\sqrt{x} + C$.)
6. $\int \frac{2xdx}{(x+1)(x^2+1)^2}$. (*Javob:* $\frac{x-1}{2(x^2+1)} - \frac{1}{2}ln|x+1| + \frac{1}{4}ln|1+x^2| + C$.)
7. $\int \frac{ln(x+1)}{\sqrt{x+1}}dx$. (*Javob:* $2\sqrt{x+1}(ln|x+1|-2) + C$.)
8. $\int e^{\sqrt[3]{x}}dx$. (*Javob:* $3e^{\sqrt[3]{x}}(\sqrt[3]{x^2}-2\sqrt[3]{x}+2) + C$.)

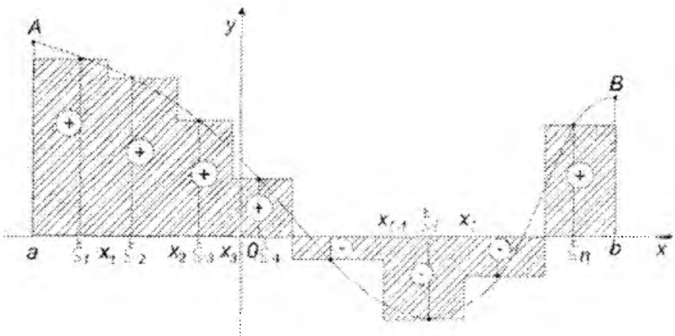
9. Aniq integral

9.1. Aniq integral tushunchasi. Aniq integrallarni hisoblash

$y=f(x)$ funksiya $[a;b]$ kesmada aniqlangan bo'lsin. Ushbu kesmani ixtiyoriy usul bilan $a=x_0< x_1< x_2 <\dots x_n=b$ nuqtalar orqali uzunligi $\Delta x_i= x_i - x_{i-1}$, $i=\overline{1,n}$ bo'lgan bo'lakchalarga bo'laylik. Har bir bo'lakchada bittadan ξ_i , $x_{i-1}< \xi_i< x_i$ nuqtani ixtiyoriy tanlaymiz (9.1-rasm). Quyidagi yig'indini

$$S_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i$$

$y=f(x)$ funksiyaning $[a,b]$ kesmadagi n-integral yig'indisi deyiladi. Geometrik nuqtai nazardan S_n-yig'indi 9.1-rasmda tasvirlangan to'g'ri to'rtburchaklar yuzalarining yig'indisi bo'lib, ularning asoslari Δx_i kesmalardan, balandligi esa $f(\xi_i)$ ga teng.



9.1- rasm

S_n integral yig'indining qismiy kesmalarning eng kattasi 0 ga intilgandagi limiti $f(x)$ funksiyaning $x=a$ dan $x=b$ gacha aniq integral deyiladi va ushbu ko'rinishda belgilanadi

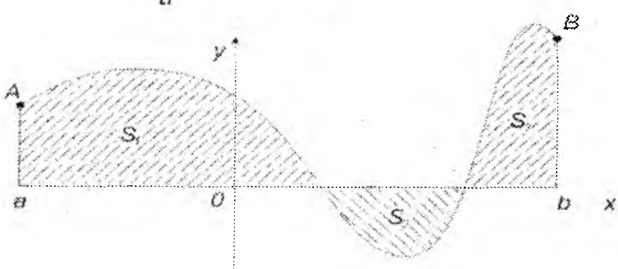
$$\lim_{max\Delta x_i\to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \int_a^b f(x)dx \qquad (9.1)$$

Bu yerda $f(x)$ integral ostidagi funksiya, $[a;b]$ -kesma integrallash oralig'i, a va b sonlar integrallashning quyi va yuqori chegaralari, x-integrallash o'zgaruvchisi deyiladi.

Teorema. Agar $f(x)$ funksiya $[a;b]$ kesmada aniqlangan, ham uzluksiz bo'lsa, u $[a,b]$ oraliqda integrallanuvchi bo'ladi, ya'ni (9.1) integral yig'indining limiti mavjud va u $[a;b]$ kesmani bo'lish usuliga, qismiy kesmalardan nuqta tanlashga bog'liq bo'lmaydi.

Agar *f(x)≥0, x∈[a;b]* bo'lsa, aniq integralning geometrik ma'nosi, *y=f(x)* funksiyaning grafigi, *x=a, x=b* to'g'ri chiziqlar va *Ox* o'qi bilan chegaralangan figuraning yuzini anglatadi. Bu figura egri chiziqli trapetsiya deyiladi. Umumiy holda, *f(x)* funksiya *[a,b]* kesmada turli ishoraga ega bo'lsa, aniq integral *Ox* o'qning yuqori qismida va quyi qismida joylashgan egri chiziqli trapetsiyalar yuzalarining ayirmasini bildiradi, *Ox* o'qidan pastda joylashgan yuzalar minus ishorasi bilan olinadi. Masalan grafigi 9.2 rasmdagi funksiya uchun

$$\int_a^b f(x)dx = S_1 - S_2 + S_3$$



9.2-rasm

Aniq integralning asosiy xossalarini keltirib o'tamiz (*f(x)* va φ(*x*) funksiyalarni mos kesmalarda integrallanuvchi deb faraz qilamiz)

1) $\int_a^b (f(x) \pm \varphi(x))dx = \int_a^b f(x)dx \pm \int_a^b \varphi(x)dx;$

2) $\int_a^b cf(x)dx = c\int_a^b f(x)dx \ (c = const);$

3) $\int_a^b f(x)dx = -\int_b^a f(x)dx;$

4) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$

5) agar[*a;b*] kesmada *f(x)≥0* va *a*<*b* bo'lsa, u holda

$$\int_a^b f(x)dx \geq 0$$

6) agar $\varphi(x) \leq f(x)$, *x[a;b], a*<*b* u holda

$$\int_a^b \varphi(x)dx \leq \int_a^b f(x)dx$$

7) agar $m=\min_{x\in[a;b]} f(x)$, $M=\max_{x\in[a;b]} f(x)$ va $a<b$ bo'lsa u holda

$$m(b-a)\le \int_a^b f(x)dx \le M(b-a)$$

8) agar *f(x)* funksiya *[a;b]* kesmada uzluksiz bo'lsa kamida bitta $x=c$, $x\le c\le b$ nuqta topiladiki, quyidagi tenglik bajariladi

$$\int_a^b f(x)dx = f(c)(b-a)$$

9) agar *f(x)* funksiyamiz uzluksiz va $\Phi(x)=\int_a^x f(t)dt$ tenglik o'rinli bo'lsa u holda

$$\Phi'(x)=f(x)$$

ya'ni aniq integraldan yuqori chegarasi *x* o'zgaruvchi bo'yicha hosila, integral ostidagi funksiyaning yuqori chegarasidagi qiymatiga teng.

10) agar *F(x)*– birorta boshlang'ich funksiya bo'lsa, quyidagi tenglik o'rinli

$$\int_a^b f(x)dx = F(b)-F(a) = F(x)\Big|_a^b$$

va bu formula Nyuton – Leybnits formulasi deyiladi. Uni *F(x)* boshlang'ich funksiya ma'lum bo'lgan holda $x=a$ va $x=b$ qiymatlarda hisoblash qiyinchilik tug'dirmaydigan shartlarda qo'llangan maqul.

1-misol. Aniq integral hisoblansin

$$\int_1^2 3(x-1)^2dx$$

► $\int_1^2 3(x-1)^2dx = (x-1)^3\Big|_1^2 = (2-1)^3-(1-1)^3 = 1$ ◄

2-misol. Hisoblang

$$\int_0^8(\sqrt{2x}+\sqrt[3]{x})dx$$

► $\int_0^8(\sqrt{2x}+\sqrt[3]{x})dx = \int_0^8\sqrt{2x}dx+\int_0^8\sqrt[3]{x}dx = \frac{1}{2}\frac{(2x)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^8+\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\Bigg|_0^8 =$

$$\frac{1}{3}(16)^{\frac{3}{2}}+\frac{3}{4}(8)^{\frac{4}{3}} = 33\frac{1}{3}$$ ◄

3-misol. Hisoblang

$$\int_0^{\frac{\pi}{2}} sin^3\,\varphi\, d\varphi$$

$$► \int_0^{\frac{\pi}{2}} sin^3 \varphi d\varphi = -\int_0^{\frac{\pi}{2}}(1-cos^2\varphi)d(cos\varphi) =$$

$$-\cos\varphi \Big|_0^{\frac{\pi}{2}} + \frac{cos^3\varphi}{3}\Big|_0^{\frac{\pi}{2}} = \frac{2}{3} ◄$$

4-misol. Hisoblang

$$\int_1^2 \frac{2x-1}{x^3+x}dx$$

Integral ostidagi funksiya to'g'ri ratsional kasr, uni sodda kasrlarga ajratib olamiz

$$\frac{2x-1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}, 2x-1 = A(x^2+1) + Bx^2 + Cx$$

$$\left| \begin{array}{ll} x^2 & 0=A+B \\ x^1 & -1=A \\ x^0 & 2=C \end{array} \right\}$$

bundan $A=-1$, $B=1$, $C=2$. Demak,

$$\int_1^2 \frac{2x-1}{x^3+x}dx = \int_1^2\left(-\frac{1}{x} + \frac{x}{1+x^2} + \frac{2}{1+x^2}\right)dx = \left(-\ln|x| + \frac{1}{2}\ln(1+x^2) + 2arctgx\right)\Big|_1^2 = -ln2 + \frac{1}{2}ln5 + 2arctg2 - \frac{1}{2}\ln 2 - 2arctg\ 1 = \frac{1}{2}\ln\frac{5}{8} + 2(arctg\ 2 - arctg\ 1) \approx\approx 0.38$$

Faraz qilaylik, $y=f(x)$ funksiya $[a;b]$ kesmada uzluksiz, $x=\varphi(t)$ funksiya o'zining hosilasi bilan $[\alpha;\beta]$ kesmada uzluksiz, monoton va $\varphi(\alpha) = a, \varphi(\beta) = b$ tenglik o'rinli, murakkab $f(\varphi(t))$ funksiya $[\alpha;\beta]$ kesmada uzluksiz bo'lsin. U holda aniq integral uchun o'zgaruvchini almashtirish formulasi o'rinli

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt \qquad (9.2)$$

5-misol. Hisoblang

$$\int_3^8 \frac{xdx}{\sqrt{1+x}}$$

► Quyidagi $\sqrt{1+x} = t$ almashtirishni bajaramiz. U holda $x=t^2-1$, $dx=2tdt$. $x=3$ bo'lganda qiymatida $t = 2 = \alpha, x = 8$ da $t = 3 = \beta$ bo'ladi. Yuqoridagi (9.2) formula uchun hamma shartlar bajarilgan. Demak,

$$\int_3^8 \frac{xdx}{\sqrt{x+1}} = \int_2^3 \frac{(t^2-1)2tdt}{t} = 2\int_2^3 (t^2-1)dt = 2\left(\frac{t^3}{3}-t\right)\Big|_2^3 =$$
$$2(9-3)-2\left(\frac{8}{3}-2\right) = = \frac{32}{3}$$ ◄

6-misol. Hisoblang

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2cosx+3}$$

► Integral ostida $u = tg\left(\frac{x}{2}\right)$ almashtirishni bajarsak $cosx = \frac{1-u^2}{1+u^2}, dx = \frac{2du}{1+u^2}, \alpha = tg\,0 = 0, \beta = tg\left(\frac{\pi}{4}\right) = 1$ bo'ladi. Demak,

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2cosx+3} = \int_0^1 \frac{\frac{2du}{1+u^2}}{\frac{2(1-u^2)}{(1+u^2)}+3} = \int_0^1 \frac{2du}{u^2+5} = \frac{2}{\sqrt{5}} arctg \frac{u}{\sqrt{5}}\Big|_0^1 =$$
$$\frac{2}{\sqrt{5}} arctg \frac{1}{\sqrt{5}} \approx 0.38$$ ◄

Agar $u(x)$ va $v(x)$ funksiyalar $[a;b]$ kesmada uzluksiz xususiy hosilalarga ega bo'lsa, u holda

$$\int_a^b u(x)dv(x) = u(x)\cdot v(x)\Big|_a^b - \int_a^b v(x)du(x) \quad (9.3)$$

7-misol. Integralni hisoblang $\int_0^{\frac{\pi}{2}} xcosxdx$

► $\int_0^{\frac{\pi}{2}} xcosxdx = \begin{vmatrix} u = x & du = dx \\ dv = cosxdx & v = sinx \end{vmatrix} =$

$$xsinx\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} sinxdx = = \frac{\pi}{2}\sin\frac{\pi}{2} - 0 + cosx\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$$ ◄

8-misol. Integralni hisoblang $\int_1^e xln^2xdx$

► $\int_1^e xln^2xdx = \begin{vmatrix} u = ln^2x & du = 2lnx\cdot\frac{1}{x}dx \\ dv = xdx & v = \frac{1}{2}x^2 \end{vmatrix} =$

$$\frac{1}{2}x^2ln^2\Big|_1^e - \int_1^e x\ln x dx = = \begin{vmatrix} u = \ln x \\ du = \frac{1}{x}dx \\ dv = xdx \\ v = \frac{1}{2}x^2 \end{vmatrix} \frac{1}{2}e^2 -$$

$$\left(\frac{x^2}{2}lnx\Big|_1^e - \int_1^e \frac{1}{2}x^2\cdot\frac{1}{2}dx\right) = \frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{2}\int_1^e xdx = \frac{1}{4}x^2\Big|_1^e ==$$
$$\frac{1}{4}(e^2-1).$$ ◄

9.1-AT

Aniq integral hisoblansin.

1. $\int_1^2 \left(2x^2 + \frac{2}{x^2}\right) dx$ $(Javob : \frac{21}{4})$
2. $\int_1^4 \sqrt{x}dx$ $(Javob : \frac{14}{3})$
3. $\int_1^{e^2} \frac{dx}{x\sqrt{1+lnx}}$ $(Javob : 2)$
4. $\int_0^1 \frac{dx}{x^2+4x+5}$ $(Javob : arctg\frac{1}{7})$
5. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{cosx - cos^3x}dx$ $(Javob : \frac{4}{3})$
6. $\int_0^4 \frac{dx}{1+\sqrt{2x+1}}$ $(Javob : 2 - ln2)$
7. $\int_0^{\sqrt{3}} x^5\sqrt{1+x^2}dx$ $(Javob : \frac{848}{105})$
8. $\int_0^2 \sqrt{4-x^2}dx$ $(Javob : \pi)$
9. $\int_1^3 \frac{dx}{x\sqrt{x^2+5x+1}}$ $(Javob : ln\frac{7+2\sqrt{7}}{9})$
10. $\int_0^5 \frac{dx}{2x+\sqrt{3x+1}}$ $(Javob : \frac{1}{5}ln112)$

Mustaqil ish

Aniq integrallarni hisoblang

1. a) $\int_1^4 \left(2x + \frac{3}{\sqrt{x}}\right) dx$; b) $\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1}dx$ *(Javob: a)21; b) 7+2ln2)*
2. a) $\int_4^9 \frac{y-1}{\sqrt{y}+1}dy$; b) $\int_0^4 \frac{xdx}{1+\sqrt{x}}$ (*Javob: a)23/3; b)16/3–2ln3)*
3. a) $\int_4^9 \frac{xdx}{(1+x^2)^3}$; b) $\int_0^9 \frac{\sqrt{x}}{1+\sqrt{x}}dx$ (*Javob:a)3/16; b) 3+4ln2)*

9.2 Xosmas integrallar

Agar $y=f(x)$ funksiya $a \le x < +\infty$, oraliqda uzluksiz bo'lsa, u holda

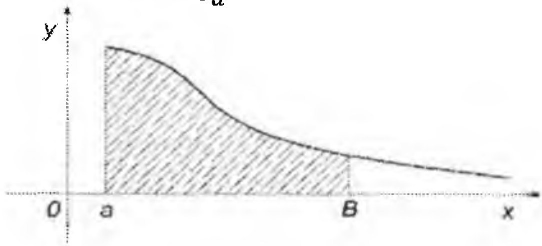
$\int_\alpha^\beta f(x)dx = I(\beta)$ integral β ning uzluksiz funksiyasi bo'ladi. (9.3-rasm)

U holda quyidagi limit

$$\lim_{\beta\to+\infty} \int_\alpha^\beta f(x)dx \qquad (9.4)$$

f(x) funksiyaning $[\alpha; +\infty]$ oraliqda yuqori chegarasi cheksiz bo‘lgan xosmas integrali deyiladi.

$$\int_{\alpha}^{+\infty} f(x)dx \qquad (9.5)$$



9.3-rasm

Demak, ta’rif bo‘yicha

$$\int_{\alpha}^{+\infty} f(x)dx = \lim_{\beta \to +\infty} \int_{\alpha}^{\beta} f(x)dx$$

Agar (9.4) limit mavjud bo‘lsa, u holda (9.5) integral yaqinlashuvchi, agar (9.4) limit mavjud bo‘lmasa, xususan cheksiz bo‘lsa uzoqlashuvchi deyiladi.

Quyi chegarasi cheksiz bo‘lgan xosmas integrallar va yuqori, quyi chegarasi cheksiz bo‘lgan xosmas integrallar ham shu kabi aniqlanadi:

$$\int_{-\infty}^{b} f(x)dx = \lim_{A \to -\infty} \int_{A}^{b} f(x)\,dx,$$

$$\int_{-\infty}^{+\infty} f(x)dx = \lim_{A \to -\infty} \int_{A}^{c} f(x)dx + \lim_{B \to +\infty} \int_{c}^{B} f(x)dx$$

bu yerda $-\infty < c < +\infty$. Agar $\int_{a}^{+\infty} |f(x)|dx$ integral yaqinlashuvchi bo‘lsa u holda (9.5) integral absolyut yaqinlashuvchi deyiladi. Xosmas (9.5) integralning yaqinlashishini tekshirish uchun quyidagi taqqoslash belgilaridan foydalanish mumkin.

1-teorema. Agar barcha $x \geq a$ uchun $0 \leq f(x) \leq \varphi(x)$ tengsizlik o‘rinli bo‘lsa, u holda:

1) agar $\int_{a}^{+\infty} \varphi(x)dx$ integral yaqinlashsa $\int_{a}^{+\infty} f(x)dx$ integral ham yaqinlashadi shu bilan birga $\int_{a}^{+\infty} f(x)dx \leq \int_{a}^{+\infty} \varphi(x)dx$

2) agar $\int_a^{+\infty} f(x)dx$ integral uzoqlashsa u holda $\int_a^{+\infty} \varphi(x)dx$ integral ham uzoqlashuvchi bo‘ladi. Absolyut yaqinlashuvchi xosmas integral yaqinlashuvchi bo‘ladi.

1-misol. Xosmas integral $\int_1^{+\infty} \frac{dx}{x^\alpha}$ $(\alpha > 0)$ berilgan. Ushbu integral α ning qanday qiymatlarida yaqinlashuvchi, qanday qiymatlarida uzoqlashuvchi bo‘ladi?

►Faraz qilaylik, $\alpha \neq 1$ bo‘lsin. U holda:

$$\int_1^\beta \frac{dx}{x^\alpha} = \frac{1}{1-\alpha} x^{1-\alpha} \Big|_1^B = \frac{1}{1-\alpha}(B^{1-\alpha} - 1),$$

$$\int_1^{+\infty} \frac{dx}{x^\alpha} = \lim_{B\to\infty} \frac{1}{1-\alpha}(B^{1-\alpha} - 1)$$

Demak, agar $\alpha > 1$, bo‘lsa

$$\int_1^{+\infty} \frac{dx}{x^\alpha} = \frac{1}{\alpha - 1}$$

ya’ni integral yaqinlashuvchi, agar $\alpha < 1$, bo‘lsa

$$\int_1^{+\infty} \frac{dx}{x^\alpha} = +\infty$$

ya’ni integral uzoqlashuvchi bo‘ladi.

Agar $\alpha = 1$, bo‘lsa

$$\int_1^{+\infty} \frac{dx}{x} = \lim_{B\to+\infty} \int_1^B \frac{dx}{x} = \lim_{B\to+\infty} lnB = +\infty$$

ya’ni integral uzoqlashuvchi bo‘ladi.

2-misol. Xosmas integralni hisoblang

$$\int_1^{+\infty} \frac{dx}{x^2+4x+13}$$

yoki uning uzoqlashuvchi ekanligini ko‘rsating

►$\int_1^{+\infty} \frac{dx}{x^2+4x+13} = \lim_{\beta\to+\infty} \int_1^{+\infty} \frac{dx}{(x+2)^2+9} = \lim_{\beta\to+\infty} \frac{1}{3} arctg \frac{x+2}{3} \Big|_1^\beta = =$

$$\frac{1}{3}\lim_{\beta\to+\infty}\left(arctg\frac{\beta+2}{3} - arctg1\right) = \frac{1}{3}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\pi}{12}$$◄

3-misol. Xosmas integralni yaqinlashishini isbotlang.

$$\int_1^{+\infty} \frac{dx}{(x^2+1)e^x}$$

► $x \geq 1$ qiymatda $\frac{1}{(x^2+1)e^x} \leq \frac{1}{(1+x^2)}$ tengsizlik o‘rinli bo‘ladi va integral

$$\int_1^{+\infty} \frac{dx}{(1+x^2)} = \lim_{\beta\to\infty} \int_1^\beta \frac{dx}{1+x^2} = \lim_{\beta\to\infty} arctgx \Big|_1^\beta =$$

$$\lim_{B\to+\infty}(arctg\beta - arctg1) = \frac{\pi}{2} - -\frac{\pi}{4} = \frac{\pi}{4}$$◄

yaqinlashuvchi, demak 1-teoremaga ko'ra berilgan integral yaqinlashuvchi bo'ladi.

Eslatma. Integrallash oralig'i cheksiz bo'lgan xosmas integrallarni hisoblashda quydagi tenglikdan foydalanamiz

$$\int_a^{+\infty} f(x)dx = F(x)\Big|_a^{+\infty}$$

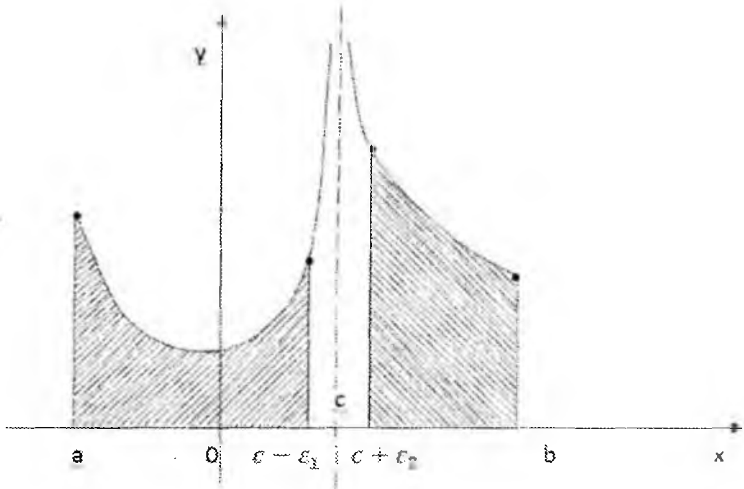
bu yerda $F'(x)=f(x)$ va $F(+\infty) = \lim_{x\to+\infty} F(x)$

Faraz qilaylik $y=f(x)$ funksiya $[a;b]$ kesmaning $x=c$ nuqtasidan tashqari barcha nuqtalarida uzluksiz bo'lsin. U holda ta'rifga asosan:

$$\int_a^b f(x)dx =$$

$$\lim_{\varepsilon_1\to 0}\int_a^{c-\varepsilon_1} f(x)dx + \lim_{\varepsilon_2\to 0}\int_{c+\varepsilon_2}^{b} f(x)dx \qquad (9.6)$$

Bu yerda, $\varepsilon_1, \varepsilon_2 > 0$ va s nuqta ikkinchi tur uzilish nuqtasi. Yuqoridagi (9.6) integral uzulishga ega bo'lgan funksiyaning xosmas integrali deyiladi. Agar (9.6) tenglikning o'ng tomonidagi limitlar mavjud bo'lsa, bu integral yaqinlashuvchi, agar ulardan kamida bittasi mavjud bo'lmasa, integral uzoqlashuvchi deyiladi. Uzilish *c* nuqtasi *s* uchun *c=a* yoki *c=b* bo'lsa, (9.6) tenglikning o'ng tomonida faqat bitta limit bo'ladi.



9.4-rasm

4-misol. Xosmas integral uchun yaqinlashish va uzoqlashish shartlarini aniqlang

$$\int_0^1 \frac{dx}{x^\alpha} \ (\alpha = const > 0)$$

►Integral ostidagi funksiya *x=0* nuqtada ikkinchi tur uzlishga ega. Agar $\alpha \neq 1$ bo'lsa u holda

$$\int_0^1 \frac{dx}{x^\alpha} = \lim_{\varepsilon\to 0+0} \int_\varepsilon^1 \frac{dx}{x^\alpha} = \lim_{\varepsilon\to 0+0} \frac{x^{-\alpha+1}}{-\alpha+1}\Big|_\varepsilon^1 =$$

$$= \lim_{\varepsilon\to 0+0}\left(\frac{1}{-\alpha+1} - \frac{\varepsilon^{-\alpha+1}}{-\alpha+1}\right) = \begin{cases}\frac{1}{1-\alpha}, \alpha < 1\\ \infty, \alpha > 1\end{cases}$$

Agar *α=1* bo'lsa $\int_0^1 \frac{dx}{x} = \lim\limits_{\varepsilon\to 0+0} \ln|x|\Big|_\varepsilon^1 = -\lim\limits_{\varepsilon\to +0} ln\varepsilon = +\infty$.
Demak, ushbu hosmas integral *0<α<1* da yaqinlashuvchi *α≥1* da esa uzoqlashuvchi bo'ladi.

5-misol. Xosmas

$$\int_0^1 \frac{dx}{\sqrt{1-x}}$$

integralni hisoblang

►Integral ostidagi funksiya *x=1* nuqtada cheksiz uzilishga ega. Demak, ta'rif bo'yicha

$$\int_0^1 \frac{dx}{\sqrt{1-x}} = \lim_{\varepsilon\to 0}\int_0^{1-\varepsilon}(1-x)^{-\frac{1}{2}}dx = lim_{\varepsilon\to 0}(-2)(1-x)^{\frac{1}{2}}\Big|_0^{1-\varepsilon} =$$

$$= -2\lim_{\varepsilon\to 0}\left(\sqrt{1-1+\varepsilon} + \sqrt{1-0}\right) = 2\lim_{\varepsilon\to 0}\left(1-\sqrt{\varepsilon}\right) =$$

2 $(\varepsilon > 0)$ ya'ni bu integral yaqinlashuvchi bo'ladi.◄

2-Teorema. Agar [a;b] kesmada x=c nuqtadan tashqar ibarcha nuqtalarda φ(x)≥f(x)≥0 tengsizlik bajarilsa, va faqat x=c nuqtada bu funksiyalar cheksiz uzilishga ega bo'lsa, u holda

1) agar

$$\int_a^b \varphi(x)dx$$

integral yaqinlashuvchi bo'lsa,

$$\int_a^b f(x)dx$$

integral ham yaqinlashuvchi bo'ladi.

2) agar

$$\int_a^b f(x)dx$$

integral uzoqlashuvchi bo'lsa,

$$\int_a^b \varphi(x)dx$$

integral ham uzoqlashuvchi bo'ladi.

Bu 1 va 2 tasdiqlar taqqoslash teoremalari deyiladi.

6-Misol. Xosmas

$$\int_0^1 \frac{dx}{\sqrt[3]{x}+2x^3}$$

integralning yaqinlashishini tekshiring:

►Integral ostidagi funksiya $x=0$ nuqtada uzilishga ega

$$\frac{1}{\sqrt[3]{x}+2x^3} \leq \frac{1}{\sqrt[3]{x}}$$

va $x \geq 0$ da yuqoridagi tengsizlik o'rinli bo'ladi. Bundan xosmas

$$\int_0^1 \frac{dx}{\sqrt[3]{x}} = \lim_{\varepsilon\to 0}\int_{0+\varepsilon}^1 \frac{1}{\sqrt[3]{x}}dx = \lim_{\varepsilon\to 0}\frac{2}{3}\sqrt{x}\Big|_\varepsilon^1 = \frac{2}{3}\lim_{\varepsilon\to 0}(1-\sqrt{\varepsilon}) = \frac{2}{3}\ (\varepsilon > 0)$$

integral yaqinlashuvchi va 2 teoremaning 1 tasdig'iga ko'ra berilgan xosmas integral yaqinlashuvchi bo'ladi.◄

9.2- AT

Berilgan xosmas integrallar hisoblansin.

1. $\int_1^e lnxdx\ (Javob:\ 1)$
2. $\int_0^\pi x^2cosxdx\ (Javob:\ -2\pi)$
3. $\int_0^{\pi^2} cos\sqrt{x}dx\ (Javob:\ -4)$
4. $\int_0^{\sqrt{3}} xarctgxdx\ (Javob:\ \frac{2\pi}{3}-\frac{\sqrt{3}}{2})$
5. $\int_0^1 x^2e^xdx\ (Javob:\ e-2)$

Xosmas integrallarni hisoblang yoki ularning uzoqlashuvchiekanini ko'rsating.

6. $\int_e^\infty \frac{dx}{x(lnx)^3}$ $(Javob$: 0,5)
7. $\int_0^\infty x^3e^{-x^2}dx$ $(Javob$: 0,5)
8. $\int_1^\infty \frac{2+sinx}{\sqrt{x}}dx$ $(Javob$: *uzoqlashuvchi)*

9. $\int_0^{\frac{1}{e}} \frac{dx}{x(\ln x)^2}$ *(Jovob: 1)*

10. $\int_1^2 \frac{xdx}{\sqrt{x-1}}$ $(Javob: \frac{8}{3})$

Mustaqil ish

1.1) Integralni hisoblang

$\int_0^1 xe^{-x}dx$ *(Javob: 1–2/e)*

2) Integralni hisoblang yoki uning uzoqlashuvchi ekanligini ko'rsating

$$\int_1^e \frac{dx}{x\sqrt{\ln x}} \ (Javob: 2)$$

2.1) Integralni hisoblang

$$\int_0^{\pi} x\sin x dx; \ (Javob: \pi)$$

2) Xosmas integralni hisoblang yoki uni uzoqlashuvchi ekanligini ko'rsating

$$\int_0^{\infty} \frac{xdx}{(1+x)^3}; \ (Javob: 0.5)$$

3.1) Integralni hisoblang

$$\int_0^1 xe^{3x}dx; \ (Javob: \frac{2e^3+1}{9})$$

2) Xosmas integralni hisoblang yoki uning uzoqlashuvchi ekanligini ko'rsating

$$\int_1^2 \frac{dx}{x\ln x}; \ (Javob: Uzoqlashuvchi)$$

9.3 Aniq integralning geometrik masalalarga tatbiqi

Yassi figuraning yuzini hisoblash. Aniq integral ($f(x)\geq 0$ $x \in [a;b]$), geometrik nuqtai nazardan (9.1§) egri chiziqli trapetsiyaning yuziga teng bo'lar edi. Yassi figuraning yuzini esa egri chiziqli trapetsiya yuzlarining yig'indisi va ayirmasi sifatida qarash mumkin. Demak, aniq integral yordamida turli yassi figuralarning yuzalarini hisoblash mumkin.

1-misol. Ushbu $y=x^2-2x$ egri chiziq, $x=-1$, $x=1$ to'g'ri chiziqlar va Ox o'qi bilan chegaralangan yassi figuraning yuzini hisoblang.

►Dastlab berilgan chiziqlar bilan chegaralangan figurani chizib olamiz (9.5-rasm). Qidirilayotgan yuza $S=|S_1|+|S_2|=S_1-S_2$ demak,

$$S=\int_{-1}^{0}(x^2-2x)dx-\int_{0}^{1}(x^2-2x)dx=\left(\frac{x^3}{3}-x^2\right)\Big|_{-1}^{0}-\left(\frac{x^3}{3}-x^2\right)\Big|_{0}^{1}=\left(\frac{1}{3}-1\right)=2$$ ◄

Umumiy holda berilgan figura $y=f_1(x)$, $y=f_2(x)$ egri chiziqlar, $x=a$, $x=b$ to‘g‘ri chiziqlar bilan chegaralangan bo‘lsa, bu yerda $f_1(x)\leq f_2(x)$, $x[a;b]$, (9.6-rasm) u holda

$$s=\int_{a}^{b}(f_2(x)-f_1(x))dx \qquad (9.7)$$

2-misol. Quyidagi $y=3x-x^2$ va $y=-x$ chiziqlar bilan chegaralangan figuraning yuzasini hisoblang.

►Egri chiziqlarning kesishish nuqtasini topib olamiz va yuzasi qidirilayotgan figuraning rasmini chizib olamiz (9.7-rasm)

$$\left.\begin{matrix} y=3x-x^2 \\ y=-x \end{matrix}\right\} \Rightarrow \begin{cases} y=-x \\ -x=3x-x^2 \end{cases}$$

Sistemani yechib: $x_1=0$, $x_2=4$, $y_1=0$, $y_2=-4$ qiymatlarga ega bo‘lamiz, u holda (9.7) formulaga ko‘ra

$$S=\int_{0}^{4}(3x-x^2-(-x))dx=\int_{0}^{4}(4x-x^2)dx=\left(2x^2-\frac{x^3}{3}\right)\Big|_{0}^{4}=\frac{32}{3}$$ ◄

Agar egri chiziqli trapetsiyani chegaralovchi AV egri chiziq parametrik ko‘rinishda berilgan bo‘lsa $x=\varphi(t)$, $y=\psi(t)$ u holda uning yuzasi

$$S=\int_{\alpha}^{\beta}\psi(t)\varphi'(t)\,dt \qquad (9.8)$$

bu yerda α va β, $\varphi(\alpha)=a$, $\psi(\beta)=b$ tenglamadan aniqlanadi ($\psi(t)\geq 0$, $[\alpha;\beta]$ kesmada), formula bilan aniqlanadi.

3-misol. Berilgan $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips bilan chegaralangan yuzani hisoblang

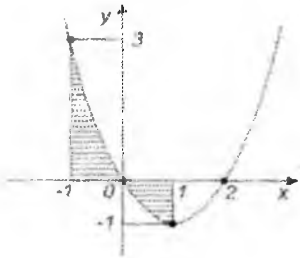
►Ellipsning parametrik tenglamasini yozib olamiz: $x=acost$, $y=bsint$. Figuraning simmetrik ekanligini hisobga olib va (9.8) formuladan (9.9-rasm)

$$S=4\int_{0}^{a}ydx=4\int_{\frac{\pi}{2}}^{0}asint(-bsint)dt=4ab\int_{0}^{\frac{\pi}{2}}sin^2tdt==$$

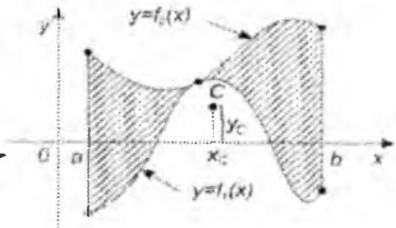
$$4ab\int_{0}^{\frac{\pi}{2}}\frac{1-cos2t}{2}dt=2ab\left(t-\frac{1}{2}sin2t\right)\Big|_{0}^{\frac{\pi}{2}}=\pi ab$$ ◄

Egri chiziq qutb koordinatalar sistemasida $\rho=\rho(\varphi)$ tenglama bilan berilgan bo‘lsa, egri chiziqli OM_1M_2 (9.10-rasm), egri chiziqning yoyi va OM_1 va OM_2 φ_1 va φ_2 qiymatlarga mos keluvchi qutb radiuslari bilan chegaralangan sektorning yuzi ushbu formula bilan hisoblanadi.

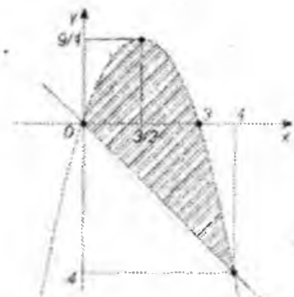
$$S = \frac{1}{2}\int_{\varphi_1}^{\varphi_2}(\rho(\varphi))^2 d\varphi \tag{9.9}$$



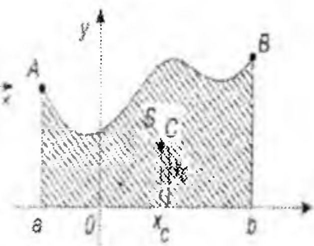
9.5-rasm



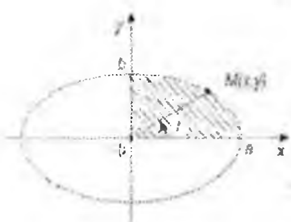
9.6-rasm



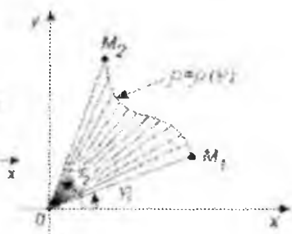
9.7-rasm



9.8-rasm



9.9-rasm



9.10-rasm

4-misol. $(x^2+y^2)^2=a^2(x^2-y^2)$ Bernulli lemniskatasi bilan chegaralangan figuraning yuzini hisoblang

►Egri chiziqning tenglamasini qutb koordinatalar sistemasida yozib olamiz. Tenglamada $x=\rho cos\varphi, y=\rho sin\varphi$ almashtirish bajarsak, $\rho^2=a^2cos2\varphi$

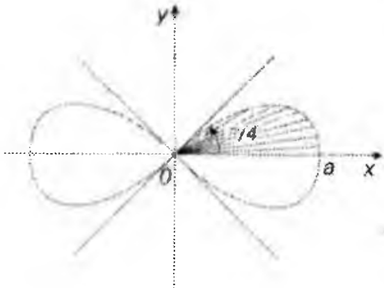
yoki $\rho=a\sqrt{cos2\varphi}$. Figuraning simmetrikligini hisobga olsak, qidirilayotgan yuza (9.9) formula bilan hisoblanadi:

$$S=4\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}}a^2cos2\varphi d\varphi=2a^2\cdot\frac{1}{2}sin2\varphi|_0^{\frac{\pi}{4}}=a^2$$◄

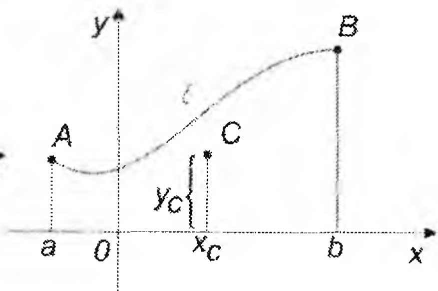
Egri chiziq yoyining uzunligini hisoblash. Agar egri chiziqning $\overset{\frown}{AB}$ yoyi (a;b) va *y=f(x)* tenglama bilan berilgan bo'lib, *f(x)*-differensiallanuvchi funksiya bo'lsa, *u* holda $\overset{\frown}{AB}$ yoyning uzunligi (9.12-rasm)

$$l=\int_a^b\sqrt{1+y'^2}dx \qquad (9.10)$$

formula bilan hisoblanadi.



9.11 rasm



9.12 rasm

Agar egri chiziq o'zining parametrik tenglamalari $x=\varphi(t), y=\psi(t)$ lar bilan berilgan bo'lib, $x=\varphi(t), y=\psi(t)$ lar differensiallanuvchi funksiyalar bo'lsa, u holda *l* yoyning uzunligi

$$l=\int_\alpha^\beta\sqrt{x_t'^2+y_t'^2}dt \qquad (9.11)$$

formula bilan hisoblanadi, parametr *t* ning *α* va *β* qiymatlari yoyning chekka nuqtalari *A* va *B* ga mos keladi.

Agar silliq egri chiziq qutb koordinatalar sistemasida $\rho = \rho(\varphi)$ tenglama bilan berilgan bo'lsa, M_1M_2 yoyning l uzunligi ushbu formula bilan hisoblanadi:

$$l=\int_{\phi_1}^{\phi_2} \sqrt{\rho^2 + p'^2}\, d\phi \qquad (9.12)$$

Bu yerda φ_1 va φ_2 qiymatlari yoyning boshi va oxirgi nuqalari M_1 va M_2 ga mos keladi.

5 – misol. Egri chiziq $y=\frac{2}{3}\sqrt{x^3}$ tenglama bilan berilgan, yoyning boshi va oxirgi nuqtalari abssissalari $x_1=\sqrt{3}$ va $x_2=\sqrt{8}$ bo'lsa, yoyning uzunligini toping.

►Yoyni uzunligini hisoblash uchun (9.10) formuladan foydalanamiz:

$$l=\int_{\sqrt{3}}^{\sqrt{8}}\sqrt{1+(\sqrt{x})^2}\,dx=\int_{\sqrt{3}}^{\sqrt{8}}\sqrt{1+x}\,dx=\frac{(1+x)^{3/2}}{\frac{3}{2}}\Bigg|_{\sqrt{3}}^{\sqrt{8}}=\frac{34}{3}$$ ◄

6-misol. Sikloidaning birinchi arkasi yoyining uzunligini toping

$$y=a(1-\cos t), x=a(t-\sin t)$$

► Sikloidaning hamma arkalari bir xil, birinchi arkada t parametr 0 dan 2π gacha o'zgaradi. U holda (9.11) formulani qo'llaymiz:

$$l=\int_0^{2\pi}\sqrt{a^2(1-\cos t)^2+a^2\sin^2 t}\;dt=a\int_0^{2\pi}\sqrt{1-2\cos t+\cos^2 t+\sin^2 t}\;dt=$$

$$=a\int_0^{2\pi}\sqrt{2(1-\cos t)}\,dt=2a\int_0^{2\pi}\sin\frac{t}{2}\,dt=-4a\cos\frac{t}{2}\Bigg|_0^{2\pi}=8a$$ ◄

7-misol. Logarifmik spiralning bitta aylanishda hosil bo'ladigan yoyining uzunligini toping. $\rho = e^{\varphi}$

▶ Qutb koordinatalarida yoyning uzunligini hisoblash (9.12) formulasidan

$$l=\int_0^{2\pi}\sqrt{e^{2\varphi}+e^{2\varphi}}\,d\varphi=\int_0^{2\pi}\sqrt{2}\,e^{\varphi}d\varphi=\sqrt{2}\,e^{\varphi}\Big|_0^{2\pi}=\sqrt{2}(e^{2\pi}-1)\approx 108{,}16$$ ◀

kelib chiqadi.

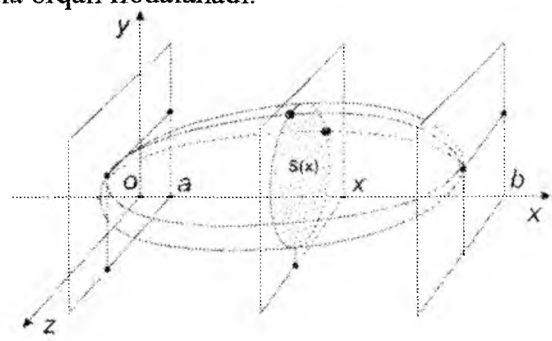
Jismlarning hajmini hisoblash. Fazoda berilgan jism Ox o‘qidagi [a, b] kesmaga proeksiyalansin. Har qanday Ox o‘qiga perpendikulyar $x\in[a,\ b]$ nuqtadan o‘tuvchi tekislik, jism bilan kesishganda, yuzi $S(x)$ ga teng figura hosil qiladi (9.13 rasm). Bunda jismning hajmi quyidagi formula bilan hisoblanadi:

$$V=\int_a^b S(x)\,dx \qquad (9.13)$$

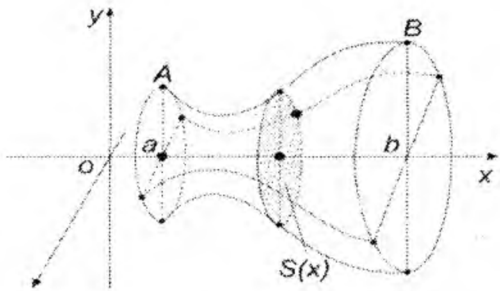
Xususan Ox o‘qi atrofida egri chiziqli $aABb$ (9.14-rasm) trapetsiyani aylantirsak, ko‘ndalang kesimning yuzi: $S(x)=\pi(f(x))^2$ ga teng bo‘ladi. Shuning uchun egri chiziqli trapetsiyani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi:

$$V_x=\pi\int_a^b (f(x))^2\,dx \qquad (9.14)$$

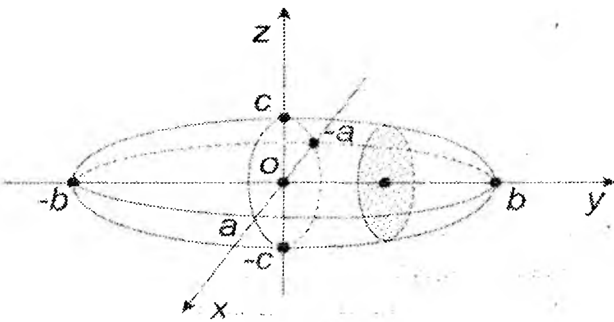
formula orqali ifodalanadi.



9.13-rasm



9.14-rasm



9.15-rasm

8-misol. Ushbu sirt bilan chegaralangan jismning hajmi hisoblansin

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

▶Berilgan tenglama bo'yicha ellipsoidning rasmini chizib olamiz (9.15-rasm). Oy o'qiga perpendikulyar va $y\in[-b;b]$ ixtiyoriy nuqtadan o'tuvchi tekislikni qaraymiz. Ko'ndalang kesimda:

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1-\frac{y^2}{b^2},\ y=const$$, yoki agar $1-\frac{y^2}{b^2}>0$ bo'lsa

$$\frac{x^2}{\left(a\sqrt{1-\frac{y^2}{b^2}}\right)^2}+\frac{z^2}{\left(c\sqrt{1-\frac{y^2}{b^2}}\right)^2}=1, \qquad y=const$$

ya'ni, yarim o'qlari $a_1=a\sqrt{1-\frac{y^2}{b^2}}$, $c_1=c\sqrt{1-\frac{y^2}{b^2}}$ bo'lgan ellipsni hosil qilamiz.

Bu kesimlarning yuzi:

$$S(y)=\pi a_1\cdot c_1=\pi a\cdot c\left(1-\frac{y^2}{b^2}\right).$$

U holda (9.13) formuladan

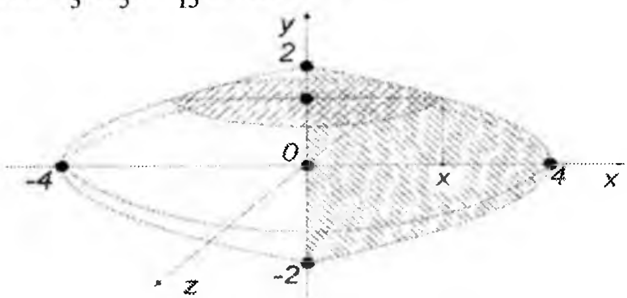
$$V=\int_{-b}^{b}\pi ac\left(1-\frac{y^2}{b^2}\right)dy=2\pi ac\int_0^b\left(1-\frac{y^2}{b^2}\right)dy=2\pi ac\left(y-\frac{y^3}{3b^2}\right)\Bigg|_0^b=\frac{4}{3}\pi abc$$

9-misol. *Oxy* tekislikda yotuvchi va $y^2=4-x, x=0$ chiziqlar bilan chegaralangan figurani *Oy* o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmini hisoblang.

▶Rasmdan (9.16-rasm) ko'rinib turibdiki:

$$V_y=\pi\int_c^d x^2dy=\pi\int_{-2}^{2}(4-y^2)^2dy=2\pi\int_0^2(4-y^2)^2dy=2\pi\int_0^2(16-8y^2+y^4)dy=2\pi\left(16y-\frac{8}{3}y^3+\frac{y^5}{5}\right)\Bigg|_0^2=$$

$$=2\pi\left(32-\frac{64}{3}+\frac{32}{5}\right)=\frac{512}{15}\pi\approx 107{,}23$$ ◀



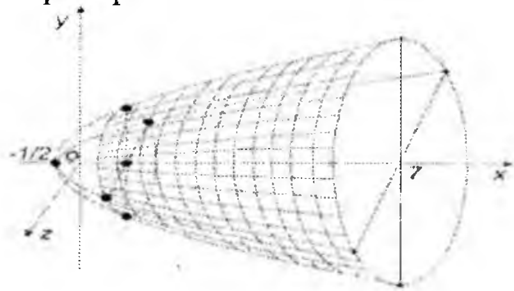
9.16-rasm

Aylanish jismlarining sirtlari yuzini hisoblash

Agar $y=f(x)$ funkssiya uzluksiz differensiallanuvchi bo'lsa, shu egrichiziqning AB qismi yoki $A(a;f(a))$, $B(b;f(b))$ Ox o'qi atrofida aylanishidan hosil bo'lgan sirtning yuzi:

$$Q_x = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx \qquad (9.15)$$

formula orqali topiladi.



9.17-rasm

10-misol. $y^2=2x+1$ parabolaning $x_1=1$ va $x_2=7$ abssissalar oralig'idagi yoyini (9.17) aylantirishdan hosil bo'lgan sirtning yuzasi hisoblansin.

▶Rasmdan va (9.15) formuladan quyidagiga ega bo'lamiz:

$$Q_x = 2\pi\int_1^7 \sqrt{2x+1}\sqrt{1+\left(\frac{1}{\sqrt{2x+1}}\right)^2}\,dx = 2\pi\int_1^7 \sqrt{2x+1+1}\,dx =$$

$$2\pi\int_1^7 \sqrt{2x+1}\,dx == 2\pi\cdot\frac{1}{2}\frac{(2x+2)^{3/2}}{3/2}\Big|_1^7 = \frac{2}{3}\pi(64-8) = \frac{112\pi}{3}$$◀

AT-9.3.

1. Ushbu $y^2=9x, y=3x$ chiziqlar bilan chegaralangan soxaning yuzasini toping (*Javob:0,5*).

2. $y=x^2+4x$, $y=x+4$ chiziqlar bilan chegaralangan soxaning yuzasini toping (*Javob:* $\frac{125}{6}$).

3. Ushbu $y = \frac{1}{(1+x^2)}$, $y = \frac{x^2}{2}$ chiziqlar bilan chegaralangan soxalarning yuzasini toping (*Javob:* $\frac{\pi}{2} - \frac{1}{3}$).

4. Yopiq $y^2 = x^2 - x^4$ chiziq bilan chegaralangan soxaning yuzasini toping (*Javob:* 4/3).

5. Sikloidaning birinchi arkasi $y = a(1-\cos t)$, $x = a(t - \sin t)$ va Ox o'qi bilan chegaralangan soxaning yuzasini hisoblang (*Javob:* $3\pi a^2$).

6. Parametrik ko'rinishdagi $x = 3t^2$, $y = 3t - t^3$ chiziq bilan chegaralangan soxaning yuzasini hisoblang (*Javob:* $\frac{72\sqrt{3}}{5}$).

7. $y = xe^{-\frac{x^2}{2}}$ egri chiziq va uning asimptotasi bilan chegaralangan soxaning yuzasini hisoblang (*Javob:* 2).

8. Kardioida bilan chegaralangan soxaning yuzasini hisoblang: $\rho = a(1 - \cos\varphi)$. (*Javob:* $3\pi a^2/2$).

9. Ushbu $x^2 + y^2 = 4$, $x^2 + y^2 = 9$, $y = x$, $y = -x\sqrt{3}$ chiziqlar bilan chegaralangan soxaning yuzasini hisoblang (*Javob:* $\frac{25\pi}{24}$).

Mustaqil ish

1. Ushbu chiziqlar bilan chegaralangan soxaning yuzasini hisoblang:

a) $y^2 = x + 5$, $y^2 = -x + 4$, *b)* $\rho = a\cos 2\phi$ (*Javob:* *a)* $9\sqrt{2}$; *b)* $\frac{\pi a^2}{2}$).

2. *a)* $y = (x-4)^2$, $y = 16 - x^2$ chiziqlar bilan chegaralangani soxaning yuzasini hisoblang:

b) Arximed spiralining birinchi va ikkinchi o'rami orasidagi, $\rho = a\phi$ $(a > 0)$ yuzani hisoblang (*Javob:* a) $\frac{64}{3}$; *b)* $\frac{8\pi^3}{a^3}$).

3) Ushbu egri chiziqlar bilan chegaralangan soxalarning yuzasini hisoblang:

3. $a)\ 4y=8x-x^2,\ 4y=x+6$; $b)\ y=4t^2-6t,\quad x=2t$ va Ox o'qi (*Javob*: $a)\ \frac{49}{24}\approx 2,04;\quad b)\frac{9}{2}$).

AT-9.4

1. Berilgan $y=2\sqrt{x}$ parabolaning $x_1=0$ va $x_2=1$ abssissalari o'rtasidagi yoy uzunligini hisoblang. *(Javob:* $\sqrt{2}+\ln(1+\sqrt{2})\approx 2,29$*)*

2. Astroidaning uzunligi hisoblansin $x=a\cos^3 t,\ y=a\sin^3 t$. *(Javob: 6a)*

3. Kardiodaning uzunligini hisoblang $\rho=a(1-\cos\varphi)$. *(Javob: 8a)*

4. $y=\frac{2}{3}\sqrt{(x-1)^3}$ egri chiziqning $x_1=1,\ x_2=9$ abssissalar o'rtasidagi yoy bo'lagining uzunligi hisoblansin.*(Javob:* $\frac{56}{3}$*)*

5. Ushbu $z=\frac{x^2}{4}+\frac{y^2}{2}$, $z=1$ sirtlar bilan chegaralangan jismning hajmi hisoblansin. *(Javob:* $\pi\sqrt{2}$ *)*

6. *Oxy* tekisligida yotgan va $y=x^2,\ x=y^2$ chiziqlar bilan chegaralangan figuraning Ox o'qi atrofida aylanishdan hosil bo'lgan jismning hajmini hisoblang

(Javob: $\frac{3\pi}{10}$*)*

7. Sikloida $x=a(t-\sin t),\ y=a(1-\cos t)$ birinchi arkasini Ox o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmini hisoblang.

(Javob: $5a^2\pi^2$*)*

8. $y=\frac{1}{2}\sqrt{4x-1}$ egri chiziqning $x_1=1$ nuqtadan $x_2=9$ nuqtagacha bo‘lgan yoy bo‘lagini aylantirishdan hosil bo‘lgansirtning yuzini toping *(Javob: 104 π/3)*

9. $y=a\, ch\frac{x}{a}$ chiziqni $x_1=0$ nuqtadan $x_2=a$ nuqtagacha bo‘lgan qismining *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan sirt, katenoidning yuzini hisoblang. *(Javob:* $\frac{\pi a^2}{4}(e^2-e^{-2}+4)$ *)*

Mustaqil ish

1. 1.Ushbu $y=\frac{1}{3}\sqrt{(2x-1)^3}$ egri chiziqning abssissalari $x_1=2$ va $x_2=8$ ga teng bo‘lgan M_1M_2 nuqtalar orasidagi yoy bo‘lagining uzunligi hisoblansin (*Javob:56/3*).

2. $y=3x$ to‘g‘ri chiziqning $x_1=0$ va $x_2=2$ abssissaga ega nuqtalari bilan chegaralangan kesmasini *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan aylanish sirtining yuzasi hisoblansin. *(Javob:* $12\sqrt{10}\,\pi$*)*

2. 1. Tenglamasi $y=\frac{4}{3}x$ bo‘lgan chiziqning $x_1=2$ va $x_2=5$ nuqtalar orasidagi yoy uzunligi hisoblansin. (*Javob: 5*).

2. Tenglamasi $y=\frac{x^2}{1}+\frac{z^2}{4}, y=1$ bo‘lgan sirtlar bilan chegaralangan jismning hajmi topilsin. *(Javob: π)*

3. 1. $y=lnx$ tenglama bilan berilgan egri chiziqning abssissalari $x_1==\sqrt{3}$ va $x_2=\sqrt{8}$ nuqtalar orasidagi yoy uzunligi hisoblansin.

(*Javob*: $1+\frac{1}{2}\ln\frac{3}{2}\approx 1,2$)

2. *Oxu* tekislikda yotgan $y=2x-x^2$ va $y=0$ chiziqlar bilan chegaralangan figurani *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi topilsin. (*Javob:* $\frac{16}{15}\pi$*)*

9.4. Aniq integralni fizik masalalarni yechishga qo'llash. Tezlik bo'yicha bosib o'tilgan yo'lni hisoblash.

Agar $\upsilon = f(t)$ moddiy nuqtaning to'g'ri chiziq bo'yicha harakatidagi tezligini ifodalasa, u holda $[t_1;t_2]$ vaqt oralig'ida bosib o'tilgan yo'l.

$$S = \int_{t_1}^{t_2} f(t)dt \qquad (9.16)$$

formula bilan ifodalanadi.

1-misol. Moddiy M nuqta $\upsilon(t) = 3t^2 + 2t + 1$ m/s tezlik bilan to'g'ri chiziqli harakat qilsin. Nuqtaning $[0;3]$ sekund oralig'ida bosib o'tgan yo'lini toping.

► (9.16) formulaga asosan

$$S = \int_0^3 (3t^2 + 2t + 1)dt = (t^3 + t^2 + t)\Big|_0^3 = 39 \text{ M.}$$ ◄

O'zgaruvchi kuchning bajargan ishini hisoblash

Moddiy M nuqta $F(s)$ kuch ta'siri ostida OS to'g'ri chiziq bo'yicha harakatlansin. Bu kuchning yo'lning $[a;b]$ qismida bajargan ishi $A = \int_a^b F(s)ds$ formula bilan hisoblanadi.

2-misol. Agar prujinani *1sm* cho'zish uchun *1kN* kuch sarf qilinsa uni *10sm* cho'zish uchun bajariladigan ishni hisoblang.

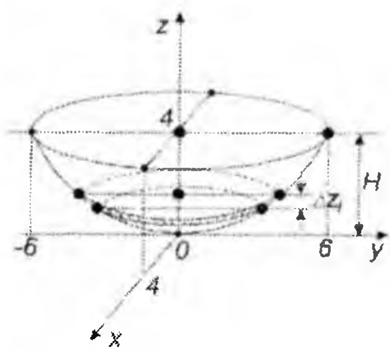
►Guk qonuniga asosan, prujinani cho'zadigan kuch, uni cho'zilishiga proporsional, ya'ni $F=kx$, bu yerda x-prujinaning cho'zilishi (*metrda*), k-proporsionallik koeffitsenti. Masala shartiga ko'ra $x=0{,}01m$, $F=1\ kn$, $1=0{,}01\ k$ tenglikdan $k=100$ ekanligi kelib chiqadi va $F=100x$ bajarilgan ish.

$$A = \int_0^{0,1} 100x dx = 50x^2 \Big|_0^{0,1} = 0{,}5\, kDj.$$ ◄

3-misol. Qozon $z = \frac{x^2}{4} + \frac{y^2}{9}$ elliptik paraboloid shaklida bo'lib, balandligi $H=4m$ va zichligi $\delta=0{,}8t/m^3$ bo'lgan suyuqlik bilan to'ldirilgan. Qozon chetidan suyuqlikni haydab chiqarishda bajarilgan ishni hisoblang.

▶ z_i balandlikda qalinligi Δz_i (9.18-rasm) bo'lgan suyuqlik qatlamini ajratamiz, ko'ndalang kesimda yarim o'qlari $a=2\sqrt{z_i}$ $b=3\sqrt{z_i}$ ga teng bo'lgan ellips hosil bo'lgani uchun, bu qatlamning massasi $\Delta mi \approx 6\pi\delta z_i \Delta z_i$ va hajmi $\Delta V_i = \pi \cdot 2\sqrt{z_i} \cdot 3\sqrt{z_i}\Delta z_i$ ga teng bo'ladi. Suyuqlikni haydab chiqarish uchun bajarilgan ish:

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \Big| 6\pi g\delta z_i(H - z_i)\Delta z_i = \int_0^H 6\pi g\delta z(H - z)\, dz = 6\pi g\delta \left(H\frac{z^2}{2} - -\frac{z^3}{3}\right)\Big|_0^H = \pi g\delta H^3 = 64 g\pi\delta \approx 1575{,}53 kDj.$ ◀



9.18- rasm

Suyuqlikning plastinkaga bosim kuchini hisoblash

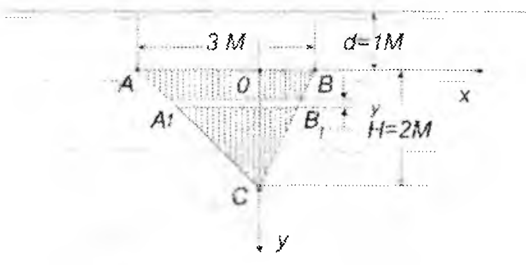
Bu masalani yechish usulini aniq misolda ko'rib o'tamiz.

4-misol. Asosi $a=3m$ va balandligi $N=2m$ bo'lgan uchburchakli plastinka suyuqlikka uchi bilan vertikal botirilgan va asosi suyuqlikning sathiga parallel bo'lib, undan $1m$ uzoqlikda joylashgan. Suyuqlikning zichligi

$\delta = 0{,}9\ t/m^3$ bo'lsa, uning plastinkaning har ikki tomoniga bosim kuchini hisoblang.

►Suyuqlikning bosim kuchini hisoblash uchun Paskal qonunidan foydalanamiz, unga ko'ra suyuqlikning h chuqirlikdagi ΔS yuziga bosim $\Delta p = \delta g h \Delta S$ formula bilan aniqlanadi. Bu yerda δ -suyuqlikning zichligi, g-jismning erkin tushishdagi tezlanishi.

Suyuqlik satxiga parallel to'g'ri chiziqlar bilan, uchburchakni eni dy ga teng bo'lgan kesimlar (9.19-rasm)



9.19- rasm

bilan bo'lib chiqamiz va u suyuqlik satxidan $y+d$ masofada bo'lsin. AVS va A, V, S, uchburchaklarning o'xshashligidan:

$\frac{|A_1B_1|}{a} = \frac{H-y}{H}, |A_1B_1| = \frac{a}{H}(H-y)$, ya'ni kesimning yuzi:

$$dS = \frac{a}{H}(H-y)dy,$$

Uchburchakli plastinkada kesilgan kesim yuzining har bir tomoniga bosimi

$dp = \frac{a}{H}\delta g\,(d+y)(H-y)dy$ ga teng bo'ladi. Oxirgi tenglikning ikki tomonini integrallab, quyidagini hosil qilamiz

$$P = \int_0^H \frac{a}{H}\delta g\,(d+y)(H-y)dy = \frac{3}{2}\delta g\int_0^2 (2+y-y^2)dy = \frac{3}{2}\delta g\left(2y+\frac{y^2}{2}-\frac{y^3}{3}\right)\Big|_0^2 = 5\,\delta g \approx 44{,}1\,KH$$

◄

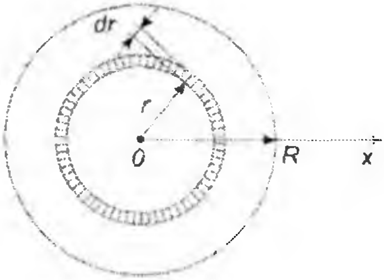
Inersiya momentini hisoblash

Aniq integral yordamida yassi figuralarning inersiya momentini hisoblash mumkin.

5-misol. Massasi M ga radiusi R ga teng bo'lgan bir jinsli doiraning markaziga nisbatan inersiya momenti hisoblansin.

▶Massasi M bo'lgan moddiy nuqtaning O nuqtaga nisbatan inersion momenti, shu nuqta massasining, undan O nuqtagacha bo'lgan masofa kvadratining ko'paytmasiga teng. Moddiy nuqtalar sistemasining inersion momenti, shu nuqtalar inersion momentlarining yig'indisiga teng.

Konsentrik aylanalar yordamida doirani n ta, eni d ga teng bo'lgan halqalarga



9.20- rasm

bo'lamiz, Bu xalqalarning yuzi $ds = 2\pi r dr$, massasi $dm = 2\pi r dr\delta$ zichligi $\delta = M(\pi R^2)$ ga teng (9.20-rasm). Ajratilgan xalqalarning elementar momenti $dI_0 = 2\pi\delta r^3 dr$. Elementar inersion momentlarni integrallab

$$I = \int_0^R 2\pi\delta r^3 dr = 2\pi\delta^r \frac{4}{4}\Big|_o^R = \frac{1}{2}\pi R^4 \frac{M}{\pi R^2} = \frac{1}{2}MR^2. \quad ◀$$

Yassi figuraning og'irlik markazini hisoblash

Quyidagi hollarni ko'rib chiqamiz.

1. $u=f(x)$ funksiya grafigining, zichligi $\delta=\delta(x)$ bo'lgan, AV yoy bo'lagining og'irlik markazi koordinatalari $c(x_c, y_c)$, quyidagi formulalar yordamida aniqlanadi (9.12-rasm):

$$x_c = \frac{\int_a^b x\delta(x)\sqrt{1+y'^2dx}}{\int_a^b \delta(x)\sqrt{1+y'^2dx}}, y_c = \frac{\int_a^b y\delta(x)\sqrt{1+y'^2dx}}{\int_a^b \delta(x)\sqrt{1+y'^2dx}}$$

2. Agar yassi figura quyidan $f_1(x)$, yuqoridan $f_2(x)$, $f_1(x) \le f_2(x)$, chiziqlar bilan $[a,b]$ kesmada chegaralangan va

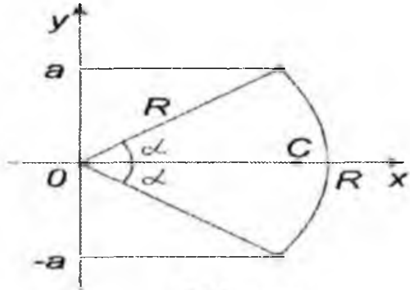
figuraning zichligi $\delta=\delta(x)$ bo‘lsa, u holda uning og‘irlik markazi $S(x_s;u_s)$

$$x_c = \frac{\int_a^b x\delta(x)(f_2(x)-f_1(x))dx}{\int_a^b \delta(x)(f_2(x)-f_1(x))dx} \quad y_c = \frac{\frac{1}{2}\int_a^b \delta(x)(f_2^2(x)-f_1^2(x))dx}{\int_a^b \delta x(f_2(x)-f_1(x))dx} \qquad (9.17)$$

formula orqali topiladi

6- masala. Markaziy burchagi 2α, radiusi R ga teng bo‘lgan aylananing bir jinsli yoy bo‘lagining og‘irlik markazi topilsin.

►Koordinatalar sistemasini 9.21 rasmda



9.21- rasm

ko‘rsatilgandek tanlab olamiz. U holda yoyning bir jinsli va simmetrik ekanligidan $u_s=0$ kelib chiqadi. Yuqoridagi formuladan x_s ni topib olamiz $(\delta = const)$

$$x_c = \frac{\int_{-a}^{a} x\sqrt{1+x'^2 dy}}{\int_{-a}^{a}\sqrt{1+x'^2 dy}}$$

Aylananing parametrik tenglamasidan foydalanamiz $x=R\cos t$, $y=R\sin t$,

$$x_c = \frac{\int_{-\alpha}^{\alpha} R^2 \cos t dt}{\int_{-\alpha}^{\alpha} R\,dt} = R\frac{\sin t\big|_{-\alpha}^{\alpha}}{t\big|_{-\alpha}^{\alpha}}\Bigg|_{-\alpha}^{\alpha} = R\frac{\sin\alpha}{\alpha}$$ ◄

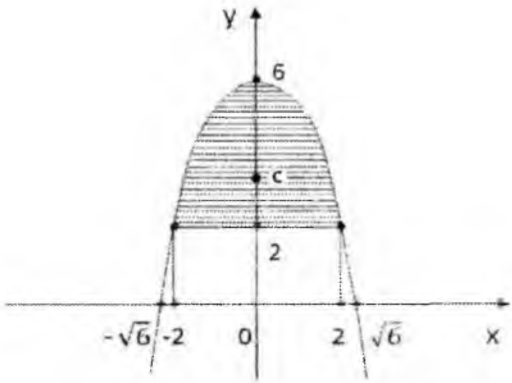
7- misol. Bir jinsli, $y=6-x^2$, $y=2$ chiziqlar bilan chegaralangan yassi figuraning og‘irlik markazi topilsin.

►Bu figuraning bir jinsli va simmetrik ekanligidan (9.22-rasm) $x_c = 0$. y_c koordinatani topish uchun (9.17) formuladan foydalanamiz:

$$y_c = \frac{1}{2}\frac{\int_{-2}^{2}((6-x^2)^2-2^2)dx}{\int_{-2}^{2}(4-x^2)dx} = \frac{1}{2}\frac{\int_{-2}^{2}(32-12x^2+x^4)dx}{2\int_{0}^{2}(4-x^2)dx}$$

$$= \frac{1}{2}\frac{\left(32x-4x^3+\frac{x^5}{5}\right)\Big|_0^2}{(4x-x^3/3)\Big|_0^2} =$$

$$= \frac{1}{2}\frac{192/5}{16/3} = 3{,}6. \blacktriangleleft$$



9.22-rasm

AT-9.5.

1. To‘g‘ri chiziqli harakat qilayotgan nuqtaning tezligi $\upsilon = t.e^{-0.01t}$ *m/s* ga teng. Nuqtaning to‘la to‘xtaguncha bosib o‘tgan masofasini hisoblang (*Javob:* $10^4 m$)

2. Uzunligi *l* ga og‘irligi *R* ga, teng bo‘lgan bir jinsli sterjenning oxiriga nisbatan inersiya momenti topilsin. (*Javob:* $\frac{1}{3}\frac{P}{g}l^2$).

3. Bir jinsli, zichligi δ=2,5 t/m^3 qurilish materialidan, radiusi *R=2m*, balandligi *H=3m* bo‘lgan konus ko‘rinishdagi

qo‘rg‘onni qurish uchun sarflangan ishni hisoblang. (*Javob:* $\frac{8}{15}\pi g\delta\, H^2\, R^2 = 48\pi g \approx 1477{,}8\,кДж$)

4. Yuqori asosi suyuqlik sathiga parallel, *5m* chuqurlikka vertikal cho‘ktirilgan, asosi *8m*, balandligi *12m* bo‘lgan to‘g‘ri to‘rt burchakka suvning bosim kuchi aniqlansin. Suvning zichligi $\delta = 1 t/m^3$. (*Javob:* $656\, g \approx 6428{,}8 kN$)

5. Zanjir $y = ach\frac{x}{a}$ chiziqning $x= -a$ nuqtadan $x=a$ nuqtagacha bo‘lgan bir jinsli yoy bo‘lagining og‘irlik markazini toping.

(*Javob:* $x_c=0,\ y_c = \frac{a}{4}\frac{2+sh^2}{sh\,1}$)

6. $x=a\ (t\text{-}sint),\ y=a\ (1\text{-}cost),\ (0 \le t \le 2\pi)$, siklonda birinchi arkasining bir jinsli yoy bo‘lagining og‘irlik markazini toping (*Javob:* $x_c= \pi a\ \ y_c=4a/3$)

7. Yassi $u=x$ va $u=x^2-2x$ chiziqlar bilan chegaralangan birjinsli figuraning og‘irlik markazini toping. (*Javob:* $(3/2, 3/5)$)

Mustaqil ish

1. 1. Asosi suv sathiga parallel, balandligi $H=3$ m, asosi $a=2m$ va *4m* chuqurlikka tik tushirilgan paralelogramm ko‘rinishdagi plastinkaga beradigan suvning bosim kuchini aniqlang. Suvning zichligi $1 t/m^3$. (*Javob:* $16g \approx 156{,}8\ кН$)

2. Radiusi R ga markazi koordinata boshida bo‘lgan aylananing birinchi kvadratda yotgan bir jinsli yoy bo‘lagining og‘irlik markazi topilsin.

(*Javob:* $2R/\pi, 2R/\pi$).

2. 1. Moddiy nuqtaning tezligi $\upsilon = 4te^{-t^2}$ m/s bo‘lsa, nuqta harakat boshlagandan to‘xtaguncha qancha yo‘l bosib o‘tadi (*Javob:2m*)

2. Bir jinsli $y=\sin x$, $y=0$ $(0 \le x \le \pi)$ chiziqlar bilan chegaralangan figuraning og'irlik markazi topilsin. (*Javob:* $\frac{\pi}{2}, \ \frac{\pi}{8}$)

3. 1. Agar suvning zichligi $\delta=1\ t/m^3$, bo'lsa diametri $20m$ yarim sferik ko'rinishdagi idishdan suvni haydab chiqarishda bajarilgan ishni hisoblang. (*Javob:* $2{,}5g10^3\pi \approx 76969$ *кДэж*)

2. Bir jinsli yassi $u^2=20x$, $x^2=20u$ chiziqlar bilan chegaralangan figuraning og'irlik markazini toping. (*Javob:* *(9,9)*).

9.5. 9 bo'limga doir indiviudal uy topshiriqlari

IUT-9.1.

Aniq integralni verguldan keyin 2 ta raqam aniqligida hisoblang.

1.1. $\int_0^{\sqrt{3}} x\sqrt[3]{1+x^2}\,dx.$ (*Javob: 1,78.*)

1.2. $\int_0^{12\sqrt{3}} \frac{12x^5dx}{\sqrt{x^6+1}}.$ (*Javob: 2,60.*)

1.3. $\int_0^1 \frac{x^2dx}{x^2+1}.$ (*Javob: 0,21.*)

1.4. $\int_0^{\pi/2} \sin x\cos^2 x dx.$ (*Javob: 0,33.*)

1.5. $\int_0^{\pi/2} \frac{\cos x}{1+\cos x}dx.$ (*Javob: 0,57.*)

1.6. $\int_{3/4}^{4/3} \frac{dx}{x^2+1}$. (*Javob: 0,41.*)

1.7. $\int\limits_0^{-3} \frac{dx}{\sqrt{25+3x}}.$ *(Javob: -0,67.)*

1.8. $\int\limits_0^{2} \frac{x^3 dx}{\sqrt{x^4+4}}.$ *(Javob: 1,24.)*

1.9. $\int\limits_1^{e} \frac{1+\ln x}{x} dx.$ *(Javob: 1,50.)*

1.10. $\int\limits_0^{1} \frac{z^3}{z^8+1} dz.$ *(Javob: 0,20.)*

1.11. $\int\limits_{\pi/4}^{\pi/2} \frac{dx}{1-\cos^2 x}.$ *(Javob: 0,50.)*

1.12. $\int\limits_2^{5} \frac{dx}{\sqrt{5+4x-x^2}}.$ *(Javob: 1,57.)*

1.13. $\int\limits_0^{1} x^3\sqrt{4+5x^4}\, dx.$ *(Javob: 0,63.)*

1.14. $\int\limits_{-\pi}^{\pi} \sin^2\frac{x}{2} dx.$ *(Javob: 3,14.)*

1.15. $\int\limits_1^{2} \frac{e^{1/x}}{x^2} dx.$ *(Javob: 1,07.)*

1.16. $\int\limits_0^{1/2} \frac{xdx}{\sqrt{1-x^2}}.$ *(Javob: 0.13.)*

1.17. $\int\limits_0^{1} 3\left(x^2+x^2e^{x^3}\right)dx.$ *(Javob: 2,72.)*

1.18. $\int\limits_{\pi^2/9}^{\pi^2} \frac{\cos\sqrt{x}}{\sqrt{x}}dx.$ *(Javob: 1,73.)*

1.19. $\int\limits_{1}^{\sqrt{3}} \frac{x^2dx}{1+x^6}.$ *(Javob: 0,20.)*

1.20. $\int\limits_{1}^{e} \frac{\sin\ln x}{x}dx.$ *(Javob: 0,46.)*

1.21. $\int\limits_{1}^{\sqrt{e}} \frac{dx}{x\sqrt{1-\ln^2 x}}.$ *(Javob: 0,52.)*

1.22. $\int\limits_{3}^{8} \sqrt{x+1}dx.$ *(Javob: 12,67.)*

1.23. $\int\limits_{\pi/6}^{\pi/2} \sin\alpha\cos^3\alpha d\alpha.$ *(Javob: 0,14.)*

1.24. $\int\limits_{\pi/18}^{\pi/6} 12ctg\,3xdx.$ *(Javob: 2,77.)*

1.25. $\int\limits_{0}^{1} \frac{dx}{\sqrt{4-3x}}.$ *(Javob: 0,67.)*

1.26. $\int\limits_{1}^{\sqrt{2}} \frac{xdx}{\sqrt{4-x^2}}.$ *(Javob: 0,32.)*

1.27. $\int\limits_{1}^{e} \frac{\ln^2 x}{x}dx.$ *(Javob: 0,33.)*

1.28. $\int\limits_{-1}^{0} \frac{dx}{4x^2-9}.$ *(Javob: -0,13)*

1.29. $\int\limits_{\pi/6}^{\pi/2} \cos\alpha \sin^3 \alpha d\alpha.$ *(Javob: 0,23)*

1.30. $\int\limits_{0}^{\sqrt{\pi}/4} \frac{xdx}{\cos^2(x^2)}.$ *(Javob: 0,50.)*

2.1. $\int\limits_{2}^{3} y\ln(y-1)dy.$ *(Javob: 1,02.)*

2.2. $\int\limits_{-2}^{0} x^2 e^{-x/2} dx.$ *(Javob: 5,76.)*

2.3. $\int\limits_{0}^{\pi/2} x\cos xdx.$ *(Javob: 0,57.)*

2.4. $\int\limits_{0}^{\pi} x^2 \sin xdx.$ *(Javob: 5,86.)*

2.5. $\int\limits_{-1/2}^{1/2} \arccos 2xdx.$ *(Javob: 3,14.)*

2.6. $\int\limits_{1}^{2} (y-1)\ln ydy.$ *(Javob: 0,25.)*

2.7. $\int\limits_{-1/2}^{0} xe^{-2x} dx.$ *(Javob: -0,25.)*

2.8. $\int\limits_{-\pi}^{\pi} x\sin x\cos xdx.$ *(Javob: 1,57.)*

2.9. $\int\limits_{-1/3}^{-2/3} \frac{x}{e^{3x}} dx.$ *(Javob: 0,82.)*

2.10. $\int_1^e \frac{\ln^2 x}{x^2}dx.$ *(Javob: 0,16.)*

2.11. $\int_1^{e^2} \sqrt{x}\ln x dx.$ *(Javob: 18,33.)*

2.12. $\int_0^1 arctg\sqrt{x}\, dx.$ *(Javob: 0,57.)*

2.13. $\int_0^{\pi} (x+2)\cos\frac{x}{2}dx.$ *(Javob: 6,28.)*

2.14. $\int_0^{\pi/8} x^2 \sin 4x dx.$ *(Javob: 0,17.)*

2.15. $\int_1^2 y^2 \ln y dy.$ *(Javob: 1,07.)*

2.16. $\int_1^2 \frac{\ln(x+1)}{(x+1)^2}dx.$ *(Javob: 0,15.)*

2.17. $\int_{3/2}^2 artg(2x-3)dx.$ *(Javob: 0,21.)*

2.18. $\int_0^{\pi/2} (x+3)\sin x dx.$ *(Javob: 4,00.)*

2.19. $\int_1^e x\ln^2 x dx.$ *(Javob: 1,60.)*

2.20. $\int_{-3}^0 (x-2)e^{-x/3}dx.$ *(Javob: -19,32.)*

2.21. $\int\limits_0^{\pi/9} \frac{xdx}{\cos^2 3x}$. *(Javob: 0,12.)*

2.22. $\int\limits_{1/2}^{1} \arcsin(1-x)dx.$ *(Javob: 0,13.)*

2.23. $\int\limits_1^{\sqrt{3}} arctg\frac{1}{x}dx$. *(Javob: 1,37.)*

2.24. $\int\limits_{-1}^{0} x\ln(1-x)dx.$ *(Javob: -0,25.)*

2.25. $\int\limits_0^{1} \frac{\arcsin(x/2)}{\sqrt{2-x}}dx.$ *(Javob: 2,32.)*

2.26. $\int\limits_1^{2} \ln(3x+2)dx.$ *(Javob: 1,87.)*

2.27. $\int\limits_0^{4} x^3\sqrt{x^2+9}dx.$ *(Javob: 282,40.)*

2.28. $\int\limits_{-1}^{0} (x+1)e^{-2x}dx.$ *(Javob: 1,10.)*

2.29. $\int\limits_0^{\pi/4} x\,tg^2 xdx.$ *(Javob: 0,13.)*

2.30. $\int\limits_0^{1} x\,arctg\,xdx.$ *(Javob: 0,29.)*

3.1. $\int\limits_0^{1} \frac{3x^4+3x^2+1}{x^2+1}dx.$ *(Javob: 1,79.)*

3.2. $\int_{2}^{3}\frac{2x^4-5x^2+3}{x^2-1}dx.$ *(Javob: 9,67.)*

3.3. $\int_{2}^{3}\frac{x+2}{x^2(x-1)}dx.$ *(Javob: 0,53.)*

3.4. $\int_{2}^{3}\frac{dx}{x^2(x-1)}.$ *(Javob: 0,12.)*

3.5. $\int_{-1}^{1}\frac{y^5dy}{y+2}.$ *(Javob: -0,09.)*

3.6. $\int_{2}^{3}\frac{3x^2+2x-3}{x^3-x}dx.$ *(Javob: 1,62.)*

3.7. $\int_{1/3}^{1/2}\frac{xdx}{(x-1)^3}.$ *(Javob: -1,25.)*

3.8. $\int_{4}^{5}\frac{dx}{(x-1)(x+2)}.$ *(Javob: 0,04.)*

3.9. $\int_{3}^{4}\frac{dx}{(x+1)(x-2)}.$ *(Javob: 0,16.)*

3.10. $\int_{0}^{1}\frac{(2x+3)dx}{(x-2)^3}.$ *(Javob: -1,63.)*

3.11. $\int_{2}^{3}\frac{dx}{(x-1)^2(x+1)}.$ *(Javob: 0,15.)*

3.12. $\int_{3}^{5}\frac{(x^2+2)dx}{(x+1)^2(x-1)}.$ *(Javob: 0,50.)*

3.13. $\int_{0}^{1} \frac{x^4+3x^3-1}{(x+1)^2}dx.$ *(Javob: -0,20.)*

3.14. $\int_{-1}^{0} \frac{x^5-2x^2+3}{(x-2)^2}dx.$ *(Javob: 9,38.)*

3.15. $\int_{0}^{1} \frac{xdx}{x^2+3x+2}.$ *(Javob: 0,12.)*

3.16. $\int_{8}^{10} \frac{(x^2+3)dx}{x^3-x^2-6x}.$ *(Javob: 0,29.)*

3.17. $\int_{1}^{\sqrt{3}} \frac{dx}{x^4+x^2}.$ *(Javob: 0,16.)*

3.18. $\int_{2}^{3} \frac{x^7dx}{1-x^4}.$ *(Javob: -15,34.)*

3.19. $\int_{2}^{3} \frac{dx}{x^4-1}.$ *(Javob: 0,02.)*

3.20. $\int_{-1}^{0} \frac{xdx}{x^3-1}.$ *(Javob: 0,37.)*

3.21. $\int_{0}^{\sqrt{3}/3} \frac{2x^2+4}{x^3-x^2+x+1}.$ *(Javob: 0,88.)*

3.22. $\int_{4}^{5} \frac{dx}{x^2(x-1)}.$ *(Javob: 0,02.)*

3.23. $\int_{0}^{2} \frac{dx}{(x+1)(x^2+4)}.$ *(Javob: 0,23.)*

3.24. $\int_{7}^{9}\frac{x^2-x+2}{x^4-5x^2+4}dx.$ *(Javob: 0,04.)*

3.25. $\int_{4}^{6}\frac{xdx}{x^3-6x^2+16-6}.$ *(Javob: 0,51.)*

3.26. $\int_{1}^{2}\frac{dx}{x^3+1}.$ *(Javob: 0,25.)*

3.27. $\int_{1}^{\sqrt{3}}\frac{x^5+1}{x^6+x^4}dx.$ *(Javob: 1,44.)*

3.28. $\int_{2}^{3}\frac{x^3+x^2+2}{x(x^2-1)^2}dx.$ *(Javob: -0,12.)*

3.29. $\int_{3}^{5}\frac{x^3-2x^2+4}{x^3(x-2)^2}dx.$ *(Javob: 0,35.)*

3.30. $\int_{0}^{1/\sqrt{3}}\frac{x^2dx}{x^4-1}.$ *(Javob: -0,08.)*

4.1. $\int_{0}^{2}x^2\sqrt{x-x^2}dx.$ *(Javob: 3,14.)*

4.2. $\int_{\sqrt{2}}^{1}\frac{\sqrt{4-x^2}}{x^2}dx.$ *(Javob: -0,47.)*

4.3. $\int_{3}^{6}\frac{\sqrt{x^2-9}}{x^4}dx.$ *(Javob: 0,02.)*

4.4. $\int_{0}^{1}\sqrt{4-x^2}dx.$ *(Javob: 1,91.)*

4.5. $\int_{1}^{\sqrt{3}} \frac{x^3+1}{x^2\sqrt{4-x^2}}dx.$ *(Javob: 1,02.)*

4.6. $\int_{0}^{\sqrt{3}} \sqrt{3-x^2}\,dx.$ *(Javob: 2,36.)*

4.7. $\int_{-3}^{3} x^2\sqrt{9-x^2}\,dx.$ *(Javob: 31,79.)*

4.8. $\int_{\sqrt{2}}^{1} \frac{\sqrt{1-x^2}}{x^6}dx.$ *(Javob: 0,53.)*

4.9. $\int_{0}^{1} \sqrt{(1-x^2)^3}\,dx.$ *(Javob: 0,59.)*

4.10. $\int_{\sqrt{3}/3}^{1} \frac{dx}{x^2\sqrt{(1+x^2)^3}}.$ *(Javob: -0,62.)*

4.11. $\int_{1}^{2} \frac{\sqrt{x^2-1}}{x}dx.$ *(Javob: 0,68.)*

4.12. $\int_{0}^{1} \frac{dx}{(x^2+3)^{3/2}}$ *(Javob: 0,27.)*

4.13. $\int_{1}^{\sqrt{2}} \sqrt{2-x^2}\,dx.$ *(Javob: 1,29.)*

4.14. $\int_{0}^{1} \frac{x^2dx}{(x^2+1)^2}.$ *(Javob: 0,14.)*

4.15. $\int_{2\sqrt{3}}^{6} \frac{dx}{x^2\sqrt{x^2-9}}.$ *(Javob: 0,04.)*

4.16. $\int_{1/\sqrt{3}}^{1} \frac{dx}{x^2\sqrt{1+x^2}}$. *(Javob: 0,59.)*

4.17. $\int_{1/2}^{\sqrt{3}/2} \sqrt{1-x^2}\,dx.$ *(Javob: 0,26.)*

4.18. $\int_{0}^{3} \frac{dx}{(9+x^2)\sqrt{9+x^2}}$ *(Javob: 0,08.)*

4.19. $\int_{2}^{4} \frac{\sqrt{x^2-4}}{x}\,dx.$ *(Javob: 0,68.)*

4.20. $\int_{-1/2}^{1/2} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$. *(Javob: 1,16.)*

4.21. $\int_{0}^{\sqrt{2,5}} \frac{dx}{(5-x^2)^3}$. *(Javob: 0,20.)*

4.22. $\int_{0}^{1/2} \frac{x^4\,dx}{\sqrt{(1-x^2)^3}}$. *(Javob: -0,20.)*

4.23. $\int_{\sqrt{3}}^{2} \frac{dx}{x^4\sqrt{x^2-3}}$. *(Javob: 0,05.)*

4.24. $\int_{2}^{4} \frac{\sqrt{16-x^2}}{x^4}\,dx.$ *(Javob: 0,11.)*

4.25. $\int_{0}^{\sqrt{7/3}} x^3\sqrt{7+x^2}\,dx.$ *(Javob: -502,09.)*

4.26. $\int_{4\sqrt{2/3}}^{\sqrt{8}} \frac{\sqrt{x^2-8}}{x^4}\,dx.$ *(Javob: 0,01.)*

4.27. $\int_{1}^{\sqrt{2}} \frac{dx}{x^5\sqrt{x^2-1}}$. *(Javob: 0,29.)*

4.28. $\int_{0}^{3} x^4\sqrt{9-x^2}\,dx$. *(Javob: 71,53.)*

4.29. $\int_{0}^{3} \frac{x^3 dx}{\sqrt{9+x^2}}$. *(Javob: 5,31.)*

4.30. $\int_{0}^{\sqrt{6}} \sqrt{6-x^2}\,dx$. *(Javob: 4,71.)*

5.1. $\int_{-\pi/2}^{-\pi/4} \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$. *(Javob: 0,26.)*

5.2. $\int_{0}^{\pi/2} \frac{dx}{2+\cos x}$. *(Javob: 0,60.)*

5.3. $\int_{0}^{\pi/4} \sin^3 2x\,dx$. *(Javob: 0,33.)*

5.4. $\int_{0}^{\pi} \sin^4 \frac{x}{2}\,dx$. *(Javob: 1,18.)*

5.5. $\int_{0}^{\pi/3} \cos^3 x \sin 2x\,dx$. *(Javob: 0,39.)*

5.6. $\int_{0}^{\pi/3} tg^2 x\,dx$. *(Javob: 0,68.)*

5.7. $\int_{\pi/2}^{\pi} \frac{\sin x}{(1-\cos x)^3}\,dx$. *(Javob: 0,38.)*

5.8. $\int\limits_0^{\pi/4} 2\cos x \sin 3x dx.$ *(Javob: 1,00.)*

5.9. $\int\limits_0^{\pi} \cos\frac{x}{2}\cos\frac{x}{3} dx.$ *(Javob: 1,80.)*

5.10. $\int\limits_0^{\pi/32} (32\cos^2 4x - 16) dx.$ *(Javob: 1,41.)*

5.11. $\int\limits_0^{\pi/2} \frac{\cos x dx}{\sin^2 x + 1}.$ *(Javob: 3,14.)*

5.12. $\int\limits_{\pi/4}^{\pi/3} tg^4 \varphi d\varphi.$ *(Javob: 0,93.)*

5.13. $\int\limits_0^{\pi} \cos\frac{x}{2}\cos\frac{3x}{2} dx.$ *(Javob: 0.)*

5.14. $\int\limits_0^{\pi/4} \sin 3x \cos 5x dx.$ *(Javob: -0,25.)*

5.15. $\int\limits_0^{\pi/3} \frac{\sin^3 x}{\cos^4 x} dx.$ *(Javob: 1,33.)*

5.16. $\int\limits_0^{\pi/6} \frac{dx}{\cos x}.$ *(Javob: 0,55.)*

5.17. $\int\limits_{\pi/6}^{\pi/2} ctg^3 x dx.$ *(Javob: 0,81.)*

5.18. $\int\limits_0^{\pi/2} \cos x \cos 3x \cos 5x dx.$ *(Javob: 0,16.)*

5.19. $\int_0^{\pi} \cos^4 x \sin^2 x dx.$ *(Javob: 0,20.)*

5.20. $\int_0^{\pi/2} \sin^6 x dx.$ *(Javob: 0,49.)*

5.21. $\int_{\pi/2}^{\pi} \sqrt{1+\sin x} dx.$ *(Javob: 2.)*

5.22. $\int_{\pi/6}^{\pi/4} \frac{1+tg\ x}{\sin 2x}.$ *(Javob: 0,38.)*

5.23. $\int_{\pi/5}^{\pi/3} \frac{\sin 2x}{\cos^3 x} dx.$ *(Javob: 1,69.)*

5.24. $\int_0^{\pi/8} \sin x \sin 3x dx.$ *(Javob: 0,05.)*

5.25. $\int_{\pi/4}^{\pi} \sin x \sin 2x \sin 3x dx.$ *(Javob: -0,21.)*

5.26. $\int_{\pi/3}^{\pi/2} \frac{dx}{\sin x}.$ *(Javob: 0,55.)*

5.27. $\int_0^{\pi/2} \cos^5 x dx.$ *(Javob: 0,53.)*

5.28. $\int_{\pi/2}^{\pi} \cos^2 x \sin^4 x dx.$ *(Javob: 0,10.)*

5.29. $\int_{\pi/3}^{\pi/2} \frac{dx}{\sin^3 x}.$ *(Javob: 0,60.)*

5.30. $\int_0^{\pi} \sin^4 \frac{x}{2} dx.$ *(Javob: 1,18.)*

6.1. $\int_2^3 \frac{dx}{2x^2 + 3x - 2}.$ *(Javob: 0,06.)*

6.2. $\int_{-2}^0 \frac{dx}{\sqrt{x^2 + 2x + 4}}.$ *(Javob: 1,10.)*

6.3. $\int_{-5}^{-2} \frac{dx}{x^2 + 4x - 21}.$ *(Javob: -0,14.)*

6.4. $\int_1^{\sqrt{5}} \frac{x^2 dx}{13 - 6x^3 + x^6}.$ *(Javob: 0,26.)*

6.5. $\int_1^2 \frac{dx}{x^2 + x}.$ *(Javob: 0,29.)*

6.6. $\int_{-1/2}^{1/2} \frac{dx}{4x^2 + 4x + 5}.$ *(Javob: 0,20.)*

6.7. $\int_{-1/2}^1 \frac{dx}{\sqrt{8 + 2x - x^2}}.$ *(Javob: 0,52.)*

6.8. $\int_1^2 \frac{dt}{t^2 + 5t + 4}.$ *(Javob: 0,07.)*

6.9. $\int_0^2 \frac{xdx}{x^2 + 3x + 2}.$ *(Javob: 0,28.)*

6.10. $\int_1^2 \frac{x - 5}{x^2 - 2x + 2}.$ *(Javob: -2,79.)*

6.11. $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$. *(Javob: 0,39.)*

6.12. $\int_{6}^{8} \frac{dx}{x^2+2x}$. *(Javob: 0,03.)*

6.13. $\int_{1/2}^{1} \frac{dx}{\sqrt{x-x^2}}$. *(Javob: 1,57.)*

6.14. $\int_{-1/2}^{0} \frac{2x-8}{\sqrt{1-x-x^2}}$. *(Javob: 3,99.)*

6.15. $\int_{3/4}^{2} \frac{dx}{\sqrt{2+3x-2x^2}}$. *(Javob: 1,11.)*

6.16. $\int_{1/6}^{2} \frac{dx}{3x^2-x+1}$. *(Javob: 0,77.)*

6.17. $\int_{3}^{4} \frac{x^2dx}{x^2-6x+10}$. *(Javob: 9,35.)*

6.18. $\int_{3,5}^{5} \frac{xdx}{x^2-7x+13}$. *(Javob: 4,94.)*

6.19. $\int_{2}^{3} \frac{3x-2}{x^2-4x+5}$. *(Javob: 3,19.)*

6.20. $\int_{-3/2}^{2} \frac{(x-1)^2}{x^2+3x+4}$. *(Javob: 2,41.)*

6.21. $\int_{4}^{5} \frac{xdx}{x^4-4x^2+3}$. *(Javob: 0,02.)*

6.22. $\int\limits_{-1/2}^{1} \frac{x^3}{x^2+x+1}dx.$ *(Javob: 0,08.)*

6.23. $\int\limits_{7}^{10} \frac{x^3dx}{x^2-3x+2}.$ *(Javob: 38,67.)*

6.24. $\int\limits_{3}^{5} \frac{x^2dx}{\sqrt{8x-x^2-15}}.$ *(Javob: 51,81.)*

6.25. $\int\limits_{0}^{1} \frac{dx}{x^2+4x+5}.$ *(Javob: 0,14.)*

6.26. $\int\limits_{-1/3}^{0} \frac{dx}{\sqrt{2-6x-9x^2}}.$ *(Javob: 0,21.)*

6.27. $\int\limits_{4}^{7} \frac{dx}{x^2+3x-10}.$ *(Javob: 0,09.)*

6.28. $\int\limits_{1/3}^{4/3} \frac{dx}{\sqrt{8+6x-9x^2}}.$ *(Javob: 0,52.)*

6.29. $\int\limits_{2}^{3} \frac{dx}{\sqrt{4x-3-x^2}}.$ *(Javob: 1,57.)*

6.30. $\int\limits_{-1}^{1} \frac{dx}{x^2+2x+3}.$ *(Javob: 0,61.)*

7.1. $\int\limits_{3}^{29} \frac{\sqrt[3]{(x-2)^2}}{3+\sqrt[3]{(x-2)^2}}dx.$ *(Javob: 16,16.)*

7.2. $\int\limits_{0}^{\ln 2} \frac{dx}{e^x(3+e^{-x})}.$ *(Javob: 0,13.)*

7.3. $\int_0^5 \frac{dx}{2x+\sqrt{3x+1}}$. *(Javob: 0,94.)*

7.4. $\int_3^8 \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx$. *(Javob: 11,77.)*

7.5. $\int_3^8 \frac{xdx}{\sqrt{x+1}}$. *(Javob: 10,67.)*

7.6. $\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}dx$. *(Javob: 0,86.)*

7.7. $\int_{\ln 2}^{2\ln 2} \frac{dx}{e^x-1}$. *(Javob: 0,41.)*

7.8. $\int_0^{\ln 2} \sqrt{e^x-1}dx$ *(Javob: 0,43.)*

7.9. $\int_0^5 \frac{xdx}{\sqrt{x+4}}$. *(Javob: 4,67.)*

7.10. $\int_0^4 \frac{dx}{1+\sqrt{2x+1}}$. *(Javob: 1,31.)*

7.11. $\int_{2/3}^{7/3} \frac{xdx}{\sqrt{2+3x}}$. *(Javob: 0,96.)*

7.12. $\int_{\ln 2}^{\ln 3} \frac{dx}{e^x-e^{-x}}$. *(Javob: 0,20.)*

7.13. $\int_0^1 \frac{x^2dx}{(1+x)^4}$. *(Javob: 0,04.)*

7.14. $\int\limits_{-1}^{0} \frac{dx}{1+\sqrt[3]{x+1}}$. *(Javob: 0,58.)*

7.15. $\int\limits_{0}^{\frac{1}{2}\ln 2} \frac{e^x dx}{e^x + e^{-x}}$. *(Javob: 0,20.)*

7.16. $\int\limits_{0}^{\sqrt[3]{7}} \frac{z^2 dz}{\sqrt{9+z^3}}$. *(Javob: 0,67.)*

7.17. $\int\limits_{0}^{5} \frac{xdx}{\sqrt{1+3x}}$. *(Javob: 4,00.)*

7.18. $\int\limits_{0}^{2} \frac{dx}{\sqrt{x+1}+\sqrt{(x+1)^3}}$. *(Javob: 0,52.)*

7.19. $\int\limits_{\ln 3}^{0} \frac{1-e^x}{1+e^x} dx.$ *(Javob: 0,29.)*

7.20. $\int\limits_{0}^{\pi/2} \frac{\cos y dy}{4+\sqrt{\sin y}}$. *(Javob: 0,22.)*

7.21. $\int\limits_{2}^{5} \frac{x^2 dx}{(x-1)\sqrt{x-1}}$. *(Javob: 8,44.)*

7.22. $\int\limits_{0}^{\ln 2} \frac{dx}{e^x\sqrt{1-e^{-2x}}}$. *(Javob: 1,05.)*

7.23. $\int\limits_{1}^{e^3} \frac{dx}{x\sqrt{1+\ln x}}$. *(Javob: 2,00.)*

7.24. $\int\limits_{\ln 2}^{\ln x} \frac{dx}{\sqrt{1+e^x}}$. *(Javob: 0,41.)*

7.25. $\int\limits_{e^2}^{e^3}\frac{\ln xdx}{x(1-\ln^2 x)}$. *(Javob: -0,49.)*

7.26. $\int\limits_{4}^{9}\frac{\sqrt{x}}{\sqrt{x}-1}dx$. *(Javob: 8,39.)*

7.27. $\int\limits_{\sqrt{7}}^{\sqrt{26}}\frac{x^3xd}{(x^2+1)^{2/3}}$. *(Javob: 22,88.)*

7.28. $\int\limits_{0}^{13}\frac{x+1}{\sqrt[3]{2x+1}}dx$. *(Javob: 38,06.)*

7.29. $\int\limits_{\ln 5}^{\ln 12}\frac{dx}{\sqrt{e^x+4}}$. *(Javob: 0,26.)*

7.30. $\int\limits_{-1}^{1}\frac{xdx}{\sqrt{5-4x}}$. *(Javob: 0,17.)*

8

Xosmas integrallar hisoblansin yoki ularning uzoqlashuvchi ekanligi isbotlansin.

8.1. a) $\int\limits_{0}^{\infty}\frac{xdx}{16x^4+1}$, b) $\int\limits_{0}^{1}\frac{dx}{\sqrt[3]{2-4x}}$.

8.2. a) $\int\limits_{1}^{\infty}\frac{16xdx}{16x^4-1}$; b) $\int\limits_{1}^{3}\frac{dx}{\sqrt{x^2-6x+9}}$.

8.3. a) $\int\limits_{0}^{\infty}\frac{x^3dx}{\sqrt{16x^4+1}}$; b) $\int\limits_{0}^{1/3}\frac{e^{3+\frac{1}{x}}}{x^2}dx$.

8.4. a) $\int\limits_{1}^{\infty}\frac{xdx}{\sqrt{16x^4-1}}$; b) $\int\limits_{1}^{3}\frac{dx}{\sqrt[3]{(3-x)^5}}$.

8.5. a) $\int\limits_{-\infty}^{0}\frac{xdx}{\sqrt{\left(x^2+4\right)^3}}$; b) $\int\limits_{1/3}^{1}\frac{\ln(3x-1)}{3x-1}dx$.

8.6. a) $\int\limits_{0}^{\infty}\frac{x^2dx}{\sqrt[3]{\left(x^3+8\right)^4}}$; b) $\int\limits_{1/4}^{1}\frac{dx}{20x^2-9x+1}$.

8.7. a) $\int\limits_{0}^{\infty}\frac{xdx}{\sqrt[4]{\left(16+x^2\right)^5}}$; b) $\int\limits_{1/2}^{1}\frac{\ln 2dx}{(1-x)\ln^2(1-x)}$.

8.8. a) $\int\limits_{4}^{\infty}\frac{xdx}{\sqrt{x^2-4x+1}}$; b) $\int\limits_{0}^{2/3}\frac{\sqrt[3]{\ln(2-3x)}}{2-3x}dx$.

8.9. a) $\int\limits_{-1}^{\infty}\frac{dx}{\pi\left(x^2+4x+5\right)}$; b) $\int\limits_{0}^{1}\frac{xdx}{1-x^4}$.

8.10. a) $\int\limits_{-1}^{\infty}\frac{xdx}{x^2+4x+5}$; b) $\int\limits_{0}^{\pi/6}\frac{\cos 3x}{\sqrt[6]{(1-\sin 3x)^5}}dx$.

8.11. a) $\int\limits_{0}^{\infty}\frac{arctg2x}{\pi\left(1+4x^2\right)}dx$; b) $\int\limits_{0}^{1}\frac{2xdx}{\sqrt{1-x^4}}$.

8.12. a) $\int\limits_{1/2}^{\infty}\frac{16dx}{\pi\left(4x^2+4x+5\right)}$; b) $\int\limits_{-1/3}^{0}\frac{dx}{\sqrt[3]{1+3x}}$.

8.13. a) $\int\limits_{0}^{\infty}\frac{xdx}{4x^2+4x+5}$; b) $\int\limits_{3/4}^{1}\frac{dx}{\sqrt[5]{3-4x}}$.

8.14. a) $\int\limits_{0}^{\infty}\frac{(x+2)dx}{\sqrt[3]{(x^2+4x+1)^4}}$; b) $\int\limits_{0}^{\pi/2}\frac{e^{tgx}}{\cos 2x}dx$.

8.15. a) $\int\limits_0^{\infty}\frac{3-x^2}{x^2+4}dx;$ b) $\int\limits_0^{1}\frac{2e^{1-\frac{2}{\pi}\arcsin x}}{\pi\sqrt{1-x^2}}dx.$

8.16. a) $\int\limits_0^{\infty}\sqrt{\frac{2}{\pi}}\frac{\sqrt{arctg\, 2x}}{1+4x^2}dx;$ b) $\int\limits_1^{2}\frac{dx}{\sqrt[5]{4x-x^2-4}}.$

8.17. a) $\int\limits_1^{\infty}\frac{4dx}{x(1+\ln^2 x)};$ b) $\int\limits_{\pi/2}^{\pi}\frac{\sin x dx}{\sqrt[7]{\cos^2 x}}.$

8.18. a) $\int\limits_0^{\infty}x\sin x dx;$ b) $\int\limits_{-3/4}^{0}\frac{dx}{\sqrt{4x+3}}.$

8.19. a) $\int\limits_{-\infty}^{-1}\frac{7dx}{(x^2-4x)\ln 5};$ b) $\int\limits_1^{2}\frac{xdx}{\sqrt{(x^2-1)^3}\ln 2}.$

8.20. a) $\int\limits_{1/3}^{\infty}\frac{\pi dx}{(1+9x^2)arctg^2 3x};$ b) $\int\limits_0^{1/3}\frac{dx}{9x^2-9x+2}.$

8.21. a) $\int\limits_2^{\infty}\frac{dx}{(4+x^2)\sqrt{\pi\, arctg\frac{x}{2}}};$ b) $\int\limits_0^{\pi/2}\frac{3\sin^3 x dx}{\sqrt{\cos x}}.$

8.22. a) $\int\limits_1^{\infty}\frac{dx}{(x^2+2x)\ln^3};$ b) $\int\limits_0^{3}\frac{\sqrt[3]{9x}dx}{\sqrt[3]{9-x^2}}.$

8.23. a) $\int\limits_0^{\infty}e^{-3x}xdx;$ b) $\int\limits_0^{1}\frac{x^4 dx}{\sqrt[3]{1-x^5}}.$

8.24. a) $\int\limits_{-\infty}^{0}\left(\frac{x^2}{x^3-1}-\frac{x}{1+x^2}\right)dx;$ b) $\int\limits_0^{2}\frac{x^2 dx}{\sqrt{64-x^6}}.$

8.25. a) $\int_0^1 \frac{dx}{2x^2-2x+1}$; b) $\int_{1/2}^1 \frac{dx}{\sqrt[9]{1-2x}}$.

8.26. a) $\int_1^\infty \frac{dx}{x^2(x+1)}$; b) $\int_1^5 \frac{x^2dx}{\sqrt{31(x^3-1)}}$.

8.27. a) $\int_{e^2}^\infty \frac{dx}{x(\ln x-1)^2}$; b) $\int_1^{3/2} \frac{dx}{\sqrt{3x-x^2-2}}$.

8.28. a) $\int_1^\infty \frac{dx}{(6x^2-5x+1)\ln\frac{3}{4}}$; b) $\int_0^4 \frac{10xdx}{\sqrt[4]{(16-x^2)^3}}$.

8.29. a) $\int_1^\infty \frac{dx}{9x^2-9x+2}$; b) $\int_0^{1/4} \frac{dx}{\sqrt[3]{1-4x}}$.

8.30. a) $\int_3^\infty \frac{dx}{x^2-3x+2}$; b) $\int_0^{1/2} \frac{dx}{(2x-1)^2}$.

Namunaviy variantni yechish

Aniq integralni verguldan keyin 2 ta raqam aniqlikda hisoblang

1. $\int_1^2 \frac{dx}{x(1+x^2)}$

▶ Nyuton –Leybnits formulosidan foydalanib $\int_a^b f(x)dx = F(b)-F(a)$, kasr-ratsional funksiyadan integral hisoblaymiz:

$$\int_1^2 \frac{dx}{x(1+x^2)}=\int_1^2\left(\frac{A}{x}+\frac{Bx+C}{1+x^2}\right)dx=\left|\begin{matrix}1=A(1+x^2)+(Bx+C)x\\ \begin{array}{c|c|c} x=0 & 1=A & A=1\\ x^2 & 0=A+B & B=-1\\ x & 0=C & C=0\end{array}\end{matrix}\right|=\int_1^2\frac{dx}{x}-\int_1^2\frac{xdx}{1+x^2}=\ln|x|\Big|_1^2-\frac{1}{2}\ln(1+x^2)\Big|_1^2=$$

$$=\ln 2-\frac{1}{2}\ln 5+\frac{1}{2}\ln 2=\frac{3}{2}\ln 2-\frac{1}{2}\ln 5=\frac{3}{2}\cdot 0,69-\frac{1}{2}\cdot 1,61=0,24$$

2. $\int_1^e \ln^2 xdx$

► Bo‘laklab integrallash formulasini ikki marta qo‘llaymiz

$$\int_1^e \ln^2 xdx=\left|\begin{matrix}u=\ln^2 x & du=2\ln x\cdot\frac{1}{x}dx\\ dv=dx & v=x\end{matrix}\right|=x\ln^2 x\Big|_1^e-2\int_1^e \ln xdx=$$

$$=\left|\begin{matrix}u=\ln x & du=\frac{1}{x}dx\\ dv=dx & v=x\end{matrix}\right|=e\ln^2 e-2(x\ln x-x)\Big|_1^e=e-2e+2e-2=0,72$$

3. $\int_3^4 \frac{9x^2-14x+1}{x^3-2x^2-x+2}dx$

► Integral ostidagi funksiya to‘g‘ri ratsional kasrni bildiradi. Mahrajni oddiy ko‘paytmalarga ajratib hosil qilingan kasrlarni oddiy kasrlarga ajratsak

$$\int_3^4\frac{9x^2-14x+1}{x^3-2x^2-x+2}dx=\int_3^4\frac{9x^2-14x+1}{(x+1)(x-1)(x-2)}dx=\int_3^4\left(\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{x-2}\right)dx=$$

$$=\left|\begin{matrix}9x^2-14x+1=A(x-1)(x-2)+B(x+1)\times\\ \times(x-2)+C(x+1)(x-1)\\ \begin{array}{c|c|c} x=-1 & 24=6A & A=4\\ x=1 & -4=-2B & B=2\\ x=2 & 9=3C & C=3\end{array}\end{matrix}\right|=\int_3^4\left(\frac{4}{x+1}+\frac{2}{x-1}+\frac{3}{x-2}\right)dx=$$

$$=(4\ln|x+1|+2\ln|x-1|+3\ln|x-2|)\Big|_3^4=4\ln 5+2\ln 3+3\ln 2-4\ln 4-2\ln 2=\ln(5^4\cdot 3^2\cdot 2)-\ln 4^4=$$

$$=\ln\frac{5^4\cdot 3^2\cdot 2}{4^4}=\ln\frac{11250}{256}=3,78.$$ ◄

4. $\int_0^1 \frac{x^3dx}{\sqrt{x^2+1}}$.

►

$$\int_0^1\frac{x^3dx}{\sqrt{x^2+1}}=\left|\begin{matrix}\sqrt{x^2+1}=t, & x^2+1=t^2, & xdx=tdt\\ t=1,\ x=0, & t=\sqrt{2}, & x=1\end{matrix}\right|=\int_1^{\sqrt{2}}\frac{(t^2-1)t}{t}dt=\int_1^{\sqrt{2}}(t^2-1)dt=\left(\frac{1}{3}t^3-t\right)\Big|_1^{\sqrt{2}}=0,20.$$ ◄

5. $\int_0^{\frac{\pi}{4}} \frac{dx}{4-3\cos^2 x+5\sin^2 x}$

► Integral ostidagi funksiya *sinx* va *cosx* ga nisbatan juft bo'lgani uchun *(sin²x* va *cos²x* ga ratsional bog'liq) *t=tgx* ((8.14) formula) almashtirish bajaramiz.

$$\int_0^{\frac{\pi}{4}} \frac{dx}{4-3\cos^2 x+5\sin^2 x} = \left| \begin{matrix} t=tgx, dx=\frac{dt}{1+t^2}, \cos^2 x=\frac{1}{1+t^2} \\ \sin^2 x=\frac{t^2}{1+t^2}, t=0, x=0, t=1, x=\frac{\pi}{4} \end{matrix} \right|$$

$$= \int_0^1 \frac{dt}{(1+t^2)\left(4-\frac{3}{1+t^2}+\frac{5t^2}{1+t^2}\right)} =$$

$$= \int_0^1 \frac{dt}{9t^2+1} = \frac{1}{3} arctg3t \Big|_0^1 = \frac{1}{3}(arctg3-arctg0)=0,42. \blacktriangleleft$$

6. $\int_0^1 \frac{2x-11}{\sqrt{3-2x-x^2}} dx.$

►Berilgan integralni, ikkita integralga shunday ajratamizki, suratida maxrajidagi ildiz ostidagi uchhadning birinchi tartibli hosilsi tursin. Natijada quyidagiga ega bo'lamiz:

$$\int_0^1 \frac{2x-11}{\sqrt{3-2x-x^2}} dx = -4\int_0^1 \frac{-2x-2}{\sqrt{3-2x-x^2}} dx - 19\int_0^1 \frac{dx}{\sqrt{4-(x+1)^2}} = -8\sqrt{3-2x-x^2}\Big|_0^1 - 19\arcsin\frac{x+1}{2}\Big|_0^1 =$$

$$= 8\sqrt{3} - \frac{19}{2}\pi + \frac{19}{6}\pi \approx -6{,}05.$$

7. $\int_{2/3}^{10/3} \frac{xdx}{(3x-1)\sqrt{3x-1}}$ ◄

► Ushbu integral $\sqrt{3x-1}=t$ almashtirish orqali ratsional funksiyadan integralga keltiriladi.

$$\int_{2/3}^{10/3} \frac{xdx}{(3x-1)\sqrt{3x-1}} =$$

$$\left| \begin{matrix} \sqrt{3x-1}=t, 3x-1=t^2, x=\frac{1}{3}(t^2+1), dx=\frac{2}{3}tdt, \\ t=1, x=2/3, t=3, x=10/3 \end{matrix} \right| =$$

$$\int_1^3 \frac{\frac{1}{3}(t^2+1)\frac{2}{3}tdt}{t^2\cdot t} = = \frac{2}{9}\int_1^3 \frac{t^3+t}{t^3}dt = \frac{2}{9}\left(t-\frac{1}{t}\right)\Big|_1^3 \approx 0{,}59. \blacktriangleleft$$

8. Xosmas integralni hisoblang yoki ularning uzoqlashuvchi ekanligini ko'rsating

a) $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{x^2+4x+9}$, b) $\displaystyle\int_{-1}^{1}\frac{3x^2+2}{\sqrt[3]{x^2}}\,dx$

$\rhd$ a) $$\int_{-\infty}^{\infty}\frac{dx}{x^2+4x+9}=\int_{-\infty}^{0}\frac{dx}{x^2+4x+9}+\int_{0}^{\infty}\frac{dx}{x^2+4x+9}=\lim_{\alpha\to-\infty}\int_{\alpha}^{0}\frac{dx}{(x+2)^2+5}+\lim_{\beta\to+\infty}\int_{0}^{\beta}\frac{dx}{(x+2)^2+5}=$$

$$=\lim_{\alpha\to-\infty}\frac{1}{\sqrt5}arctg\frac{x+2}{\sqrt5}\Big|_{\alpha}^{0}+\lim_{\beta\to+\infty}\frac{1}{\sqrt5}arctg\frac{x+2}{\sqrt5}\Big|_{0}^{\beta}=\lim_{\alpha\to-\infty}\left(\frac{1}{\sqrt5}arctg\frac{2}{\sqrt5}-\frac{1}{\sqrt5}arctg\frac{\alpha+2}{5}\right)+$$

$$+\lim_{\beta\to\infty}\left(\frac{1}{\sqrt5}arctg\frac{\beta+2}{\sqrt5}-\frac{1}{\sqrt5}arctg\frac{2}{\sqrt5}\right)=\frac{1}{\sqrt5}arctg\frac{2}{\sqrt5}-\frac{1}{\sqrt5}\left(-\frac{\pi}{2}\right)+\frac{1}{\sqrt5}\frac{\pi}{2}-\frac{1}{\sqrt5}arctg\frac{2}{\sqrt5}=\frac{\pi}{\sqrt5};$$

b)

$$\int_{-1}^{1}\frac{3x^2+2}{\sqrt[3]{x^2}}dx=\int_{-1}^{0}\frac{3x^2+2}{\sqrt[3]{x^2}}dx+\int_{0}^{1}\frac{3x^2+2}{\sqrt[3]{x^2}}dx=\lim_{\beta\to-0}\int_{-1}^{\beta}\left(3x^{4/3}+2x^{-2/3}\right)dx+\lim_{\alpha\to0+}\int_{\alpha}^{1}\left(3x^{4/3}+2x^{-2/3}\right)dx=$$

$$=\lim_{\beta\to0-}\left(\frac97x^{\frac73}+6x^{\frac13}\right)\Big|_{-1}^{\beta}+\lim_{\alpha\to0+}\left(\frac97x^{\frac73}+6x^{\frac13}\right)\Big|_{\alpha}^{1}=\lim_{\beta\to0-}\left(\frac97\beta^{\frac73}+6\beta^{\frac13}+\frac97+6\right)+\lim_{x\to0+}$$

$$\left(\frac97+6-\frac97\alpha^{\frac73}-6\alpha^{\frac13}\right)=14\frac47 \lhd$$

IUT-9.2

1. Berilgan chiziqlar bilan chegaralangan figuraning yuzini verguldan keyin ikkita raqam aniqligida hisoblang

1.1. $\rho=3\sqrt{\cos2\phi}$. *(Javob: 9,00.)*

1.2. $y=x^2$, $y=3-x$. *(Javob: 10,67.)*

1.3. $y=\sqrt{x}$, $y=x^3$. *(Javob: 0,42.)*

1.4. $x=7\cos^3t$, $y=7\sin^3t$. *(Javob: 57,70.)*

1.5. $\rho=4\cos3\phi$. *(Javob: 12,56.)*

1.6. $\rho=3\cos2\phi$. *(Javob: 14,13.)*

1.7. $\rho=2(1-\cos\phi)$. *(Javob: 18,84.)*

1.8. $\rho^2=2\sin2\varphi$. *(Javob: 1,00.)*

1.9. $x=4(t-\sin t)$, $y=4(1-\cos t)$. *(Javob: 150,72.)*

1.10. $\rho=2(1+\cos\phi)$. *(Javob: 18,84.)*

1.11. $\rho=2\sin3\phi$. *(Javob: 3,14.)*

1.12. $\rho = 2 + \cos\phi.$ *(Javob: 14,13.)*

1.13. $y = 1/(1 + x^2),\ y = x^2/2.$ *(Javob: 1,23.)*

1.14. $y^2 = x + 1,\ y^2 = 9 - x.$ *(Javob: 29,87.)*

1.15. $y^2 = x^3,\ x = 0,\ y = 4.$ *(Javob: 6,05.)*

1.16. $\rho = 4\sin^2\phi.$ *(Javob: 18,84.)*

1.17. $x = 3\cos t,\ y = 2\sin t.$ *(Javob: 18,84.)*

1.18. $y^2 = 9x,\ y = 3x.$ *(Javob: 0,50.)*

1.19.

$x = 3(\cos t + t\sin t),\ y = 3(\sin t - t\cos t),\ y = 0\ (0 \le t \le \pi).$
(Javob: 29,25.)

1.20. $y^2 = 4x,\ x^2 = 4y.$ *(Javob: 5,33.)*

1.21. $y^2 = x^3,\ x = 2.$ *(Javob: 4,51.)*

1.22. $y = x^2,\ y = 2 - x^2.$ *(Javob: 2,67.)*

1.23. $y^2 = (4 - x^3),\ x = 0.$ *(Javob: 25,60.)*

1.24. $\rho = 3\sin 4\phi.$ *(Javob: 14,13.)*

1.25. $y = x^3,\ y = 1,\ x = 0.$ *(Javob: 0,75.)*

1.26. $xy = 6,\ x + y - 7 = 0.$ *(Javob: 6,76.)*

1.27. $y = 2^x,\ y = 2x - x^2,\ x = 0,\ x = 2.$ *(Javob: 3,02.)*

1.28. $x^2 = 4y,\ y = 8/(x^2 + 4).$ *(Javob: 4,95.)*

1.29. $y = x + 1,\ y = \cos x,\ y = 0.$ *(Javob: 1,50.)*

1.30. $x = 2\cos^3 t,\ y = 2\sin^3 t.$ *(Javob: 4,71.)*

2. Berilgan chiziqning uzunligini verguldan keyin ikkita raqam aniqligida hisoblang.

2.1. $x = 2\cos^3 t,\ y = 2\sin^3 t.$ *(Javob: 12,00.)*

2.2. $x = 2(\cos t + t\sin t),\ y = 2(\sin t - t\cos t)\ (0 \le t \le \pi).$
(Javob: 9,86.)

2.3. $\rho = \sin^3(\phi/3)\ (0 \le \phi \le \pi/2).$ *(Javob: 0,14.)*

2.4. $\rho = 2\sin^3(\phi/3)(0 \le \phi \le \pi/2).$ *(Javob: 0,27.)*

2.5. $\sqrt[3]{x^2} + \sqrt[3]{y^2} = \sqrt[3]{9}.$ *(Javob: 18,00.)*

2.6. $x^{2/3} + y^{2/3} = 4^{2/3}.$ *(Javob: 24,00.)*

2.7. $y^2 = (x+1)^3,\ x = 4$ *to'g'ri chiziq bilan kesilgan (Javob: 24,81.)*

2.8. $y = 1 - \ln\cos x\ \ (0 \le x \le \pi/6).$ *(Javob: 0,55.)*

2.9. $\rho = 6\cos^3(\phi/3)\ (0 \le \phi \le \pi/2).$ *(Javob: 8,60.)*

2.10. $x = 4\cos^3 t,\ y = 4\sin^4 t.$ *(Javob: 24,00.)*

2.11. $y^2 = (x-1)^3$ *A (1,0) nuqtadan V* $(6, \sqrt{125})$ *nuqtagacha (Javob: 8,27.)*

2.12. $y^2 = x^5,\ x = 5$ *to'g'ri chiziq bilan kesilgan (Javob: 24,81.)*

2.13. $\rho = 3\cos\phi.$ *(Javob: 9,42.)*

2.14. $\rho = 3(1 - \cos\phi).$ *(Javob: 24,00.)*

2.15. $\rho = 2\cos^3(\phi/3).$ *(Javob: 9,42.)*

2.16. $x = 5\cos^2 t,\ y = 5\sin^2 t(0 \le t \le \pi/2).$ *(Javob: 7,05.)*

2.17. $9y^2 = 4(3-x)^3$ *(Oy o'qi bilan kesishgan nuqtalar orasida) (Javob: 9,33.)*

2.18. $\rho = 3\sin\phi.$ *(Javob: 9,42.)*

2.19. $y = \ln\sin x(\pi/3 \le x \le \pi/2).$ *(Javob: 0,55.)*

2.20. $x = 9(t - \sin t),\ y = 9(1 - \cos t)(0 \le t \le 2\pi).$ *(Javob: 72,00.)*

2.21. $\rho = 2(1 - \cos\phi).$ *(Javob: 16,00.)*

2.22. $y^2 = (x-1)^3$ *A (2,-1) nuqtadan V (5,-8). (Javob: 7,63.)*

2.23. $x = 7(t - \sin t),\ y = 7(1 - \cos t)\ (2\pi \le t \le 4\pi)$. *(Javob: 56,00.)*

2.24. $y = e^{x/2} + e^{-x/2}\ (0 \le x \le 2)$. *(Javob: 2,35.)*

2.25. $x = 4\cos^3 t,\ y = 4\sin^3 t$. *(Javob: 24,00.)*

2.26. $x = \sqrt{3}t^2,\ y = t - t^3$ *(sirtmoq). (Javob: nuqtagacha 4,00.)*

2.27. $\rho = 5\sin\phi$. *(Javob: 15,70.)*

2.28. $\rho = 4\cos\phi$. *(Javob: 12,56.)*

2.29. $\rho = 5(1 + \cos\phi)$. *(Javob: 40,00.)*

2.30. $y^2 = x^3$ *A (0, 0)nuqtadan V (4, 8) nuqtagacha. (Javob: 9,07.)*

3. Berilgan o‘q bo‘yicha F shaklni aylantirishdan hosil bo‘lgan jismning hajmini (verguldan keyin 2 raqam aniqligida) hisoblang.

3.1. *F:* $y^2 = 4 - x,\ x = 0,\ Oy$. *(Javob: 107,17.)*

3.2. *F:* $\sqrt{x} + \sqrt{y} = \sqrt{2},\ x = 0,\ y = 0,\ Ox$. *(Javob: 1,68.)*

3.3. *F:* $x^2/9 + y^2/4 = 1,\ Oy$. *(Javob: 150,72.)*

3.4. *F:* $y^3 = x^2,\ y = 1,\ Ox$. *(Javob: 3,59.)*

3.5. *F:* $x = 6(t - \sin t),\ y = 6(1 - \cos t),\ Ox$. *(Javob: 1064,88.)*

3.6. *F:* $x = 3\cos^2 t,\ y = 4\sin^2 t\ (0 \le t \le \pi/2),\ Oy$. *(Javob: 37,68.)*

3.7. *F:* $y^2 = x,\ x^2 = y,\ Ox$. *(Javob: 0,94.)*

3.8. *F:* $y^2 = (x-1)^3,\ x = 2,\ Ox$. *(Javob: 0,78.)*

3.9. *F:* $x=\sqrt{1-y^2},\ y=\sqrt{\frac{3}{2}}x,\ y=0,\ Ox.$ *(Javob: 1,24.)*

3.10. *F:* $y=\sin x,\ y=0\ (0\le x\le \pi),\ Ox.$ *(Javob: 4,93.)*

3.11. *F:* $y^2=4x,\ x^2=4y,\ Ox.$ *(Javob: 60,29.)*

3.12. *F:* $x=2\cos t,\ y=5\sin t,\ Oy.$ *(Javob: 83,73.)*

3.13. *F:* $y=x^2,\ 8x=y^2,\ Oy.$ *(Javob: 15,07.)*

3.14. *F:* $y=e^x,\ x=0,\ y=0,\ x=1,\ Ox.$ *(Javob: 10,05.)*

3.15. *F:* $y^2=4x/3,\ x=3,\ Ox.$ *(Javob: 90,43.)*

3.16. *F:* $y=2x-x^2,\ y=0,\ Ox.$ *(Javob: 3,35.)*

3.17. *F:* $\rho=2(1+\cos\phi)$, *polyarnaya os.* *(Javob: 66,99.)*

3.18. *F:* $x=7\cos^3 t,\ y=7\sin^3 t,\ Oy.$ *(Javob: 328,23.)*

3.19. *F:* $x^2/16+y^2/1=1,\ Ox.$ *(Javob: 16,75.)*

3.20. *F:* $x^3=(y-1)^2,\ x=0,\ y=0,\ Ox.$ *(Javob: 6,44.)*

3.21. *F:* $xy=4,\ 2x+y-6=0,\ Ox.$ *(Javob: 4,19.)*

3.22. *F:* $x=\sqrt{3}\cos t,\ y=2\sin t,\ Oy.$ *(Javob: 25,12.)*

3.23. *F:* $y=2-x^2,\ y=x^2,\ Ox.$ *(Javob: 16,75.)*

3.24. *F:* $y=-x^2+8,\ y=x^2,\ Ox.$ *(Javob: 535,89.)*

3.25. *F:* $y^2=(x+4)^3,\ x=0,\ Ox.$ *(Javob: 200,96.)*

3.26. *F:* $y=x^3,\ x=0,\ y=8,\ Oy.$ *(Javob: 60,29.)*

3.27. *F:* $x=\cos^3 t,\ y=\sin^3 t,\ Ox.$ *(Javob: 0,96.)*

3.28. *F:* $2y=x^2,\ 2x+2y-3=0,\ Ox.$ *(Javob: 57,10.)*

3.29. *F:* $y=x-x^2,\ y=0,\ Ox.$ *(Javob: 0,10.)*

3.30. *F:* $y=2-x^2/2,\ x+y=2,\ Oy.$ *(Javob: 4,17.)*

4. Berilgan o‘q bo‘yicha L egri chiziq yoyini aylantirishdan hosil bo‘lgan jismning hajmini (verguldan keyin 2 raqam aniqligida) hisoblang.

4.1. $L: y = x^3/3 \left(-1/2 \le x \le 1/2\right),\ Ox.$ *(Javob: 4,25.)*

4.2. $L: \rho = 2\cos\phi,$ *qutb o‘qi. (Javob: 12,57.)*

4.3. $L: x = 10(t - \sin t),\ y = 10(1 - \cos t)\,(0 \le t \le 2\pi),\ Ox.$ *(Javob: 6698,67.)*

4.4. $L: y = x^2/2, Oy.$ *y=3/2 to‘g‘ri chiziq bilan kesilgan.(Javob: 14,65.)*

4.5. $L: 3y = x^2\ (0 \le x \le 2),\ Ox.$ *(Javob: 24,09.)*

4.6. $L: y = \sqrt{x},\ Ox.\ y = x$ *to‘g‘ri chiziq bilan kesilgan. (Javob: 5,34.)*

4.7. $L: x = 2(t - \sin t),\ y = 2(1 - \cos t)\,(0 \le t \le 2\pi),\ Ox.$ *(Javob: 267,95.)*

4.8. $L: x = \cos t,\ y = 3 + \sin t,\ Ox.$ *(Javob: 118,32.)*

4.9. $L: 3x = y^3\,(0 \le y \le 2),\ Oy.$ *(Javob: 24,09.)*

4.10. $L: y = x^3/3(-1 \le x \le 1),\ Ox.$ *(Javob: 1,27.)*

4.11. $L: x = \cos t,\ y = 1 + \sin t,\ Ox.$ *(Javob: 32,28.)*

4.12. $L: x^2 = 4 + y,\ Oy.\ y = 2$ *to‘g‘ri chiziq bilan kesilgan.(Javob: 259,57.)*

4.13. $L: x = 3(t - \sin t),\ y = 3(1 - \cos t)\,(0 \le t \le 2\pi),\ Ox.$ *(Javob: 602,88.)*

4.14. $L: x = \cos^3 t,\ y = \sin^3 t,\ Ox.$ *(Javob: 7,54.)*

4.15. $L: \rho = \sqrt{\cos 2\varphi},$ *qutb o‘qi. (Javob: 14,82.)*

4.16. $L: y^2 = 4 + x,\ Ox.\ x = 2$ *to‘g‘ri chiziq bilan kesilgan.(Javob: 64,89.)*

4.17. $L: y^2 = 2x,\ Ox.\ 2x = 3$ *to‘g‘ri chiziq bilan kesilgan.(Javob: 14,65.)*

4.18. $L: 3y = x^3\ (0 \le x \le 1),\ Ox.$ *(Javob: 0,63.)*

4.19. $L: \rho^2 = 4\cos 2\varphi,$ *qutb o'qi. (Javob: 14,80.)*

4.20. $L: \rho = 6\sin\varphi,$ *qutb o'qi.(Javob: 354,96.)*

4.21. $L: x = t - \sin t,\ y = 1 - \cos t\ (0 \le t \le 2\pi),\ Ox.$ *(Javob: 66,99.)*

4.22. $L: \rho = 2\sin\varphi,$ *qutb o'qi. (Javob: 39,44.)*

4.23. $L: \rho = \frac{2}{3}\cos\varphi,$ *qutb o'qi. (Javob: 7,07.)*

4.24. $L: x = 3\cos^3 t,\ y = 3\sin^3 t,\ Ox.$ *(Javob: 67,82.)*

4.25. $L: x = 2\cos t,\ y = 3 + 2\sin t,\ Ox.$ *(Javob: 236,64.)*

4.26. $L: \rho^2 = 9\cos 2\varphi,$ *qutb o'qi. (Javob: 16,38.)*

4.27. $L: y = x^3,\ Ox.\ x = \pm 2/3$ *to'g'ri chiziq bilan kesilgan. (Javob: 0,84.)*

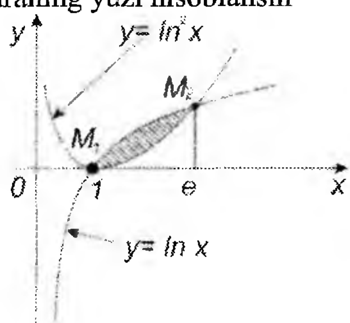
4.28. $L: x = 2\cos^3 t,\ y = 2\sin^3 t,\ Ox.$ *(Javob: 30,14.)*

4.29. $L: x = \cos t,\ y = 2 + \sin t,\ Ox.$ *(Javob: 78,88.)*

4.30. $L: \rho = 4\sin\varphi,$ *qutb o'qi. (Javob: 157,76.)*

Namunaviy variantni yechish

1. $y = \ln x$ va $y = \ln^2 x$ (9.23-rasm) egri chiziqlar bilan chegaralangan figuraning yuzi hisoblansin



9.23- rasm

▶ Egri chiziqlarning kesishish nuqtasini topamiz: $M_1(1,0)$, $M_2(e,1)$ va (9.7) formuladan foydalanamiz. Natijada:

$$S = \int_1^e (\ln x - \ln^2 x)dx$$

$$\int \ln^2 x dx = \begin{vmatrix} u = \ln^2 x, du = 2\ln x \cdot \frac{1}{x} dx \\ dv = dx, v = x \end{vmatrix} = x\ln^2 x - 2\int \ln x\, dx,$$

$$\int \ln x dx = \begin{vmatrix} u = \ln x, \ du = \frac{1}{x} dx \\ dv = dx \quad v = x \end{vmatrix} = x\ln x - \int dx = x\ln x - x + C.$$

U holda

$$S = \int_1^e \ln x dx - \int_1^e \ln^2 x dx = (x\ln x - x)\Big|_1^e - (x\ln^2 x - 2x\ln x + 2x)\Big|_1^e =$$

$$= e\ln e - e + 1 - (e\ln^2 e - 2e\ln e + 2e) + 2 = 3 - e \approx 0{,}28.$$ ◀

2. Parametrik ko'rinishda berilgan $x = (t^2 - 2)sint + 2tcost$, $y = (2 - t^2)sost + 2tsint\,(0 \le t \le \pi)$ chiziqning yoy uzunligi verguldan keyin ikki raqam aniqlikda hisoblang.

▶(9.11) formuladan foydalanamiz:

$$l = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Integral ostidagi funksiyani topamiz:

$$\frac{dx}{dt} = 2t\sin + (t^2 - 2)\cos t + 2\cos t - 2t\sin t = t^2 \cos t,$$

$$\frac{dy}{dt} = -2t\cos t - (2 - t^2)\sin t + 2\sin t + 2t\cos t = t^2 \sin t,$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{t^4\cos^2 t + t^4 \sin^2 t} = t^2$$

Yakunda quyidagiga ega bo'lamiz:

$$l = \int_0^\pi t^2\, dt = \frac{t^3}{3}\Big|_0^\pi = \frac{\pi^3}{3} \approx 10{,}32.$$

◀

3. Yassi $y=3-x^2$ va $y=x^2+1$ parabolalar bilan chegaralangan figuraning Ox o'qi atrofida aylantirishdan hosil bo'lgan jismning hajmini hisoblang

▶ Parabolalarning kesishish nuqtalarini topamiz: M_1 *(-1;2)*, M_2 *(1,2)*. Jismning hajmini (9,14) formulaga ko‘ra hajmlar ayirmasi V_2-V_1 ko‘rinishda hisoblaymiz.

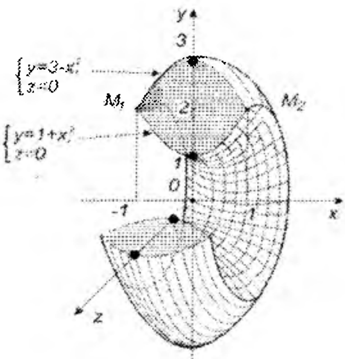
$$V_2 = \pi\int_{-1}^{1}(3-x^2)^2 dx, \quad V_1 = \pi\int_{-1}^{1}(x^2+1)^2 dx$$

Demak

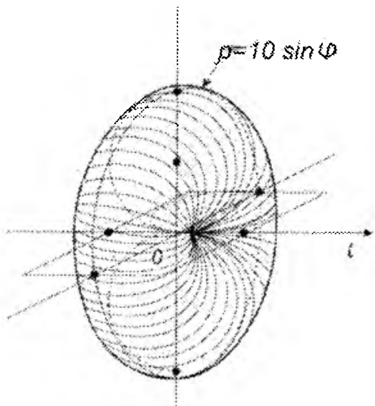
$$V = V_2 - V = \pi\int_{-1}^{1}(3-x^2)^2 dx - \pi\int_{-1}^{1}(x^2+1)^2 dx = \pi\int_{-1}^{1}\left((3-x^2)^2 - (x^2+1)^2 \right)dx = \pi\int_{-1}^{1}(8-8x^2)dx =$$

$$= 8\pi\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = 16\pi\left(1-\frac{1}{3}\right) \approx 33,50 \triangleleft$$

Quyidagi 9.24 rasmda *Oxu* tekislikda yassi figura va uning *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jism (uning chorak qismi kesib olingan) keltirilgan.



9.24- rasm



9.25- rasm

Qutb koordinatalar sistemasida $\rho = 10 \sin \varphi$ aylananing qutb o‘qi atrofida aylantirishdan hosil bo‘lgan sirt (9.25 rasm) yuzini verguldan keyin 2 ta raqam aniqligida hisoblang.

▶ Qutb koordinatalar sistemasidagi (9.18) va (9.15) formulalardan foydalanamiz.

$$S = 2\pi\int_{\phi_1}^{\phi_2} y\sqrt{\rho_\phi'^2 + \rho^2} \quad d\phi,$$

Bu yerda $y = \rho \sin \varphi$. Bundan $\rho'_\phi = 10\cos\phi;$ $y = \rho \sin\varphi = 10\sin^2\varphi$,

$$\varphi_1 = 0, \quad \varphi_2 = \pi,$$

$$S = 2\pi\int_0^\pi 10\sin^2\varphi\sqrt{100\cos^2\varphi + 100\sin^2\varphi}\, d\varphi = 200\pi\int_0^\pi \sin^2\varphi d\varphi =$$

$$= 200\pi\int_0^\pi \frac{1-\cos 2\varphi}{2} d\varphi = 100\pi\left(\varphi - \frac{1}{2}\sin 2\varphi\right)\Bigg|_0^\pi \approx 985,96 \quad \triangleleft$$

IUT-9.3

1. R rezervuardan suv haydab chiqarish uchun bajarilagn ishni hisoblang. Suvning solishtirma og'irligi *9,81 kH/m³*, π=3,14. (natijani butun soniga yaxlitlang).

1.1. R: Asosi *2m* va balandligi *5m* bo'lgan to'rtburchakli muntazam piramida *(Javob: 245kDj)*

1.2. R: Uchi pastga yo'naltirilagan asosi *2m,* balandligi *6m* bo'lgan to'rtburchakli muntazam piramida *(Javob: 118 kDj)*

1.3. R: Balandligi *1,5m,* radiusi *1m* bo'lgan sferik segment ko'rinishdagi qozon *(Javob: 22 kDj)*

1.4. R: Asosining radiusi *1m,* balandligi *5m* bo'lgan yarim silindr. *(Javob: 33kDj)*

1.5. R: Yuqori asosining radiusi *1m,* quyi asosining radiusi *2m,* balandligi *3m* bo'lgan kesik konus *(Javob: 393 kDj)*

1.6. R: Ko'ndalang kesim parabola, uzunligi *5m,* eni *4m,* chuqurligi *4m* bo'lagan tarnov. *(Javob: 837 kDj)*

1.7. R: Asosning radiusi *1m,* uzunligi *5m* bo'lgan silindrik sisterna *(Javob: 154 kDj)*

1.8. R: Asosi *2m,* balandligi *5m* bo'lagn muntazam uchburchakli piramida *(Javob: 106kDj)*

1.9. R: Asosi *4m,* balandligi *6m* uchi pastga qaratilagn uchburchakli muntazam piramida. *(Javob: 204 kDj)*

1.10. R: Asosning radiusi *3m,* balandligi *5m* uchi pastga qaratilagn konus. *(Javob: 578 kDj)*

1.11. R: Yuqori asosning radiusi *3m,* quyi asosning radiusi *1m,* balandligi *3m* bo'lgan kesik konus. *(Javob: 416kDj)*

1.12. R: Asosning radiusi *2m,* balandligi *5m* bo'lagn konus. *(Javob: 770 kDj)*

1.13. R:Yuqori asosning tomonlari *8m,* quyi asosning tomonlari *4m,* balandligi *2m* bo'lagn muntazam kesik piramida. *(Javob: 576kDj)*

1.14. R:Asosining radiusi *2m,* chuqurligi *4m* bo'lgan aylanma paraboloid. *(Javob: 329 kDj)*

1.15. R: Asosning radiusi *1m,* chuqurligi *2m* bo'lgan aylanma ellipsoid. *(Javob: 31 kDj)*

1.16. R: Yuqori asosning tomoni *2m,* quyi asosning tomoni *4m,* balandligi 1m bo'lgan muntazam to'rtburchaklikesik piramida. *(Javob: 56kDj)*

1.17. Asosning tomoni *1m,* balandligi *2m* bo'lgan muntazam olti burchakli piramida *(Javob: 26kDj)*

1.18. R: Asosning tomoni *2m*, balandligi *6m,* uchi pastga qaragan olti burchakli muntazam piramida. *(Javob: 306 kDj)*

1.19. R: Asosning radiusi *1m*, balandligi *3m* bo'lgan silindr. *(Javob: 139kDj)*

1.20. Yuqori asosning tomoni *1m,* quyi asosning tomoni *2m,* balandligi *2m* bo'lagn muntazam oltiburchakli kesik piramida. *(Javob: 144 kDj)*

1.21. R: Ko'ndalang kesim radiusi *1m* ga teng yarim aylana, uzunligi *10 m* bo'lgan tarnov. *(Javob: 65kDj)*

1.22. R: Yuqori asosning tomoni *2m,* quyi asosning tomni *1m*, balandligi *2m* bo'lgan muntazam oltiburchakli piramida. *(Javob: 93kDj)*

1.23. R: radius *2m* bo'lgan yarim sfera. *(Javob: 123kDj)*

Solishtirma og'irligi γ bo'lgan materialdan Q inshootni qurishda og'irlik kuchini bartaraf etish uchun bajarilgan ishni hisoblang.

1.24. Q: yuqori asosning tomoni *2m,* quyi asosning tomoni *4m,* balandligi *2m* bo'lagn muntazam to'rtburchakli kesik piramida: $\gamma=24\ kn/m^3$. *(Javob: 352kDj)*

1.25. Asosning tomoni *1m,* balandligi *2m* bo'lgan muntazam oltiburchakli piramida; $\gamma=24\ kn/m^3$. *(Javob: 21kDj)*

1.26. Q: Asosning tomoni *2m*, balandligi *4m* bo'lgan muntazam to'rtburchakli piramida: γ=*24 kn/m³*. *(Javob: 128kDj)*

1.27. Q: Yuqori asosning tomoni *1m*, quyi asosning tomoni *2m*, balandligi *2m* bo'lgan muntazam oltiburchakli kesik piramida: $\gamma=24\ kn/m^3$. *(Javob: 229kDj)*

1.28. Q: Asosining tomoni *3m*, balandligi 6m bo'lgan, muntazam uchburchakli piramida; $\gamma=24\ kn/m^3$. *(Javob: 234kDj)*

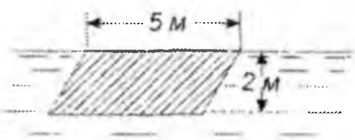
1.29. Q: asosning radiusi *2m*, balandligi 3*m* bo'lgan konus $\gamma=20\ kn/m^3$. *(Javob: 188kDj)*

1.30. Q: Yuqori asosning radiusi *1m*, quyi asosning radiusi *2m*, balandligi *2m* bo'lagn kesik konusi; $\gamma=21\ kn/m^3$. *(Javob: 88kDj)*

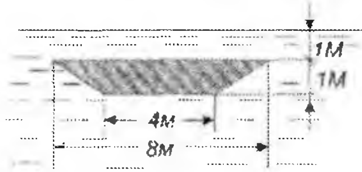
2. Vertikal cho'ktirilgan plastinkaga, suvning solishtirma og'irligi $9{,}81\ kH/m^3$ deb hisoblab, suvning bosim kuchini aniqlang (natijada butun soniga yaxlitlang). Plastinkaning joylashishi, shakli va o'lchami rasmda ko'rsatilgan.

2.1. *9.26. rasm. (Javob: 98 kH)*
2.2. *9.27. rasm. (Javob: 85 kH)*
2.3. *9.28. rasm (Javob: 248 kH)*
2.4. *9.29. rasm (Javob: 105 kH)*
2.5. *9.30 rasm (Javob: 167 kH)*
2.6. *9.31 rasm (Javob: 26 kH)*
2.7. *9.32 rasm (Javob: 131 kH)*
2.8. *9.33 rasm (Javob: 23 kH)*
2.9. *9.34 rasm (Javob: 523 kH)*
2.10. *9.35 rasm (Javob: 33 kH)*
2.11. *9.36 rasm (Javob: 31 kH)*
2.12. *9.37 rasm (Javob: 62kH)*
2.13. *9.38 rasm (Javob: 24 kH)*
2.14. *9.39 rasm (Javob: 22 kH)*
2.15. *9.40 rasm (Javob: 239 kH)*
2.16. *9.41 rasm (Javob: 123 kH)*
2.17. *9.42 rasm (Javob: 78 kH)*
2.18. *9.43 rasm (Javob: 13 kH)*
2.19. *9.44 rasm (Javob: 52 kH)*
2.20. *9.45 rasm (Javob: 3 kH)*

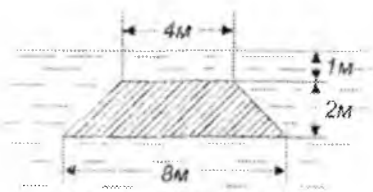
2.21. *9.46 rasm (Javob: 23 kH)*
2.22. *9.47 rasm (Javob: 16 kH)*
2.23. *9.48 rasm (Javob: 251 kH)*
2.24. *9.49 rasm (Javob: 31 kH)*
2.25. *9.50 rasm (Javob: 13kH)*
2.26. *9.51 rasm (Javob: 6kH)*
2.27. *9.52 rasm (Javob: 6 kH)*
2.28. *9.53 rasm (Javob: 39 kH)*
2.29. *9.54 rasm (Javob: 20 kH)*
2.30. *9.55 rasm (Javob: 272 kH)*



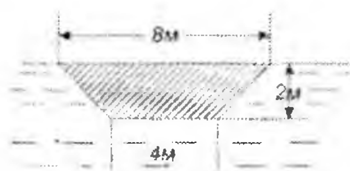
Parallelogramm
9.26- rasm



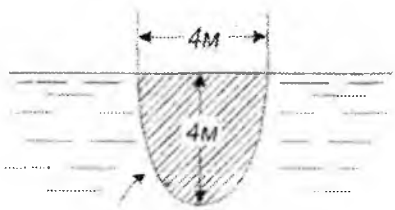
Teng yonli trapetsiya
9.27- rasm



Teng yonli trapetsiya
9.28- rasm

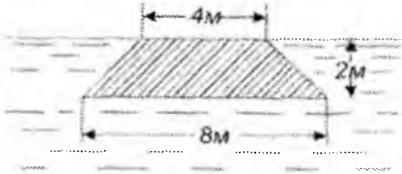


Teng yonli trapetsiya
9.29- rasm

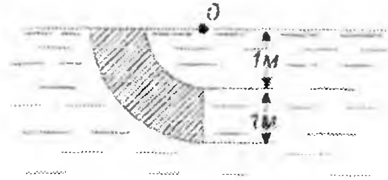


Parabola
9.30- rasm

Yarim ellips
9.31- rasm

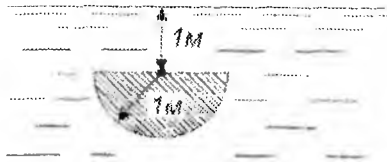


Teng yonli trapesiya
9.32- rasm



Halqaning to'rtdan bir qismi
9.33- rasm

Teng yonli uchburchak
9.34- rasm

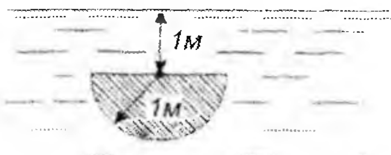


Teng yonli uchburchak
9.35- rasm

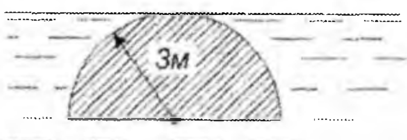
doira
9.36- rasm

doira
9.37- rasm

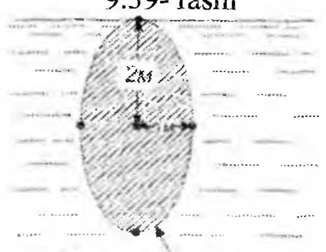
Yarim aylana
9.38- rasm



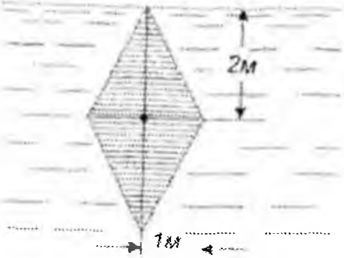
Yarim aylana
9.39- rasm



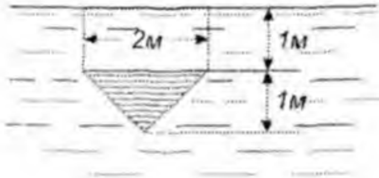
Yarim aylana
9.40- rasm



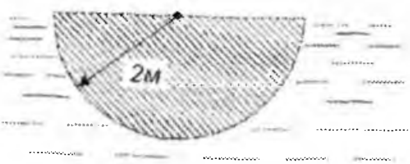
ellips
9.41- rasm



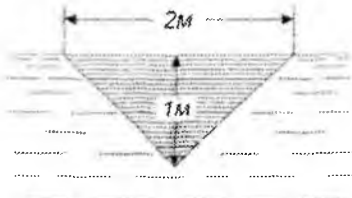
Romb
9.42- rasm



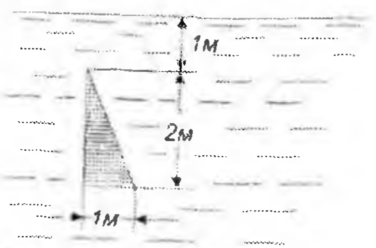
To'g'ri burchakli uchburchak
9.43- rasm



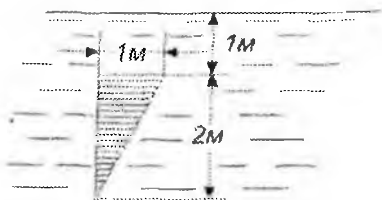
Yarim aylana
9.44- rasm



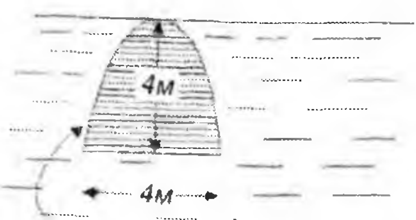
To'g'ri burchakli uchburchak
9.45- rasm



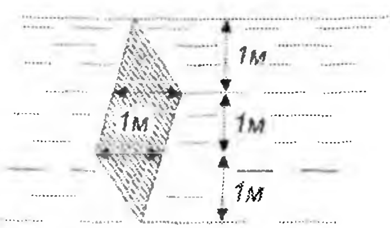
To'g'ri burchakli uchburchak
9.46- rasm



To'g'ri burchakli uchburchak
9.47- rasm



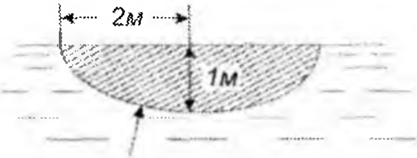
Parabola



Parallelogram

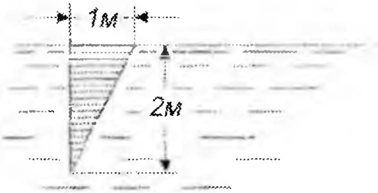
9.48- rasm

9.49 rasm



Yarim ellips

9.50- rasm

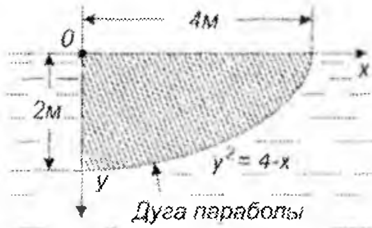


To‘g‘ri burchakli uchburchak

9.51- rasm

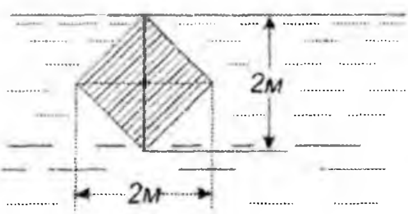
To‘g‘ri burchakli uchburchak

9.52- rasm



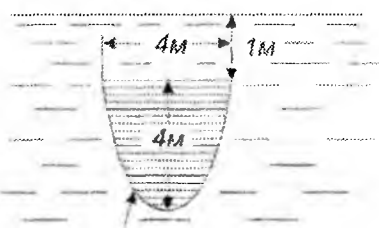
Parabola yoyi

9.53- rasm



Kvadrat

9.54- rasm



Parabola

9.55- rasm

3. Bir jinsli yassi egri chiziqning og‘irlik markazi topilsin.

3.1. *L:Ox* o‘qi ustida joylashgan yarim aylana: $x^2 + y^2 = R^2$.

(Javob: $X_c = 0,\ Y_c = 2R/\pi$

3.2. *L:* Sikolidaning birinchi arkasi *x=a (t-sint), y=a (1-cost)* $(0 \le t \le 2\pi)$. *(Javob:* $x_c = \pi a,\ \ y_c = \frac{4}{3}a$)

3.3. *L:* astroidaning $x^{\frac{2}{3}} + x^{\frac{2}{3}} = a^{\frac{2}{3}}$ uchinchi kvadrantidagi bo'lagi.

(Javob: $x_s=y_c= - 0,4a$.)

3.4. *L:* Radius R ga teng bo'lgan aylananing x ga teng markaziy burchakni tortib turgan yoy bo'lagi. *(Javob:* og'irlik markazi yoy tortib turuvchi markaziy burchakning bissektrisasida markazdan $\frac{2R\sin(x/2)}{x}$ masofada joylashgan).

3.5. *L:* Zanjir chiziqning yoy bo'lagi y=a (x-a), $(-a \le x \le a)$.

(Javob: $x_2 = 0,\ y_c = \frac{a}{4}\ \frac{2+sh2}{sh1}$)

3.6. *L:* Kordiodaning yoyi ρ=a$(1+\cos\varphi)(0 \le \varphi \le \pi)$. *(Javob:* $x_c = y_c = \frac{4}{5}a$)

3.7. *L:* Logarifmik spiralining yoyi $\rho = a \cdot e^{\varphi}\ \left(\frac{\pi}{2} \le \varphi \le \pi\right)$.

(Javob: $x_c = -\frac{a}{5}\ \frac{2\cdot e^{2\pi}+e^{\pi}}{e^{\pi}-e^{\pi/2}},\ y_c = \frac{a}{5}\ \frac{e^{2\pi}-2e^{\pi}}{e^{\pi}-e^{\pi/2}}$)

3.8. *L:* Sikloidaning bitta arkasi x=3(t-sint), y=3 (1-cost). *(Javob:* $x_c = 3\pi,\ y_c = 4$)

3.9. *L:* Astroidaning yoyi $x = 2\cos^3\left(\frac{t}{4}\right),\ y = 2\sin^3\left(\frac{t}{4}\right)$

(Javob: $x_c = y_c = \frac{4}{5}$)

3.10. $L: x=e^t \sin t \; y=e^t \cos t$ $\left(0 \le t \le \frac{\pi}{2}\right)$ egri chiziqning yoyi

$$\left(Javob: x_c = \frac{2e^{\pi}+1}{5\left(e^{\frac{\pi}{2}}-1\right)}, y_c = \frac{e^{\pi}-1}{5\left(e^{\frac{\pi}{2}}-1\right)} \right)$$

3.11. *L:* kardioida $\rho = 2(1+\cos\varphi)$. $(x_c = 1{,}6,\ y_c = 0)$

3.12. *L:* $\rho = 2\sin\varphi$ egri chiziq (0:0) nuqtadan $\left(\sqrt{2}; \frac{\pi}{4}\right)$ nuqtagacha. *(Javob:* $x_c = \frac{2}{\pi}$, $y_c = (\pi - 2)/\pi$)

3.13. *L:* Aylana oʻramasining yoyi *x=a (cost+ t sint), y=a (sint-t cost),* $(0 \le t \le \pi)$.*(Javob:* $x_c = 2(\pi^2+4)/a\pi^2, y_c = \frac{6a}{\pi}$)

3.14. *L:* $\varphi = 0$ va $\varphi = \frac{\pi}{4}$ nurlar orasidagi $\rho = 2\sqrt{3}\cos\varphi$ egri chiziqning yoy boʻlagi. *(Javob:* $x_c = \sqrt{3}(\pi+2)/\pi,\ y_c = 2\sqrt{3}/\pi.$)

3.15. *L:*Egri chiziq $x = \sqrt{3}\ t^2, y = t - t^3$ $(0 \le t \le 1)$.

(*Javob*: $x_c = \frac{7\sqrt{3}}{15}, y_c = \frac{1}{4}, (0 \le t \le 1)$)

3.16. *L:*-tomonlari*x+y=a, x=0,* va *y=0* toʻgʻri chiziqlarda yotgan uchburchak. (*Javob:* $x_c = y_c = \frac{a}{3}$)

3.17. *L:* koordinata oʻqlari $(x \ge 0,\ y \ge 0)$ va $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellips bilan chegaralangan.

(Javob: $x_c = \frac{4a}{(3\pi)},\ y_c = \frac{4b}{(3\pi)}$).

3.18. *L:*- Sikloidaning birinchi arkasi *x=a (t-sint), y=a (1-cost)* va *Ox* oʻqi bilan chegaralangan. *(Javob* $x_c = \pi a, y_c = \frac{5a}{6}$).

3.19. *L:* $y = x^2;\ y = \sqrt{x}$ egri chiziqlar bilan chegaralangan: $\left(Javob: x_c = y_c = \frac{9}{20} \right)$.

3.20. F: y=sinxSinusoidaning yoyi va OX o'qining $(0 \le x \le \pi)$kesmasi bilan chegaralangan: $\left(Javob: x_c = \frac{\pi}{2}, y_c = \frac{\pi}{8} \right)$.

3.21. F: Yarim aylana $y = \sqrt{R^2 - x^2}$ va OX o'qi bilan chegaralangan. $\left(Javob: x_c = 0,\ y_c = \frac{4R}{3\pi} \right)$.

3.22. F: Parabolaning yoyi $y = b\sqrt{x/a}$ $(a > 0, b > 0)$, OX o'qi va $x = b$ to'g'ri chiziq bilan chegaralangan. $\left(Javob: x_c = \frac{3a}{5}, y_c = \frac{3в}{8} \right)$.

3.23. . F: Parabolaning yoyi $y = b\sqrt{x/a}$ $(a > 0, b > 0)$, OU o'qi va $y = b$ to'g'ri chiziq bilan chegaralangan. $\left(Javob: x_c = \frac{3a}{10}, y_c = \frac{3в}{4} \right)$.

3.24. F:Yopiq $y^2 = ax^3 - x^4$ chiziq bilan chegaralangan. $\left(Javob: x_c = \frac{5a}{8}, y_c = 0 \right)$

3.25. F: Koordinata o'qlari va astroidaning birinchi kvadrantga joylashgan yoy bo'lagi bilan chegaralangan. $\left(Javob: x_c = y_c = \frac{256a}{(315\pi)} \right)$.

3.26. F: Radiusi R markaziy burchagi 2α bo'lgan doiraning sektori: (*Javob:* Og'irlik markazi sektorning simmetrik o'qida doiraning markazidan $\frac{2}{3} R \frac{Sin\alpha}{\alpha}$ masofada bo'ladi. Agar doiraning markazi koordinata boshida, sektorning simmetrik o'qi *Ou* o'qida bo'lsa, $x_c = 0,\ y_c = \frac{2}{3}R\ \frac{Sin\alpha}{\alpha}$ ga teng).

3.27. F: Kardoida $\rho = a(1+\cos\varphi)$ bilan chegaralangan. $\left(Javob: x_c = \frac{5a}{6}, y_c = 0\right)$

3.28. F: Bernulli lemniskatasining birinchi bo'lagi $\rho^2 = a^2\cos 2\varphi \left(Javob: x_c = \sqrt{2}\ \pi a/8,\ \ y_c = 0\right)$.

3.29. F: Koordinata o'qlari va $\sqrt{x}+\sqrt{y}=\sqrt{a}$ parabola bilan chegaralangan. $\left(Javob: x_c = y_c = \frac{a}{5}\right)$.

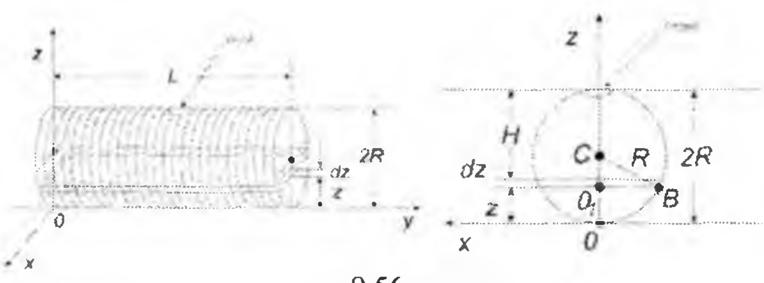
3.30. F:Yarim kubikparabola $ay^2 = x^3$ va $x = a$ to'g'ri chiziq bilan chegaralangan (a> 0). $\left(Javob: x_c = \frac{5a}{7},\ y_c = 0\right)$.

Namunaviy variant yechish.

1. Uzunligi *L*, asosining radiusi *R* bo'lgan doiraviy silindr (9.56-rasm) ko'rinishidagi rezervuarning yuqoridagi suvni haydab chiqarish uchun bajarilgan *A* ishni aniqlang. Suvning solishtirma og'irligi $\gamma = 9{,}81\ kN/m^3$. Bajarilgan *A* ishni *L=5m, R=1m* bo'lgan hol uchun hisoblang.

▶ z balandlikda suvning dz qatlamini ajratamiz (9.56-rasm). Uning hajmi:

$$dv = 2|O_1B|Ldz = 2L\sqrt{R^2-(R-z)^2}dz = 2L\sqrt{z(2R-z)}dz.$$



9.56- rasm

Bu qatlamni $H=2R-z$ balandlikga ko'tarish kerak, dz qatlamdagi suvni haydab chiqarish uchun bajariladigan elementar dA ish quyidagi formula orqali topiladi:

$$dA = H\gamma d\upsilon = 2\gamma L(2R-z)\sqrt{z(2R-z)}\,dz.$$

Butun suvni haydab chiqarish uchun bajariladigan ish elementar ishlarning yig'indisiga teng:

$$A=\int_0^{2R} dA=\int_0^{2R} 2\gamma L(2R-z)\sqrt{z(2R-z)}\,dz=2\gamma L\int_0^{2R} z^{\frac{1}{2}}(2R-z)^{\frac{3}{2}}\,dt \qquad (1)$$

Yuqoridagi differensial binomdan olingan (1) integralni hisoblaymiz $m=\frac{1}{2},\ n=1,\ p=\frac{3}{2}.\ \frac{(m+1)}{n}+p=3\in Z$ Bo'lgani uchun (1) integralni hisoblashda $a+bx^n=u^s x^n$ (8.7 paragraf) almashtirish bajaramiz. Almashtirishni qo'llab:

$$A=2\gamma L\int_0^{2R} z^{\frac{1}{2}}(2R-z)^{\frac{3}{2}}dz=\left|\begin{array}{c}2R-z=u^2z,\ dz=-4Ru(u^2+1)^{-2}du\\ z=2R/u^2+1,\ agar\ z=0, u=\infty\\ agar\ z=2R,\ u=0\end{array}\right|=32\gamma LR^3\int_0^{\infty}\frac{u^4du}{(u^2+1)^4}$$ ga

ega bo'lamiz.

Oxirgi xosmas integralda, integral ostidagi funksiya to'g'ri ratsional kasr bo'lib, (8.10) formulaga ko'ra uni soda kasrlarning yig'indisi ko'rinishida yozilishi mumkin (8.6). Bu kasrlardan integrallar (8.4) rekkurent formula orqali oson topiladi. Rekkurent formulani qo'llab:

$$\int_0^{\infty}\frac{u^4du}{(u^2+1)^4}=\int_0^{\infty}\left(\frac{1}{(u^2+1)^2}-\frac{2}{(u^2+1)^3}+\frac{1}{(u^2+1)^4}\right)du=\frac{\pi}{4}-2\cdot\frac{3}{4}\cdot\frac{\pi}{4}+\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{\pi}{4}=\frac{\pi}{32}.$$

u holda: $A=32jLR^3\pi/32=\pi jLR^3$. Agar: $L=5m$, $R=1m$ bo'lsa $A=3{,}14\cdot 9{,}81\cdot 5\cdot 1\approx 154\,\text{кж}$ ◁

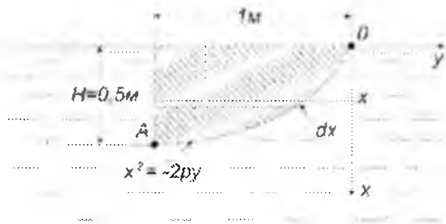
2. Suvning solishtirma og'irligini $9.18\ \text{кн}/\text{м}^3$ deb hisoblab, suvga vertikal cho'ktirilgan plastinkaga ta'sir etayotgan suvning bosim kuchini hisoblang. Plastinkaning joylashishi, o'lchovi, formasi (9.57 rasm) da keltirilgan.

▶ Koordinata sistemasini 9.57 rasmda ko'rsatilgandek tanlab olamiz. U holda parabolaning sodda tenglamasi $x^2=-2py$

ko‘rinishda bo‘ladi. Parabola $A(1/2,-1)$ nuqtadan o‘tgani uchun, $p=1/8$, $x^2=-y/4$ ga teng bo‘ladi.

Kengligi dx va yuzi $ds=(1-|y|)dx$ bo‘lgan gorizontal kesimni x chuqurlikda ajratamiz. Suvning bu kesimga bosimi:

$\Delta p=\gamma x(1-|y|)dx=\gamma x(1-4x^2)dx$ ga teng bo‘ladi.



9.57- rasm

U holda suvning butun plastinkaga bosimi ushbu formula orqali topiladi:

$$P=\gamma\int_0^H x(1-4x^2)dx=\gamma\left(\frac{x^2}{2}-x^4\right)\Bigg|_0^H=\gamma\left(\frac{H^2}{2}-H^4\right)$$

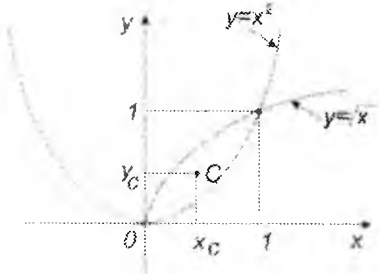
Agar $H=\frac{1}{2}$ va $\gamma=9.81\ \text{кн}/\text{м}^3$ bo‘lsa bosim

$P=9{,}81\left(\frac{1}{8}-\frac{1}{16}\right)=\frac{9{,}81}{16}\approx 0{,}61\text{кн}$ ga teng bo‘ladi. ◀

3. Bir jinsli figura $y=x^2$ *ва* $y=\sqrt{x}$ egri chiziqlar bilan chegaralangan bo‘lsa, uning og‘irlik markazini toping.

▶ Figuraning og‘irlik markazi (9.58 – rasm) (9.17) formula orqali hisoblanadi, bu yerda:

$f_1(x)=x^2,\ \ f_2(x)=\sqrt{x}.$



9.58- rasm

Egri chiziqlarning kesishish nuqtalari 0 (0,0) va B (1,1) bo‘lgani uchun *a=1, b=1*. U holda:

$$\int_0^1 (y_2 - y_1)dx = \int_0^1 \left(\sqrt{x} - x^2\right)dx = \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Bigg|_0^1 = \frac{1}{3},$$

$$\int_0^1 x(y_2 - y_1)dx = \int_0^1 x\left(\sqrt{x} - x^2\right)dx = \left(\frac{2}{5}x^{\frac{5}{2}} - \frac{x^4}{4}\right)\Bigg|_0^1 = \frac{3}{20},$$

$$\frac{1}{2}\int_0^1 (y_2 + y_1)(y_2 - y_1)dx = \frac{1}{2}\int_0^1 \left(x - x^4\right)dx = \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Bigg|_0^1 = \frac{3}{20},$$

bundan $x_c = y_c = \dfrac{9}{20}$. ◄

9.6 9 – bo‘limga qo‘shimcha masalalar.

1. Tenglamani yeching:

a) $\displaystyle\int_{\sqrt{2}}^{x} \frac{dx}{x\sqrt{x^2 - 1}} = \frac{\pi}{12}$; *b)* $\displaystyle\int_{\ln 2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. (*Javob: a) x=2; b) x= ln 4*)

2. Tenglikni isbotlang:

$$\int_x^1 \frac{dt}{1+t^2} = \int_1^{1/x} \frac{dt}{1+t^2} \quad (x > 0).$$

3. Agar $I_n = \displaystyle\int_0^{\frac{\pi}{4}} tg^n x dx \; (n > 1,\, n - butun)$ bo‘lsa, ushbu $I_n + I_{n-2} = \dfrac{1}{n-1}$ tenglikni isbotlang.

4. Berilgan chiziqlar bilan chegaralangan egri chiziqli trapetsiya yoki figuraning yuzini toping?

a) $y=\frac{x^2}{\sqrt{(x-3)(5-x)}},\ x\in(3;5);$

b) $y=\frac{(\arcsin\sqrt{x})}{\sqrt{1-x}},\ x\in[0;1);$

c) $p=tg\varphi,\ p=\frac{1}{\cos\varphi},\ \varphi\in\left[0,\frac{\pi}{2}\right);$

d) $y=x\cdot e^{-\frac{x^2}{2}},\ x\in[0;\infty);$

e) $y=\frac{\sqrt{x}}{(x+1)^2},\ x\in[1;\infty);$

f) $xy^2=8-4x$ va uning asimptotasi;

g) $(x+1)y^2=x^2$ $(x<0)$ va uning asimptotasi;

(*Javob:* a) $39\pi/2$; b) 2; c) $\pi/4$; d) 1; e) $\frac{\pi}{4}+\frac{1}{2}$; f) 4π; g) 8/3.)

5. Berilgan chiziqlarni aylanishidan hosil bo'lgan sirt bilan chegaralangan jismning hajmini toping:

a) $y=e^{-x^2}$ *va* $y=0, Oy$ o'qi atrofida;

b) $(4-x)y^2-x^3=0$ uning asimptotasi atrofida;

c) $y=\frac{1}{1+x^2}$ uning asimptotasi atrofida;

d) $y=e^{-x}\operatorname{Sin}\pi x$ va $x\geq 0$, Ox o'qi atrofida.

(*Javob:* a) π; b) $16\pi^2$; c) $\pi^2/2$; d) $\pi^3/(4(1+\pi^2))$)

6. Silindrik bak vertikal joylashgan bo'lib suv bilan to'ldirilgan, tubida kichik tirqish bor. Bakdan suvning yarim "t" vaqt ichida oqib chiqdi. Hamma suv qancha vaqt ichida oqib chiqadi?

Bu yerda $\mu=1$ va $\upsilon=\mu\sqrt{2gh}$, υ - tirqishdan oqib chiqayotgan suvning tezligi. $\left(Javob: (2+\sqrt{2})T \text{ min}\right)$.

7. Qarshiligi o'zgarmas R bo'lgan rezistorga o'zgaruvchan $U = U_0 \sin \omega t$ kuchlanish berilgan. Rezistorga qanchalik o'zgarmas kuchlanish berish kerakki, $T = \frac{2\pi}{\omega}$ vaqt ichida ajralib chiqqan issiqlik, o'zgaruvchi kuchlanish bergandagi shu vaqt ichidagi ajralib chiqqan issiqlikga teng bo'lishi kerak. (*Javob:* $U_0 / \sqrt{2}$).

8. Elektr zanjir boshlang'ich paytida R om qarshilikga ega va u tekis $\upsilon o м / c$ tezlik bilan o'sadi. Zanjirga o'zgarmas $U в$ kuchlanish berilgan. Elektr zanjirdan tc. vaqt miqdorida o'tgan zaryadni aniqlang (*Javob :* $\frac{U}{a} \ln \frac{R + at}{R}$).

9. Yer atmosferasi massasini uning zichligi, balandlik oshishi bilan $p = p_0 \; e^{-ah}$ qonun bilan o'zgarsa, bu yerda h- yer sirtidan qaralayotgan nuqtagacha bo'lsa, hisoblab toping. (Yer radiusi "R" bo'lgan shar deb hisoblanadi).
(*Javob:* $\left(4\pi\rho_0\left(a^2R^2 + 2aR + 2\right)\right)/a^3$).

10. Jism temperaturasi $T = 20°C$ bo'lgan muhit bilan qoplangan. Sovutish natijasida jismning temperaturasi $100^o дан 60^o$ ga tushgan. Sovutish boshlanishidan qancha vaqt keyin jismning temperaturasi $30^o C$ ga tushadi? (*Javob: 1soat*).

11. Massasi "m" bo'lgan moddiy nuqta chiziqli zichligi ρ bo'lgan cheksiz sterjendan " ℓ " masofada joylashgan. Qanday kuch bilan sterjen nuqtani tortadi? (*Javob:* $\pi\gamma\rho m/l$, γ – gravitatsion o'zgarmas).

12. O'q qalinligi "h" bo'lgan taxtani teshib o'tgandan keyin tezligi v_1 *dan*v_2 ga o'zgaradi. Qarshilikni tezlikning kvadratiga proporsional deb hisoblab, o'qning taxta ichidagi vaqtini toping?

(*Javob:* $h(\upsilon_1 - \upsilon_2)/(\upsilon_1 \upsilon_2 \ln \frac{\upsilon_1}{\upsilon_2})$.

10. BIR NECHA O'ZGARUVCHILI FUNKSIYALARNING DIFFERENTSIAL HISOBI

10.1. BIR NECHA O'ZGARUVCHILI FUNKSIYA TUSHUNCHASI. XUSUSIY HOSILALAR

Aytaylik, biror D*(x,y)* sohada har bir tartiblangan *(x,y)* juftlikka aniq $z \in E \subset R$ son mos qo'yilgan bo'lsin. U holda z, *x va y larga bog'liq bo'lgan ikki o'zgaruvchili funksiya* deyiladi. *x* va y o'zaro bog'liq bo'lmagan o'zgaruvchilar yoki argumentlar deyiladi. *D* to'plam funksiyaning mavjudlik yoki aniqlanish sohasi, *E* to'plam esa *funksiyaning qiymatlari sohasi* deyiladi. Simvolik ravishda ikki o'zgaruvchili funksiya $z = f(x,y)$ ko'rinishda yoziladi, bu yerda *f* moslik qonuniyatini belgilaydi. Bu qonuniyat analitik ko'rinishda (formula orqali), jadval yordamida yoki grafik ko'rinishda berilishi mumkin.

Umuman olganda dekart koordinatalari sistemasi *Oxyz* kiritilgan fazoda har qanday $z = f(x,y)$ tenglama biror sirtni aniqlaydi, ya'ni ikki o'zgaruvchili funksiyaning grafigi deganda koordinatalari $z = f(x,y)$ tenglamani qanoatlantiruvchi fazodagi *M(x,y,z)* nuqtalar to'plamidan hosil qilingan sirtni tushunamiz. (10.1 – rasm).

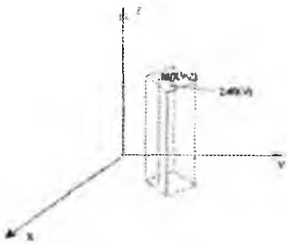
Geometrik nuqtai nazardan, funksiyaning aniqlanish sohasi *D*, odatda shu sohaga tegishli yoki tegishli bo'lmagan chiziqlar bilan chegaralangan *Oxy* tekislikning biror qismini tasvirlaydi. Birinchi holatda *D* soha yopiq soha deyiladi va $\bar{D}$ bilan belgilanadi, ikkinchi holatda esa ochiq soha deyiladi.

1 – misol. $z = \ln(y - x^2 + 2x)$ funksiyaning *D* aniqlanish sohasini va *E* – qiymatlar sohasini toping.

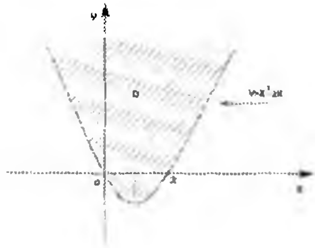
► Berilgan funksiya *Oxy* tekislikning $y - x^2 + 2x > 0$ yoki $y > x^2 - 2x$ o'rinli bo'ladigan nuqtalaridagina aniqlangan.

Tekislikning $y = x^2 - 2x$ tenglikni qanoatlantiradigan nuqtalari *D* sohaning chegarasini tashkil qiladi. $y = x^2 - 2x$ paraboladir. (10.2 – rasm.) Parabola *D* sohada yotmaganligi uchun shtrixli chiziqlar bilan tasvirlangan). $y > x^2 - 2x$ tengsizlik o'rinli bo'ladigan nuqtalar paraboladan yuqorida yotishini tekshirish oson. *D* soha ochiq atrof bo'lib, (10.2 – rasmda y shtrixlangan) uni quyidagi tengsizliklar sistemasi bilan aniqlash mumkin:

D: $\{-\infty < x < +\infty,\ x^2 - 2x < u < +\infty\}$



10.1. rasm



10.2. rasm

Ikki o'zgaruvchili funksiyaning tarifini uch va undan ko'p o'zgaruvchilar uchun umumlashtirish qiyin emas.

Agar biror n – o'lchamli fazoda $x_1,...,x_n$ o'zgaruvchilarning har bir $(x_1,...,x_n)$ to'plamiga, y ning biror aniq qiymati mos qo'yilsa, u holda y kattalik $x_1,...,x_n$ o'zgaruvchilarning funksiyasi deyiladi va simvolik ravishda $y=f(x_1,...,x_n)$ ko'rinishda yoziladi.

O'zaro bog'liq bo'lmagan $x_1,...,x_n$ o'zgaruvchilarning qiymatlari to'plami n o'lchamli fazoda $M(x_1,...,x_n)$ nuqtani aniqlaydi, u holda har qanday ko'p o'zgaruvchili funksiyani odatda mos o'lchamli fazodagi M nuqtaning funksiyasi deb qaraladi: $y=f(M)$

Agar har qanday $\varepsilon > 0$ son uchun, shunday $\delta > 0$ son mavjud bo'lib, $|x - -x_0| < \delta$ va $|u - u_0| < \delta$ shartlarni qanoatlantiruvchi x va y lar uchun

$$|f(x,y) - A| < \varepsilon$$

tengsizlik o'rinli bo'lsa, u holda A soni $z=f(x,y)$ funksiyaning $M_0(x_0,u_0)$ nuqtadagi limiti deyiladi.

Agar A soni $f(x,y)$ funksiyaning $M_0(x_0,y_0)$ nuqtadagi limiti bo'lsa, u holda quyidagicha yoziladi:

$$A = \lim_{\substack{x\to x_0 \\ y\to y_0}} f(x,y) = \lim_{M\to M_0} f(x,y)$$

2 – misol. $A = \lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$ limitni hisoblang.

► Limit belgisi ostidagi ifodada almashtirishlar bajarib, quyidagiga ega bo'lamiz.

$$A = \lim_{\substack{x\to 0\\ y\to 0}} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+1}+1\right)}{\left(\sqrt{x^2+y^2+1}-1\right)\left(\sqrt{x^2+y^2+1}+1\right)} =$$

$= \lim_{\substack{x\to 0\\ y\to 0}} \frac{(x^2+y^2)(\sqrt{x^2+y^2+1}+1)}{x^2+y^2+1-1} = \lim_{\substack{x\to 0\\ y\to 0}}\left(\sqrt{x^2+y^2+1}+\right.$
$\left.1\right) = 2$◄

Agar $\lim_{\substack{x\to x_0\\ y\to y_0}} f(x,y) = f(x_0,y_0)$ tenglik o'rinli bo'lsa, $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ nuqtada uzluksiz deyiladi.

Masalan, $z=1/(2x^2+y^2)$ funksiya cheksiz ko'p uzilishga ega bo'ladigan $M(0,0)$ nuqtadan tashqari, tekislikning barcha nuqtalarida uzluksizdir.

Biror D atrofning barcha nuqtalarida uzluksiz bo'lgan funksiya, berilgan D sohada uzluksiz deyiladi.

Agar y ni o'zgarmas deb olib, x o'zgaruvchiga biror Δx orttirma bersak, u holda $z= f(x,y)$ funksiya x o'zgaruvchi bo'yicha z funksiyaning xususiy orttirmasi deb ataluvchi $\Delta_x z$ orttirma oladi.

$$\Delta_x z = f(x+\Delta x, y) - f(x,y)$$

Xuddi shu kabi, $z= f(x,y)$ funksiyada x ni o'zgarmas deb olib, u ga Δy orttirma bersak, u holda y o'zgaruvchi bo'yicha z funksiyaning xususiy orttirmasi quyidagicha bo'ladi.

$$\Delta_y z = f(x, y+\Delta y) - f(x,y)$$

Agar

$$\lim_{\Delta x\to 0} \frac{\Delta_x z}{\Delta_x} = \frac{\partial z}{\partial x} = z'_x = f'_x(x,y)$$

$$\lim_{\Delta y\to 0} \frac{\Delta_y z}{\Delta y} = \frac{\partial z}{\partial y} = z'_y = f'_y(x,y)$$

limitlar mavjud bo'lsa, bu ifodalar $z= f(x,y)$ funksiyaning mos ravishda x va y o'zgaruvchilar bo'yicha *xususiy hosilalari* deb ataladi.

Ixtiyoriy sondagi o‘zaro bog‘liq bo‘lmagan o‘zgaruvchilarga ega bo‘lgan funksiyaning xususiy hosilalari ham shu kabi aniqlanadi.

Ixtiyoriy o‘zgaruvchi bo‘yicha olingan xususiy hosila, qolgan o‘zgaruvchilarni o‘zgarmas degan shartda shu o‘zgaruvchidan olingan hosilaga teng bo‘lgani uchun bir o‘zgaruvchili funksiyani differensiallashning barcha qoidalari va formulalari ko‘p o‘zgaruvchili funksiyaning xususiy hosilalarini topish uchun o‘rinlidir.◄

3 – misol. $Z=arctg\frac{y}{x}$ funksiyaning xususiy hosilalarini toping.

► Xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x}=\frac{1}{1+(y/x)^2}\times\left(-\frac{y}{x^2}\right)=-\frac{y}{x^2+y^2}$$

$$\frac{\partial z}{\partial y}=\frac{1}{1+(y/x)^2}\times\frac{1}{x}=\frac{x}{x^2+y^2}$$ ◄

4 – misol. $W=ln^2(x^2+y^2+z^2)$ funksiyaning xususiy hosilalarini toping.

► Xususiy hosilalarini topamiz.

$$\frac{\partial w}{\partial x}=2\ln(x^2+y^2+z^2)\times\frac{1}{x^2+y^2+z^2}\times 2x$$

$$\frac{\partial w}{\partial y}=2\ln(x^2+y^2+z^2)\times\frac{1}{x^2+y^2+z^2}\times 2y$$

$$\frac{\partial w}{\partial z}=2\ln(x^2+y^2+z^2)\times\frac{1}{x^2+y^2+z^2}\times 2z$$ ◄

Bog‘liq bo‘lmagan o‘zgaruvchilardan biri o‘zgarmas, ikkinchisi o‘zgaradi degan shartdagi $z=f(x,y)$ funksiyaning differensiali xususiy differensial deb ataladi, yani tarif bo‘yicha

$$d_xz=f_x'(x,y)dx,\, d_yz=f_y'(x,y)dy$$

bu yerda, $dx=\Delta x$, $dy=\Delta y$ lar o‘zaro bog‘liq bo‘lmagan o‘zgaruvchilarning ixtiyoriy orttirmalari bo‘lib, ularning differensiallari deb ataladi. Bu uch o‘zgaruvchili $w=f(x,y,z)$ funksiya uchun ham o‘rinlidir.

5–misol. $w=(xy^2)^{z^3}$ funksiyaning xususiy differensiallarini toping.

► Berilgan funksiyaning xususiy differensiali

$$d_xw=z^3(xy^2)^{z^3-1}\times y^2dx,\, d_yw=z^3(xy^2)^{z^3-1}\times 2xydy$$

$$d_z w = (xy^2)^{z^3} \times \ln(xy^2) \times 3z^2 dz$$ ◀

6 –misol. $w = \sqrt{x^2 + y^2 + z^2} - xyz$ funksiyaning $M(2,-2,1)$ nuqtadagi xususiy hosilalarining qiymatlarini toping.

▶ Xususiy hosilalarni topamiz:

$$\frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}} - yz, \frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}} - xz,$$

$$\frac{\partial w}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}} - xy$$

Hosil qilingan ifodalarga berilgan nuqtaning koordinatalarini qo'yamiz.

$$\left.\frac{\partial w}{\partial z}\right|_{M_0} = \frac{2}{3} + 2 = \frac{8}{3}, \left.\frac{\partial w}{\partial z}\right|_{M_0} = -\frac{2}{3} - 2 = -\frac{8}{3}$$

$$\left.\frac{\partial w}{\partial z}\right|_{M_0} = \frac{1}{3} + 4 = \frac{13}{3}$$ ◀

10.1.- AT

1. Quyidagi funksiyalarning aniqlanish sohasini toping.

a) $z = \sqrt{u^2 - 2x + 4)}$ b) $z = \frac{1}{\sqrt{x+u}} + \sqrt{x - u}$

v) $z = \ln x + \ln \cos y$ c) $z = \sqrt{x^2 + y^2 - 9}$

2. Ko'rsatilgan funksiyalarning xususiy hosilalarini toping.

a) $z = (x^2 + y^2 - xy^2)^3$ b) $z = \arcsin\frac{y}{x}$

c) $z = x\sqrt{y} + \frac{y}{\sqrt{x}}$ d) $z = \ln(x + \sqrt{x^2 + y^2})$

g) $z = \ln(x \times y + \ln x \times y)$ e) $u = arctg(xy/z)$

f) $u = \ln\sqrt{(x^2 + y^2)(x^2 + z^2)}$ z) $u = (xy)^{z^2} - 1$

3. Agar $u = \ln(1 + x + y^2 + z^2)$ bo'lsa $U'_x + U'_y + U'_z$ ning $M_0(1,1.1)$ nuqtadagi qiymatini hisoblang. (*Javob*: 3/2)

4. $z = x + u + \sqrt{x^2 + y^2}$ funksiyaning xususiy hosilalarining $M_0(3,4)$ nuqtadagi qiymatlarini hisoblang. (*Javob*: 2/5, 1/5)

5. Quyidagi funksiyalarning xususiy differensiallarini toping:

a) $z = \ln\sqrt{x^2 + y^2}$ b) $z = arctg\frac{x+y}{1-xy}$

c) $U = x^{yz}$ d) $U = \frac{x^2 + y^2 - z^2}{z^2 - x^2 - y^2}$

Mustaqil ish

1. Quyidagilarni toping:

a) funksiyaning aniqlanish va qiymatlar sohasini:

$$z=ln(4-x^2+y^2);$$

b) funksiyaning xususiy hosilalarini

$$z=sin^2(x\,cos^2y+y\,sin^2x);$$

v) funksiyaning xususiy differensiallarini

$$u=ln\frac{xyz}{x^2+y^2+z^2}.$$

2. Quyidagilarni toping:

a) funksiyaning aniqlanish va qiymatlar sohasini

$$z=\sqrt{4-x^2+u};$$

b) funksiyaning xususiy hosilalarini

$$u=\arcsin\sqrt{xy^2z^3};$$

v) funksiyaning xususiy differensiallarini

$$z=\sqrt{(x^2+y^2)/(x^2-y^2)}.$$

3. Quyidagilarni toping:

a) funksiyaning aniqlanish sohasini va qiymatlarini

$$z=\sqrt{x\times u}+\sqrt{x-u};$$

b) funksiyaning xususiy hosilalarini

$$u=tg^2(x-y^2+z^3);$$

v) funksiyaning xususiy differensiallarini

$$z=\sqrt[3]{(x^2-y^2)^2}.$$

10.2. TO'LA DIFFERENSIAL. OSHKORMAS VA MURAKKAB FUNKSIYALARNI DIFFERENSIALLASH

$$\Delta z=f(x+\Delta x,y+\Delta x)-f(x,y)$$

ayirmaga $z=f(x,y)$ *funksiyaning to'la orttirmasi* deb ataladi.

$z=f(x,y)$ funksiyaning to'la differensialning, o'zaro bog'liq bo'lmagan Δx *va* Δy o'zgaruvchilarning orttirmasiga chiziqli bog'liq bo'lgan bosh qismi, *funksiyaning to'la differensiali* deb ataladi va quyidagicha belgilanadi *dz*.

Agar funksiya uzluksiz xususiy hosilalarga ega bo‘lsa, u holda to‘la differensial mavjud bo‘ladi va quyidagiga teng bo‘ladi.

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \qquad (10.1)$$

bu yerda $dx = \Delta x, dy = \Delta y$ – *o‘zaro bog‘liq bo‘lmagan o‘zgaruvchilarning differensiallari* deb ataluvchi ixtiyoriy orttirmalardir.

n o‘zgaruvchili u= $f(x_1, x_2, \ldots x_n)$ funksiya uchun to‘la differensial quyidagi ifoda bilan aniqlanadi.

$$du = \frac{\partial u}{\partial x_1}dx_1 + \frac{\partial u}{\partial x_2}dx_2 + \cdots + \frac{\partial u}{\partial x_n}dx_n \qquad (10.2)$$

1–misol. $z = x^2 - xy + y^2$ funksiyaning to‘la orttirmasi va to‘la differensialini toping

►Tarifga asosan

$$\Delta z = (x + \Delta x)^2 - (x+\Delta x)(y+\Delta y) + (y + \Delta y)^2 - x^2 + xy - y^2$$

$$x^2+2x\Delta x+\Delta x^2-xy-x\Delta y-u\Delta x-\Delta x\Delta y+y^2+2y\Delta y + \Delta y^2-$$

$$-\quad x^2+xy-y^2=2x\Delta x-x\Delta y+2y\Delta y-y\Delta x+\Delta x^2-\Delta x\Delta y+\Delta y^2=(2x-y)\,\Delta x+$$

$$+(2y-x)\,\Delta y+\Delta x^2-\Delta x\Delta y+\Delta y^2$$

$(2x-y)\,\Delta x+(2y-x)\,\Delta y$ ifoda Δx *va* Δy larga nisbatan chiziqli bo‘lib dz нинг дифференциалидир, $\alpha=\Delta x^2-\Delta x\Delta y+\Delta y^2$ kattalik esa $\Delta\rho = \sqrt{\Delta x^2 + \Delta y}$ ga nisbatan yuqori tartibli cheksiz kichikdir. Shunday qilib, $\Delta z = dz+\alpha$◄

2 – misol. $u=ln^2(x^2+ y^2 - z^2)$ funksiyaning to‘la differensialini toping.

►Avval xususiy hosilalarni topamiz:

$$\frac{\partial u}{\partial x} = 2ln(x^2 + y^2 - z^2) \times \frac{2x}{x^2 + y^2 - z^2};$$

$$\frac{\partial u}{\partial y} = 2\ln(x^2 + y^2 - z^2) \times \frac{1}{x^2 + y^2 - z^2} \times 2y;$$

$$\frac{\partial u}{\partial z} = 2\ln(x^2 + y^2 - z^2) \times \frac{1}{x^2 + y^2 - z^2} \times (-2z).$$

(10.2) formulaga asosan quyidagiga ega bo‘lamiz.

$$du = 4\ln(x^2 + y^2 - z^2) \times \frac{1}{x^2+y^2-z^2} \times (xdx + ydy - zdz)$$◄

$\Delta z \approx dz$ bo‘lgani uchun ko‘p hollarda to‘la differensial funksiyaning qiymatini taqribiy hisoblashda qo‘llaniladi, yani

$$f(x_0+\Delta x, y+\Delta y) \quad f(x_0, y_0)+dz(x_0, y_0)$$

3 – misol $(1{,}02)^{3{,}01}$ ni taqribiy hisoblang.

► $z=xy$ funksiyani qaraymiz. $x_0=1$ *va* $y_0=3$ da quyidagilarga ega bo‘lamiz.

$z_0=1^3=1$; $\Delta x=1{,}02-1=0{,}02$, $\Delta y = 3{,}01 - 3 = 0{,}01$

$z=x^u$ funksiyaning ixtiyoriy nuqtadagi to‘la differensialini topamiz.

$$dz = yx^{u-1}\Delta x+x^u \ln x\,\Delta y$$

Berilgan $\Delta x=0{,}02$ va $\Delta y = 0{,}01$ orttirmalarni etiborga olgan holda buning M(1;3) nuqtadagi qiymatini hisoblaymiz.

$$dz =3\cdot 12\cdot 0{,}02+13\ln 1\cdot 0{,}02=0{,}06$$

U holda $z=(1{,}02)^{3{,}01} \quad z_0+dz=1+0{,}06=1{,}06$ ◄

$z = f(u,v)$ *funksiya, bu yerda* $u=\varphi(x,y)$, $v=\varphi(x,y)$ x *va* y *o‘zgaruvchilarning murakkab funksiyasi* deb ataladi. Murakkab funksiyaning xususiy hosilalarini topish uchun quyidagi formulalardan foydalaniladi:

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}, \qquad (10.3)$$
$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}.$$

$u=\varphi(x)$, $v=\psi(x)$ bo‘lgan holda (10.3) formulaning ikkinchisi aynan no‘lga aylanadi. Birinchisi esa quyidagi ko‘rinishga ega bo‘ladi.

$$\frac{dz}{dx}=\frac{\partial z}{\partial x}\frac{du}{dx}+\frac{\partial z}{\partial v}\frac{dv}{dx}.$$

Oxirgi formuladagi $\frac{dz}{dx}$ ifoda funksiyaning to‘la hosilasi deb ataladi ($\frac{\partial z}{\partial x}$ xususiy hosiladan farqli ravishda.)

4 – misol. $z=\sin(u\,v)$ funksiyaning xususiy hosilalarini toping, bu yerda $u=2x+3y$, $v=x+y$.

► Quyidagiga ega bo‘lamiz:

$\frac{\partial z}{\partial x}=v\cos(uv)\cdot 2+u\cos(u\times v)\cdot y=\cos \quad (2x^2y+3xy^2)$ $(4xy+3y^2)$

$\frac{\partial z}{\partial y}= v\cos(uv)\cdot 3+u\cos(u\times v)\cdot x=\cos \quad (2x^2y+3xy^2)$ $(6xy+2x^2)$ ◄

5 – misol. $U=x+u^2+z^3$ funksiyaning to'la hosilasini toping, bu yerda $y=sinx;\ z=cosx$

► Quyidagiga ega bo'lamiz:

$$\frac{\partial z}{\partial x}=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\times\frac{dy}{dx}+\frac{\partial u}{\partial z}\times\frac{dz}{dx}=1+2y\cos x+3z^2(-\sin x)=$$

$$=1+2\sin x\cos x-3\cos^2 x\sin x$$ ◄

Agar $F(x,y)=0$ tenglama oshkormas ravishda ikki o'zgaruvchili $z(x,y)$ funksiyani ifodalasa va $F_z'(x,y,z)\neq 0$ bo'lsa, u holda quyidagi formulalar o'rinlidir:

$$\frac{\partial z}{\partial x}=\frac{F_x'(x,y,z)}{F_z'(x,y,z)},\ \frac{\partial z}{\partial u}=-\frac{Fy_u(x,y,z)}{F_z'(x,y,z)}, \qquad (10.7)$$

6-misol. Oshkormas ravishda berilgan $x^3+y^3-e^{xy}-5=0$ tenglamadan y funksiyaning hosilasini toping.

► (10.6) formulaga asosan, quyidagiga ega bo'lamiz.

$$\frac{\partial u}{\partial x}=-\frac{3x^2-e^{xy}\times y}{3y^2-e^{xy}\times x}$$ ◄

7 – misol. Oshkormas ko'rinishda berilgan $xyz+x^3-y^3-z^3+5=0$ tenglamadan z funksiyaning xususiy hosilalarini toping.

► (10.7) formuladan foydalanib, quyidagiga ega bo'lamiz:

$$\frac{\partial z}{\partial x}=-\frac{yz+3x^2}{xy-3x^2},\ \frac{\partial z}{\partial y}=-\frac{xz-3y^2}{xy-3x^2},$$ ◄

10.2 – AT

1. Quyidagi funksiyalarning to'la differensialini toping.

a) $z=x^3+xy^2+x^{2y}$; *b)* $z=e^{x^3}$; *v)* $U=\sin^2(xy^2z^3)$

2. Funksiyalarning mos orttirmalarini ularning to'la differensiallari bilan almashtirib, berilgan ifodani taqribiy hisoblang:

a) $(1,02)^3\cdot(0,97)^3$; *b)* $\sqrt{(4,05)^2+(2,93)^2}$ *(Javob: a)0,97; b)4,998.)*

3. Agar $u=x\sin y,\ v=y\cos x$ bo'lsa, $z=\sqrt{u^2+v^2}$ funksiyaning xususiy hosilalarini toping.

4. Agar $u=xy,\ v=x/u,\ t=e^{xy}$ bo'lsa, $w=\ln(u^3+v^3-t^3)$ funksiyaning xususiy hosilalarini toping.

5. Agar $y=\sin\sqrt{x}$ bo'lsa, $z=tg^2(x^2-y^2)$ funksiyaning hosilasini toping.

6. $sinxy-x^2-y^2=5$ tenglama bilan oshkormas ravishda berilgan u funksiyaning hosilasini toping.

7. $xyz- sin^2 xuz+x^3+y^3+ z^3=7$ tenglama bilan oshkormas ravishda berilgan z funksiyaning xususiy hosilalarini toping.

8. $x^2 + y^2 + z^2 - xyz = 2$ tenglama bilan oshkormas holda berilgan z funksiyaning xususiy hosilalarining $M_0(1,1,1)$ nuqtadagi qiymatlarini hisoblang.

Mustaqil ish

1. Quyidagilarni toping:

a) $u=z \cdot arctg(x/y)$ funksiyaning to'la differensialini;

b) $sin^3xy^2+cos^3yx^2=1$ tenglama bilan berilgan y funksiyaning hosilasini.

2. Quyidagilarni toping:

a) $z=ctg^2(xy^2-y^3+x^2y)$ funksiyaning to'la differensialini; agar $y = e^{-x^2}$ bo'lsa,

b) $z = arctg\sqrt{x^2 + y^2}$ funksiyaning hosilasini.

3. Quyidagilarni toping:

a) $z = e^{cos^2 (x^2-y^2)}$ funksiyaning to'la differensialini;

b) $x^2y^2z^2 + 7y^4 - 8xz^3 + z^4 = 10$ tenglama bilan berilgan z funksiyaning xususiy hosilalarini.

10.3. YUQORI TARTIBLI XUSUSIY HOSILALAR URINMA TEKISLIK VA SIRTNING NORMALI

Birinchi tartibli xususiy hosiladan olingan hosila *ikkinchi tartibli xususiy hosila* deb ataladi.

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = f''_{xx}(x,y),$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = f''_{yy}(x,y),$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = f''_{xy}(x,y),$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = f''_{yx}(x,y).$$

Uchinchi va undan yuqori xususiy hosilalar aynan shu kabi aniqlanadi. $\frac{\partial^n z}{\partial x^k \partial y^{n-k}}$ yozuv z funksiya x o'zgaruvchi bo'yicha k marta, y o'zgaruvchi bo'yicha n-k marta differensiyallanganligini bildiradi. $f''_{xy}(x,u)$ *va* $f''_{yx}(x,y)$ xususiy hosilalar *aralash xususiy hosilalar* deb ataladi.

Aralash xususiy hosilalar uzluksiz bo'lgan barcha nuqtalarda ularning qiymatlari teng bo'ladi.

1 – misol. $z = e^{x^2y^2}$ funksiyaning ikkinchi tartibli xususiy hosilalarini toping.

► Avval birinchi tartibli xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x} = e^{x^2y^2} \cdot 2xy^2, \frac{\partial z}{\partial x} = e^{x^2y^2} \cdot 2x^2y.$$

Yana bir marta differensiallab quyidagiga ega bo'lamiz:

$$\frac{\partial^2 z}{\partial x^2} = e^{x^2y^2} \cdot 4x^2y^4 + e^{x^2y^2} \cdot 2y^2,$$

$$\frac{\partial^2 z}{\partial y^2} = e^{x^2y^2} \cdot 4x^4y^2 + e^{x^2y^2} \cdot 2x^2,$$

$$\frac{\partial^2 z}{\partial x \partial y} = e^{x^2y^2} \cdot 4x^3y^3 + e^{x^2y^2} \cdot 4xy,$$

$$\frac{\partial^2 z}{\partial u \partial x} = e^{x^2y^2} \cdot 4x^3y^3 + e^{x^2y^2} \cdot 4xy.$$

Oxirgi ikki ifodani solishtirib, $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ ekannligini ko'ramiz. ◄

2 – misol $z = arctg\frac{y}{x}$ funksiya $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ Laplas tenglamasini qanoatlantirishini isbotlang.

► Quyidagilarni topamiz:

$$\frac{\partial z}{\partial x} = -\frac{y}{x^2+y^2}, \frac{\partial z}{\partial y} = \frac{x}{x^2+y^2}$$

U holda $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2yx}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2} = 0$ ◄

$z=f(x,y)$ funksiyaning *ikkinchi tartibli to'la differensiali* d^2z quyidagi formula bilan ifodalanadi.

$$d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dxdy + \frac{\partial^2 z}{\partial y^2}dy^2$$

3 – misol $z=x^3+y^3+x^2y^2$ funksiyaning ikkinchi tartibli to'la differensialini toping.

▶ Ikkinchi tartibli xususiy hosilalarni topamiz.

$$\frac{\partial z}{\partial x}=3x^2+2xy^2;\ \frac{\partial z}{\partial y}=3y^2+2x^2y;$$

$$\frac{\partial^2 z}{\partial x^2}=6x+2y^2;\ \frac{\partial^2 z}{\partial y^2}=6y+2x^2;\ \frac{\partial^2 z}{\partial x\partial y}=4xy.$$

Shunday qilib,

$$\partial^2 z=(6x+2y^2)dx+8xydxdy+(6y+2x^2)dy^2 ◀$$

Agar sirt $z=f(x,y)$ tenglama bilan berilgan bo'lsa, berilgan sirtga $M_0(x_0,y_0,z_0)$ nuqtada *o'tkazilgan urinma tekislik tenglamasi* quyidagicha bo'ladi:

$$z-z_0=f_x'(x_0,y_0)(x-x_0)+f_y'(x_0,y_0)(y-y_0). \quad (10.8)$$

Sirtga $M_0(x_0,y_0,)$ nuqta orqali o'tkazilgan *normalning kanonik tenglamasi* esa quyidagicha bo'ladi.

$$\frac{x-x_0}{f_x'(x_0,y_0)}=\frac{y-y_0}{f_y'(x_0,y_0)}=\frac{z-z_0}{-1}. \quad (10.9)$$

Silliq sirt tenglamasi oshkormas holda $F(x,y,z)=0$ va $F(x_0,y_0,z_0)=0$ ko'rinishda berilgan bo'lsa, u holda $M_0(x_0,y_0,)$ nuqtadagi urinma tekislik tenglamasi quyidagi ko'rinishda bo'ladi.

$$F_x'(x_0,y_0,z_0)(x-x_0)-F_y'(x_0,y_0,z_0)(y-y_0)+$$
$$F_z'(x_0,y_0,z_0)(z-z_0)=0, \quad (10.10)$$

normalning tenglamasi esa:

$$\frac{x-x_0}{F_x'(x_0,y_0,z_0)}=\frac{y-y_0}{F_y'(x_0,y_0,z_0)}=\frac{z-z_0}{F_z'(x_0,y_0,z_0)} ◀ \quad (10.11)$$

4–misol. $x^3+y^3+z^3+xyz-6=0$ sirtga $M_0(1,2,-1)$ nuqtadagi xususiy hosilalarning qiymatlarini hisoblaymiz:

$$▶ F_x'(x_0,y_0,z_0)=(3x^2+yz)|_{M_0}=1,$$
$$F_y'(x_0,y_0,z_0)=(3y^2+xz)|_{M_0}=11,$$
$$F_z'(x_0,y_0,z_0)=(3z^2+yx)|_{M_0}=5.$$

Bularni (10.10) va (10.11) tenglamalarga qo'yib, mos ravishda urinma tekislik tenglamasi

$$(x-1)+11(y-2)+5(z-1)=0$$

va normalning kanonik tenglamasini topamiz:

$$\frac{x-1}{1}=\frac{y-2}{11}=\frac{z+1}{5} ◀$$

10.3 – AT

1. Quyida ko'rsatilgan funksiyalarning ikkinchi tartibli xususiy hosilalarini toping va ularning aralash hosilalarini tengligini tekshiring.

$$a)\ z = \frac{1}{3}\sqrt{(x^2+y^2)^3};$$

$$b)\ z = \ln\left(x + \sqrt{x^2+y^2}\right);$$

$$v)\ z = e^x(\sin y + \cos x);$$

$$g)\ z = arctg\frac{x+y}{1-xy}.$$

2. $z = e^x(x\cos y - y\sin y)$ funksiyaning $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial u^2} = 0$ tenglamani qanoatlantirishini isbotlang.

3. $z = e^{\cos(x+3y)}$ funksiyaning $9 \times \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$ tenglamani qanoatlantirishini isbotlang.

4. $xy\,z^2 + 2\,y^2 + 3y\,z + 4 = 0$ sirtga $M_0(0.2,-2)$ nuqtada o'tkazilgan urinma tekislik tenglamasi va normal tenglamasini toping.

5. $z = \frac{1}{2}x^2 - \frac{1}{2}y^2$ sirtga $M_0(3,1,4)$ nuqtada o'tkazilgan urinma tekislik va normal tenglamasini toping.

(*Javob:* $3x-y-z=4$, $\frac{x-3}{3} = \frac{y-1}{-1} = \frac{z-4}{-1}$.)

6. $x^2+2y^2+z^2=1$ ellipsoid uchun $x-y+2z=0$ tekislikka parallel urinma tekislik tenglamasini yozing.

Mustaqil ish

1. *1.* $z = \ln(x^2+y)$ funksiyaning ikkinchi tartibli hosilalarini toping.

2. $x^2+2y^2+3z^2=6$ sirtga $M_0(1,-1,1)$ nuqtada o'tkazilgan urinma tekislik va normalning tenglamasini yozing.

2. 1. $z = e^{xy^2}$ funksiyaning ikkinchi tartibli hosilalarini toping.

2. $z = 1 + x^2+y^2$ sirtga $M_0(1,1,z_0)$ nuqtada o'tkazilgan urinma tekislik va normalining tenglamasini yozing.

3. *1.* $z=(x+y)/(x-y)$ funksiyaning ikkinchi tartibli xususiy hosilalarini toping.

2. $x^2z-xyz+y^2-x-3=0$ sirtga $M_0(-2,3,z_0)$ nuqtada o'tkazilgan urinma tekislik va normalining tenglamasini yozing.

10.4. IKKI O'ZGARUVCHILI FUNKSIYANING EKSTREMUMI

Agar $M_0(x_0,y_0)$ nuqtadan farqli va uning yyetarlicha kichik atrofiga tegishli barcha $M(x,y)$ nuqtalar uchun

$$f(x_0,y_0)\geq f(x,y) \quad (f(x_0,y_0)\leq f(x,y))$$

tengsizlik o'rinli bo'lsa, $M_0(x_0,y_0)$ nuqta $z=f(x,y)$ *funksiyaning lokal maksimumi (minimumi)* deb ataladi. Funksiyaning maksimum yoki minimumi uning ekstremumi deyiladi. Funksiya ekstremumga erishadigan nuqta, funksiyaning ekstremum nuqtasi deb ataladi.

1 – teorema. (Ekstremum zaruriy sharti).

Agar $M_0(x_0,y_0)$ nuqta $f(x,y)$ funksiyaning ekstremum nuqtasi bo'lsa u holda $f'_x(x_0,y_0)=f'_y(x_0,y_0)=0$ bo'ladi yoki bu hosilalardan birontasi mavjud bo'lmaydi.

Shu shart bajariladigan nuqtalar *statsionar yoki kritik nuqtalar* deb ataladi. Ekstremum nuqtasi har doim statsionar nuqta bo'ladi, ammo statsionar nuqta ekstremum nuqtasi bo'lmasligi ham mumkin. Statsionar nuqta ekstremum nuqtasi bo'lishi uchun, *ekstremum mavjudligining yyetarli sharti* bajarilishi kerak. Ikki o'zgaruvchili funksiya ekstremumining mavjudligining yyetarli shartini tariflash uchun quyidagicha belgilashlar kiritamiz:

$$A=f''_{xx}(x_0,y_0),\ B=f''_{xy}(x_0,y_0),\ C=f''_{yy}(x_0,y_0),\ \Delta=A\times C-B^2.$$

2- teorema. (Ekstremum yyetarli sharti).

Aytaylik $z=f(x,y)$ funksiya $M_0(x_0,y_0)$ statsionar nuqtani o'z ichiga olgan biror sohada uchinchi tartibli uzluksiz xususiy hosilalarga ega bo'lsin. U holda:

1) agar $\Delta>0$ bo'lsa u holda $M_0(x_0,y_0)$ nuqta berilgan funksiya uchun ekstremum nuqtasi bo'ladi, bunda M_0 nuqta

$A<0 (C<0)$ bo'lganda maksimum nuqtasi va $A>0 (C>0)$ bo'lganda minimum nuqtasi bo'ladi;

2) agar $\Delta < 0$ *bo'lsa, u holda* $M_0(x_0,y_0,)$ *nuqtada ekstremum yo'q;*

3) agar $\Delta = 0$ *bo'lsa, u holda ekstremum bo'lishi ham, bo'lmasligi ham mumkin.*

Ko'rinib turibdiki, uchinchi holda qo'shimcha tekshirish talab etiladi.

1 – misol. $z = x^3 + y^3 - 3xy$ funksiyani ekstremumga tekshiring.

►Qaralayotgan misolda $\frac{\partial z}{\partial x}$ *va* $\frac{\partial z}{\partial y}$ doimo mavjud bo'ladi, shuning uchun statsionar (kritik) nuqtalarni topish uchun quyidagi tenglamalar sistemasiga ega bo'lamiz (1- teoremaga qarang):

$$\frac{\partial z}{\partial x} = 3x^2 - 3y = 0,$$
$$\frac{\partial z}{\partial y} = 3y^2 - 3x = 0.$$

Tenglamalar sistemasini yechamiz:

$\begin{cases} x^2 - y = 0 \\ y^2 - x = 0 \end{cases}$ bundan $x_1 = 0$, x_2 -1, $y_1 = 0$, y_2 1.

Shunday qilib, $M_1(0,0)$ va , $M_2(1,1)$ ikkita statsionar nuqtalarga ega bo'lamiz.

Quyidagilarni topamiz:

$$A = \frac{\partial^2 z}{\partial x^2} = 6x, B = \frac{\partial^2 z}{\partial x \partial y} = -3, C = \frac{\partial^2 z}{\partial y^2} = 6y$$

U holda $=AC-B^2=36xy-9$.

$M_1(0,0)$ nuqtada $=-9$, yani bu nuqtada ekstremum yo'q. $M_2(1,1)$ nuqtada $= 27$ 0 *va* $A = 6 > 0$, bundan kelib chiqadiki, bu nuqtada berilgan yunksiya lokal minimumga erishadi: $z_{min} = -1$. ◄

$z=f(x,y)$ funksiyaning $\varphi(x,y) = 0$ shartda topilgan ekstremumi shartli ekstremum deb ataladi. $\varphi(x,y) = 0$ tenglama bog'lanish tenglamasi deb ataladi.

Shartli ekstremumni topishning geometrik masalasi $z=f(x,y)$ sirtning $\varphi(x,y) = 0$ silindr bilan kesishgandagi egri chiziqning ekstremal nuqtalarini topishga keltiriladi.

Agar $\varphi(x,y) = 0$ bog'lanish tenglamasidan y $y(x)$ ni topib $z=f(x,y)$ funksiyaga qo'ysak, u holda shartli ekstremumni topish masalasi bir o'zgaruvchili $z=f(x,y(x))$ funksiyaning ekstremumini topishga keltiriladi.

2 – misol. $z=x^2-y^2$ funksiyaning y $2x-6$ shartni qanoatlantiruvchi ekstremumini toping.

► y $2x-6$ ifodani berilgan funksiyaga qo'yib bir o'zgaruvchili funksiyaga ega bo'lamiz:

$$z=x^2-(2x-6)^2,\ z=-3x^2+24x-36.$$

Quyidagini topamiz $z' =-6x+24$; bundan $x=4$. Shunday qilib $z'' =-6$ 0, u holda berilgan funksiya $M_1(4,2)$ nuqtada shartli maksimumga erishadi: $z_{max}=12$. ◄

Differensiallanuvchi funksiya, chegaralangan yopiq $\overline{D}$ sohada o'zining eng katta (eng kichik) qiymatiga yoki $\overline{D}$ sohaning ichida yotuvchi statsionar nuqtada yoki shu sohaning chegarasida erishadi.

Funksiyaning yopiq $\overline{D}$ sohadagi eng katta va eng kichik qiymatlarini topish uchun, uning berilgan sohaning ichida va chegarasida yotuvchi barcha kritik nuqtalarni topish zarur, funksiyaning shu nuqtalardagi va shuningdek, chegaralarning qolgan barcha nuqtalaridagi qiymatlari hisoblanadi, so'ngra solishtirish yo'li bilan hosil qilingan sonlardagi eng katta va eng kichiklari tanlanadi.

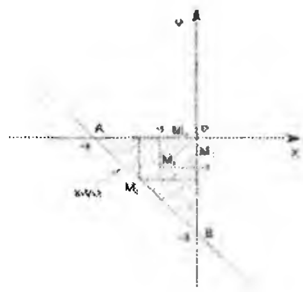
3 – misol. $z=x^2+y^2-xy+x+y$ funksiyaning $x=0$, $y=0$, $x+y=-3$ chiziqlar bilan chegaralangan sohadagi eng katta va eng kichik qiymatlarini toping.

► Quyidagi tenglamalar sistemasidan M_1 statsionar nuqtani topamiz:

$$\left.\begin{aligned} \frac{\partial z}{\partial x} &= 2x - y + 1 = 0, \\ \frac{\partial z}{\partial y} &= 2y - x + 1 = 0. \end{aligned}\right\}$$

Bundan $x=-1$, $u=-1$. $z1=z(-1,-1)=1$ bo'lgan $M_1(-1,-1)$ nuqtani hosil qilamiz.

Berilgan funksiyani chegaralarida tekshiramiz.



x=0 bo‘lgan *OB* to‘g‘ri chiziqda $z=y^2+y$ ga ega bo‘lamiz va masala bir o‘lchovli funksiyaning [−3,0] oraliqdagi eng katta va eng kichik qiymatlarini topishga keltiriladi.

Quyidagilarni topamiz:

$$z'_y = 2y + 1 = 0, x = -\frac{1}{2}, z''_{yy} = 2$$

$z_2 = z\left(0; -\frac{1}{2}\right) = -\frac{1}{4}$ bo‘ladigan $M_2(0,-\frac{1}{2})$ shartli lokal minimum nuqtani hosil qilamiz. *OV* kesmaning chekka nuqtalarida $z_3 = z(0;-3) = 6, z_4 = z(0;0) = 0$ bo‘ladi.

Xuddi shu kabi y=0 bo‘ladigan *OA* kesmada quyidagilarga ega bo‘lamiz: $z=x^2+x$, $z'_x=2x+1$, $x=-1/2$, $z''_{xx} = 2$, yani $M_3(-\frac{1}{2},0)$ – lokal minimum nuqta bo‘lib, bu nuqtada $z_5=(-\frac{1}{2},0)=-\frac{1}{4}$ bo‘ladi. A nuqtada $z_6=z(-3;0)=6$ bo‘ladi. $x+u=3$ to‘g‘ri chiziqdagi AB kesmada $y=-x-3$ ifodani z funksiyaga qo‘yib, quyidagilarni hosil qilamiz.

$$z=3x^2+9x+6,\ z'_x=6x+9=0,\ x=-3/2.$$

Bundan, $z_4 = z\left(-\frac{3}{2}, -\frac{3}{2}\right) = -\frac{3}{4}$ bo‘ladigan $M_4(-\frac{3}{2}; -\frac{3}{2})$ nuqtani topamiz. *AB* kesmaning chetki nuqtalaridagi funksiyaning qiymatlari topilgan. *z* funksiyaning barcha topilgan qiymatlarini solishtirib, quyidagi xulosaga kelamiz, *A(-3,0) va B(0,-3)* nuqtalarda o‘zining eng katta qiymatiga erishadi $z_{max} = 6$, $M_1(-1,-1)$ statsionar nuqtada esa z_{min} =-1 bo‘ladi. ◄

4 – misol. To‘g‘ri burchakli parallelepipedning to‘la sirtining yuzi *S* ga teng. Eng katta hajmga ega bo‘ladigan o‘lchamlarini toping.

► To‘g‘ri burchakli parallelepipedning hajmi $V=x\times y\times z$ ga teng, bu yerda x, y, z – parallelepipedning o‘lchamlari, uning to‘la sirtining yuzi esa $S=2(xy+xz+yz)$ ga teng.

Bundan $z=\frac{S-2xy}{2(x+y)}, V = \frac{S\times x\times y-2x^2y^2}{2(x+y)} = V(x,y)$

$V = V(x, y)$ funksiyaning ekstremumini topamiz:

$$\left.\begin{aligned} \frac{\partial V}{\partial x} &= \frac{y^2(S - 2x^2 - 4xy)}{2(x+y)^2} = 0, \\ \frac{\partial V}{\partial u} &= \frac{x^2(S - 2y^2 - 4xy)}{2(x+y)^2} = 0. \end{aligned}\right\}$$

$x > 0, y > 0$ bo'lgani uchun oxirgi sistemadan $x=y=\sqrt{\frac{S}{6}}$ ekanligi kelib chiqadi. $V=V(x,y)$ funksiyaning maksimumi bo'ladigan yagona $M(\sqrt{\frac{S}{6}}, \sqrt{\frac{S}{6}})$ nuqtaga ega bo'ldik. (yani masala yechimga ega!), shuning uchun maksimum mavjudligining yyetarli shartini tekshirishning hojati yo'q.

Quyidagini topamiz:

$$z = \frac{S - \frac{S}{3}}{4\sqrt{\frac{S}{6}}} = \frac{2 \times \frac{S}{3}}{4\sqrt{\frac{S}{6}}} = \sqrt{S/6}$$

Shunday qilib, qirrasi $\sqrt{\frac{S}{6}}$ ga teng bo'lgan kub eng katta hajmga ega bo'lar ekan. ◄

10.4 – AT.

1. Quyida berilgan funksiyalarni lokal ekstremumga tekshiring:

a) $z=x^3+3xy^2-15x-12y;$

b) $z=x^2+xy+y^2-2x-y;$

c) $z=3xy-x^2-y^2-10x+5y;$

(*Javob:* a) $z_{min} = z(2,1) = -28, z_{max} = z(-2,-1) = 28;$

b) $z_{min} = z(1,0) = -1;$ s) ekstremum nuqtalari yo'q.)

2. $z=x+2y$ funksiyaning $x^2+y^2=5$ shartni qanoatlantiruvchi ekstremumlarni toping. (*Javob:* $x=-1$, $y=-2$ bo'lganda $z_{min} = -5$; $x=1$, $y=2$ bo'lganda $z_{max} = 5$).

3. $z=x^2-y^2+4xy-6x+5$ funksiyaning, $x=0$, $y=0$, $x+y=3$ to'g'ri chiziqlar bilan chegaralangan sohadagi eng katta va eng kichik qiymatlarini toping. (*Javob:*

$z_{e.kichik} = z(3,0) = -9, z_{e.katta} = z(0,0) = 5.$)

4. $z=x^2y(4-x-y)$ funksiyaning, $x=0$, $y=0$, $x+y=6$ to‘g‘ri chiziqlar bilan chegaralangan sohadagi eng katta va eng kichik qiymatlarini toping. (*Javob:* $z_{e.kichik} = z(4,2) = -64$, $z_{e.katta} = z(2,1) = 4.$)

5. Hajmi V ga teng to‘g‘ri burchakli parallelepipedning, sirti eng kichik yuzaga ega bo‘ladigan o‘lchamlarini toping.

(*Javob:* qirrasi $\sqrt[3]{V}$ ga teng bo‘lgan kub.)

Mustaqil ish

1. $z=x^3+y^3-3x+2y$ funksiyani ekstremumga tekshiring. (*Javob*: $z_{min} = z(1,-1) = 3.$)

2. $z=x\sqrt{y}-x^2-y+6x+3$ funksiyani ekstremumga tekshiring. (*Javob*: $z_{max} = z(4,4) = 15.$)

3. $z=3x^2-x^3+3y^2+4y$ funksiyani ekstremumga tekshiring. (*Javob*: $z_{min} = z(0,-2/3) = -\frac{4}{3}.$)

10.5. 10 – BOBGA DOIR INDIVIDUAL UY TOPSHIRIQLARI

1. Ko‘rsatilgan funksiyalarning aniqlanish sohasini toping.

1.1. $z=3xy/(2x-5y)$
1.2. $z=\arcsin(x-y)$
1.3. $z=\sqrt{y^2-x^2}$
1.4. $z=\ln(4-x^2-y^2)$
1.5. $z=2/(6-x^2-y^2)$
1.6. $z=\sqrt{x^2+y^2-5}$
1.7. $z=\arccos(x+y)$
1.8. $z=3x+y/(2-x+y)$
1.9. $z=\sqrt{9-x^2-y^2}$
1.10. $z=\ln(x^2+y^2-3)$
1.11. $z=\sqrt{2x^2-y^2}$
1.12. $z=4xy/(x-3y+1)$
1.13. $z=\sqrt{xy}/(x^2+y^2)$
1.14. $z=\arcsin(x/y)$
1.15. $z=\ln(y^2-x^2)$
1.16. $z=x^3y/(3+x-y)$
1.17. $z=\arccos(x+2y)$
1.18. $z=\arcsin(2x-y)$
1.19. $z=\ln(9-x^2-y^2)$
1.20. $z=\sqrt{3-x^2-y^2}$
1.21. $z=1/\sqrt{x^2+y^2-5}$
1.22. $z=4x+y/(2x-5y)$
1.23. $z=\sqrt{3x-2}/(x^2+y^2+4)$
1.24. $z=5/(4-x^2-y^2)$
1.25. $z=\ln(2x-y)$
1.26. $z=7x^3y/(x-4y)$
1.27. $z=\sqrt{1-x-y}$
1.28. $z=e^{\sqrt{x^2+y^2-1}}$
1.29. $z=1/(x^2+y^2-6)$
1.30. $z=4xy/(x^2-y^2)$

2. Quyidagi funksiyalarning xususiy hosilalarini va xususiy differensiallarini toping.

2.1. $z = ln(u^2 - e - x)$

2.2. $z = arcsin\sqrt{xy}$

2.3. $z = arctg\,(x^2 + y^2)$

2.4. $z = cos(x^3 - 2xy)$

2.5. $z = sin\,\sqrt{y/x^3}$

2.6. $z = tg(x^3 + y^2)$

2.7. $z = ctg\,\sqrt{xy^3}$

2.8. $z = e^{-x^2+y^2}$

2.9. $z = ln(3x^2 - y^4)$

2.10. $z = arcos(y/x)$

2.11. $z = arcctg(xy2)$

2.12. $z = cos\,\sqrt{x^2 + y^2}$

2.13. $z = sin\,\sqrt{x - y^3}$

2.14. $z = tg(x^3 y^4)$

2.15. $z = ctg(3x - 2x)$

2.16. $z = e^{2x^2 - y^5}$

2.17. $z = ln\,(\sqrt{xy - 1}\,)$

2.18. $z = arcsin(2x^3 y)$

2.19. $z = arctg\,(x^2/y^3)$

2.20. $z = cos\,(x - \sqrt{xy^3}\,)$

2.21. $z = sin\frac{x+y}{x-y}$

2.22. $z = tg\frac{2x+y^2}{x}$

2.23. $z = ctg\,\sqrt{\frac{x}{x-y}}$

2.24. $z = e^{-\sqrt{x^2+y^2}}$

2.25. $z = ln(3x^2 - y^2)$

2.26. $z = arccos\,(x - y^2)$

2.27. $z = arcctg\frac{x^3}{y}$

2.28. $z = cos\,\frac{x-y}{x^2+y^2}$

2.29. $z = sin\,\sqrt{\frac{y}{x+y}}$

2.30. $z = e^{-(x^3+y^3)}$

3. $f(x,y,z)$ berilgan funksiyaning $M_0(x_0,y_0,z_0)$ nuqtada $f'_x\,(M_0)$, $f'_y\,(M_0)$, $f'_z\,(M_0)$ xususiy hosilalarining qiymatini verguldan keyin ikki xonagacha aniqlikda hisoblang.

3.1. $f(x,y,z) = Z/\sqrt{x^2 + y^2}$, $M_0\,(0,-1,1)$. (*Javob*: $f'_x\,(0,-1,1) = 0$, $f'_u\,(0,-1,1) = 1$, $f'_z(0,-1,1) = 1$.)

3.2. $f(x,v,z) = ln(x + \frac{y}{2z})$, $M_0\,(1,2,1)$. (*Javob*: $f'_x\,(1,2,1) = 0.5$,

f_y' (1,2,1)=0.25, f_z' (1,2,1)=-0,5.)

3.3. $f(x,y,z)=(\sin x)^{yz}$, M_0 $(\frac{\pi}{6}, 1,2)$. (*Javob*: $f_x'(\frac{\pi}{6}, 1,2)=0,87$, f_y' $(\frac{\pi}{6}, 1,2)=-0,35$, $f_z'(\frac{\pi}{6}, 1,2)=-0,17$.)

3.4. $f(x,y,z)=\ln(x^3+2y^3-z^3)$, M_0 $(2,1,0)$.

(*Javob*: f_x' (2,1,0)=1,2, f_y' (2,1,0)=0,6, f_z' (2,1,0)=0.)

3.5. $f(x,y,z)=x/\sqrt{y^2+z_2}$. $M_0(1,0,1)$.

(*Javob*: f_x' (1,0,1)=1, f_y' (1,0,1)=0, f_z'(1,0,1)=-1.)

3.6. $f(x,y,z)=\ln\cos(x^2+y^2+z)$, M_0 $(0,0,\frac{\pi}{4})$.

(*Javob*: f_x' $(0,0,\frac{\pi}{4})=0$, f_y' $(0,0,\frac{\pi}{4})=0$, f_z' $(0,0,\frac{\pi}{4})=-1$)

3.7. $f(x,y,z)=27\sqrt[3]{x+y^2+z^3}$, M_0 $(3,4,2)$.

(*Javob*: f_x' (3,4,2)=1, f_y' (3,4,2)=8, f_z' (3,4,2)=12.)

3.8. $f(x,y,z)=\mathrm{arctg}(xy^2+z)$, M_0 $(2,1,0)$.

(*Javob*: f_x' (2,1,0)=0,2, f_y' (2,1,0)=0,8, f_z' (2,1,0)=0,2.)

3.9. $f(x,y,z)=\arcsin(x^2/y-z)$, $M_0(2,5,0)$.

(*Javob*: f_x'(2,5,0)=1,33, f_y'(2,5,0)=-0,27, f_z'(2,5,0)=-1,67.)

3.10. $f(x,y,z)=\sqrt{z}\sin(y/x)$, M_0 $(2,0,4)$.

(*Javob*: f_x'(2,0,4)=0, f_y'(2,0,4)=1, f_z'(2,0,4)=0.)

3.11. $f(x,y,z)=y/\sqrt{x^2+z^2}$, M_0 $(-1,1,0)$.

(*Javob*: f_x'(-1,1,0)=1, f_y'(-1,1,0)=1, f_z'(-1,1,0)=0.)

3.12. $f(x,y,z)=\mathrm{arctg}(xz/y^2)$, M_0 $(2,1,1)$.

(*Javob*: f_x'(2,1,1)=0,2, f_y'(2,1,1)=-0,8, f_z'(2,1,1)=0,4.)

3.13. $f(x,y,z)=\ln\sin(x-2y+z/4)$. M_0 $(1,1/2,\pi)$.

(*Javob*: f_x'(1,1/2,π)=1, f_y'(1,1/2,π)=-2, f_z'(1,1/2,π)=0,25.)

3.14. $f(x,y,z)=\frac{y}{x}+\frac{z}{y}-\frac{x}{z}$. M_0 $(1,1,2)$.

(*Javob*: f_x'(1,1,2)=-1,5, f_y'(1,1,2)=-1, f_z'(1,1,2)=1,25.)

3.15. $f(x,y,z)=1/\sqrt{x^2+y^2-z^2}$, M_0 $(1,2,2)$.

(*Javob*: f_x'(1,2,2)=-1, f_y'(1,2,2)=-2, f_z'(1,2,2)=2.)

3.16. $f(x,y,z)=\ln(x+y^2)-\sqrt{x^2z^2}$, M_0 $(5,2,3)$.

(*Javob*: f_x'(5,2,3)=-1,14, f_y'(5,2,3)=0,44, f_z'(5,2,3)=0,75.)

3.17. $f(x,y,z)=\sqrt{z}x^y$, M_0 $(1,2,4)$.

(*Javob*: $f'_x(1,2,4)=4$, $f'_y(1,2,4)=0$, $f'_z(1,2,4)=0,25$).

3.18. $f(x,y,z)=z/\sqrt{x^2+y^2}$, $M_0\,(\sqrt{2}, \sqrt{2}, \sqrt{2})$.

(*Javob*: $f'_x(\sqrt{2}, \sqrt{2}, \sqrt{2})=0,25$, $f'_y(\sqrt{2}, \sqrt{2}, \sqrt{2})=0.25$, $f'_z(\sqrt{2}, \sqrt{2}, \sqrt{2})=-0,5$.

3.19. $f(x,y,z)=\ln(x^3+\sqrt[3]{y}-z)$, $M_0\,(2,1,8)$.

(*Javob*: $f'_x\,(2,1,8)=12$, $f'_y\,(2,1,8)=0,33$, $f'_z\,(2,1,8)=-1$.)

3.20. $f(x,y,z)=z/(x^4+y^2)$, $M_0\,(2,3,25)$.

(*Javob*: $f'_x(2,3,25)=-1,28$, $f'_y(2,3,25)=-0,24$, $f'_z\,(2,3,25)=0,04$.)

3.21. $f(x,y,z)=8\sqrt[5]{x^3+y^2+z}$, $M_0\,(3,2,1)$.

(*Javob*: $f'_x(3,2,1)=2,7$, $f'_y(3,2,1)=0,4$, $f'_z(3,2,1)=0,1$.)

3.22. $f(x,y,z)=\ln(\sqrt[5]{x}+\sqrt[4]{y}-z)$, $M_0(1,1,1)$.

(*Javob*: $f'_x(1,1,1)=0,2$, $f'_y(1,1,1)=0,25$, $f'_z(1,1,1)=-1$.)

3.23. $f(x,y,z)=-2x/\sqrt{y^2+z^2}$, $M_0\,(3,0,1)$.

(*Javob*: $f'_x(3,0,1)=-2$, $f'_y(3,0,1)=0$, $f'_z(3,0,1)=6$.)

3.24. $f(x,y,z)=ze^{-(x^2+y^2)/2}$, $M_0\,(0,0,1)$.

(*Javob*: $f'_x(0,0,1)=0$, $f'_y(0,0,1)=0$, $f'_z(0,0,1)=1$.)

3.25. $f(x,y,z)=\frac{\sin(x-y)}{z}$, $M_0\,(\frac{\pi}{2}, \frac{\pi}{2}, \sqrt{3})$.

(*Javob*: $f'_x(\frac{\pi}{2}, \frac{\pi}{2}, \sqrt{3})=0,5$, $f'_y(\frac{\pi}{2}, \frac{\pi}{2}, \sqrt{3})=-0,5$, $f'_z(\frac{\pi}{2}, \frac{\pi}{2}, \sqrt{3})=-0,17$.)

3.26. $f(x,y,z)=\sqrt{z}\,\ln(\sqrt{x}+\sqrt{y})$, $M_0\,(4,1,4)$.

(*Javob*: $f'_x(4,1,4)=0,17$, $f'_y(4,1,4)=0,33$, $f'_z(4,1,4)=0,27$.)

3.27. $f(x,y,z)=xz/(x-y)$, $M_0(3,1,1)$.

(*Javob*: $f'_x(3,1,1)=-0,25$, $f'_y(3,1,1)=0,75$, $f'_z(3,1,1)=1,5$.)

3.28. $f(x,y,z)=\sqrt{x^2+y^2-2xy\cos z}$, $M_0\,(3,4,\frac{\pi}{2})$.

(*Javob*: $f'_x(3,4,\frac{\pi}{2})=0,6$, $f'_y(3,4,\frac{\pi}{2})=0,8$, $f'_z(3,4,\frac{\pi}{2})=2,4$.)

3.29. $f(x,y,z)=ze^{-xy}$, $M_0\,(0,1,1)$.

(*Javob*: $f'_x(0,1,1)=-1$, $f'_y(0,1,1)=0$, $f'_z(0,1,1)=1$.)

3.30. $f(x,y,z)=\arcsin(x\sqrt{y}-yz^2)$, $M_0\,(0,4,1)$.

(*Javob*: $f'_x(0,4,1)=2$, $f'_y(0,4,1)=-1$, $f'_z(0,4,1)=-8$.)

4. Ko'rsatilgan funksiyalarning to'la differensialini toping.

4.1. $z=2x^3y-4xy^5$ **4.2.** $z=x^2y\sin x-3y$

4.3. $z=\mathrm{arctg}\,x+\sqrt{y}$ **4.4.** $z=\arcsin(xy)-3xy^2$

4.5. $z=5xy^4+2x^2y^7$ **4.6.** $z=\cos(x^2-y^2)+x^3$

4.7. $z=\ln(3x^2-2y^2)$ **4.8.** $z=5xy^2-3x^3y^4$

4.9. $z=\arcsin(x+y)$ **4.10.** $z=\mathrm{arctg}\,(2x-y)$

4.11. $z=7x^3y-\sqrt{xy}$ **4.12.** $z=\sqrt{x^2+y^2-2xy}$

4.13. $z=e^{x+y-4}$ **4.14.** $z=\cos(3x+y)-x^2$

4.15. $z=\mathrm{tg}((x+y)/(x-y))$ **4.16.** $z=\mathrm{ctg}(y/x)$.

4.17. $z=xy^4-3x^2y+1$ **4.18.** $z=\ln(x+xy-y^2)$

4.19. $z=2x^2y^2+x^3-y3$ **4.20.** $z=\sqrt{3x^2-2y^2+5}$

4.21. $z=\arcsin((x+y)/x)$ **4.22.** $z=\mathrm{arcctg}\,(x-y)$

4.23. $z=\sqrt{3x^2-y^2+x}$ **4.24.** $z=y^2-3xy-x^4$

4.25. $z=\arccos(x+y)$ **4.26.** $z=\ln(y^2-x^2+3)$

4.27. $z=2-x^3-y^3+5x$ **4.28.** $z=7x-x^3y^2+y^4$

4.29. $z=e^y-x$ **4.30.** $z=\mathrm{arctg}\,(2x-y)$

5. $u=u(x,y)$, bu yerda $x=x(t)$, $y=y(t)$ murakkab funksiyaning $t=t_0$ nuqtadagi hosilasining qiymatini verguldan keyin 2 xonagacha aniqlikda hisoblang.

5.1. $u=e^x-2y$, $x=\sin t$, $y=t^3$, $t_0=0$. (Javob: 1.)

5.2. $u=\ln(e^x+e^{-y})$, $x=t^2$, $y=t^3$, $t_0=-1$. (Javob: -2,5.)

5.3. $u=y^x$, $x=\ln(t-1)$, $y=e^t/2$, $t_0=2$. (Javob: 1.)

5.4. $u=e^y-2x+2$, $x=\sin t$, $y=\cos t$, $t_0=\frac{\pi}{2}$. (Javob:-1.)

5.5. $u=x^2e^y$, $x=\cos t$, $y=\sin t$, $t_0=\pi$. (Javob: -1.)

5.6. $u=\ln(e^x+e^y)$, $x=t^2$, $y=t^3$, $t_0=1$. (Javob 2,5.)

5.7. $u=x^y$, $x=e^t$, $y=\ln t$, $t_0=1$. (Javob:1.)

5.8. $u=e^y-2x$, $x=\sin t$, $y=t^3$, $t_0=0$. (Javob:-2.)

5.9. $u = x^2e^{-y}$, $x=\sin t$, $y=\sin^2 t$, $t_0=\frac{\pi}{2}$. (Javob:0.)

5.10. $u = \ln(e^{-x}+e^{y})$, $x=t^2$, $y=t^3$, $t_0=-1$. (Javob:2,5.)

5.11. $u = e^{y}-2x-1$, $x=\cos t$, $y=\sin t$, $t_0=\frac{\pi}{2}$. (Javob: 2.)

5.12. $u = \arcsin (x/y)$, $x=\sin t$, $y=\cos t$, $t_0=\pi$. (Javob: 1.)

5.13. $u = \arccos (2x/y)$, $x=\sin t$, $y=\cos t$, $t_0=\pi$. (Javob: -2.)

5.14. $u = x^2/(y+1)$, $x=1-2t$, $y=\operatorname{arctg} t$, $t_0=0$. (Javob: -5.)

5.15. $u = x/y$, $x=e^t$, $y=2-e^{2t}$, $t_0=0$. (Javob: 3.)

5.16. $u = \ln(e^{x}+e^{-2y})$, $x=t^2$, $y=\frac{1}{3}t^3$, $t_0=1$. (Javob: -2.)

5.17. $u = \sqrt{x+y^2+3}$, $x=\ln t$, $y=t^2$, $t_0=1$. (Javob: 1,25.)

5.18. $u = \arcsin x^2/y$, $x=\sin t$, $y=\cos t$, $t_0=\pi$. (Javob: 0.)

5.19. $u = y^2/x$, $x=1-2t$, $y=1+\operatorname{arctg} t$, $t_0=0$. (Javob: 4.)

5.20. $u = \frac{y}{x} - \frac{x}{y}$, $x=\sin t$, $y=\cos t$, $t_0=\frac{\pi}{4}$. (Javob: -4.)

5.21. $u = \sqrt{x^2+y+3}$, $x=\ln t$, $y=t^2$, $t_0=1$. (Javob: 0,5.)

5.22. $u = \arcsin \frac{x}{2y}$, $x=\sin t$, $y=\cos t$, $t_0=\pi$. (Javob: 0,5.)

5.23. $u = \frac{x}{y} - \frac{y}{x}$, $x=\sin 2t$, $y=\operatorname{tg}^2 t$, $t_0=\frac{\pi}{4}$. (Javob: -8.)

5.24. $u = \sqrt{x+y+3}$, $x=\ln t$, $y=t^2$, $t_0=1$. (Javob: 0,75.)

5.25. $u = y/x$, $x=e^t$, $y=1-e^{2t}$ t, $t_0=0$. (Javob: -2.)

5.26. $u = \arcsin 2x/y$, $x=\sin t$, $y=\cos t$, $t_0=\pi$. (Javob: 2.)

5.27. $u = \ln(e^{2x}+e^{y})$, $x=t^2$, $y=t^4$, $t_0=1$. (Javob: 4.)

5.28. $u = \operatorname{arctg} (x+y)$, $x=t^2+2$, $y=4-t^2$, $t_0=1$. (Javob: 0.)

5.29. $u = \sqrt{x^2+y^2+3}$, $x=\ln t$, $y=t^3$, $t_0=1$. (Javob: 1,5.)

5.30. $u = \operatorname{arctg} (xy)$, $x=t+3$, $y=e^t$, $t_0=0$. (Javob: 0,4.)

6. Oshkormas funksiya ko'rinishida berilgan $z(x,y)$ funksiyaning xususiy hosilalarining M_0 (x_0, y_0, z_0) nuqtadagi qiymatlarini verguldan keyin ikki xonagacha aniqlikda hisoblang.

6.1. $x^3+y^3+z^3-3xyz=4$, $M_0(2,1,1)$.

(Javob: $z'_x(2,1,1)=3$, $z'_u(2,1,1)=-1$.)

6.2. $x^2+y^2+z^2-xy=2$, $M_0(-1,0,1)$.

(Javob: $z'_x(-1,0,1)=-1$, $z'_u(-1,0,1)=0,5$.)

6.3. $3x-2y+z=xz+5$, $M_0(2,1,-1)$.

(Javob: $z'_x(2,1,-1)=4$, $z'_u(2,1,-1)=-2$.)

6.4. $e^z+x+2y+z=4$, $M_0(1,1,0)$.
(*Javob:* $z'_x(1,1,0)=-0,5$, $z'_u\ (1,1,0)=-1$.)
6.5. $x^2+y^2+z^2-z-4=0$, $M_0(1,1,-1)$.
(*Javob:* $z'_x(1,1,-1)=0,67$, $z'_u\ (1,1,-1)=0,67$.)
6.6. $z^3+3xyz+3y=7$, $M_0(1,1,1)$.
(*Javob:* $z'_x(1,1,1)=-0,5$, $z'_u\ (1,1,1)=-0,5$.)
6.7. $\cos^2x+\cos^2y+\cos^2z=\frac{3}{2}$, $M_0(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4})$.
(*Javob:* $z'_x(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4})=-1$, $z'_y\ (\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4})=1$.)
6.8. $e^{z-1}-1=\cos x\cos y+1$, $M_0(0, \frac{\pi}{2},1)$.
(*Javob:* $z'_x(0, \frac{\pi}{2},1)=0$, $z'_y\ (0, \frac{\pi}{2},1)=-1$.)
6.9. $x^2+y^2+z^2-6x=0$, $M_0(1,2,1)$.
(*Javob:* $z'_x(1,2,1)=2$, $z'_y\ (1,2,1)=-2$.)
6.10. $xy=z^2-1$, $M_0(0,1,-1)$.
(*Javob:* $z'_x(0,1,-1)=-0,5$, $z'_y\ (0,1,-1)=0$.)
6.11. $x^2+2y^2+3z^2-yz+y=2$, $M_0(1,1,1)$.
(*Javob:* $z'_x(1,1,1)=-0,4$, $z'_y\ (1.1,1)=0,8$.)
6.12. $x^2+y^2+z^2+2xz=5$, $M_0(0,2,1)$.
(*Javob:* $z'_x(0,2,1)=-1$, $z'_y\ (0,2,1)=-2$.)
6.13. $x\cos y+y\cos z+z\cos x=\frac{\pi}{2}$, $M_0(0, \frac{\pi}{2}, \pi)$
(*Javob:* $z'_x(0, \frac{\pi}{2}, \pi)=0$, $z'_y(0, \frac{\pi}{2}, \pi)=1$.)
6.14. $3x^2y^2+2xyz^2-2x^3z+4y^3z=4$, $M_0(2,1,2)$.
(*Javob:* $z'_x(2,1,2)=7$, $z'_y\ (2,1,2)=-16$.)
6.15. $x^2-2y^2+z^2-4x+2z+2=0$, $M_0(1,1,1)$.
(*Javob:* $z'_x(1,1,1)=0,5$, $z'_y\ (1,1,1)=1$.)
6.16. $x+y+z+2=xyz$, $M_0(2,-1,-1)$.
(*Javob:* $z'_x(2,-1,-1)=0$, $z'_y\ (2,-1,-1)=-1$.)
6.17. $x^2+y^2+z^2-2xz=2$, $M_0(0,1,-1)$.
(*Javob:* $z'_x(0,1,-1)=1$, $z'_y\ (0,1,-1)=1$.)
6.18. $e^z-xyz-x+1=0$, $M_0(2,1,0)$.
(*Javob:* $z'_x(2,1,0)=-1$, $z'_y\ (2,1,0))=0$.)
6.19. $x^3+2y^3+z^3-3xyz-2y-15=0$, $M_0(1,-1,2)$.
(*Javob:* $z'_x(1,-1,2)=-0,6$, $z'_y\ (1,-1,2)=0,13$.)

6.20. $x^2-2xy-3y^2+6x-2y+z^2-8z+20=0$, $M_0(0,-2,2)$.
(Javob: $z'_x(0,-2,2)=2,5$, $z'_y\ (0,-2,2)=2,5$.*)*
6.21. $x^2+y^2+z^2=y-z+3$, $M_0(1,2,0)$.
(Javob: $z'_x(1,2,0)=-2$, $z'_y\ (1,2,0)=-3$.*)*
6.22. $x^2+y^2+z^2+2xy-yz-4x-3y-z=0$, $M_0(1,-1,1)$.
(Javob: $z'_x(1.-1,1)=2$, $z'_y\ (1,-1,1)=2$.*)*
6.23. $x^2-y^2-z^2+6z+2x-4y+12=0$, $M_0(0,1,-1)$.
(Javob: $z'_x(0,1,-1)=-0,25$, $z'_y\ (0,1,-1)=0,75$.*)*
6.24. $\sqrt{x^2+y^2}+z^2-3z=3$, $M_0(4,3,1)$.
(Javob: $z'_x(4,3,1)=0,8$, $z'_y\ (4,3,1)=0,6$.*)*
6.25. $x^2+2y^2+3z^2=59$, $M_0(3,1,4)$.
(Javob: $z'_x(3,1,4)=-0,25$, $z'_y\ (3,1,4)=-0,17$.*)*
6.26. $x^2+y^2+z^2-2xy-2xz-2yz=17$, $M_0(-2,-1,2)$.
(Javob: $z'_x(-2,-1,2)=0,6$, $z'_y\ (-2,-1,2)=0,2$.*)*
6.27. $x^3+3xyz-z^3=27$, $M_0(3,1,3)$.
(Javob: $z'_x(3,1,3)=2$, $z'_y\ (3,1,3)=1,5$.*)*
6.28. $\ln z=x+2y-z+\ln 3$, $M_0(1,1,3)$.
(Javob: $z'_x(1,1,3)=3/4$, $z'_u\ (1,1,3)=3/2$.*)*
6.29. $2x^2+2y^2+z^2-8xz-z+6=0$, $M_0(2,1,1)$.
(Javob: $z'_x(2,1,1)=0$, $z'_y\ (2,1,1)=0,27$.*)*
6.30. $z^2=xy-z+x^2-4$, $M_0(2,1,1)$.
(Javob: $z'_x(2,1,1)\quad 1,67$, $z'_y\ (2,1,1)\quad 0,67$.*)*

Namunaviy variantning yechimi

1. $z=\ln(x^2-3y+6)$ funksiyaning aniqlanish sohasini toping.

►Logorifmik funksiya argumentning faqat musbat qiymatlariga aniqlangan, shuning uchun $x^2-3y+6\quad 0$ yoki $3y\quad x^2+6$. Demak, sohaning chegarasi $x^2-3y+6\quad 0$ yoki $x^2=3y-6$ chiziqdan iborat parabola bo'ladi.

Berilgan funksiyaning aniqlanish sohasi parabolaning tashqi nuqtalaridan iborat bo'ladi. (104 – rasm.)

2. $z=e^{-\sqrt[3]{x^2+5y^2}}$ funksiyaning xususiy hosilalari va xususiy differensiallarini toping.

► Bir o'zgaruvchili murakkab funksiyani differensiallash formulalaridan foydalanib, avval xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x} = e^{-\sqrt[3]{x^2+5y^2}}\left(-\frac{1}{3}(x^2+5y^2)^{-\frac{2}{3}} \cdot 2x\right) = -\frac{2x}{3} e^{-\sqrt[3]{x^2+5y^2}} \cdot \frac{1}{\sqrt[3]{(x^2+5y^2)^2}}$$

$$\frac{\partial z}{\partial y} = e^{-\sqrt[3]{x^2+5y^2}}\left(-\frac{1}{3}(x^2+5y^2)^{-\frac{2}{3}} \cdot 10y\right) = -\frac{10y}{3} e^{-\sqrt[3]{x^2+5y^2}} \cdot \frac{1}{\sqrt[3]{(x^2+5y^2)^2}}$$

Endi xususiy differensiallarni topamiz:

$$d_x z = \frac{\partial z}{\partial x} dx = -\frac{2x}{3} e^{-\sqrt[3]{x^2+5y^2}} \cdot \frac{1}{\sqrt[3]{(x^2+5y^2)^2}} dx,$$

$$d_y z = \frac{\partial z}{\partial y} dy = -\frac{10y}{3} e^{-\sqrt[3]{x^2+5y^2}} \cdot \frac{1}{\sqrt[3]{(x^2+5y^2)^2}} dy, \blacktriangleleft$$

3. $f(x,y,z)=\sqrt{xy}\cos z$ funksiyaning $M_0(1.1, \frac{\pi}{3})$ nuqtadagi f'_x (M_0), f'_u (M_0), f'_z (M_0), xususiy hosilalarining qiymatlarini verguldan keyin ikki xona aniqlikda hisoblang.

► Berilgan funksiyaning xususiy hosilalarini topamiz, so'ngra ularning $M_0(1,1, \frac{\pi}{3})$ nuqtadagi qiymatlarini hisoblaymiz:

$$f'_x(x,y,z) = \frac{y}{2\sqrt{xy}} \cos z, f'_x\left(1,1,\frac{\pi}{3}\right) = 0{,}25,$$

$$f'_y(x,y,z) = \frac{y}{2\sqrt{xy}} \cos z, f'_y\left(1,1,\frac{\pi}{3}\right) = 0{,}25,$$

$$f'_z(x,y,z) = -\sqrt{xy}\sin z, \quad f'_z\left(1,1,\frac{\pi}{3}\right) = -0{,}86 \blacktriangleleft$$

4. $z=arctg\sqrt{\frac{x}{y}}$ funksiyaning to'la differensialini toping.

► Berilgan funksiyaning xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x} = \frac{1}{1+\frac{x}{y}} \cdot \frac{1}{2\sqrt{\frac{x}{y}}} \cdot \frac{1}{y} = \frac{y}{x+y} \cdot \frac{\sqrt{y}}{2\sqrt{x}} \cdot \frac{1}{y} = \frac{\sqrt{\frac{y}{x}}}{2(x+y)},$$

$$\frac{\partial z}{\partial y} = \frac{1}{1+\frac{x}{y}} \cdot \frac{1}{2\sqrt{\frac{x}{y}}} \cdot \left(-\frac{x}{y^2}\right) = \frac{y}{x+y} \cdot \frac{\sqrt{y}}{2\sqrt{x}} \cdot \left(-\frac{x}{y^2}\right) = -\frac{\sqrt{\frac{x}{y}}}{2(x+y)},$$

(10.1) formulaga asosan, quyidagiga ega bo'lamiz

$$\partial z = \frac{\sqrt{\frac{y}{x}}}{2(x+y)} dx - \frac{\sqrt{\frac{x}{y}}}{2(x+y)} dy \blacktriangleleft$$

5. $z = arccos\frac{x^2}{y}$, bu yerda $x=1+lnt$, $y=-2e^{-t^2+1}$, murakkab funksiyaning $t_0=1$ bo‘lgandagi qiymatini verguldan keyin ikki xona aniqlikda hisoblang.

► (10.4) formulaga asosan quyidagiga ega bo‘lamiz:

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=-\frac{1}{\sqrt{1-\frac{x^4}{y^2}}}\cdot\frac{2x}{y}\cdot\frac{1}{z}-$$

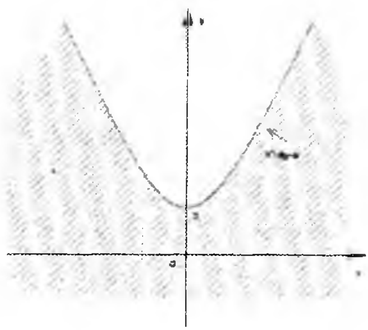
$$-\frac{1}{\sqrt{1-\frac{x^4}{y^2}}}\cdot\left(-\frac{x^2}{y^2}\right)\cdot\left(-2e^{-t^2+1}\right)(-2t).$$

$t_0=1$ bo‘lganda, $x=1$, $u=-2$ bo‘ladi.

Bundan,

$$\left.\frac{dz}{dt}\right|_{t=1}=\frac{4}{\sqrt{3}}$$ ◄

6. $4x^3-3y^3+2xy\,z-4x\,z=3$ tenglama bilan oshkormas ravishda berilgan $z(x,y)$ funksiyaning xususiy hosilalarining $M_0(0,1,-1)$ nuqtadagi qiymatlarini verguldan keyin ikki xonagacha aniqlikda hisoblang.



► Shartga asosan

$F(x,y,z)=5x^3-3y^3+2xyz-4xz-3$,

Shuning uchun

$$F_x'=12x^2+2yz-4z,$$

$$F_y'=-9y^2+2xz,$$

$$F_z'=2xy-4x+2z.$$

(10.7.) formulaga asosan,

$$\frac{\partial z}{\partial x}=-\frac{F_x'}{F_z'}=-\frac{12x^2+2yz-4z}{2xy-4x+2z},$$

$$\frac{\partial z}{\partial y}=-\frac{F_y'}{F_z'}=-\frac{-9y^2+2xz}{2xy-4x+2z}.$$

$\frac{\partial z}{\partial x}$ *va* $\frac{\partial z}{\partial y}$ larning *$M_0(0,1,-1)$* nuqtadagi qiymatlarini hisoblaymiz:

$$\frac{\partial z(0,1,-1)}{\partial x} = 1, \frac{\partial z(0,1,-1)}{\partial u} = -4{,}5 \blacktriangleleft$$

10.2 Individual uy topshiriqlari

1. Berilgan S sirtga $M_0(x_0,y_0,z_0)$ nuqtada o‘tkazilgan urinma tekislik va normal tenglamasini toping.

1.1. *S:* $x^2+y^2+z^2+6z-4x+8=0$, $M_0\ (2,1,-1)$.
1.2. *S:* $x^2+z^2-4y^2=-2xy$, $M_0\ (-2,1,2)$.
1.3. *S:* $x^2+y^2+z^2-xy+3z=7$, $M_0\ (1,2,1)$.
1.4. *S:* $x^2+y^2+z^2+6y+4x=8$, $M_0\ (-1,1,2)$.
1.5. *S:* $2x^2-y^2+z^2-4z+y=13$, $M_0\ (2,1,-1)$.
1.6. *S:* $x^2+y^2+z^2-6y+4z+4=0$, $M_0\ (2,1,-1)$.
1.7. *S:* $x^2+z^2-5yz+3y=46$, $M_0\ (1,2,-3)$.
1.8. *S:* $x^2+y^2-xz-yz=0$, $M_0\ (0,2,2)$.
1.9. *S:* $x^2+y^2+2yz-z^2+y-2z=2$, $M_0\ (1,1,1)$.
1.10. *S:* $y^2-z^2+x^2-2xz+2x=z$, $M_0\ (1,1,1)$.
1.11. *S:* $z=x^2+y^2-2xy+2x-y$, $M_0\ (-1,-1,-1)$.
1.12. *S:* $z=y^2-x^2+2xy-3y$, $M_0\ (1,-1,1)$.
1.13. *S:* $z=x^2-y^2-2xy-x-2v$, $M_0\ (-1,1,1)$.
1.14. *S:* $x^2-y^2+z^2+xz-4y=13$, $M_0\ (3,1,2)$.
1.15. $4y^2-z^2+4xy-xz+3z=9$, $M_0\ (1,-2,1)$.
1.16. *S:* $z=x^2+y^2-3xy-x+y+2$, $M_0\ (2,1,0)$.
1.17. *S:* $2x^2-y^2+2z^2+xy+xz=3$, $M_0\ (1,2,1)$.
1.18. *S:* $x^2-y^2+z^2-4x+2y=14$, $M_0\ (3,1,4)$.
1.19. *S:* $x^2+y^2-z^2+xz+4y=4$, $M_0\ (1,1,2)$.
1.20. *S:* $x^2-y^2-z^2+xz+4x=-5$, $M_0\ (-2,1,0)$.
1.21. *S:* $x^2+y^2-xz+yz-3x=11$, $M_0\ (1,4,-1)$.
1.22. *S:* $x^2+2y^2+z^2-4xz=8$, $M_0\ (0,2,0)$.
1.23. *S:* $x^2-y^2-2z^2-2y=0$, $M_0(-1,-1,1)$.
1.24. *S:* $x^2+y^2-3z^2+xy=-2z$, $M_0(1,0,1)$.
1.25. *S:* $2x^2-y^2+z^2-6x+2y+6=0$, $M_0(1,-1,1)$.
1.26. *S:* $x^2+y^2-z^2+6xy-z=8$, $M_0(1,1,0)$.
1.27. *S:* $z=2x^2-3y^2+4x-2y+10$, $M_0\ (-1,1,3)$.
1.28. *S:* $z=x^2+y^2-4xy+3x-15$, $M_0\ (-1,3,4)$.

1.29. *S:* $z=x^2+2y^2+4xy-5y-10$, M_0 (-7,1,8).

1.30. *S:* $z=2x^2-3y^2+xy+3x+1$, M_0 (1,-1,2).

2. Ko'rsatilgan funksiyalarning 2-tartibli xususiy hosilalarini toping. $z''_{xu} == z''_{ux}$ ekanligiga ishonch hosil qiling.

2.1. $z=e^{x^2-y^2}$.

2.2. $z=ctg(x+y)$.

2.3. $z=tg(x/y)$.

2.4. $z=cos(xy2)$.

2.5. $z=sin(x^2-y)$.

2.6. $z=arctg(x+y)$.

2.7. $z=arcsin(x-y)$.

2.8. $z=arccos(2x+y)$.

2.9. $z=arctg(x-3y)$.

2.10. $z=ln(3x^2-2y^2)$.

2.11. $z=e^{2x^2+y^2}$.

2.12. $z=ctg(y/x)$.

2.13. $z=tg\sqrt{xy}$.

2.14. $z=cos(x^2y^2-5)$.

2.15. $z=sin\sqrt{x^2y}$.

2.16. $z=arcsin(x-2y)$.

2.17. $z=arccos(4x-y)$.

2.18. $z=arctg(5x+2y)$.

2.19. $z=arctg(2x-y)$.

2.20. $z=ln(4x^2-5y^3)$.

2.21. $z=e^{\sqrt{x+y}}$.

2.22. $z=arcsin(4x+y)$.

2.23. $z=arccos(x-5y)$.

2.24. $z=sin\sqrt{xy}$.

2.25. $z=cos(3x^2-y^3)$.

2.26. $z=arctg(3x+2y)$.

2.27. $z=ln(5x^2-3y^4)$.

2.28. $z=arctg(x-4y)$.

2.29. $z=ln(3xy-4)$.

2.30. $z=tg(xy^2)$.

3. Berilgan u funksiyaning ko'rsatilgan tenglamani qanoatlantirishini tekshiring.

3.1. $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0, u=\frac{y}{x}$.

3.2. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3(x^3-y^3)$, $u=ln\frac{y}{x}+(x^3-y^3)$.

3.3. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}, u=ln\,(x^2+(y+1)^2)$.

3.4. $y\frac{\partial^2 u}{\partial x\partial y}=(1+y\,ln\,x)\frac{\partial u}{\partial x}, y=x^y$.

3.5. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u, u=\frac{xy}{x+y}$.

3.6. $x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}=0, u=e^{xy}$.

3.7. $a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}, u=sin^2(x-ay)$.

3.8. $x^2\frac{\partial^2 u}{\partial x^2}-y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=y\sqrt{\frac{y}{x}}$.

3.9. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0,\ u=\frac{1}{\sqrt{x^2+y^2+z^2}}$.

3.10. $a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2},\ u=e^{-\cos(x+2y)}$.

3.11. $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0,\ u=(x-y)(y-z)(z-x)$.

3.12. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u.\ u=x\ln\frac{y}{x}$.

3.13. $y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}=0,\ u=\ln(x^2+y^2)$.

3.14. $x^2\frac{\partial u}{\partial x}-xy\frac{\partial u}{\partial y}+y^2=0,\ u=\frac{y^2}{3x}+\arcsin(xy)$.

3.15. $x^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+2xy,\ u=0,\ u=e^{xy}$.

3.16. $\frac{\partial^2 u}{\partial x\partial y}=0,\ u=\operatorname{arctg}\frac{x+y}{1-xy}$.

3.17. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=\ln(x^2+y^2+2x+1)$.

3.18. $x\frac{\partial y}{\partial x}+y\frac{\partial u}{\partial y}+u=0,\ u=\frac{2x+3y}{x^2+y^2}$.

3.19. $(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2+(\frac{\partial u}{\partial z})^2=1,\ u=\sqrt{x^2+y^2+z^2}$.

3.20. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=24, u=(x^2+y^2)\operatorname{tg}\frac{x}{y}$.

3.21. $9\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, u=e^{-(x+3y)}\sin(x+3y)$.

3.22. $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=xe^{y/x}$.

3.23. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=xe^{y/x}$.

3.24. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0,\ u=\operatorname{arctg}\frac{x}{y}$.

3.25. $\frac{\partial u}{\partial x}\cdot\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial u}{\partial y}\cdot\frac{\partial^2 u}{\partial y\partial x^2}=0,\ u=\ln(x+e^{-y})$.

3.26. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0, u=\arcsin\frac{x}{x+y}$.

3.27. $\frac{1}{x}\cdot\frac{\partial u}{\partial x}+\frac{1}{y}\cdot\frac{\partial u}{\partial y}=\frac{u}{y^2},\ u=\frac{y}{(x^2+y^2)^5}$.

3.28. $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=\frac{x+y}{x-y}, u=\frac{x^2+y^2}{x-y}$.

3.29. $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=\frac{2y}{u}$, $u=\sqrt{2xy+y^2}$.

3.30. $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$, $u=ln(x^2-y^2)$.

4. Quyidagi funksiyalarni ekstremumga tekshiring.

4.1. $z=y\sqrt{x}-2y^2-x+14y$. (*Javob:* $z_{max}(4,4)=28$.)
4.2. $z=x^3+8y^{3-}6xy+5$. (*Javob:* $z_{min}(1,0,5)=4$.)
4.3. $z=1+15x-2x^2-xy-2y^2$. (*Javob:* $z_{max}(-4,-1)=-97$.)
4.4. $z=1+6x-x^2-xy-y^2$. (*Javob:* $z_{max}(4,-2)=13$.)
4.5. $z=x^3+y-6xy-39x+18y+20$. (*Javob:* $z_{min}(5,6)=-86$.)
4.6. $z=2x^3+2y^{3-}6xy+5$. (*Javob:* $z_{min}(1,1)=3$.)
4.7. $z=3x^3+3y^{3-}9xy+10$. (*Javob:* $z_{min}(1,1)=7$.)
4.8. $z=x^2+xy+y^2+x-y+1$. (*Javob:* $z_{min}(-1,1)=0$.)
4.9. $z=4(x-y)-x^2-y^2$. (*Javob:* $z_{max}(2,-2)=8$.)
4.10. $z=6(x-y)-3x^{3-}3y^3$. (*Javob:* $z_{max}(1,-1)=6$.)
4.11. $z=x^2+xy+y^{2-}6x-9y$. (*Javob:* $z_{min}(1,4)=-21$.)
4.12. $z=(x-2)2+2y^{2-}10$. (*Javob:* $z_{min}(2,0)=-10$.)
4.13. $z=(x-5)2+y^2+1$. (*Javob:* $z_{min}(5,0)=1$.)
4.14. $z=x^3+y^{3-}3xy$. (*Javob:* $z_{min}(1,1)=-1$.)
4.15. $z=2xy-2x^{2-}4y^2$. (*Javob:* $z_{max}(0,0)=0$.)
4.16. $z=x\sqrt{u}-x^2-u+6x+3$. (*Javob:* $z_{max}(4,4)=15$.)
4.17. $z=2xy-5x^{2-}3y^2+2$. (*Javob:* $z_{max}(0,0)=2$.)
4.18. $z=xy(12-x-u)$. (*Javob:* $z_{max}(4,4)=64$.)
4.19. $z=xy-x^2-y^2+9$. (*Javob:* $z_{max}(0,0)=9$.)
4.20. $z=2xy-3x^{2-}2y^2+10$. (*Javob:* $z_{max}(0,0)=10$.)
4.21. $z=x^3+8y^{3-}6xy+1$. (*Javob:* $z_{min}(1,0,5)=0$.)
4.22. $z=u\sqrt{x}-y^2-x+6y$. (*Javob:* $z_{max}(4,4)=12$.)
4.23. $z=x^2-xy+y^2+9x-6y+20$. (*Javob:* $z_{min}(-4,1)=-1$.)
4.24. $z=xy(6-x-y)$. (*Javob:* $z_{max}(2,2)=8$.)
4.25. $z=x^2+y^2-xy+x+y$. (*Javob:* $z_{min}(-1,-1)=-1$.)
4.26. $z=x^2+xy+y^{2-}2x-y$. (*Javob:* $z_{min}(1,0)=-1$.)
4.27. $z=(x-1)^2+2y^2$. (*Javob:* $z_{min}(1,0)=0$.)
4.28. $z=xy-3x^{2-}2y^2$. (*Javob:* $z_{max}(0,0)=0$.)
4.29. $z=x^2+3(y+2)^2$. (*Javob:* $z_{min}(0,-2)=0$.)
4.30. $z=2(x+y)-x^2-y^2$. (*Javob:* $z_{max}(1,1)=2$.)

5. Berilgan chiziqlar bilan chegaralangan $\overline{D}$ sohadagi $z = z(x,y)$ funksiyaning eng katta va eng kichik qiymatlarini toping.

5.1. $z = 3x+y-xy$, $\overline{D}: y=x,\ y=4,\ x=0$. (*Javob:* $z_{eng\ katta}(2,2)=4$, $z_{eng\ kichik}(0,0)= z(4,4)=0$.)

5.2. $z = xy-x-2y$, $\overline{D}: x=3,\ y=x,\ y=0$. (*Javob:* $z_{eng\ katta}(0,0)= z(3,3)=0$, $z_{eng\ kichik}(3,0)=-3$.)

5.3. $z = x^2+2xy-4x+8y$, $\overline{D}: x=0,\ x=1,\ y=0,\ y=2$ (*Javob:* $z_{eng\ katta}(1,2)=17$, $z_{eng\ kichik}(1,0)=-3$.)

5.4. $z = 5x^2-3xy+y^2$, $\overline{D}: x=0,\ x=1,\ y=0,\ y=1$.
(*Javob:* $z_{eng\ katta}(1,0)=5$, $z_{eng\ kichik}(0,0)=0$.)

5.5. $z = x^2+2xy-y^2-4x$, $\overline{D}: x-y+1=0,\ x=0,\ x=3,\ y=0$,
(*Javob:* $z_{eng\ katta}(3,3)=6$, $z_{eng\ kichik}(2,0)=-4$.)

5.6. $z = x^2+y^2-2x-2y+8$, $\overline{D}: x=0,\ y=0,\ x+y-1=0$, .
(*Javob:* $z_{eng\ katta}(0,0)=8$, $z_{eng\ kichik}(0{,}5,0{,}5)=6{,}5$.)

5.7. $z = 2x^3-xy^2+y^2$, $\overline{D}: x=0,\ x=1,\ y=0,\ y=6$.
(*Javob:* $z_{eng\ katta}(0,6)=36$, $z_{eng\ kichik}(0.0)=0$.)

5.8. $z = 3x+6y-x^2-xy-y^2$, $\overline{D}: x=0,\ x=1,\ y=0,\ y=1$.
(*Javob:* $z_{eng\ katta}(1,1)=6$, $z_{eng\ kichik}(0,0)=0$.)

5.9. $z = x^2-2y^2+4xy-6x-1$, $\overline{D}: x=0,\ y=0,\ x+y-3=0$, .
(*Javob:* $z_{eng\ katta}(0,0)=-1$, $z_{eng\ kichik}(0,0,3)=-19$.)

5.10. $z = x^2+2xy-10$, $\overline{D}: y=0,\ y=x^2-4$,
(*Javob:* $z_{eng\ katta}\left(-\frac{4}{3},-\frac{2}{5}\right)=-\frac{62}{27}$, $z_{eng\ kichik}(1,-3)=-15$.)

5.11. $z = xy-2x-y$, $\overline{D}: x=0,\ x=3,\ y=0,\ y=4$ (*Javob:* $z_{eng\ katta}(3,4)=2$, $z_{eng\ kichik=}=(3,0)=-6$.)

5.12. $z = \frac{1}{2}x^2-xy$, $\overline{D}: y=8,\ y=2x^2$ (*Javob:* $z_{eng\ katta}(-2,8)=18$, $z_{eng\ kichik}(2,8)=-14$.)

5.13. $z = 3x^2+3y^2-2x-2y+2$, $\overline{D}: x=0,\ y=0,\ x+y-1=0$, .
(*Javob:* $z_{eng\ katta}(0,1)= z(1,0)=3$, $z_{eng\ kichik}\left(\frac{1}{3},\frac{1}{3}\right)=\frac{4}{3}$.)

5.14. $z = 2x^2+3y^2+1$, $\overline{D}: y=\sqrt{9-\frac{9}{4}x^2}$, $y=0$.
(*Javob:* $z_{eng\ katta}(0,3)=28$, $z_{eng\ kichik}(0,0)=1$.)

5.15. $z = x^2-2xy-y^2+4x+1$, $\overline{D}: x=-3,\ y=0,\ x+y+1=0$, (*Javob*: $z_{eng\ katta}(-3,2)=6$, $z_{eng\ kichik}(-2,0)=-3$.)

5.16. $z=3x^2+3y^2-x-y+1$, $\overline{D}$: $x=5$, $y=0$, $x-y-1=0$, .

(*Javob*: $z_{eng\ katta}(5,4)=115$, $z_{eng\ kichik}(1,0)=3$.)

5.17. $z=2x^2+2xy-\frac{1}{2}y^2-4x$, $\overline{D}$: $y=2x$, $y=2$, $x=0$,

(*Javob*: $z_{eng\ katta}(0,0)=z(1,2)=0$, $z_{eng\ kichik}(0,2)=-2$.)

5.18. $z=x^2-2xy+\frac{5}{2}y^2-2x$, $\overline{D}$: $x=0$, $x=2$, $y=0$, $y=2$.

(*Javob*: $z_{eng\ katta}(0,2)=10$, $z_{eng\ kichik}(\frac{5}{2},\frac{2}{3})=-1{,}67$.)

5.19. $z=xy-3x-2y$, $\overline{D}$: $x=0$, $x=4$, $y=0$, $y=4$

(*Javob*: $z_{eng\ katta}(0,0)=0$, $z_{eng\ kichik}(4,0)=-12$.)

5.20. $z=x^2+xy-2$, $\overline{D}$: $y=4x^2-4$, $y=0$.

(*Javob*: $z_{eng\ katta}(-\frac{2}{3},-2{,}22)=-0{,}07$, $z_{eng\ kichik}(0{,}5;-3)=-3{,}25$.)

5.21. $z=x^2y(4-x-y)$, $\overline{D}$: $x=0$, $y=0$, $y=6-x$.

(*Javob*: $z_{eng\ katta}(2,1)=4$, $z_{eng\ kichik}(4,2)=-64$.)

5.22. $z=x^3+y^3-3xy$, $\overline{D}$: $x=0$, $x=2$, $y=-1$, $y=6$.

(*Javob*: $z_{eng\ katta}(2,-1)=13$, $z_{eng\ kichik}(0,-1)=-1$.)

5.23. $z=4(x-y)-x^2-y^2$, $\overline{D}$: $x+2y=4$, $x-2y=4$, $x=0$.

(*Javob*: $z_{eng\ katta}(\frac{8}{5},\frac{6}{5})=\frac{36}{5}$, $z_{eng\ kichik}(0,2)=-12$.)

5.24. $z=x^2+2xy-y^2-4x$, $\overline{D}$: $x=3$, $y=0$, $y=x+1$.

(*Javob*: $z_{eng\ katta}(3,3)=6$, $z_{eng\ kichik}(2,0)=-4$.)

5.25. $z=6xy-9x^2-9y^2+4x+4y$, $\overline{D}$: $x=0$, $x=1$, $y=0$, $y=2$

(*Javob*. $z_{eng\ katta}(\frac{1}{3},\frac{1}{3})=\frac{4}{3}$, $z_{eng\ kichik}(0,2)=-28$.)

5.26. $z=x^2+2xy-y^2-2x+2y$, $\overline{D}$: $y=x+2$, $y=0$, $x=2$

(*Javob*: $z_{eng\ katta}(2,3)=9$, $z_{eng\ kichik}(1,0)=-1$.)

5.27. $z=4-2x^2-y^2$, $\overline{D}$: $y=0$, $y=\sqrt{1-x^2}$.

(*Javob*. $z_{eng\ katta}(0,0)=4$, $z_{eng\ kichik}(-1,0)=z(1,0)=2$.)

5.28. $z=5x^2-3xy+y^2+4$, $\overline{D}$: $x=-1$, $x=1$, $y=-1$, $y=1$.

(*Javob*: $z_{eng\ katta}(-1,1)=z(1,-1)=13$, $z_{eng\ kichik}(0,0)=4$.)

5.29. $z=x^2+2xy+4x-y^2$, $\overline{D}$: $x+y+2=0$, $x=0$, $y=0$

(*Javob*: $z_{eng\ katta}(0,0)=0$, $z_{eng\ kichik}(-2,0)=z(0,-4)=-4$.)

5.30. $z=2x^2y-x^3y-x^2y^2$, $\overline{D}$: $x=0$, $y=0$, $x+y=6$

(*Javob*: $z_{eng\ katta}(1,0.5)=0.25$, $z_{eng\ kichik}(4,2)=-128$.)

Namunaviy variantlar yechimi

1. S: $z=x^2+y^2+3xy-4x+2y-4$ sirtga $M_0(-1,0,1)$ nuqtada o'tkazilgan urinma tekislik va normalning tenglamasini toping.

►Xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x}=2x+3y-4, \frac{\partial z}{\partial y}=2y+3x+2$$

$M_0(-1,0,1)$ nuqtaning koordinatalarini hosil qilingan ifodaga qo'yib, berilgan nuqtada (10,8) formulaga asosan S sirtga perpendikulyar bo'lgan II vektorning koordinatalarini hisoblaymiz.

$$A=\frac{\partial z}{\partial x}\Big|_{M0}=-6, B=\frac{\partial z}{\partial y}\Big|_{M0}=-1\ C=-1$$

Bundan, urinma tekislik tenglamasi quyidagicha bo'ladi.

$-\ 6(x+1)-y-(z-1)=0$ yoki $6x+y+\ +5=0.$

(10.9) formulaga asosan normalning tenglamasi

$$\frac{x+1}{6}=\frac{y}{1}=\frac{z-1}{1}$$ ◄

ko'rinishda yoziladi.

2. $z=arccos\sqrt{\frac{x}{y}}$ funksiyaning ikkinchi tartibli xususiy hosilasini toping.

$z''_{xy}=z''_{yx}$ ekanligiga ishonch hosil qiling.

► Avval berilgan funksiyaning birinchi tartibli xususiy hosilalarini topamiz:

$$z'_x=-\frac{1}{\sqrt{1-\frac{x}{y}}}\cdot\frac{1}{2\sqrt{\frac{x}{y}}}\cdot\frac{1}{y}=-\frac{1}{2\sqrt{x}\sqrt{y-x}},$$

$$z'_y=-\frac{1}{\sqrt{1-\frac{x}{y}}}\cdot\frac{1}{2\sqrt{\frac{x}{y}}}\left(-\frac{x}{y^2}\right)=\frac{\sqrt{x}}{2\sqrt{x}\sqrt{y-x}}.$$

Olingan hosilalarning har birini x va y bo'yicha differensiallab, berilgan funksiyaning ikkinchi tartibli xususiy hosilalarini topamiz:

$$z''_{xx}=\frac{\frac{1}{2\sqrt{x}}\sqrt{y-x}-\frac{\sqrt{x}}{2\sqrt{y-x}}}{2x(y-x)}=\frac{y-x-x}{4x\sqrt{y}\sqrt{y-x}(y-x)}=\frac{y-2x}{4\sqrt{x}(y-x)\sqrt{y-x}}$$

$$z''_{xy}=-\frac{1}{2\sqrt{x}}\left(-\frac{1}{2}\right)(y-x)^{-\frac{3}{2}}=\frac{1}{4\sqrt{x}(y-x)\sqrt{y-x}}$$

$$z''_{yy}=\frac{\sqrt{x}}{2}\left(-\frac{\sqrt{y-x}+\frac{y}{2\sqrt{y-x}}}{y^2(y-x)}\right)=-\frac{\sqrt{x}(2x+3y)}{2y^2(y-x)}$$

$$z''_{yx}=\frac{1}{2y}\times\frac{\frac{\sqrt{y-x}}{2\sqrt{x}}+\frac{\sqrt{x}}{2\sqrt{y-x}}}{y-x}=\frac{y-x+x}{4y(y-x)\sqrt{x}\sqrt{y-x}}=$$

$$=\frac{1}{4\sqrt{x}(y-x)\sqrt{y-x}}.$$

Ko‘rinib turibdiki, aralash xususiy hosilalar teng bo‘ladi, yani $z''_{yx}=z''_{xy}$ ◀

3. $u=ln\ (x^2+y^2)$ funksiyaning $\frac{\partial^2u}{\partial x^2}-2xy\frac{\partial^2u}{\partial x\partial y}+\frac{\partial^2u}{\partial y^2}=\frac{4y^2}{x^2+y^2}\times\frac{\partial u}{\partial x}$ tenglamani qanoatlantirishini tekshiring.

▶ Birinchi va ikkinchi tartibli xususiy hosilalarini topamiz.

$$\frac{\partial u}{\partial x}=\frac{2x}{x^2+y^2},\ \frac{\partial u}{\partial y}=\frac{2y}{x^2+y^2},$$

$$\frac{\partial^2u}{\partial x^2}=\frac{2(y^2-x^2)}{(x^2+y^2)^2},\ \frac{\partial^2u}{\partial x\partial y}=\frac{4xy}{(x^2+y^2)^2},\ \frac{\partial^2u}{\partial y^2}=\frac{2(x^2-y^2)}{(x^2+y^2)^2}$$

Olingan hosilalarning qiymatlarini dastlabki tenglamaning chap tomoniga qo‘yamiz:

$$\frac{2(y^2-x^2)}{(x^2+y^2)^2}+\frac{8x^2y^2}{(x^2+y^2)^2}+\frac{2(x^2-y^2)}{(x^2+y^2)^2}=\frac{8x^2y^2}{(x^2+y^2)^2}$$

U holda tenglamaning o‘ng tomonida quyidagiga ega bo‘lamiz.

$$\frac{4y^2}{x^2+y^2}\times\frac{2x}{x^2+y^2}=\frac{8xy^2}{x^2+y^2}$$

Olingan natijalarni solishtirib, berilgan funksiya dastlabki tenglamani qanoatlantirmasligini ko‘ramiz.

4. $z=xy(x+y-2)$ funksiyani lokal ekstremumga tekshiring.

Berilgan funksiyaning birinchi tartibli xususiy hosilalarini topamiz:

$$z'_x=2xy+y^2-2y,\ z'_y=x^2+2xy-2x$$

Bularni no'lga tenglab, quyidagi tenglamalar sistemasiga ega bo'lamiz.

$$\left.\begin{aligned} y(2x+y-2)&=0,\\ x(x+2y-2)&=0, \end{aligned}\right\}$$

Bu sistemani yechib, berilgan funksiyaning $M_1(0,0,)$, $M_2(2,0,)$, $M_3(0,2,)$, $M_4(3,2/3,)$ statsionar nuqtalarini aniqlaymiz.

10.4 dagi 2 teoremadan foydalanib, bu nuqtalarning qaysilari ekstremum nuqtalari ekanligini aniqlaymiz.

Buning uchun avval berilgan funksiyaning ikkinchi tartibli xususiy hosilasini topamiz:

$$z''_{xx}=2y, z''_{xy}=2x+2y-2, z''_{yy}=2x$$

Hosilalar uchun olingan ifodaga statsionar nuqtalarning koordinatalarini qo'yib va ekstremum mavjudligining yyetarli shartidan foydalanib, (§ 10.4 ga qarang) quyidagilarga ega bo'lamiz:

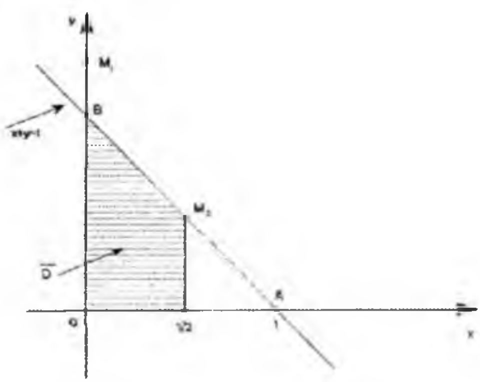
M_1 nuqta uchun $\Delta=-4<0$, yani ekstremum yo'q,

M_2 nuqta uchun $\Delta=-4<0$, yani ekstremum yo'q,

M_3 nuqta uchun $\Delta=-4<0$, yani ekstremum yo'q.

M_4 nuqta uchun $\Delta=\frac{12}{9}>0, A=4/3>0$, yani $z_{min}=z(2/3,2/3)=-8/27$ bo'lgan funksiyaning lokal minimumiga ega bo'lamiz.

5. $x=0$, $y=0$, $x+y-1=0$ chiziqlar bilan chegaralangan $\bar{D}$ sohadagi $z=xy-y^2+3x+4y$ funksiyaning eng katta va eng kichik qiymatlarini toping. (10.5 – rasm.)



▶Berilgan $\overline{D}$ soha ichida yotuvchi, yani OAB uchburchak ichida statsionar nuqtalarning mavjudligini aniqlaymiz. Quyidagiga ega bo'lamiz.

$$\left.\begin{aligned} z'_x &= y + 3 = 0, \\ z'_u &= x - 2y + 4 = 0, \end{aligned}\right\}$$

Olingan tenglamalar sistemasini yechib, *M(-10,-3)* statsionar nuqtani topamiz. Bu nuqta $\overline{D}$ sohadan tashqarida yotganligidan, masalani yechishda bu nuqtani hisobga olmaymiz.

Funksiya qiymatlarini $\overline{D}$ soha chegaralarida tekshiramiz. *z* funksiya OAB burchakning *OA* $(y=0,\ 0 \le x \le 1)$ tomonida $z=3x$ ko'rinishga ega. $z'=3$ bo'lganligidan, OA kesmada statsionar nuqtlar yo'q.

O va *A* nuqtalarda mos ravishda $z\ (0,0)=0,\ z\ (1,0)=3$ Uchburchakning OB $(x=0,\ 0 \le y \le 1)$ tomonida *z* funksiya quyidagi ko'rinishga ega. $z = -y^2+4y,\ z'=-2y+4=0; 2y+4=0$ tenglamadan $y=2$ statsionar nuqta topamiz.

· Shunday qilib, $M_1(0,2)$ nuqta $\overline{D}$ sohada yotmaydi. Funksiyaning B nuqtadagi qiymati $z(0,1) = 3$. AB tomondagi eng katta va eng kichik qiymatlarini topamiz. *AB:* $x+y=1$, bundan, $y=1-x,\ z=-2x^2+2x+3$, u holda $z'=-4x+2$ va $z'=0$ dan $x=1/2$ bo'ladi, yani $M_2(1/2,\ 1/2)$ statsionar nuqta $\overline{D}$ sohaning chegarasida yotadi. Bu nuqtada funksiyaning qiymati $z(1/2,1/2)=3,5$ bo'ladi. Funksiyaning barcha olingan qiymatlarini solishtirib, *z eng katta*= $z\ (1/2,1/2)\ )=3,5$, *z eng kichik*= $z\ (0,0)=0$ ekanligini ko'ramiz.

10.6. 10-bobga qo'shimcha masalalar

1. $U=\sqrt{z(2-z)} + ln(4-x^2) - 3y$ funksiyaning aniqlanish sohasini toping. *(Javob:* $|x| < 2, 0 \le z \le 2$*)*

2. $f(x,y)=\begin{cases} \frac{x^3y}{x^6+y^2}, agar\ x^6+y^2 \neq 0 \\ 0, agar\ x=y=0 \end{cases}$

funksiyaning $x=u=0$ nuqtada uzilishga ega ekanligini, ammo O(0,0) nuqtada xususiy hosilaga ega ekanligini isbotlang.

3. $f(x,y)=\begin{cases} \frac{xy(x^2-u^2)}{x^2+y^2}, agar\ x^2+y^2 \neq 0 \\ 0, agar\ x=y=0 \end{cases}$

funksiya uchun $f''_{xy}(0,0) \neq f''_{yx}(0,0)$ tengsizlik bajarilishini isbotlang.

4. $z \quad x^u y^x$ funksiya $x\frac{\partial z}{\partial x} + u\frac{\partial z}{\partial y} = (x+y+lnz)z$ tenglamani qanoatlantirishini isbotlang.

5. $z= |x+y| - \sqrt{1-x^2-y^2}$ funksiyaning uzluksizlik sohasidagi eng katta va eng kichik qiymatlarini toping. (*Javob*: *z* eng katta=$\sqrt{2}$, *z* eng kichik=-1)

6. Fazoda *A (4,1,5)* nuqtadan *2x+6y+3 -12=0* tekislikka parallel tekislik o'tkazilgan. $z = x^2 + y^2$ aylanish paraboloididan shu tekislik bilan ajratilgan sohani, tengsizliklar sistemasi orqali ifodalang.

(*Javob*: $x^2 + y^2 \leq z \leq 2x + 6y + 3z - 29$)

7. $yz''_{yy} + 2z'_y = z/x$ tenglamani *u=x/y* va *v=x-y* yangi o'zgaruvchilar bilan ifodalangan.

(*Javob*: $\frac{u^2(u-1)}{v} z''_{uu} + 2uz''_{uv} + \frac{v}{u-1} z''_{vv} - \frac{2u(u-1)}{v} z'_u - 2z'_v = \frac{2(u-1)}{uv}$)

8. $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ ifodani qutb koordinatalarida yozing.

(*Javob*: $\frac{\partial^2 z}{\partial \rho^2} + \frac{1}{\rho^2} \cdot \frac{\partial^2 z}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial z}{\partial \rho}$)

9. Koordinata o'qlaridan bir xil kesma ajratuvchi $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{c^2} = 1$ urinma tekislik tenglamasini toping.

(*Javob*: $\pm x \pm y \pm z = \sqrt{a^2+b^2+c^2}$)

10. $xyz=a^3$ sirtga urinma tekislikning sirtning ixtiyoriy nuqtasida koordinata tekisliklari bilan o'zgarmas hajmli tetraedr hosil qilishini isbotlang va bu hajmni hisoblang. (*Javob*: $V=\frac{9}{2}a^3$)

11. Perimetri 2r ga teng uchburchakni biror tomoni orqali aylantirishdan eng katta hajmli jism hosil bo'ladi. Shu uchburchakning tomonlarini toping. (*Javob*: *a=b=3p/4, c=p/2*)

12. $x^2+4y^2=4$ ellipsda ikkita $A(-\sqrt{3},1/2)$ va $B(1,\sqrt{3}/2)$ nuqtalar berilgan. Bu ellipsda, shunday C nuqtani topingki, ABC uchburchakning yuzi eng katta bo'lsin. (*Javob:* C $(\frac{\sqrt{3}-1}{2},\frac{-\sqrt{3}-1}{2})$)

13. $z=x^3+y^3-9xy+27$ funksiyani ekstremumga tekshiring. (*Javob:* $z_{min}(3,3)=0$.)

14. Agar $u=zx+e^{yz}+y$ bo'lsa, $\frac{\partial^4 u}{\partial x^2 \partial u \partial z}=\frac{\partial^4 u}{\partial x \partial u \partial z\, \partial x}$ ekanligini isbotlang.

15. $xyz=8,\ xy/z=8$ shartlarni qanoatlantiruvchi $u=x+y+z$ funksiyaning shartli ekstremumini toping. (*Javob:* $x=y=2\sqrt[4]{6}$, $z=\sqrt{\frac{2}{3}}$)

16. Oshkormas ko'rinishda $3x^2y^2+2xyz^2-2x^3+4y^3-4=0$ tenglama bilan berilgan funksiyaning $(2,1,2)$ nuqtadagi ikkinchi tartibli d^2z differensialini toping.

(*Javob:* $-31,5\,dx^2+206\,dxdy-306\,dy^2$)

17. Kvadrat taxta, shaxmat tartibida joylashtirilgan 2 ta oq va 2 ta qora kataklardan iborat. Har bir katakning tomoni uzunlik birligiga teng. Tomonlari taxtaning tomonlariga parallel, bitta burchagi taxtaning qora burchagi bilan ustma-ust tushadigan to'g'ri to'rtburchakni qaraymiz. Bu to'g'ri to'rtburchakning qora qisimning yuzi S bo'lib, uning tomonlarining uzunliklari x va u ning funksiyasi bo'ladi. Shu funksiyaning analitik ko'rinishini yozing.

(*Javob:*

$$S(x,y)=\begin{cases} xy, agar\ 0\le x\le 1, 0\le y\le 1 \\ x, agar\ 0\le x\le 1, 1\le y\le 2 \\ y, agar\ 0\le x\le 2, 0\le y\le 1 \\ 1+(x-1)(y-1), agar\ 1\le x\le 2, 1\le y\le 2 \end{cases}$$

11. ODDIY DIFFERENSIAL TENGLAMALAR

11.1. ASOSIY TUSHUNCHALAR. BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

Izoklin usuli.

Agar tenglamada izlanayotgan funksiyaning hech bo'lmaganda bitti hosilasi qatnashsa, bunday tenglama differensial tenglama deyiladi.

Differensial tenglamaning tartibi tarifga asosan tenglama tarkibiga kiruvchi eng yuqori hosila tartibi bilan ustma-ust tushadi.

Agar izlanayotgan u funksiya bitta argumentli funksiya bo'lsa, u holda differensial tenglama *oddiy differensial tenglama* deyiladi.

Agar izlanayotgan *y* funksiya bir necha argumentli funksiya bo'lsa, u holda differensial tenglama *xususiy hosilali tenglama* deyiladi.

Masalan: $2xy'-3y=0$ tenglama, bu yerda $y=y(x)$, birinchi tartibli oddiy differensial tenglama bo'ladi. $u'_x - u'_u + xy + 1 = 0$, bu yerda $u=u(x,y)$ esa birinchi tartibli xususiy hosilali differensial tenglama deyiladi. (Bu bobda faqat oddiy differensial tenglamalar qaraladi shuning uchun, keyinchalik, qisqalik uchun "oddiy" degan so'zni qoldirib ketamiz.)

Umumiy holda n tartibli differensial tenglama quyidagi ko'rinishda yoziladi.

$$F(x,y,y',y'',\dots y^{(n-1)}, y^{(n)})=0 \qquad (11.1)$$

Agar (11.1) tenglamani eng yuqori hosilaga nisbatan yecha olsak, u holda normal formadagi tenglamani olamiz.

$$y^{(n)}=f(x,y,y',y'',\dots y^{(n-1)}) \qquad (11.2)$$

Differensial tenglama yechimlarini topish jarayoni *tenglamani integrallash* deb ataladi.

(11.1) yoki (11.2) differensial tenglamaning yechimi (yoki integrali) deb, biror (a,b) oraliqda aniqlangan va o'zining hosilalari bilan berilgan differensial tenglamani ayniyatga aylantiruvchi ixtiyoriy haqiqiy $y=y(x)$ funksiyaga aytiladi. (Shu bilan birgalikda $y=y(x)$ funksiyaning hosilasi mavjud deb faraz qilinadi.)

1-misol. Sonlar o‘qida aniqlangan $y=xe^{2x}$ funksiya $y''-4y'+4y=0$ differensial tenglamaning yechimi ekanligini isbotlang.

▶Funksiyaning o‘zini va uning hosilalarini

$y'=e^{2x}(1+2x),\ y''=4e^{2x}(1+x)$

berilgan tenglamaga qo‘yib, quyidagi ayniyatni hosil qilamiz.

$4e^{2x}(1+x)-4e^{2x}(1+2x)+4x^{2x}=4e^{2x}(1+x-1-2x+x)=0.$

2-misol. $F(x,y)=ln\frac{y}{x}-5+xy=0$ oshkormas ko‘rinishda berilgan $y=y(x)$ funksiya $(x+x^{2y})y'=y-xy^2$ differensial tenglamani ayniyatga aylantirilishni yani uning yechimi ekanligini isbotlang.

Haqiqatan ham , $F(x,y)=0$ (10.6 formulaga qarang) oshkormas funksiyani diffensiallash qoidasiga asosan, quydagiga ega bo‘lamiz.

$$y'=-\frac{F'_x}{F'_y}=-\frac{(y-\frac{1}{x})}{(x+\frac{1}{y})'}=\frac{y}{x}\cdot\frac{1-xy}{1+xy}=\frac{1-xy^2}{x+x^2y}$$

Olingan hosila y' ni dastlabki differensial tenglamaga qo‘yib, ayniyat hosil qilamiz.

Agar $F(x,y)=0$ oshkormas ko‘rinishda berilgan funksiya differensial tenglamaganing yechimi bo‘lsa, u holda $F(x,y)=0$ berilgan differensial tenglamaning integrali (*yechim emas)* deyiladi. Shunday qilib, 1 va 2 misollarda berilgan differensial tenglamalarning mos ravishda yechimi va integraliga ega bo‘lamiz.

(11.1) differensial tenglama yechimining (*yoki integralining) Oxy* tekislikdagi grafigi integral chiziq deyiladi. Shunday qilib har bir yechimga yoki integralga integral chiziq mos keladi.

(11.2) differensial tenglama yechimining mavjudligi va yagonaligi qo‘ydagicha hal qilinadi.

1-Teorema (Koshi).

Agar (11.2) tenglamaning o‘ng tomoni

$$x_0,\ y_0,\ y_0,\ \dots\dots,\ y_0^{(n-1)} \qquad (11.3)$$

Qiymatlarning biror atrofida uzluksiz funksiya bo‘lsa, u holda (11.2) tenglama

$$y(x_0)=y_0,\ y(x_0)=y_0\ ,\ \dots,\ y_0^{(n-1)}=y_0^{(n-1)} \qquad (11.4)$$

bo'lgan x_0 nuqtani o'z ichiga olgan biror *(a,b)* oraliqda $y=y(x)$ yechimiga ega bo'ladi.

Agar ko'rsatilgan atrofda y , y , ..., $y^{(n-1)}$ argumentlari bo'yicha bu funksiyaning xususiy hosilalari ham uzluksiz bo'lsa, u holda $y= y(x)$ yechim yagona yechim bo'ladi.

(11.3) dagi sonlar to'plami boshlang'ich qiymatlar, (11.4) tenglik esa, *boshlang'ich shartlar* deyiladi.

n- tartibli differensial tenglama uchun Koshi masalasi qo'ydagicha ta'riflanadi.

(11.1) yoki (11.2) differensial tenglamaning, (11.3) boshlang'ich qiymatlarini va (11.4) boshlang'ich shartlarini qanoatlantiruvchi, $y=y(x)$ yechimni toping.

Koshi teoremasini qanoatlantiruvchi sohada (11.2) ko'rinishdagi ixtiyoriy differensial tenglama cheksiz ko'p yechimga ega bo'ladi. Umuman olganda bu (11.1) differensial tenglama uchun ham o'rinlidir.

Bu yechimlar to'plamini tavsiflash uchun umumiy yechim tushunchasini kiritamiz.

(11.1) yoki (11.2) differensial tenglamaning umumiy yechimi deb $y = \varphi\ (x,C_1,C_2,.........,C_n)$ yoki qisqacha $y= \varphi\ (x,C_i)$ ko'rinishdagi funksiyaga aytiladi. Bu yerda C_i $(i=\overline{1,n})$ qo'yidagi ikkita shartni qanoatlantiruvchi ixtiyoriy o'zgarmas:

1. $y=\varphi(x,c)$ funksiya C ning ixtiyoriy qiymatida (11.1) yoki (11.2) differensial tenglamaning yechimi bo'ladi.
2. Differensial tenglama yechimga ega bo'ladigan har qanday $x_0, y_0, y_0^1, ..., y_0^{(n-1)}$ boshlang'ich qiymatlardan $\varphi(x_0, c_0) = y_0^1, ..., \varphi^{n-1}(x_0, c_0) = \varphi_0^{n-1}$ shartlarni qanoatlantiruvchi, $C_i=C_{i0}$ o'zgarmaslarning qiymatlarini ko'rsatish mumkin.

$F(x,y,C_i)=0$ oshkormas ko'rinishda olingan, umumiy yechim differensial tenglamaning umumiy integrali deyiladi.

Umumiy yechim yoki umumiy integraldan, ixtiyoriy o'zgarmas C_i ning fiksirlangan qiymatlarida olingan yechim mos ravishda differensial tenglamaning xususiy yechimi yoki xususiy integrali deyiladi.

Eslatma: Differensial tenglamaning, ixtiyoriy o'zgarmas C_i ning hech qanday qiymatlarida umumiy yechimdan olib bo'lmaydigan yechimi (integrali) mavjud bo'lishi mumkin. Bunday yechim shu ma'noda maxsus deyiladiki, uning ixtiyoriy nuqtasida Koshi teoremasining qandaydir shartlari bajarilmaydi.

Masalan: $y'' = 3\sqrt[3]{(y'-1)^2}$ differensial tenglama $y = x + \frac{1}{4}(x + C_1)^4 + +C_2$ umumiy yechimga ega, bu yerda C_1 C_2 –lar ixtiyoriy o'zgarmaslar. $y=x+C$ funksiya ham berilgan tenglamaning yechimi bo'ladi, bu yerda C – ixtiyoriy o'zgarmas, ammo bu yechimni C_1 va C_2 ning hech qanday qiymatlarida umumiy yechimdan olib bo'lmaydi. Bundan tashqari, $y' =1$, yechimlarning ixtiyoriy nuqtalarida, Koshi teoremasidagi yagonalik shartining buzilishiga olib keladi yoki berilgan tenglamaning o'ng tomonidan y' bo'yicha olingan xususiy hosila $y' =1$ da uzilishga ega bo'ladi. Shunday qilib, $y=x+C$ yechim mahsus yechim bo'ladi. Bundan keyin, qoida bo'yicha, mahsus yechimlar qaralmaydi.

Aniqmas integrallar nazariyasi, umumiy yechimi $y = \int f(x)dx = F(x) + C$, (*bu erda* $F(x) - 1(x)$ funksiya uchun boshlang'ich funksiya, ya'ni $F'(x)=f(x)$; C – ixtiyoriy o'zgarmas) bo'lgan oddiy differensial tenglamalar sinfining nazariyasi hisoblanadi.

Birinchi tartibli differensial tenglama, umumiy holda

$$F(x,y,y')=0 \qquad (11.5)$$

yoki, agar uni y' ga nisbatan yechsak, qo'yidagi normal ko'rinishda yozilishi mumkin.

$$y' = f(x, y). \qquad (11.6)$$

2- teorema (Koshi teoremasi).

Agar $f(x, y)$ funksiya M_0 (x_0, y_0) nuqta va uning atrofida uzluksiz bo'lsa, u holda (11.6) tenglamaning $y(x_0)=y_0$ shartni qanoatlantiruvchi $y=y(x)$ yechimi mavjud bo'ladi. Agar berilgan funksiyaning $\frac{\partial f}{\partial x}$ xususiy hosilasi ham uzluksiz bo'lsa, u holda bu yechim yagonadir.

Ba'zi hollarda birinchi tartibli differensial tenglamalarni differensial formada yozish qulaydir:

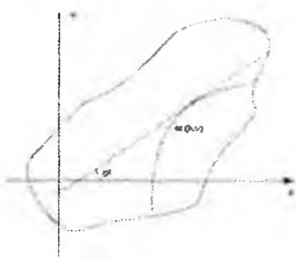
$$P(x,y)dx + Q(x,y)dy=0 \qquad (11.7)$$

Birinchi tartibli differensial tenglamalar uchun Koshi masalasi quydagicha ta'rifga ega.

(11.5) yoki (11.6) differensial tenglamalarning $\varphi(x, C_i) = y_0(\Phi(x_0, y_0) = 0)$ boshlang'ich shartni qanoatlantiruvchi $y = \varphi(x)$ $(\Phi(x, y)=0$ integral) yechimini toping. Bu geometrik nuqtai nazardan shuni anglatadiki, berilgan tenglamaning barcha integral chiziqlari orasidan berilgan M_0 (x_0,y_0) nuqtadan o'tuvchi integral chiziqni topish kerak.

(11.6) differensial tenglamaning geometrik talqini quydagidan iborat. U 2- teorema (Koshi teoremasi)ning barcha shartlarini qanoatlantiruvchi, D sohaga tegishli har bir $M(x,y)$ (11.6) tenglama yagona integral chizig'iga, $M(x,y)$ nuqtadan o'tuvchi $y = tg\alpha = k$ urinmasining yo'nalishini, ya'ni D sohadagi maydon yo'nalishini aniqlaydi. (11.1-rasm)

(11.6) tenglama uchun D sohada har biri izoklin deb ataluvchi bir parametrli $f(x,y) = K = const$ chiziqlar oilasini ajratish mumkin.



11.1-rasm

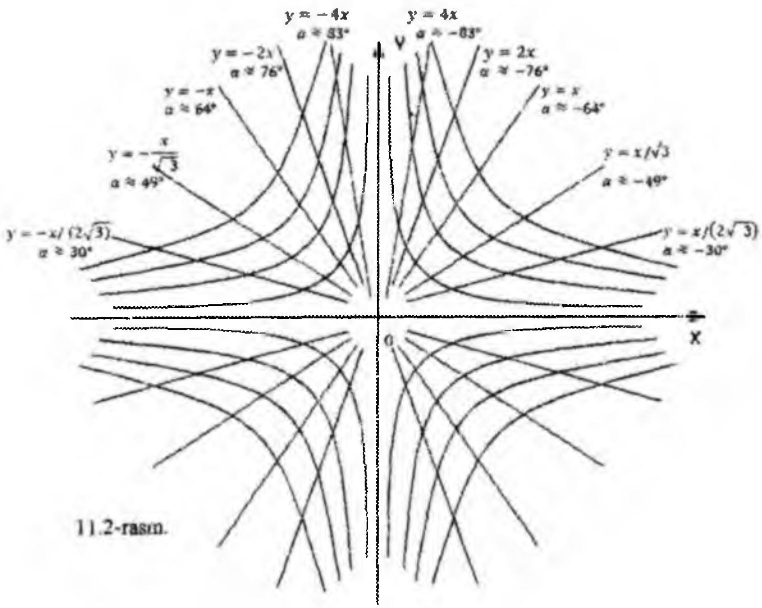
Izoklinni va u bo'yicha yo'nalishni topish, yo'nalishlar maydonini hosil qilishga va berilgan differensial tenglamaning integral chiziqlarini tahminan qurish ya'ni bu tenglamani grafik ko'rinishda integrallash imkonini beradi.

3-Misol. $y = -2y/x$ differensial tenglamaning integral chiziqlarini izoklin usulida taxminan yasang.

▶ $-\frac{2y}{x} = K(K = const)$ deb olib, berilgan tenglamaning $y = -\frac{K}{2}x$ izoklinini topamiz. Bular koordinatalar boshidan o'tuvchi to'g'ri chiziqlarni ifodalaydi. Bu chiziqlar bo'yicha $y' = K = tg\alpha$ tenglik bilan yo'nalishlar maydoni aniqlanadi. K ga turli qiymatlar berib, ularga mos izoklinlarni topamiz. Bu izoklinlar bo'ylab integral chiziqqa urinmaning Ox uqiga og'ishgan α *burchagi bilan* tavsiflanuvchi yo'nalishlar maydoni aniqlanadi. Kerak bo'lgan hisoblashlarni jadval ko'rinishida yozamiz. (1-jadvalga qarang).

K	0	$\pm 1/\sqrt{3}$	± 1	$\pm\sqrt{3}$	± 2	± 3	$\pm\infty$
α	0	$\pm 30°$	$\pm 45°$	$\approx \pm 60°$	$\approx \pm 64°$	$\approx \pm 72°$	$\approx \pm 90°$
$y = -\frac{k}{2}x$	$y = 0$	$y = \pm\frac{x}{2\sqrt{3}}$	$y = \mp\frac{1}{2}x$	$y = \mp\frac{\sqrt{3}}{2}x$	$y = \mp x$	$y = \mp\frac{3}{2}x$	$x = 0$

Shu jadvalga asosan yo'nalishlar maydonini yasaymiz va so'ngra taxminiy ravishda integrallar chizig'ini chizamiz. (11.2 rasm) OX o'qida soat stelkasi yo'nalishi bo'yicha yoki unga teskari yo'nalishda hisoblanishi α burchakning qiymatlari mos ravishda musbat yoki manfiy bo'lishini ko'rsatadi.



11.2-rasm.

11.2. O‘ZGARUVCHILARI AJRALADIGAN DIFFERENSIAL TENGLAMALAR. BIR JINSLI TENGLAMALAR

Quyidagi ko‘rinishdagi tenglamalar.

$$P(x)dx + Q(y)dy = 0 \qquad (11.8^*)$$

O‘zgaruvchilari ajraladigan differensial tenglamalar deyiladi. Uning umumiy integrali qo‘yidagi ko‘rinishda bo‘ladi.

$$\int P(x)dx + \int Q(y)dy = C \qquad (11.8)$$

bu yerda C – ixtiyoriy o‘zgarmas.

Qo‘yidagi ko‘rinishdagi tenglamalar

$$M1(x)\, N1(y)dx + M2(x)N2(y)dy = 0 \qquad (11.9)$$

yoki

$$y' = \frac{dy}{dx} = f_1(x)f_2(y) \qquad (11.10)$$

shuningdek, algebraik almashtirishlar yordamida (11.9) yoki (11.10) tenglamalarga keltiriluvchi tenglamalar ham *o‘zgaruvchilari ajraladigan tenglamalar* deyiladi.

Ushbu tenglamalarda o‘zgaruvchilarni ajratish qo‘ydagicha bajariladi. $N1(y)\quad 0$, $M2(x) \neq 0$ deb faraz qilamiz va (11.9) tenglamaning ikkala qismini $N1(y)\, M2(x)$ ga bo‘lamiz. (11.10) tenglamaning ikkala qismini dx ko‘paytiramiz va $f_2(y) \neq 0$ ga bo‘lamiz. Natijada o‘zgaruvchilari ajraladigan (ya’ni (11.8*) ko‘rinishdagi) tenglamani hosil qilamiz.

$$\frac{M_1(x)}{M_2(x)}dx + \frac{N_2(y)}{N_1(y)}dy = 0, f_1(x)dx - \frac{dy}{f_2(y)} = 0$$

Bu tenglama (11.10) formulaga asosan qo‘ydagicha integrallanadi.

$$\int \frac{M_1(x)}{M_2(x)}dx + \int \frac{N_2(y)}{N_1(y)}dy = C,\ \int f_1(x)dx - \int \frac{dy}{f_2(y)} = C$$

1-Misol. Differensial tenglamaning umumiy yechimini toping.

$$(xy+y)dx+(xy+x)dy=0 \quad (1)$$

► $x \neq 0$, $y \neq 0$ deb faraz qilamiz va berilgan tenglamaning ikkala qismini xy ga bo‘lib, o‘zgaruvchilari ajraladigan tenglamani hosil qilamiz.

$$(1 + \frac{1}{x})dx + \left(1 + \frac{1}{y}\right)dy = 0$$

buni (11.8) formulaga asosan integrallab,

$$\int (1+\frac{1}{x})dx + \int \left(1+\frac{1}{y}\right)dy = \ln|C|$$

$$x+ln\,|x| + y + ln\,|y| = ln\,|c|$$

$ln\,|xy| + \ ln\ ex+y = ln\,|c|, xye^{x+y} = C$

larni topamiz. (ixtiyoriy o‘zgarmasni $l\,n|c|$ ko‘rinishida yozish mumkin.)

Oxirgi tenglik (1) tenglamaning umumiy integrali bo‘ladi. Buni topishda $x\neq 0$, $y\neq 0$ degan shartlar qo‘yilgan edi. Ammo, x=0 va y=0 funksiyalar ham, dastlabki tenglamaning yechimlari bo‘la oladi, buni tekshirish oson, ikkinchi tarafdan ular C=0 da umumiy integraldan hosil qilinadi.

Shunday qilib, $x=0$, $y=0$ (1) tenglamaning xususiy yechimlaridir.

2-Misol. $(1+e^{2x})y2y\ \ -e^{x}$ tenglamaning $y=(0)$ boshlang‘ich shartini qanoatlantiruichi xususiy yechimini toping.

► Berilgan tenglamani differensial ko‘rinishda yozib olamiz. ((11.7) formaga qarang.)

$$(1+e^{2x})\,y^{2}\,dy - e^{x}dx = 0$$

Endi o‘zgaruvchilarini ajratamiz.

$$y'dy - \frac{e^{x}}{1+e^{2x}}dx = 0\ \cdot$$

Oxirgi tenglamani integrallaymiz va dastlabki tenglamaning umumiy yechimini hosil qilamiz.

$$\int y'dy - \int \frac{e^{x}}{1+e^{2x}}dx = \frac{C}{2}, \frac{y^{3}}{3} - arctge^{x} = \frac{C}{3},$$

$$y = \sqrt[3]{C + 3arctge^{x}}$$

Boshlang‘ich shartlardan foydalanib ixtiyoriy o‘zgarmasning qiymatini aniqlaymiz.

$$1 = \sqrt[3]{C + \frac{3}{4}\pi}, C = 1 - \frac{3}{4}\pi$$

Shunday qilib, dastlabki tenglamaning xususiy yechimi quyidagi ko‘rinishda bo‘ladi.

Agar har qanday $t\epsilon R$ uchun, $f(tx\,,ty)$ funksiya aniqlangan bo‘lib, α-$const$ va $f(tx,ty) = t^{\alpha}f(x,y)$ tenglik o‘rinli bo‘lsa , *f(x,y)* funksiya, *x* va *y* argumentlarga nisbatan *α o'lchamli bir jinsli funksiya* deyiladi.

Masalan, $f(x,y)=3x^4-x^2y^2+5y^4$ funksiya to'rt o'lchamli ($\alpha = 4$) bir jinsli bo'ladi, chunki $f(tx,ty)= 3\cdot(tx)^4 - (tx)^2+5\cdot(ty)^4= t^4(3x^4-x^2y^2+5y^4)= t^4 f(x,y)$.

$f(x,y)=\sqrt[3]{x^2}-2\sqrt[3]{xy}+4\sqrt[3]{y^2}$ funksiya $f(tx,ty)=\sqrt[3]{(tx)^2}-2\sqrt[3]{(tx)ty)}+4\sqrt[3]{(ty)^2}=\sqrt[3]{t^2}(\sqrt[3]{x^2}-2\sqrt[3]{xy}+4\sqrt[3]{y^2})=t^{2/3}f(x,y)$ bo'lganligidan $\alpha=2/3$ o'lchamli bir jinsli bo'ladi.

Agar $\alpha = 0$ bo'lsa u holda funksiya nol o'lchamli bo'ladi. Masalan, $f(x,y)=\frac{x-y}{x+y}\ln\left(\frac{x^2}{y^2}+1\right)$ – nol o'lchamli bir jinsli funksiya, chunki $f(tx,ty)=\frac{tx-ty}{tx+ty}\ln\left(\frac{(tx)^2}{(ty)^2}+1\right)=\frac{t(x-y)}{t(x+y)}\ln\left(\frac{t^2x^2}{t^2x^2}+1\right)=\frac{x-y}{x+y}\ln\left(\frac{x^2}{y^2}+1\right)=f(x,y)$, bu yerda $t\neq 0$.

Agar *f(x,y)* funksiya o'zining argumentlariga nisbatan nol o'lchamli bir jinsli funksiya bo'lsa, u holda normal ko'rinishdagi qo'yidagi differensial tenglama *x* va *y* o'zgaruvchilarga nisbatan *bir jinsli* deyiladi.

$$y'=\frac{dy}{dx}=f(x,y) \tag{11.11}$$

P(x,y) , *Q(x,y)* funksiyalarning har biri α – o'lchamli bir jinsli funksiya bo'lsa, ya'ni $P(tx,ty)=t^{\alpha}P(x,y), Q(tx,ty)=t^{\alpha}Q(x,y)$ bo'lganda, faqat shu holdagina differensial formadagi $P(x,y)dx+Q(x,y)dy=0$ differensial tenglama bir jinsli bo'ladi.

Haqiqatan ham $f(tx,ty)=\frac{P(tx,ty)}{Q(tx,ty)}=-\frac{t^{\alpha}P(x,y)}{t^{\alpha}Q(x,y)}=f(x,y)$ bo'lganligidan, uni qo'ydagicha normal formada yozib, $y'=-\frac{P(x,y)}{Q(x,y)}=f(x,y)$

f(x,y) funksiya nol o'lchamli bir jinsli funksiya ekan degan xulosaga kelamiz. Bir jinsli differensial tenglama (11.11) ni har doim normal formada $y'=f(x,y)=f(tx,ty)$ ko'rinishida yozish mumkin, u holda $t=1/x$ deb olib, qo'ydagini hosil qilamiz.

$$y' = \frac{dy}{dx} = f\left(1, \frac{y}{x}\right) = \varphi\left(\frac{y}{x}\right)$$

Bundan kelib chiqadiki, $y = xu(u = \frac{y}{x}, y' = u + xu)$ almashtirish yordamida (11.11) tenglama va yangi funksiya $u(x)$ ga nisbatan o'zgaruvchilari ajraladigan tenglamaga keltiriladi.

$u+xu = \varphi(u)$, $x\frac{dU}{dx} = \varphi(u) - u$

3-Misol. $2x^2\, y' = x^2 + y^2$ differensial tenglamani integrallab, uning $y(1)=0$ boshlang'ich shartini qanoatlantiruvchi xususiy yechimni toping.

► $2x^2$ va $x^2 + y^2$ ikki o'lshovli funksiyalar bo'lganligidan, berilgan tenglama ham bir jinslidir.

$y=xu$, $y'=u+xu'$ almashtirish bajaramiz.

U holda, $2x^2(u+xu') = x2 + (xU)2$, $2x2\ (u+xu') = x^2(1+U^2)$.
$x \neq 0$ deb faraz qilib, tenglamaning ikkala qismini $x2$ ga qisqartiramiz. So'ngra quydagiga ega bo'lamiz.

$$2u + 2x\frac{du}{dx} = 1 + u^2, 2xdu = (1 + u^2 - 2u)dx.$$

O'zgaruvchilarni ajratib, qo'ydagilarni topamiz.

$$\frac{du}{1 + u^2 - 2u} = \frac{dx}{2x}, \int \frac{du}{1 + u^2 - 2u} = \int \frac{dx}{2x}, \int \frac{d(u-1)}{(u-1)^2} = \frac{1}{2}\ln|x|,$$

$$-\frac{1}{u-1} = \frac{1}{2}\ln|x| + lnC, 1 = (1 - u)\ln(C\sqrt{|x|})$$

Ohirgi ifodada u ning o'rniga y/x qiymatni qo'yamiz va quydagi umumiy integralni hosil qilamiz.

$$1 = \left(1 - \frac{y}{x}\right)\ln(C\sqrt{|x|}), x = (x - y)ln\sqrt{|x|}$$

Buni y ga nisbatan yechib, dastlabki differensial tenglamaning umumiy yechimini topamiz. $y = x - \frac{x}{\ln(C\sqrt{|x|})}$

$y(1)=0$ boshlang'ich shartdan foydalanib, C ning qiymatini aniqlaymiz.

$$0 = 1 - \frac{1}{lnC}, lnC = 1, C = e.$$

Shunday qilib, dastlabki tenglamaning xususiy yechimi qo'ydagi ko'rinishga ega bo'ladi.

$$y = x - \frac{x}{1 - ln\sqrt{|x|}}$$ ◄

11.1 – AT

1. $y=(x,c)$ funksiya, bu yerda C- ixtiyoriy o‘zgarmas, qo‘ydagi differensial tenglamalarning yechimi (integrali) bo‘lishini aniqlang.

$$a)\ y=x^2(1+ce^{\frac{1}{x}}),\quad x^2y'+(1-2x)y=x^2,$$

$$b)\ y=ce^x+e^{-x},\quad xy''+22y'-xy=0,$$

$$v)\ x^2+y''=cy^2,\ xydx=(x^2-y'')dy.$$

(*Javob: a) ha; b) yo‘q; v) ha.*)

2. Qo‘ydagi differensial tenglamalarning har birining tahminiy integral chiziqlarini chizing va izoklin usulida maydoni yo‘nalishini yasang.

a) $y' = x + y$; *b)* $2x' = y^2/x$; *v)* $xy' = 1 - y$

3. Qo‘ydagi differensial tenglamalarning umumiy yoki xususiy yechimini (umumiy yoki xususiy integralini) toping.

a) $xy' = y^2 + 1$

b) $(x + xy)dy + (y - xy)dx = 0\ ,y(1) = 1;$

v) $3y' = \frac{y^2}{x^2} + 9\frac{y}{x} + 9\ ;$

g) $xy' = y + \sqrt{x^2+y^2}\ ,y(1) = 0$

Mustaqil ish.

1. $y = Cx + 1/c$ funksiya $xy' - y + 1/y = 0$ differensial tenglamalarning yechimi bo‘la oladimi ? (*Javob: yo‘q.*)

2. Qo‘ydagi differensial tenglamaning yechimini toping.

$$4(x^2 + y)dy + \sqrt{5 + y^2}\,dx = 0$$

(*Javob:* $y = \pm\frac{1}{16}((c - arctgx)^2 - 5)$

3. Qo‘ydagi differensial tenglama uchun Koshi masalasini yeching.

$$xy' = xsin\frac{y}{x} + y\ ,y(2) = \pi$$

(*Javob:* $y = 2xarctg\ (x/2)$.)

1. 1 $e^{y/x} = Cy$ tenglama bilan oshkormas holda berilgan $y = y(x)$ funksiya $xyy' - y^2 = xy$ differensial tenglamaning integrali bo‘la oladimi? (*Javob: ha.*)

2. Quydagi differensial tenglamaning umumiy integralini toping? $ydx + \left(\sqrt{xy} - \sqrt{x}\right)dy = 0.$ (*Javob:* $\sqrt{x} + \sqrt{y} = lnC\sqrt{y}$(c> 0).)

3. $ydx + \left(\sqrt{xy} - x\right)dy = 0$, $y(1) = 1$ differensial tenglama uchun Koshi masalasini yeching? (*Javob:* $2 - \ln|y| = 2\sqrt{y/x}.$)

2. 1 $y = \frac{2+Cx}{1+2x}$ funksiya differensial tenglamaning yechimi bo'ladimi? (*Javob: ha)*

2. Differensial tenglamaning umumiy yechimini toping? $(1 + e')y' = ye^x$ (*Javob:* $y = c(1 + e^x)$.)

3. $xy' = y(1 + lny - lnx)$, $y(1) = e^2$ differensial tenglama uchun Koshi masalasini yeching ? (*Javob:* $y = xe^{2x}$)

11.3 Birinchi tartibli chiziqli differensial tenglamalar. Bernulli tenglamasi

Noma'lum funksiya y va uning hosilasi y' ga nisbatan chiziqli bo'lgan quydagi tenglama

$$y' + P(x)y = Q(x) \qquad (11.13)$$

(algebraik shakl almashtirishlar yordamida (11.13) ko'rinishga keltirish mumkin bo'lgan har qanday tenglamalar ham) birinchi tartibli bir jinsli bo'lmagan differensial tenglama deb ataladi. Yechimi mavjudligi va yagonaligi haqidagi Koshi teoremasining shartlari bajarilishi uchun $P(x) \neq 0$ va $Q(x) \neq 0$ funksiya biror sohada, masalan $[a, b]$ kesmada o'zluksiz bo'lishi kerak. (11.11 dagi 2 teoremaga qarang.) (11.13) ko'rinishdagi tenglamaning umumiy yechimini har doim quydagi ko'rinishda yozish mumkin

$$y = e^{-\int P(x)dx}(\int Q(x)e^{\int P(x)dx}dx + C \qquad (11.14)$$

bu yerda C – ixtiyoriy o'zgarmas. Shunday qilib (11.13) tenglamaning umumiy yechimi har doim P(x) va Q(x) ma'lum funksiyalarning integrallar orqali ifodalanadi. (11.14) tenglamadagi integrallarni hisoblayotganda ixtiyoriy o'zgarmaslarni, ixtiyoriy o'zgarmas C ning ichiga kirgan degan farazda nolga tenglab olish mumkin.

Agar (11.13) tenglamada $Q(x) \equiv 0$ yoki $P(x) \equiv 0$ bo'lsa, u holda, umumiy yechimi $Q(x) \equiv 0$ yoki $P(x) \equiv 0$ bo'lgan holda mos ravishda (11.14) tenglamadan aniqlanuvchi o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz. $Q(x) = 0$ bo'lgan holda (11.13) tenglama, bir jinsli tenglama deb ataladi.

1-misol $(x^2 - x)y' + y = x^2(2x - 1)$. Tenglamani umumiy yechimini toping? Koshi masalasini $y(-2) = 2$ boshlang'ich shartda yeching.

►Berilgan tenglamaning ikkala qismini $x^2 - x \neq 0$ ga bo'lib, (11.13) ko'rinishga keltiramiz.

$$y' + \frac{y}{x^2-x} = \frac{x^2(2x-1)}{x^2-x},$$

Bu yerda $P(x) = \frac{1}{x^2-x} = \frac{1}{x(x-1)}, Q(x) = \frac{x^2(2x-1)}{x(x-1)} = \frac{x(2x-1)}{x-1},$

(11.4) formulaga asosan dastlabki tenglamaning umumiy yechimi quyidagi ko'rinishda bo'ladi.

$$y = e^{-\int\frac{dx}{x(x-1)}}\left(\int \frac{x(2x-1)}{x-1} e^{\int\frac{dx}{x(x-1)}}dx + C\right) \quad (11.15)$$

Bu yechimga kiradigan integrallarni topamiz. Quyidagiga ega bo'lamiz. $\int\frac{dx}{x(x-1)} = \left|\frac{A}{x} + \frac{B}{x-1} = \frac{1}{x(x-1)}, A = -1, B = 1\right| = \int\left(-\frac{1}{x} + \frac{1}{x-1}\right)dx = -\ln|x| + +\ln|x-1| = \ln\left|\frac{x-1}{x}\right|$

$\int\frac{x(2x-1)}{x-1}e^{\ln\left|\frac{x-1}{x}\right|}dx = \int\frac{x(2x-1)}{x-1}\left|\frac{x-1}{x}\right|dx = \pm\int(2x-1)dx = \pm(x^2 - x),$

Bu yerda modul ichida $\left|\frac{x-1}{x}\right| = \pm\frac{x-1}{x}$ tenglamaga ko'ra «+» va «-» belgilari paydo bo'ladi.

Topilgan integralni (11.15) ga qo'yib, dastlabki tenglamani umumiy yechimini topamiz.

$$y = e^{-\ln\left|x-\frac{1}{x}\right|}(\pm(x^2-x)+C) = \left|\frac{x}{x-1}\right|(\pm(x^2-x)+C)$$

$$= \pm\frac{x}{x-1}(\pm x(x--1)+C) = x^2 + \frac{Cx}{x-1}$$

Bundan $y(-2) = 2$ boshlang'ich shartga mos keluvchi xususiy yechimni ajratamiz.

$2 = 4 - \frac{2C}{-2-1}. C = 3, y = x^2 - \frac{3x}{x-1}$.◄

Ba'zida differensial tenglama y ning funksiyasi bo'lgan, x ga nisbatan ham chiziqli bo'lishini bilish foydadan holi emas, ya'ni quyidagi ko'rinishga keltirilishi ham mumkin.

$$\frac{dx}{dy} + P(y)x = q(y) \qquad (11.16)$$

Buning umumiy yechimi quyidagi formula bo'yicha topiladi.

$$x = e^{-\int P(y)dy}\left(\int q(y)e^{\int P(y)dy}\,dy + C\right)$$

2-misol. $y' = \frac{dy}{dx}$, $(2x - y^2)y' = 2y$ tenglamaning umumiy integralini toping.

►Berilgan tenglama $x(y)$ funksiyaga nisbatan chiziqlidir. Haqiqatan ham, $(2x - y^2)\frac{dy}{dx} = 2y$, $2x - y^2 = 2y\frac{dx}{dy}$, $\frac{dx}{dy} = \frac{x}{y} - \frac{y}{2}$, $\frac{dx}{dy} - \frac{x}{y} = -\frac{y}{2}$, $P(y) = -\frac{1}{y}$, $q(y) = -\frac{y}{2}$

ya'ni (11.16) ko'rinishidagi tenglamani hosil qildik. (11.17) formulaga asosan, dastlabki tenglama quyidagi ko'rinishiga ega.

$$x = e^{\int\frac{dy}{y}}\left(\int \frac{y}{2}e^{-\int\frac{dy}{y}}dy + C\right) = e^{\ln|y|}\left(-\int \frac{y}{2}e^{-\ln|y|}dy + C\right) =$$

$$|y|\left(-\frac{1}{2}\int\frac{y}{|y|}dy + C\right) = -\frac{1}{2}\int dy + Cy = Cy - \frac{1}{2}y^2$$ ◄

(11.13) chiziqli differensial tenglamani Bernulli usulida ham integrallash mumkin. Bu usulning ma'nosi quyidagicha: $y = u(x)\vartheta(x)$ formula bo'yicha (o'rniga quyishning Bernulli usuli) $u(x)$ va $\vartheta(x)$ ikkita noma'lum funksiya kiritamiz. U holda $y' = u'\vartheta + u\vartheta$. y va y' lar uchun hosil qilingan ifodalarni (11.13) tenglamaga quyib, $u'\vartheta + u\vartheta' + P(x)u\vartheta = Q(x)$ tenglamani hosil qilamiz. Bu tenglamani quyidagi ko'rinishda yozish mumkin.

$$(\vartheta' + P(x)\cdot\vartheta)u + u'\vartheta = Q(x) \qquad (11.18)$$

Noma'lum funksiyalardan birini, masalan, ϑ ni ixtiyoriy tanlashimiz mumkin (chunki dastlabki (11.13) tenglamani, faqat $u\cdot\vartheta$ ko'paytma qanoatlantirishi kerak). ϑ funksiya sifatida (11.18) tenglamadagi u ning koeffitsientini nolga aylantiruvchi $\vartheta' + P(x)\vartheta = 0$ tenglamaning ixtiyoriy xususiy yechimi $\vartheta = \vartheta(x)$ ni tanlaymiz. Shundan so'ng (11.18) tenglama $u'v = Q(x)$ ko'rinishga keladi. Bu tenglamaning $u = u(x, C)$ umumiy yechimini topamiz. So'ngra (11.13) tenglamaning $y=u(x,C)\cdot v(x)$ ko'rinishdagi umumiy yechimini hosil qilamiz. Shunday qilib, (11.13) tenglamani integrallash o'zgaruvchilari ajraladigan ikkita tenglamani integrallashga keltiriladi.

3–Misol. Quyidagi tenglamani

$$y'+ytgx=\frac{1}{\cos x}$$

Bernulli usulida integrallang va $y(\pi)=1$ boshlang'ich shartda Koshi masalasini yeching.

► O'rniga qo'yishning Bernulli usulidan foydalanamiz. $y=uv,\quad y'=u'v+uv'$ va quyidagiga ega bo'lamiz.

$$u'v+uv'+uvtgx=\frac{1}{\cos x}.$$

$$(v'+vtgx)u+u'v=\frac{1}{\cos x}.$$

$v'+vtgx=0$ tenglamaning xususiy yechimini topamiz.

$$dv+vtgxdx=0,\qquad \frac{dv}{v}+tgxdx=0\qquad ,$$

$\int\frac{dv}{v}+\int tgxdx=0,\ \ \ln|v|-\ln|\cos x|=\ln c_1\ \ .\ \ c_1=1$ deb olib, $v=\cos x$ xususiy yechimni tanlaymiz. Endi, $u'v=1/\cos x$, bu yerda $v=\cos x$, tenglamaning umumiy yechimini izlaymiz. Quyidagiga ega bo'lamiz

$$u'=\frac{1}{\cos^2 x},\quad u=\int\frac{dx}{\cos^2 x}+c=tgx+c.$$

Dastlabki tenglamning umumiy yechimi quyidagicha bo'ladi: $y=u\cdot v=(tgx+c)\cdot\cos x$. Bundan $y(\pi)=1,\ 1=(0+c)(-1),\ \ c=-1$ boshlang'ich shartlarni qanoatlantiruvchi, xususiy yechimni ajratib olamiz. $c=-1$ qiymatni umumiy yechimga qo'yib, dastlabki tenglamaning xususiy yechimini hosil qilamiz:

$y=(tgx-1)\cos x=\sin x-\cos x$. ◀

Quyidagi ko'rinishdagi differensial tenglama

$$y'+P(x)y=Q(x)y^{\alpha}. \qquad (11.19)$$

bu yerda $\alpha=const\in R,\ \alpha\neq 0,\ \alpha\neq 1$, shuningdek, algebraik shakl almashtirishlar yordamida (11.19) tenglamaga keltiriluvchi har qanday tenglama, Bernulli tenglamasi deyiladi.

$Z=y^{1-\alpha}$ formula bo'yicha yangi $Z(x)$ funksiya kiritish yo'li bilan Bernulli tenglamasi shu funksiyaga nisbatan chiziqli tenglamaga keltiriladi:

$$Z'+(1-\alpha)P(x)Z=(1-\alpha)Q(x). \qquad (11.20)$$

Oxirgi tenglamani yuqorida keltirilgan biror bir usul bilan yechib, $Z=Z(x)$ ni topamiz, so'ngra $y=Z^{1/(1-\alpha)}$ ni topamiz.

Bernulli tenglamasini, (11.13) chiziqli tenglama kabi, $y=u(x)\cdot v(x)$ Bernullining o'rniga qo'yish usuli yordamida ham yechish mumkin (3-misolga qarang).

4-misol. Bernulli tenglamasining umumiy yechimini toping.

$y'+2e^{x}y=2e^{x}\sqrt{y}$.

▶ Berilgan tenglama uchun $\alpha=1/2$ bo'lgani uchun, $Z=y^{1-\alpha}=\sqrt{y}$ almashtirishni bajaramiz. (11.20) tenglamaga asosan, $Z'+e^{x}Z=e^{x}$ tenglamani hosil qilamiz. Bu tenglamaning umumiy yechimi (11.14) formulaga asosan quyidagi ko'rinishda bo'ladi.

$$Z=e^{-\int e^{x}dx}\left(\int e^{x}e^{\int e^{x}dx}dx+c\right)=e^{-e^{x}}\left(\int e^{x}e^{e^{x}}dx+c\right)=$$

$$=e^{-e^{x}}\left(\int e^{e^{x}}de^{x}+x\right)=e^{-e^{x}}\left(e^{e^{x}}+c\right)=1+ce^{-e^{x}}.$$

Dastlabki tenglamaning umumiy yechimi quyidagicha bo'ladi.

$$y=Z^{2}=\left(1+ce^{-e^{x}}\right)^{2}. \ ◀$$

5-misol. Tenglamaning umumiy yechimini toping.

$xy' + y = xy^2 \ln x$.

► Berilgan tenglamaning ikkala qismini $x \neq 0$ ga bo‘lib, yuborib, $\alpha = 2$ bo‘lgan Bernulli tenglamasini hosil qilamiz. Uni Bernullining o‘rniga qo‘yish usuli bilan yechamiz. $(y = uv,\ y' = u'v + uv')$:

$$x(u'v + uv') + uv = x(uv)^2 \ln x.$$

$xv' + v = 0$ tenglamaning xususiy yechimi $v = x^{-1}$ ni osongina topamiz. Endi $xvu' = xu^2v^2 \ln x$ tenglamaning, bu yerda $v = x^{-1}$, ya'ni $u' = v^2 \dfrac{\ln x}{x}$ tenglamaning, umumiy yechimini topishimiz kerak. Oxirgi tenglamda o‘zgaruvchilarini ajratamiz va uni integrallab quyidagini hosil qilamiz.

$$\frac{du}{u^2} = \ln x \frac{dx}{x}, \quad \int \frac{du}{u^2} = \int \ln x \frac{dx}{x},$$

$$-\frac{1}{u} = \frac{\ln^2 x}{2} + \frac{c}{2}, \quad u = -\frac{2}{c + \ln^2 x}.$$

Shunday qilib, dastlabki tenglamaning umumiy yechimi quyidagicha bo‘ladi. $y = uv = \dfrac{2}{x(c + \ln^2 x)}$. ◄

11.2. AT

1. Differensial tenglamalarning turlarini aniqlang va ularni yechish usullarini ko‘rsating.

a) $xy' + 2\sqrt{xy} = y$;

b) $y' \cos x = \dfrac{y}{\ln y}$;

v) $y' = \dfrac{y}{2x \ln y + y - x}$;

g) $(1 + e^{2x}) y^2 dy - e^x dx = 0$;

d) $y' = e^{2x} - e^x y$;

e) $xy' + y - y^2 = 0$;

j) $2x \cos^2 y dx + (2y - x^2 \sin 2y) dy = 0$;

z) $y^2 + x^2 y' = xyy'$

2. Differensial tenglamaning umumiy yechimini toping.

a) $y' + \dfrac{y}{x} = 1 + 2\ln x$ b) $y' + 4xy = 2xe^{-x^2}\sqrt{y}$.

(*Javob:* a) $y = x\ln x + c/x$; b) $y = \pm e^{-x^2}\left(c + x^2/2\right)$.)

3. Koshi masalasini yeching.

a) $2xydx + \left(y + x^2\right)dy = 0, \quad y(-2) = 4;$

b) $y' = 2y - x + e^x, \quad y(0) = -1$.

(*Javob:* a) $x^2 - y\ln(4e/y)$; b) $y = \dfrac{1}{2}x - e^x + \dfrac{1}{4}\left(1 - e^{2x}\right)$.)

Mustaqil ish

Koshi masalasini yeching.

1. a) $y' + 3y = e^{2x}y^2, \quad y(0) = 1;$

b) $y' + ytgx = 1/\cos x, \quad y(\pi) = 5$.

(*Javob:* a) $y = e^{-2x}$; b) $y = -5\cos x + \sin x$.)

2. a) $y^2dx = \left(x + ye^{-1/y}\right)dy, \quad y(0) = -3;$

b) $y' - 7y = e^{3x}y^2, \quad y(0) = 2$.

(*Javob:* a) $x = e^{-1/y}(3 - y)$; b) $y = 10e^{7x}/\left(e^{10x} - 6\right)$.)

3. a) $xdy = \left(e^{-x} - y\right)dx, \quad y(1) = 1;$

b) $y' - \dfrac{y}{x-3} = \dfrac{y^2}{x-3}, \quad y(1) = -2$.

(*Javob:* a) $y = \dfrac{1}{x}\left(1 + \dfrac{1}{e} - \dfrac{1}{e^2}\right)$; b) $y = \dfrac{x-3}{2-x}$.)

11.4. TO‘LA DIFFERENSIAL TENGLAMALAR

Agar D sohada $P(x,y), Q(x,y)$ funksiyalarning aniqlanishidan va (11.21) tenglamaning mavjudligidan quyidagi tenglik bajarilsa,

$$\frac{\partial P(x,y)}{\partial y}=\frac{\partial Q(x,y)}{\partial x}. \tag{11.22}$$

u holda

$$P(x,y)dx+Q(x,y)dy=0. \tag{11.21}$$

ko‘rinishidagi tenglama *to‘la differensialli tenglama* deyiladi.

(11.21) tenglamaning umumiy integrali quyidagi formulalarning biri bilan aniqlanadi.

$$\int_{x_0}^{x} P(x,y_0)dx+\int_{y_0}^{y} Q(x,y)dy=c, \tag{11.23}$$

$$\int_{x_0}^{x} P(x,y)dx+\int_{y_0}^{y} Q(x_0,y)dy=c. \tag{11.24}$$

bu yerda $M_0(x_0,y_0)\in D$.

Misol. Tenglamaning umumiy integralini toping.

$(x^2+y-4)dx+(x+y+e^y)dy=0$.

► $P=x^2+y-4,\ Q=x+y+e^y$ belgilashlar kiritamiz. $\frac{\partial P}{\partial y}=1,\ \frac{\partial Q}{\partial x}=1$, ya’ni (11.22) shart bajariladi, u holda berilgan tenglama to‘la differensialli tenglama bo‘ladi. Soddalik uchun $x_0=0,\ y_0=0$ deb olib, uning umumiy integralini (11.23) yoki (11.24) formulalar orqali topish mumkin. x_0, y_0 qiymatlarni tanlashimiz o‘rinli, chunki bu nuqtada $P(x,y)$ va $Q(x,y)$ funksiyalar va ularning xususiy hosilalari aniqlangan, ya’ni $M_0(0;0)\in D$. (11.23) formulaga asosan quyidagiga ega bo‘lamiz.

$$\int_0^x (x^2+0-4)\,dx+\int_0^y (x+y+e^y)\,dy=c,$$

$$\frac{x^3}{3}-4x+xy+\frac{y^2}{2}+e^y-1=c.$$

Umumiy integralni (11.24) formula bo'yicha topamiz.

$$\int_0^x (x^2+y-4)\,dx+\int_0^y (0+y+e^y)\,dy=c,$$

$$\frac{x^3}{3}+xy-4x+\frac{y^2}{2}+e^y-1=c.$$

Bu yechim avval topilgan yechim bilan ustma-ust tushadi. ◄

11.3. Auditoriya topshiriqlari

1. Differensial tenglamalarning umumiy integralini toping.

a) $(e^x+y+\sin y)dx+(e^y+x+x\cos y)dy=0;$

b) $(2x+e^{x/y})dx+\left(1-\frac{x}{y}\right)e^{x/y}dy=0;$

v) $y'=(y-3x^2)/(4y-x).$

(*Javob:* a) $e^x+e^y+xy+x\sin y=c;$ b) $x^2+ye^{x/y}=c;$ v) $x^3-xy+2y^2=c$.)

2. Koshi masalasini yeching.

a) $e^{-y}dx+(2y-xe^{-y})dy=0,\quad y(-3)=0;$

b) $xdx+ydy=(xdy-ydx)/(x^2+y^2),\quad y(1)=1;$

v) $x+ye^x+(y+e^x)y'=0,\quad y(0)=4.$

(*Javob:* a) $xe^{-y}+y^2+3=0;$

b) $\frac{1}{2}(x^2+y^2)+arctg\frac{x}{y}=1+\frac{\pi}{4};$ v) $x^2+y^2+2ye^x=24$.)

3. Ixtiyoriy M nuqtasiga o‘tkazilgan urinmaning burchak koeffitsienti, M nuqtani koordinata boshi bilan tutashtiruvchi to‘g‘ri chiziqning burchak koeffitsientidan uch marta katta ekanligini bilgan holda $A(2,4)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini toping (*Javob*: $y = \frac{1}{2}x^3$.).

4. Nyuton qonuniga ko‘ra, jismning sovush tezligi jism va atrof-muhit temperaturalari ayirmasiga proporsional. Pechkadan olingan non temperaturasi 20 minut davomida 100^0 C dan 60^0 C ga kamayadi. Havoning temperaturasi 25^0 C. Qanday vaqt oralig‘ida (sovish boshlanishidan hisoblab) nonning temperaturasi 30^0 C ga pasayadi. (*Javob: 71 min.)*

Mustaqil ish

1. Koshi masalasini eching.

$$\left(2x + y + 3x^2 \sin y\right)dx + \left(x + x^3 \cos y + 2y\right)dy = 0;$$

$y(0) = 2$

(*Javob:* $x^2 + xy + \frac{1}{2}y^2 + x^3 \sin y = 2$.)

2. Boshlang‘ich tezligi $v(0) = 0$ bo‘lgan m massali jism yuqoridan tushmoqda. Agar jismga $p = mg$ og‘irlik kuchidan tashqari, proporsionallik koeffitsienti 3/2 ga teng, $v(t)$ tezlikka proporsional havoning qarshilik kuchi ham ta’sir qilsa, jismning ixtiyoriy t vaqtdagi $v = v(t)$ tezligini toping.

(*Javob:* $v = \frac{2}{3}mg\left(1 - e^{3/2\, t/m}\right)$.)

2. 1. Differensial tenglamaning umumiy integralini toping.

$\left(3x^2 y + \sin x\right)dx + \left(x^3 - \cos y\right)dy = 0$.(*Javob:* $x^3 y - \cos x - \sin y = c$.)

2. Radiyning tarqalish tezligi uning tarqalmagan soniga proporsional. Agar 1600-yilda radiyning dastlabki miqdorining yarmi tarqalishi aniq bo'lsa, 1 kg radiydan 650 g qolishi uchun necha yil kerakligini hisoblang.

(*Javob:* 1000-yildan keyin.)

3. 1. Differensial tenglamaning xususiy yechimini toping.

$$\left(2x\ln y+\frac{y^2}{\cos^2 x}\right)dx+\left(\frac{x^2}{y}+tgx+e^y\right)dy=0,\ y(0)=1$$

.(*Javob:* $x^2\ln y+ytgx+e^y=e$.)

2. Agar to'g'ri chiziqning ixtiyoriy nuqtasiga o'tkazilgan urinmasining O_y o'qidan ajratgan kesmasi, urinish nuqtasidan koordinata boshigacha bo'lgnan masofaga teng bo'lsa, $A(1,0)$ nuqtadan o'tuvchi shu to'g'ri chiziq tenglamasini yozing. (*Javob:* $y=\frac{1}{2}(1-x^2)$)

11.5. TARTIBI PASAYTIRILADIGAN YUQORI TARTIBLI DIFFERENSIAL TENGLAMALAR

Tartibi pasaytiriladigan yuqori tartibli differensial tenglamalarning ba'zi turlarini ko'rib chiqamiz.

$$y^{(n)}=f(x). \tag{11.25}$$

ko'rinishdagi tenglamaning umumiy yechimini n – marta integrallash orqali topamiz. Uning ikkala qismini dx ga ko'paytiramiz va integrallaymiz, natijada, $(n-1)$ tartibli tenglamani hosil qilamiz.

$$y^{(n-1)}=\int y^{(n)}dx=\int f(x)dx=\varphi_1(x)+\tilde{c}_1. \tag{11.26}$$

Shu amalni takrorlab, $(n-2)$ tartibli tenglamaga ega bo'lamiz.

$$y^{(n-2)} = \int y^{(n-1)}dx = \int(\varphi_1(x)+\tilde{c}_1)dx = \int\varphi_1(x)dx + \int\tilde{c}_1 dx = \varphi_2(x)+\tilde{c}_1 x+\tilde{c}_2$$
. (11.27)

n – marta integrallashdan keyin esa, (11.25) tenglamaning quyidagi umumiy yechimini hosil qilamiz.

$$y = \varphi_n(x) + c_1x^{n-1} + c_2x^{n-2} + \ldots + c_{n-1}x + c_n \,. \quad (11.28)$$

bu yerda $c_i\left(i=\overline{1,n}\right)$, $\tilde{c}_1,\tilde{c}_2,\ldots,\tilde{c}_n$ – ixtiyoriy o'zgarmaslar bilan ma'lum ma'noda bog'liq bo'lgan, ixtiyoriy o'zgarmaslardir.

1-misol. $y^{IV} = 8/(x-3)^5$ tenglamaning umumiy yechimini toping.

(11.26) formulaga va integrallash qoidalariga asosan, quyidagiga ega bo'lamiz:

$$y''' = \int y^{IV}dx = \int\frac{8dx}{(x-3)^5} = -\frac{2}{(x-3)^5} + \tilde{c}_1 \,.$$

(11.27) yechimga mos ravishda quyidagini topamiz.

$$y'' = \int y'''dx = \int\left(-\frac{2}{(x-4)^4} + \tilde{c}_1\right)dx = \frac{2}{3(x+3)^3} + \tilde{c}_1 x + \tilde{c}_2 \,.$$

Oxirgi tenglikni yana ikki marta integrallab, dastlabki tenglamaning umumiy yechimini hosil qilamiz:

$$y' = \int y''dx = \int\left(\frac{2}{3(x-3)} + \tilde{c}_1 x + \tilde{c}_2\right)dx = -\frac{1}{3(x-3)^2} + \frac{1}{2}\tilde{c}_2 x^2 + \tilde{c}_2 x + \tilde{c}_3 \,.$$

$$y = \int y'dx = \int\left(-\frac{1}{3(x-3)^2} + \frac{1}{2}\tilde{c}_1 x^2 + \tilde{c}_2 x + \tilde{c}_3\right)dx =$$

$$= \frac{1}{3(x-3)} + \frac{1}{6}\tilde{c}_1 x^3 + \frac{1}{2}\tilde{c}_2 x^2 + \tilde{c}_3 x + \tilde{c}_4 = \frac{1}{3(x-3)} + c_1x^3 + c_2x^2 + c_3x + c_4 \,.$$

◄

II. Faraz qilamiz, n – tartibli differensial tenglama izlanayotgan funksiya va uning $(k-1)$ – tartibli hosilalarini o'z ichiga olmasin. $(1 \le k \le n)$:

$$F\left(x, y^k,\ y^{(k+1)},...,y^{(n)}\right)=0. \qquad (11.29)$$

$Z(x)=y^k$ formula bo'yicha yangi noma'lum $Z(x)$ funksiyani kiritamiz va $y^{(k+1)}=Z',\ y^{(k+2)}=Z'',...,y^{(n)}=Z^{(n-1)}$ ekanligini e'tiborga olib, $Z(x)$ funksiyaga nisbatan $(n-k)$ – tartibli tenglamaga ega bo'lamiz.

$$F\left(x, Z, Z', Z'',...,Z^{(n-k)}\right)=0. \qquad (11.30)$$

ya'ni (11.29) tenglamaning tartibini k ga pasaytiramiz. Agar (11.30) tenglamaning umumiy yechimini $Z=\varphi(x,c,...,c_{n-k})$ ko'rinishida izlashga erishsak, yechimi k – marta integrallash bilan topiladigan (11.25) ko'rinishdagi quyidagi tenglamani hosil qilamiz.

$$Z=y^{(k)}=\varphi(x,c_1,c_2,...,c_{n-k}).$$

Xususiy holda, agar $n=2,\ k=1$ bo'lsa, u holda (11.30) tenglama birinchi tartibli tenglama bo'ladi.

2-misol. Tenglamaning xususiy yechimini toping.

$$xy''=y'\ln\frac{y'}{x}\cdot y(1)=e,\quad y'(1)=e^2.$$

► Berilgan tenglama II tur tenglama bo'ladi. $(n=2, k=1)$, ya'ni tarkibida u qatnashmaydi. $Z=y'$ ni qo'yib, bu tenglamaning tartibini bittaga pasaytiramiz. U holda $y''=Z''$ bo'ladi va berilgan tenglama izlanayotgan Z funksiyaga nisbatan birinchi tartibli bir jinsli differensial tenglamaga aylanadi.

$$xZ'=Z\ln(Z/x). \qquad (1)$$

Bu tenglamani ma'lum usullardan biri bilan yechamiz. $Z=xu(x)$ o'rniga qo'yishini bajaramiz. U holda $Z'=u+xu'$ bo'ladi va (1) tenglama quyidagi ko'rinishga keladi

$$x+xu'=u\ln x. \qquad (2)$$

(2) tenglamada o'zgaruvchilarni ajratamiz, ketma-ket quyidagilarni topamiz.

$$\frac{du}{u(\ln u-1)}=\frac{dx}{x},\quad \ln|\ln u-1|=\ln x+\ln c_1 \qquad ;$$

$$\ln u-1=c_1x,\ u=e^{1+c_1x},\quad Z=xe^{1+c_1x}.$$

$Z=y$ ekanligidan, oxirgi tenglama bir marta integrallab yechiladigan birinchi tartibli differensial tenglama bo'ladi.

$$y'=xe^{1+c_1x},\ y=\int xe^{1+c_1x}dx=\frac{1}{c_1}\int xd\left(e^{1+c_1x}\right)=$$

$$=\frac{1}{c_1}\left(xe^{1+c_1x}-\int e^{1+c^1x}dx\right)=\int\frac{c_1x-1}{c_1^2}e^{1+c_1x}+c_2.$$

Dastlabki tenglamaning umumiy yechimini hosil qildik. $y(1)=e,\ \ y'(1)=e^2$ boshlang'ich shartlardan foydalanib, c_1 va c_2 ixtiyoriy o'zgarmaslarning qiymatlarini aniqlaymiz. Quyidagi tenglamalar sistemasiga ega bo'lamiz.

$$e=\frac{c_1-1}{c_1^2}e^{1+c_1}+c_2,\quad e^2=e^{1+c_1}.$$

Bundan, $c_1=1,\quad c_2=e$ ekanligini topamiz.

Shunday qilib, dastlabki tenglamaning xususiy yechimi quyidagi formuladan topiladi.

$$y=(x-1)e^{1+x}+e. \blacktriangleleft$$

3-misol. $y'''ctgx+y''=2$ tenglamaning umumiy yechimini toping.

► Bu yerda $n=3,\ \ k=2$ bo'lganligidan berilgan tenglama II turdagi tenglama bo'ladi. $Z=y''$ yangi funksiya kiritamiz va berilgan tenglamadan $Z'+Ztgx=2tgx$ ko'rinishda yoziladigan, ya'ni $Z'ctgx+Z=2$ chiziqli tenglamani hosil qilamiz. Uning umumiy yechimi (§ 11.2 qarang)

$$Z = e^{-\int tgx dx}\left(\int 2tgx e^{\int tgx dx} dx + c_1\right) = e^{\ln|\cos x|} \times$$

$$\times\left(2\int tgx e^{-\ln|\cos x|} dx + c_1\right) = |\cos x|\left(2\int \frac{tgx}{|\cos x|} dx + c_1\right) =$$

$$= 2\cos x \int \frac{\sin x}{\cos^2 x} dx + c_1 \cos x = 2\cos x \cdot \frac{1}{\cos x} + c_1 \cos x = 2 + c_1 \cos x$$

$Z = y''$ bo‘lganligidan, yechimini oson bo‘lgan, quyidagi I tip differensial tenglamani hosil qilamiz.

$$y'' = 2 + c_1 \cos x, \quad y' = \int (2 + c_1 \cos x) dx = 2x + c_1 \sin x + c_1 .$$

$$y' = \int (2x + c_1 \sin x + c_2) dx = x^2 - c_1 \cos x + c_2 x + c_3 . \blacktriangleleft$$

III. x argumentni oshkor holda o‘z ichiga olmagan n – tartibli differensial tenglamani qaraymiz.

$$F\left(y, y', y'', \dots, y^{(n)}\right) = 0. \tag{11.31}$$

Bu holda har doim, $P(y) = y'$ yangi funksiya kiritib, tenglamaning tartibini bittaga pasaytirish mumkin, bu yerda y uning argumenti sifatida qaraladi. Buning uchun $y', y'', \dots, y^{(n)}$ larni yangi funksiyaning y bo‘yicha hosilalari orqali ifodalaymiz va murakkab funksiyani differensiallash qoidasidan foydalanib quyidagini hosil qilamiz.

$$y' = \frac{dy}{dx} = p; \quad y'' = \frac{dp}{dx} = \frac{dp}{dy} \cdot \frac{dy}{dx} = p\frac{dp}{dy}. \tag{11.32}$$

$$y''' = \frac{dy''}{dx} = \frac{d}{dx}\left(p\frac{dp}{dy}\right) = \frac{dp}{dx} \cdot \frac{dp}{dy} + p\frac{d^2p}{dxdy} = p\left(\frac{dp}{dy}\right)^2 + p^2\frac{d^2p}{dy^2}.$$

va h.k. Yuqoridagi hisoblashlardan ko‘rinib turibdiki, $y^{(k)}$, tartibi $k-1$ dan oshmaydigan, p va y larning hosilalari orqali ifodalanadi. Natijada, (11.31) tenglama o‘rniga quyidagi ko‘rinishdagi tenglamani hosil qilamiz.

$$\Phi\left(y, p, \frac{d^2 p}{dy^2}, \ldots, \frac{d^{(n-1)} p}{dy^{n-1}}\right) = 0. \qquad (11.33)$$

Agar (11.33) tenglama quyidagicha umumiy yechimga ega bo‘lsa, $p = \varphi(y, c_1, c_2, \ldots, c_{n-1})$, bu yerda $p = \frac{dy}{dx}$, u holda (11.31) tenglamaning umumiy integralini topish uchun, oxirgi tenglamaning o‘zgaruvchilarini ajratib yechish kerak

$$\int \frac{dy}{\varphi(y, c_1, c_2, \ldots, c_{n-1})} = \int dx, \; \psi(y, c_1, c_2, \ldots, c_{n-1}) = x + c_n.$$

Agar (11.31) tenglamada $n = 2$ bo‘lsa, u holda (11.33) tenglama birinchi tartibli tenglama bo‘ladi.

4-misol. Koshi masalasini yeching.

$$y^3 y' y'' + 1 = 0, \quad y(1) = 1, \quad y'(1) = \sqrt[3]{3/2}.$$

► Bu tenglama, $n = 2$ va x argument oshkor holda qatnashmaganidan, III tip tenglama bo‘ladi. Shuning uchun, (11.32) formulaga asosan, $P(y) = y'$ almashtirishni bajarib, uning tartibini bittaga kamaytiramiz va yechilishi oson bo‘lgan o‘zgaruvchilari ajraladigan birinchi tartibli tenglamani hosil qilamiz. Quyidagiga ega bo‘lamiz.

$$y^2 p^2 \frac{dp}{dy} + 1 = 0, \quad p^2 dp = -y^{-3} dy, \; \int p^2 dp = -\int y^{-3} dy,$$

$$\frac{p^3}{3} = \frac{1}{2}\frac{1}{y^2} + c_1, \quad p = \sqrt[3]{\frac{3}{2}\frac{1}{y^2} + 3c_1}.$$

$p = y' = \frac{dy}{dx}$ ekanligini e’tiborga olib, oxirgi tenglamani quyidagi ko‘rinishda qayta yozib olamiz.

$$y' = \sqrt[3]{\frac{3}{2}\frac{1}{y^2} + 3c_1}. \qquad (1)$$

Bu tenglamani yechishdan avval, boshlang'ich shartlardan foydalanib, c_1 ixtiyoriy o'zgarmasning qiymatini aniqlaymiz. Ularni (1) tenglamaga qo'yib, quyidagini hosil qilamiz.

$$\sqrt[3]{\frac{3}{2}}=\sqrt[3]{\frac{3}{2}+3c_1}, \quad c_1=0.$$

Shunday qilib, o'zgaruvchilarini ajratish yo'li bilan oson yechiladigan, $y'=\left(\frac{3}{2}y^2\right)^{1/3}$ tenglamaga kelamiz.

$$dy=\left(\frac{3}{2}y^2\right)^{1/3}dx, \quad \frac{dy}{\left(\frac{3}{2}y^2\right)}=dx\cdot$$

$$\sqrt[3]{\frac{3}{2}}\int y^{-2/3}dy=\int dx\cdot$$

$$\sqrt[3]{\frac{2}{3}}\cdot 3y^{1/3}=x+c_2, \quad y=\frac{(x+c_2)^3}{18}.$$

$y(1)=1$ boshlang'ich shartidan c_2 ni topamiz.

$$1=(1+c_2)^3/18, \quad c_2=\sqrt[3]{18}-1.$$

bundan kelib chiqadiki, izlanayotgan xususiy yechim quyidagi formuladan topiladi.

$$y=\frac{1}{18}\left(x+\sqrt[3]{18}-1\right)^3. \blacktriangleleft$$

5-misol. Koshi masalasini yeching.

$$y'''-(y'')^2/y'=6(y')^2y, \quad y(2)=0, \quad y'(2)=1, \quad y'''(2)=0$$

► $n=3$ bo'lgan, (11.31) ko'rinishdagi tenglamaga ega bo'lamiz. (11.32) tenglamaga mos ravishda, $p(y)$ yangi funksiyani kiritamiz va ketma-ket quyidagilarni topamiz.

$$p^2\frac{d^2p}{dy^2}+p\left(\frac{dp}{dy}\right)^2-\left(p\frac{dp}{dy}\right)^2/p=6p^2y;$$

$$p^2\left(\frac{d^2p}{dy^2}-6y\right)=0 \qquad (p=0).$$

bu yerdan $\frac{d^2p}{dy^2}=6y$. Bu tenglama birinchi tur tenglama bo'lib, ikki marta integrallash yo'li bilan oson yechiladi.

$$\frac{dp}{dy}=\int 6ydy=3y^2+c_1, \quad p=\int\left(3y^2+c_1\right)dy=y^3+c_1y+c_2,$$

$$y'(2)=p(0)=1, \; y''(2)=p(0)\frac{dp(0)}{dy}=0.$$

munosabatlarni va boshlang'ich shartlarni hisobga olib, $y'=y^3+c_1y+c_2$ tenglamani hosil qildik, bundan, $c_1=0, \quad c_2=1$ ekanligini topamiz.

Endi $y'=y^3+1$ tenglamani integrallaymiz.

$$\frac{dy}{dx}=y^3+1, \quad \frac{dy}{y^3+1}=dx, \quad \int\frac{dy}{y^3+1}=\int dx,$$

$$\frac{1}{\sqrt{3}}arctg\frac{2y-1}{\sqrt{3}}+\frac{1}{3}\ln\frac{|y+1|}{\sqrt{y^2-y+1}}=x+c_3.$$

$y(2)=0$ boshlang'ich shartlardan foydalanib, $c_3=-2-\frac{\pi}{6\sqrt{3}}$ ni topamiz. Bundan quyidagi izlanayotgan xususiy yechimni hosil qilamiz.

$$x=\frac{1}{\sqrt{3}}arctg\frac{2y-1}{\sqrt{3}}+\frac{1}{3}\ln\frac{|y+1|}{\sqrt{y^2-y+1}}+2+\frac{\pi}{6\sqrt{3}}. \blacktriangleleft$$

11.4. AT

1. Quyidagi tenglamalarni integrallang.

a) $y'''=x^2-\sin x$; b) $y^{IV}=y'''/x$; v) $yy''=y'^2$.

2. Koshi masalasini yeching.

a) $y''=\frac{\ln x}{x^2},\ y(1)=3,\ y'(1)=1;$

b) $xy'''-y^1=x^2+1,\ y(1)=0,\ y'(-1)=1,\ y''(-1)=0;$

v) $y''=e^{2y},\ y(0)=0,\ y'(0)=1.$

3. Avtomobil yo'lning gorizontal qismida $v=90$ km/s tezlikda harakterlanmoqda. Vaqtning biror qismida sekinlashishni boshlaydi. Sekinlashish kuchi avtomobil og'irligining 0,3 qismiga teng.

Sekinlashishi boshlanishidan bekatgacha bo'lgan masofani va bu masofani bosib o'tish uchun ketgan vaqtni toping.

(*Javob: 8,5 sek; 106,3 m.*)

Mustaqil ish

1. 1.Tenglamani integrallang. $x^2y'''=y''^2$.

2.. Koshi masalasini yeching. $2y'^2=(y-1)y'',\quad y(0)=0,\quad y'(0)=1$

2. 1.Tenglamani integrallang. $xy''-y'=x^2e^x$

2. Koshi masalasini yeching. $y^3y''+1=0,\ y(1)=1,\quad y'(1)=0$

3. 1.Tenglamani integrallang. $xy''+y'=y'^2$.

3. Koshi masalasini yeching. $2y''=3y^2,\ y(2)=1,\ y'(2)=-1$

11.6. IKKINCHI VA YUQORI TARTIBLI CHIZIQLI DIFFERENSIAL TENGLAMALAR

Umumiy hol. Quyidagi ko'rinishdagi tenglamalarni

$$y^{(n)}+a_1(x)y^{(n-1)}+a_2(x)y^{(n-2)}+\ldots+a_{n-1}(x)y'+a_n(x)y=f(x)\quad (11.34)$$

bu yerda $a_i(x)\left(i=\overline{1,n}\right)$, $f(x)$ – biror D sohada berilgan funksiyalar n -tartibli *bir jinsli bo'lmagan chiziqli differensial tenglamalar* deyiladi. Agar D sohada (11.34) tenglamaning o'ng tomoni $f(x)=0$ bo'lsa, u holda (11.34) tenglamaga mos keluvchi, *bir jinsli chiziqli differsial tenglama* deb ataluvchi quyidagi tenglamani hosil qilamiz.

$$y^{(n)}+a_1(x)y^{(n-1)}+a_2(x)y^{(n-2)}+...+a_{n-1}(x)y'+a_n(x)y=0. \quad (11.35)$$

Agar $a_i(x)$, $f(x)$ funksiyalar D sohadagi (a,b) oraliqda uzluksiz bo'lsa, u holda $y(x_0)=y_0$, $y'(x_0)=y_0',...,$ $y^{(n-1)}\left(x_0\right)=y_0^{(n-1)}$, $x_0\in(a,b)$) (bu yerda $y_0,y_0',...,y_0^{(n-1)}$ – ixtiyoriy sonlar) boshlang'ich shartlar bilan berilgan (11.34), (11.35) ko'rinishdagi har qanday tenglamalar uchun yechimning mavjudligi va yagonaligi haqidagi Koshi teoremasi o'rinlidir.

(11.34) va (11.35) ko'rinishdagi tenglamalarning umumiy va xususiy yechimlarini topishda $y_1(x)$, $y_2(x),...,y_n(x)$ funksiyalarning o'zaro *chiziqli bog'liq* yoki *chiziqli bog'liq emasligi* tushunchalari muhim ahamiyatga ega.

Agar bir vaqtda hammasi nolga teng bo'lmagan $\mu_1,\mu_2,...,\mu_n$ o'zgarmas sonlar uchun ixtiyoriy $x\in(a,b)$ da $\sum_{i=1}^{n}\mu_i y_i(x)=0$ munosabat o'rinli bo'lsa, $y_1,y_2,...,y_n$ funksiyalar (a,b) oraliqda o'zaro *chiziqli bog'liq* deyiladi. Agar yuqoridagi munosabat faqat barcha – $\mu_i=0$ lar uchun o'rinli bo'lsa, u holda $y_i(x)$ funksiyalar (a,b) oraliqda o'zaro chiziqli bog'liq bo'lmagan funksiyalar deyiladi.

Quyidagi ko'rinishdagi determinant Vrnoskiy determinanti (yoki vronskian) deb ataladi.

$$W=(y_1,y_2,\ldots,y_n)=\begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \cdots & \cdots & \cdots & \cdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}. \qquad (11.36)$$

Funksiyalarning chiziqli bog'liq yoki chiziqli bog'liq bo'lmaslik mezonlari.

Agar $y_i^{(x)}\left(i=\overline{1,n}\right)$ *funksiyalar* $c^{(n-1)}$ *fazolar sinfida* (a,b) *oraliqda o'zaro bog'liq bo'lsa, (ya'ni,* (a,b) *oraliqda* $(n-1)$ *tartibgacha uzluksiz hosilaga ega bo'lgan funksiyalar bo'lsa), u holda* (a,b) *oraliqda* $W\equiv 0$ *bo'ladi. Agar* $W\neq 0$ *bo'lsa, u holda* $y_i(x)$ *funksiyalar chiziqli bog'liq bo'lmaydi.*

Masalan: $1,x,x^2,\ldots,x^{n-1}$ funksiyalar uchun $W\neq 0$, shuning uchun ular chiziqli bog'liq bo'lmagan funksiyalardir.

(11.35) tenglamaning chiziqli bog'liq bo'lmagan n ta $y_1(x),\ y_2(x),\ldots,y_n(x)$ yechimlar to'plami fundamental yechimlar sistemasi deb ataladi. Uning yordamida (11.35) ko'rinishdagi bir jinsli tenglamaning umumiy yechimi tuziladi. Quyidagi teorema o'rinlidir.

1-teorema. *Agar* $y_1,y_2,\ldots,y_n$ *lar (11.35) tenglamaning ixtiyoriy fundamental yechimlari sistemasi bo'lsa, u holda quyidagi funksiya (11.35) tenglamaning umumiy yechimi bo'ladi.*

$$\overline{y}=c_1y_1+c_2y_2+\ldots+c_ny_n=\sum_{i=1}^{n}c_iy_i(x), \qquad (11.37)$$

bu yerda c_i *– ixtiyoriy o'zgarmas.*

1-misol. e^x,e^{-x},e^{2x} funksiyalar sistemasi $y'''-2y''-y'+2y=0$ tenglamaning fundamental yechimi ekanligini ko'rsating va uning umumiy yechimini yozing.

► $y_1=e^x,\ y_2=e^{-x},\ y_3=e^{2x}$ funksiyalar va ularning hosilalarini berilgan tenglamaga qo'ysa, ularning tenglamaning yechimlari ekanligini ko‘ramiz. Ularning vronskiani quyidagi ko‘rinishiga ega bo‘ladi (11.36).

$$W\left(e^x,e^{-x},e^{2x}\right)=\begin{vmatrix} e^x & e^{-x} & e^{2x} \\ e^x & -e^{-x} & 2e^{2x} \\ e^x & e^{-x} & 4e^{2x} \end{vmatrix}=e^x e^{-x} e^{2x}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 4 \end{vmatrix}=-6e^{2x}\neq 0.$$

Bundan kelib chiqadiki, $e^x e^{-x} e^{2x}$ funksiyalar chiziqli bog‘liqmas va ular berilgan tenglamaning fundamental yechimlar sistemasini tashkil qiladi. Uning umumiy yechimi, (11.37) formulaga asosan, quyidagi ko‘rinishga ega.

$$y=c_1e^x+c_2e^{-x}+c_3e^{2x}. \blacktriangleleft$$

2-teoerma. ((11.34) tenglamaning umumiy yechimining tuzilishi haqida). *Chiziqli bir jinsli bo‘lmagan (11.34) tenglamaning umumiy yechimi quyidagi ko‘rinishga ega.* $y=y+y^*$, *bu yerda* y *– unga mos keluvchi bir jinsli (11.35) tenglamaning (11.37) ko‘rinishdagi umumiy yechimi* y^* *esa (11.34) tenglamaning xususiy yechimlaridan biri.*

2-misol. Xususiy yechimlaridan biri $y^*=x+1$ funksiyadan iborat bo‘lgan, $y'''-2y''-y'+2y=2x+1$ tenglamaning umumiy yechimini yozing.

► 1-misolda berilgan tenglamaga mos bir jinsli tenglamaning y umumiy yechimi topilgan edi, u holda berilgan tenglamaning umumiy yechimi quyidagicha bo‘ladi.

$$y=y+y^*=c_1e^x+c_2e^{-x}+c_3e^{2x}+x+1. \blacktriangleleft$$

Agar (11.35) tenglamaning fundamental yechimlar sistemasi ma’lum bo‘lsa, u holda (11.34) tenglamaning y^* xususiy yechimini har qanday holatda ham *ixtiyoriy o‘zgarmaslarni variatsiyalash usuli (Lagranj usuli)* bilan topish mumkin va y^* har doim quyidagi ko‘rinishda ifodalanadi.

$$y^*=c_1(x)y_1(x)+c_2(x)y_2(x)+...+c_n(x)y_n(x). \quad (11.38)$$

bu yerda $y_i(x)$ (11.35) tenglamaning fundamental yechimlari sistemasini tashkil qiladi, c_i noma'lum funksiyalar esa quyidagi n – ta c_i' noma'lumlarga nisbatan chiziqli algebraik tenglamalar sistemasidan topiladi.

$$\left.\begin{array}{llll} c_1'y_1 + & c_2'y_2 + \ldots + & c_n'y_n = & 0 \\ c_1'y_1' + & c_2'y_2' + \ldots + & c_n'y_n' = & 0 \\ \ldots & \ldots & \ldots & \ldots \\ c_1'y_1^{(n-1)} + & c_2'y_2^{(n-2)} + \ldots + & c_n'y_n^{(n-1)} = & 0 \end{array}\right\}.$$

Sistemaning determinanti, $y(x)$ fundamental yechimlar sistemasi noldan farqli bo'lgan holda, Vronskiy determinanti bo'ladi. ((11.36) ga qarang). Shuning uchun (11.39) sistema $c_i' = \varphi_i(x)$ ko'rinishdagi yagona yechimga ega. Bu birinchi tartibli differensial tenglamani integrallab, $c_i(x) = \int \varphi_i(x)dx$ ni topamiz.

Shunday qilib, (11.34) tenglamaning y^* xususiy yechimi quyidagicha bo'ladi.

$$y^* = y_1\int \varphi_1(x)dx + y_2\int \varphi_2(x)dx + \ldots + y_n\int \varphi_n(x)dx. \quad (11.40)$$

1-eslatma. (11.40) formuladan integrallarni topishda n ta ixtiyoriy o'zgarmaslar paydo bo'ladi. Ularni nolga teng deb hisoblash mumkin.

3-misol. Quyidagi tenglamaning umumiy yechimini toping.

$$y''' - 2y'' - y' + 2y = \frac{2x}{e^x + 1}. \quad (1)$$

► (1) tenglamaga mos, bir jinsli tenglamaning umumiy yechimi quyidagicha bo'ladi.

$$y = c_1e^x + c_2e^{-x} + c_3e^{2x}.$$

(1-misolga qarang.) (1) tenglamaning umumiy yechimini topish uchun, Lagranj usulida uning y^* xususiy yechimini topamiz. (11.38) formulaga asosan quyidagiga ega bo'lamiz.

$$y^* = c_1(x)e^x + c_2(x)e^{-x} + c_3(x)e^{2x}$$

(11.39) sistema bizning misolimizda quyidagi ko‘rinishda bo‘ladi.

$$\left.\begin{aligned} &c_1'e^x + c_2'e^{-x} + c_3'e^{2x} = 0, \\ &c_1'e^x - c_2'e^{-x} + 2c_3'e^{2x} = 0, \\ &c_1'e^x + c_2'e^{-x} + 4c_3'e^{2x} = e/\left(e^x + 1\right). \end{aligned}\right| \qquad (2)$$

Buning determinanti $W = -6e^{2x} \neq 0$ (1-misolga qarang). (2) sistemani Kramer usuli bilan yechib, quyidagilarni topamiz.

$$c_1' = -\frac{1}{2}\cdot\frac{e^x}{e^x+1}, \quad c_2' = \frac{1}{6}\frac{e^{3x}}{e^x+1}, \quad c_3' = \frac{1}{3}\frac{1}{e^x+1}, \qquad (3)$$

(3) ifodani integrallab, quyidagilarni hosil qilamiz (1-eslatmaga qarang).

$$c_1 = -\frac{1}{2}\int\frac{e^x dx}{e^x+1} = -\frac{1}{2}\int\frac{d\left(e^x+1\right)}{e^x+1} = -\frac{1}{2}\ln\left(e^x+1\right),$$

$$c_2 = \frac{1}{6}\int\frac{e^{3x}dx}{e^x+1} = \frac{1}{6}\int\frac{e^{2x}d\left(e^x\right)}{e^x+1} = \frac{1}{6}\int\left(e^x - 1 + \frac{1}{e^x+1}\right)de^x = \frac{1}{6}\left(\frac{e^{2x}}{2} - e^x + \ln\left(e^x+1\right)\right),$$

$$c_3 = \frac{1}{3}\int\frac{dx}{e^x+1} = \frac{1}{3}\int\frac{e^x+1-e^x}{e^x+1}dx = \frac{1}{3}\left(1 - \frac{e^x}{e^x+1}\right)dx =$$

$$= \frac{1}{3}\left(x - \int\frac{d\left(e^x+1\right)}{e^x+1}\right) = \frac{1}{3}\left(x - \ln\left(e^x+1\right)\right).$$

(1) tenglamaning xususiy yechimini yozamiz.

$$y^* = -\frac{1}{2}e^x\ln\left(e^x+1\right) + \frac{1}{6}e^{-x}\left(\frac{1}{2}e^{2x} - e^x + \ln\left(e^x+1\right) + \frac{1}{3}e^{2x}\left(x - \ln\left(e^x+1\right)\right)\right) =$$

$$= \frac{1}{12}e^x - \frac{1}{6} + \frac{1}{3}xe^{2x} + \left(\frac{1}{6}e^{-x} - \frac{1}{2}e^x - \frac{1}{3}e^{2x}\right)\ln\left(e^x+1\right).$$

(1) tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi.

$$y = y + y^* = c_1e^x + c_2e^{-x} + c_3e^{2x} + \frac{1}{12}\left(4xe^{2x} + e^x - 2\right) + \frac{1}{6}\left(e^{-x} - 2e^{2x}\right)\ln\left(e^x+1\right)$$

. ◄

2-eslatma. (11.35) tenglamaning fundamental echimlar sistemasini topish usullari mavjud emas. Shuning uchun umumiy holda (11.34) tenglamaning y^* xususiy yechimini topish mumkin emas, demak uning umumiy yechimini ham topish mumkin emas. (11.34) tenglamani yechimining boshqa usullari mavjud emas. Faqat xususiy holda, (11.34) tenglamaning barcha $a_i(x)$ koeffitsientlari o'zgarmas sonlar bo'lganda, (11.34) tenglamaning fundamental yechimlari sistemasini va umumiy yechimini topish usuli mavjud.

O'zgarmas koeffitsientli chiziqli differensial tenglamalar

(11.34) va (11.35) tenglamalarga $a_i(x) = p_i = const \in R$ ni qo'yamiz. U holda mos ravishda quyidagilarga ega bo'lamiz.

$$y^{(n)} + p_1 y^{(n-1)} + p_2 y^{(n-2)} + \ldots + p_{n-1} y' + p_n y = f(x). \quad (11.41)$$

$$y^n + p_1 y^{(n-1)} + p_2 y^{(n-2)} + \ldots + p_{n-1} y' + p_n y = 0. \quad (11.42)$$

(11.42) tenglamaning fundamental echimlar sistemasini, faqat algebraik usullardan foydalanib, quyidagicha topish mumkin. (11.42) tenglamaning *xarakteristik tenglamasi* deb ataluvchi quyidagi algebraik tenglamani tuzamiz:

$$\lambda^n + p_1 \lambda^{n-1} + p_2 \lambda^{n-2} + \ldots + p_{n-1} \lambda + p_n = 0. \quad (11.43)$$

Bu tenglama n ta ildizga ega bo'lib, bular orasida oddiy va karrali haqiqiy ildizlar, shuningdek qo'shma-kompleks (oddiy va karrali) ildizlar ham bo'lishi mumkin.

Agar (11.43) harakteristik tenglamaning barcha λ_i ildizlari haqiqiy va oddiy bo'lsa, u holda (11.42) tenglamaning quyidagi fundamental yechimlar sistemasini hosil qilamiz.

$$e^{\lambda_1 x}, e^{\lambda_2 x}, \ldots, e^{\lambda_n x}. \quad (11.44)$$

Ma'lumki, (11.43) harakteristik tenglamasining har bir k karrali ildiziga (11.42) tenglamaning quyidagi ko'rinishidagi, chiziqli bo'lmagan yechimlari mos keladi.

$$y_1 = e^{\lambda x}, \quad y_2 = xe^{\lambda x}, \ldots, y_k = x^{k-1} e^{\lambda x}. \quad (11.45)$$

(11.43) harakteristik tenglamaning m karrali qo‘shma-kompleksi har bir $\alpha \pm \beta_i$ juft ildizga quyidagi ko‘rinishdagi (11.42) tenglamaning o‘zaro chiziqli bog‘liq bo‘lmagan $2m$ ga teng yechimlari mos keladi.

$$\begin{aligned} \bar{y}_1 &= e^{\alpha x}\cos\beta x, & \bar{y}_2 &= e^{\alpha x}\sin\beta x, \\ \bar{y}_3 &= xe^{\alpha x}\cos\beta x, & \bar{y}_4 &= xe^{\alpha x}\sin\beta x, \\ \bar{y}_5 &= x^2e^{\alpha x}\cos\beta x, & \bar{y}_5 &= x^2e^{\alpha x}\sin\beta x, \\ &\ldots\ldots\ldots\ldots & &\ldots\ldots\ldots\ldots \\ \bar{y}_{2m-1} &= x^{m-1}e^{\alpha x}\cos\beta x, & \bar{y}_{2m} &= x^{m-1}e^{\alpha x}\sin\beta x. \end{aligned} \qquad (11.46)$$

Yuqoridagilarni umumlashtirib, quyidagiga ega bo‘lamiz. (11.43) ning harakteristik tenglamasining ildiziga bir jinsli (11.42) tenglamaning, ixtiyoriy koeffitsientlar bilan chiziqli kombinatsiya (11.37) formulaga asosan, (11.42) tenglamaning umumiy yechimini beruvchi, fundamental yechimlar sistemasini hosil qiluvchi, n ta chiziqli bog‘liq bo‘lmagan yechimi mos keladi.

4-misol. O‘zgarmas koeffitsientli 4-tartibli bir jinsli chiziqli tenglamaning umumiy yechimini toping.

$$y^{IV} - 16y = 0.$$

► Berilgan tenglamaning harakteristik tenglamasini tuzamiz va uning ildizlarini topamiz. $\lambda^4 - 16 = 0$, $(\lambda^2 - 4)(\lambda^2 + 4) = 0$, $\lambda^2 = 4$, $\lambda_2 = \pm 2$, $\lambda^2 = -4$, $\lambda_{3,4} = \pm 2i$ 2 tasi haqiqiy va 2 tasi qo‘shma-kompleksli, 4 ta ildizni hosil qildik $(\alpha = 0, \beta = 2)$ bo‘lgan (11.44) – (11.46) xususiy yechimlarni e’tiborga olib, quyidagi undamental yechimlar sistemasini hosil qilamiz:

$$y_1 = e^{2x},\ y_2 = e^{-2x},\ y_3 = e^{0x}cos2x = \cos 2x,\ y_4 = e^{0x}\sin 2x = \sin 2x$$

(11.37) formulaga asosan, berilgan tenglamaning umumiy yechimi quyidagi ko‘rinishga ega bo‘ladi.

$$y = c_1e^{2x} + c_2e^{-2x} + c_3\cos 2x + c_4\sin 2x. \blacktriangleleft$$

Agar (11.42) tenglamada $n=2$ bo'lsa, u holda *o'zgrmas koeffitsientli 2 tartibli bir jinsli chiziqli differensial tenglama*ni hosil qilamiz.

$$y'' + p_1 y' + p_2 y = 0 . \qquad (11.47)$$

Uning xarakteristik tenglamasi

$$\lambda^2 + p_1\lambda + p^2 = 0 . \qquad (11.48)$$

ko'rinishda bo'ldi.

Bu tenglamaning ildizlari quyidagicha bo'lishi mumkin.

a) haqiqiy va turli: $\lambda_1 \neq \lambda_2$

b) haqiqiy va o'zaro teng: $\lambda_1 = \lambda_2 = \lambda$

v) qo'shma kompleksli: $\lambda_{1,2} = \alpha \pm \beta i$.

Ularga (11.47) tenglamaning quyidagi fundamental yechimlar sistemasi va umumiy yechimlar mos keladi.

1. $y_1 = e^{\lambda_1 x}$, $y_2 = e^{\lambda_1 x}$, $y = c_1 e^{\lambda_1 x} + c_2 c^{\lambda_2 x}$;

2. $y_1 = e^{\lambda x}$, $y_2 = xe^{\lambda x}$, $y = c_1 e^{\lambda x} + c_2 xe^{\lambda x}$;

3. $y_1 = e^{\alpha x}\cos\beta x$, $y_2 = e^{\alpha x}\sin\beta x$, $y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$.

5-misol. Quyidagi tenglamalarning umumiy yechimini toping.

a) $y'' - 15y' + 26y = 0$;

b) $y'' + 6y' + 9y = 0$;

v) $y'' - 2y' + 10y = 0$.

► Har bir tenglama uchun xarakteristik tenglamasini tuzamiz va uning ildizlarini, fundamental yechimlar sistemasini va umumiy yechimlarini topamiz.

a). $\lambda^2 - 15\lambda + 26 = 0$, $\lambda_1 = 2$, $\lambda_2 = 13$,

$$y_1 = e^{2k}, \ y_2 = e^{13x},$$

$$= c_1 e^{2x} + c_2 e^{13x}.$$

б) $\lambda^2+6\lambda+9=0, \quad \lambda_1=\lambda_2=-3;$

$$y_1=e^{-3x},\ y_2=xe^{-3x}$$

$$y=e^{-3x}\left(c_1+c_2x\right).$$

в) $\lambda^2-2\lambda_2+10=0, \quad \lambda_{1.2}=1\pm 3i$

$$y_1=e^{x}\cos 3x,\ \ y_2=e^{x}\sin 3x.$$

$y=e^{x}\left(c_1\cos 3x+c_2\sin 3x\right).$ ◄

Shunday qilib, o'zgarmas koeffitsientli chiziqli tenglamalarni yechish uchun quyidagilar zarurdir:

1. mos fundamental yechimlar sistemasini topish;

2. bir jinsli (11.42) tenglamaning y umumiy yechimini tuzish;

3. Lagranj usuli bo'yicha (11.41) tenglamaning y^* xususiy yechimini topish;

4. $y=y+y^*$ formula bo'yicha (11.41) tenglamaning y umumiy yechimini hosil qilish;

Turli injenerlik masalalarini yechishda (11.41) tenglamaning $f(x)$ o'ng qismi ko'p hollarda quyidagi maxsus ko'rinishga ega bo'ladi.

$$f(x)=e^{ax}\left(p_r(x)\cos bx+Q_S(x)\sin bx\right), \qquad (11.49)$$

bu yerda $p_r(x)$, $Q_S(x)$ – mos ravishda r va s darajali ko'phadlar. a,b – biror o'zgarmas sonlar $f(x)$ funksiyaning xususiy hollari quyidagicha bo'ladi.

$$f(x)=P_r(x)e^{ax}(b=0); \qquad (11.50)$$

$$f(x)=P_r(x)\cos bx+Q_S(x)\sin bx(\alpha=0), \qquad (11.51)$$

$$f(x)=e^{ax}\left(A\cos bx+B\sin bx\right)\left(A=const,\ B-const\right). \quad (11.52)$$

$$f(x)=A\cos bx+B\sin bx\left(a=0,\ P_r(x)=A,\ Q(x)=B\right): \quad (11.53)$$

$$f(x)=P_r(x)=P_r(x)\quad (a=0,\ b=0). \qquad (11.54)$$

Bu hollarning barchasida, shuningdek, umumiy holda ((11.49) formulaga qarang), (11.41) tenglamaning xususiy yechimi, bu o‘ng qismlarning tuzilishiga aynan o‘xshashligi isbotlangan. Umumiy hol uchun $f(x)$ funksiyaning ko‘rinishi quyidagicha bo‘ladi.

$$y^*=x^k e^{\alpha x}\left(P_m(x)\cos\beta x+Q_m(x)\sin\beta x\right), \qquad (11.55)$$

bu yerda $P_m(x)$, $Q_m(x)-m=\max(r,s)$ darajali ko‘phadlar; k (11.43) $z=\alpha+\beta_i$ – songa mos keluvchi xarakteristik tenglamaning ildizlari soniga teng.

Shunday qilib, agar $\lambda_i\left(i=\overline{1,n}\right)$ ildizlar orasida Z son bo‘lmasa, $k=0$ agar Z bilan mos keluvchi bitta ildiz mavjud bo‘lsa, $k=1$, agar Z son bilan mos keluvchi ikki karrali ildiz mavjud bo‘lsa, $k=2$, va h.k. Bundan kelib chiqadiki, (11.55) formulaga asosan, faqatgina $P_m(x)$ va $Q_m(x)$ ko‘phadlarning koeffitsientlarigina noma’lum bo‘lgan, y^* xususiy yechimning tuzilishini birdaniga aniqlash mumkin ekan. (11.44) tenglamaga y^* yechimni va uning hosilalarini qo‘yib, o‘ng va chap tomonlarining o‘xshash koeffitsientlarini tenglashtirib, shu noma’lum kojeffitsientlarni topish uchun yyetarlicha sondagi chiziqli algebraik tenglamalarni hosil qilamiz. Koeffitsientlarni va y^* ni bunday usulda topish, *aniqmas koeffitsientlar usuli* deb ataladi. Bundan kelib chiqadiki, y^* ning tuzilishini bilgan holda ((11.55) formulaga qarang), tenglamani Lagranj usulida yechishda hosil bo‘luvchi integrallash amalini qo‘llamasdan, differensiallash va chiziqli tenglamalar sistemasini yechish kabi elementar amallar yordamida xususiy yechimni topish mumkin ekan.

6-misol. Tenglamaning umumiy yechimini toping.

$$y^{IV}-3y''=9x^2. \quad (1)$$

► Xarakteristik tenglamasini tuzamiz va uning ildizini, fundamental yechimlar sistemasini, bir jinsli tenglamaga mos keluvchi y umumiy yechimini topamiz.

$$\lambda^4-3\lambda^2=0,\quad \lambda^2\left(\lambda^2-3\right)=0,\ \lambda_1=\lambda_2=0,\ \lambda_3=\lambda_4=\pm\sqrt{3};$$

$$y_1=e^{0x}=1,\quad y_2=xe^{0x}=x,\quad y_3=e^{\sqrt{3}x},\ y_4=e^{-\sqrt{3}x},$$

$$y=c_1+c_2x+c_3\sqrt{3}x+c_1e^{-\sqrt{3}x}.$$

(1) tenglamaning o'ng tomoni, maxsus (11.54) xususiy holga tegishli, shuning uchun $Z=0$. Xarakteristik tenglamaning ikki karrali $\lambda_1=\lambda_2=0$ ildizlari, $Z=0$ bilan ustma-ust tushadi, bundan $k=2$ ekanligi kelib chiqadi. (1) tenglamaning o'ng tomoni ikkinchi darajali ko'phad bo'lganligi uchun (11.55) formulaga asosan, $y*$ xususiy yechim quyidagi ko'rinishga ega bo'ladi:

$$y*=x^2\left(Ax^2+Bx+c\right),$$

$y*$ larni (1) tenglamaga qo'yib, ayniyat hosil qilamiz ($y*-(1)$ tenglamaning yechimi).

Bu yerda va keyinchalik hisoblash qulay bo'lishi uchun $y*, y*', y*'', y*''', y*^{IV},...$ ifodalarning har birini alohida qator yozamiz va vertikal chiziqning chap tomoniga tenglamadagi ularning oldida turgan koeffitsientlarni mos ravishda joylashtiramiz. Bu ifodalarni koeffitsientlarga ko'paytirib, qo'shib va o'xshash hadlarni ixchamlab quyidagiga ega bo'lamiz:

0	$y*=Ax^4+Bx^3+cx^2,$
0	$y*'=4Ax^3+3Bx^2+2x$
-3	$y*''=12Ax^2+6Bx+2c,$
0	$y*'''=24Ax+6B,$
1	$y*^{IV}=24A.$

$$y*^{IV}-3y*'=-36A^2-18Bx+6c+24A\equiv 9x^2$$

Oxirgi ayniyatning o'ng va chap tomonlaridagi x ning bir xil darajalari oldidagi koeffitsientlarini tenglab, A, B, C larni aniqlash uchun algebraik tenglamalar sistemasini hosil qilamiz:

$$\begin{array}{l|l|} x^2 & -36A = 9, \\ x^1 & -18B = 0, \\ x^0 & -6C + 24A = 0. \end{array}$$

bu yerdan $A = -1/4,\ B = 0,\ C = -1$. Bundan kelib chiqadiki,

$$y'^* = x^2\left(-\frac{1}{4}x^2 - 1\right).$$

(1) tenglamaning umumiy yechimi quyidagicha bo'ladi.

$$y = y + y^* = c_1 + c_2x + c_3e^{\sqrt{3x}} + c_4e^{-\sqrt{3x}} - \frac{1}{4}x^4 - x^2. \blacktriangleleft$$

7-misol. Koshi masalasini eching

$$y'' - 7y' + 6y = (x-2)e^x,\ y(0) = 1,\ y'(0) = 3. \ (1)$$

► Xarakteristik tenglamasi $\lambda_1 = 1,\ \lambda_2 = 6$ yechimlarga ega, u holda $y'' - 7y' + 6y = 0$ bir jinsli tenglamaga mos keluvchi tenglamaning umumiy yechimi quyidagicha bo'ladi.

$$y = c_1e^x + c_2e^{6x}.$$

(1) tenglamaning o'ng tomoni, (11.50) ko'rinishdagi maxsuslikka ega, bu yerda $\alpha = 1,\ \beta = 0;\ P_1(x) = x - 2, r = 1.$ r xarakteristik tenglamaning ildizi bo'ladi, u holda $k = 1$ va (1) tenglamaning xususiy yechimi quyidagi formuladan topiladi.

$y^* - xe^x(Ax + B)$. (2)

Endi, 6-misoldagi kabi, quyidagilarni topamiz:

$$\begin{array}{r|l} 6 & y^* = e^x\left(Ax^2 + Bx\right) \\ -7 & y^{*\prime} = e^x\left(Ax^2 + Bx\right) + e^x(2Ax + B), \\ 1 & y^{*\prime\prime} = e^x\left(Ax^2 + (2A + B)x + B\right) + e^x(2Ax + 2A + B). \end{array}$$

$$y^{**}-7y^{*}+6y^{*}=e^{x}\left(\left(6A-7A+A\right)x^{2}+\left(6B-7B-14A+2A+B+2A\right)x7B+2A+2B\right)=e^{x}\left(x-2\right)$$

Oxirgi ayniyatning ikkali tomomnini $e^{x}\neq 0$ ga bo'lib yuboramiz va o'ng va chap tomonlaridagi x ning bir xil darajalari oldidagi koeffitsientlarini tenglab, quyidagiga ega bo'lamiz.

$$\begin{array}{l|l} x^{2} & 0=0, \\ x^{1} & -10A=1, \\ x^{0} & 2A-5B=-2. \end{array}$$

bu yerdan $A=-1/10,\quad B=9/25$, quyidagi funksiya (1) tenglamaning umumiy yechimi bo'ladi.

$$y=y+y^{*}=c_{1}e^{x}+c_{2}e^{6x}+e^{x}\left(-\frac{1}{10}x^{2}+\frac{9}{25}x\right).$$

Koshi masalasini yechish uchun y' ni topamiz.

$$y'=c_{1}e^{x}+6c_{2}e^{x}+e^{x}\left(-\frac{1}{10}x^{2}+\frac{9}{25}x\right)+e^{x}\left(-\frac{1}{5}x+\frac{9}{25}\right).$$

Boshlang'ich shartlardan foydalanib, c_1 va c_2 ixtiyoriy o'zgarmaslarning qiymatlarini aniqlash uchun chiziqli tenglamalar sistemasini hosil qilamiz.

$$y(0)=c_{1}+c_{2}=1,\quad y'(0)=c_{1}+6c_{2}+9/25=3.$$

Bu yerdan $c_{1}=84/125,\quad c_{2}=41/125$ larni topamiz.

Shunday qilib, berilgan boshlang'ich shartlarni qanoatlantiruvi xususiy yechimning ko'rinishi quyidagicha bo'ladi.

$$y=\frac{84}{125}e^{x}+\frac{41}{125}e^{6x}+e^{x}\left(-\frac{1}{10}x^{2}+\frac{9}{25}x\right).\ ◄$$

(11.41) ko'rinishdagi chiziqli differensial tenglamalar uchun ma'nosi quyidagicha bo'lgan, *yechimlarning superpozitsiya prinsipi* o'rinlidir. Agar (11.41) tenglamada $f(x)=f_{1}(x)+f_{2}(x)$ bo'lib, $y_{1}^{*}(x)$ va $y_{2}^{*}(x)$ lar o'ng tomoni mos ravishda $f_1(x)$ va $f_2(x)$ bo'lgan quyidagi (11.41) ko'rinishdagi tenglamani $y_1^*(x)$ va $y_2^*(x)$ lar o'ng tomoni mos ravishda $f_1(x)$ va $f_2(x)$ bo'lgan quyidagi (11.44) ko'rinishdagi tenglamalarning yechimlari bo'lsa,

$$y^{(n)} + P_1 y^{(n-1)} + ... + P_n y = f_1(x). \tag{11.56}$$

$$y^{(n)} + P y^{(n-1)} + ... + P_n y = f_2(x). \tag{11.57}$$

u holda $y^* = y_1^* + y_2^*$ funksiya o'ng tomoni $f(x)$ bo'lgan (11.41) tenglamaning yechimi bo'ladi.

$f_1(x)$ va $f_2(x)$ funksiyalar ((11.49) ko'rinishdagi faqat turli turdagi (11.50)-(11.54)) maxsus ko'rinishda bo'lish mumkin. U holda, har bir turdagiga qo'llash mumkin bo'lgan va aniqmas koeffitsientlar usulida (11.56), (11.57) tenglamalarning xususiy yechimlarini topish imkonini beruvchi (11.55) ko'rinishdagi xususiy yechimning tuzilishidan foydalanish mumkin. Shu bilan birgalikda $f_1(x)$ – maxsus ko'rinishda, $f_2(x)$ esa maxsus ko'rinishda bo'lasligi mumkin. Bunday hollarda, (11.41) tenglamaning xususiy yechimi y^* ni, Lagranj usulidan foydalanib birdanig topish mumkin yoki ikkita bosqichga bo'lib, (11.56) tenglamani yechish uchun (11.55) ning tuzilishidan foydalanib, (11.57) tenglamani yechish uchun esa Lagranj usulidan foydalanib topish mumkin.

8-misol. Tenglamaning umumiy yechimini toping.

$$y'' + y = x \sin x + \cos 2x. \tag{1}$$

► Ma'lumki, xarakteristik tenglamasi $\lambda_1 = i, \quad \lambda_2 = -i$ yechimlarga ega. U holda $y'' + y = 0$ bir jinsli teoremaning umumiy yechimi quyidagi funksiya bilan aniqlanadi.

$$y = c_1 \cos x + c_2 \sin x.$$

(1) tenglamaning o‘ng tomonini (11.51) va (11.53) ko‘rinishdagit maxsus turdagi ikkita funksiyaning yig‘indisi shaklida yozish mumkin:

$f_1(x)=x\sin x,\quad f_2(x)=\cos 2x$. Shuning uchun (11.55) ning tuzilishidan foydalanib, aniqmas koeffitsientlar usuli bilan

$$y''+y=x\sin x. \tag{2}$$

tenglamaning y_1^* xususiy yechimini, va

$$y''+y=\cos 2x. \tag{3}$$

tenglamaning y_2^* xususiy yechimini topamiz. (2) tenglama uchun $a=0,\ b=1,\ z=i=\lambda_1$, shuning uchun $k=1$ va.

$y_1^*=x\big((Ax+B)\cos x+(cx+D)\sin x\big)$

6-misolda keltirilgan sxema bo‘yicha A,B,C,D aniqmas koeffitsientlarni hisoblaymiz.

Quyidagiga esa bo‘lamiz:

$$\begin{array}{c|l}
1 & y_1^*=\left(Ax^2+Bx\right)\cos x+\left(cx^2+Dx\right)\sin x,\\
0 & y_1^{*\prime}=(2Ax+B)\cos x-\left(Ax^2+Bx\right)\sin x+\\
 & +(2cx+D)\sin x+\left(cx^2+Dx\right)\cos x=\\
 & =\left(cx^2+2Ax+Dx+B\right)\cos x+\left(-Ax^2-Bx+\right.\\
 & \left.2cx+D\right)\sin x,\\
1 & y_1^{*\prime\prime}=(2cx+2A+D)\cos x-\left(cx^2+2Ax+Dx+B\right)\sin x+\\
 & +(-2Ax-B+2c)\sin x+\left(-Ax^2-Bx+2cx+D\right)\cos x,
\end{array}$$

$$y_1^{*\prime\prime}+y^*=\left(Ax^2+Bx+2cx+2A+D-Ax^2-Bx+2cx+D\right)\cos x+$$
$$+\left(Cx^2+Dx-Cx^2-2Ax-Dx-B-2Ax-B+2c\right)\sin x\equiv x\sin x$$

Oxirgi ayniyatda o'ng va chap tomonlaridagi o'xshash hadlar oldidagi koeffitsientlarni tenglashtirib, A, B, C, D va y_1^* larni topamiz:

$$\begin{array}{r|c|} x\cos x & 4c = 0, \\ \cos x & 2A + 2D = 0, \\ x\sin x & -4A = 1, \\ \sin x & -2B + 2c = 0, \end{array}$$

bu yerdan $A = -1/4,\ B = 0, c = 0, D = 1/4$.

Bundan kelib chiqadiki,

$$y_1^* = x\left(-\frac{1}{4}x\cos x + \frac{1}{4}\sin x\right) = \frac{1}{4}x\left(\sin x - x\cos x\right).$$

(3) tenglama uchun $a = 0,\ b = 2,\ z = 2i$, shuning uchun $k = 0$ va

$$y_2^* = M\cos 2x + N\sin 2x.$$

Endi quyidagilarni topamiz:

$$\begin{array}{|l} y_2^* = M\cos 2x + N\sin 2x, \\ y_2^{*\prime} = -2M\sin 2x + 2N\cos 2x, \\ y_2^{*\prime\prime} = -4M\cos 2x - 4N\sin 2x. \end{array}$$

$$y_2^{*\prime\prime} + y_2^* = -3M\cos 2x - 3N\sin 2x = \cos 2x.$$

bundan ko'rinib turibdiki, $-3M = 1,\ -3N = 0$, shuning uchun

$$y_2^* = -\frac{1}{3}\cos 2x.$$

Natijada quyidagini hosil qilamiz:

$$y^* = y_1^* + y_2^* = \frac{1}{4}x\left(\sin x - x\cos x\right) - \frac{1}{3}\cos 2x.$$

va berilgan (1) tenglamaning umumiy yechimi quyidagi formula bilan aniqlanadi.

$$y = y + y^* = c_1 \cos x + c_2 \sin x + \frac{1}{4} x(\sin x - x\cos x) - \frac{1}{3}\cos 2x$$

. ◀

9-misol. Koshi masalasini yeching.

$$y'' - 2y' + 5y = 3y^x + e^x tg2x, \quad y(0) = 3/4, \quad y'(0) = 2. \ (1)$$

► Avval berilgan tenglamaning umumiy yechimini topamiz: tenglamaga mos keluvchi $\lambda^2 - 2\lambda + 5 = 0$ xarakteristik tenglamaning ildizlari $\lambda_{1,2} = 1 \pm 2i$ bo'ladi. $y'' - 2y + 5y = 0$ bir jinsli tenglamaning umumiy yechimi quyidagi funksiya bilan aniqlanadi.

$$y = e^x (c_1 \cos 2x + c_2 \sin 2x).$$

(1) Tenglamaning o'ng tomoni ikkita funksiyaning yig'indisi ko'rinishidan iborat. Ulardan birinchisi $f_1(x) = 3e^x$, maxsus turdagi (11.50) ga tegishli bo'lib, bu yerda $P_r(x) = 3$, $a = 1$, $b = 0$, $z = 1 \neq \lambda_{1,2}$ bo'ladi. Shuning uchun, $y'' - 2y' + 5y = 3e^x$ tenglamaning xususiy yechimi quyidagi ko'rinishga ega $y_1^* = Ae^x$, bu yerda A quyidagi ayniyatdan aniqlanadi. $(A - 2A + 5A)e^x \equiv 3e^x$; bundan $A = \frac{3}{4}$, $y_1^* = \frac{3}{4}e^x$.

Ikkinchi, $f_2(x) = e^x tg2x$ funksiya maxsuslikka ega emas va $y'' - 2y + 5y = e^x tg2x$ tenglamaning y_2^* xususiy yechimini ixtiyoriy o'zgarmasni variatsiyalash usuli bilan izlash zarurdir (Lagranj usuli). (11.38) formulaga asosan, quyidagiga ega bo'lamiz.

$$y_2^* = e^x (c_1(x)\cos 2x + c_2(x)\sin 2x).$$

Bizning misolimizda, (11.39) ko'rinishdagi sistema ikkita tenglamadan tashkil topgan $(y_1 = e^x \cos 2x, \quad y_2 = e^x \sin 2x)$.

$$\left.\begin{aligned} &c_1'e^x\cos 2x + c_2'e^x\sin 2x = 0,\\ &c_1e^x(\cos 2x - 2\sin 2x) + c_2'e^x(\sin 2x + 2\cos 2x) = e^x tg2x. \end{aligned}\right\}$$

Sistemaning tenglamalarini e^x ga qisqartirib, quyidagini hosil qilamiz.

$$\left.\begin{aligned} &c_1'\cos 2x + c_2'\sin 2x = 0,\\ &c_1'(\cos 2x - 2\sin 2x) + c_2'(\sin 2x + 2\cos 2x) = tg2x. \end{aligned}\right\}$$

Oxirgi sistemaning determinanti (vronskiani) quyidagicha bo‘ladi.

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ \cos x - 2\sin 2x & \sin 2x + 2\cos 2x \end{vmatrix} = 2$$

Kramer formulasi bo‘yicha quyidagini topamiz.

$$C_1' = \frac{1}{2}\begin{vmatrix} 0 & \sin 2x \\ tg2x & \sin 2x + 2\cos 2x \end{vmatrix} = -\frac{1}{2}\sin 2x tgx.$$

$$C_2' = \frac{1}{2}\begin{vmatrix} \cos 2x & 0 \\ \cos 2x - 2\sin 2x & tg2x \end{vmatrix} = \frac{1}{2}\sin 2x.$$

Topilgan tengliklarni integrallaymiz.

$$c_1 = -\frac{1}{2}\int\frac{\sin^2 2x}{\cos 2x}dx = -\frac{1}{2}\int\frac{1-\cos^2 2x}{\cos 2x}dx = -\frac{1}{2}\int\frac{dx}{\cos 2x} +$$

$$+\frac{1}{2}\int\cos 2x dx = \frac{1}{4}\ln\left|tg\left(\frac{\pi}{4} - x\right)\right| + \frac{1}{4}\sin 2x$$

$$c_2 = \frac{1}{2}\sin 2x dx = -\frac{1}{4}\cos 2x.$$

Bundan quyidagi kelib chiqadi.

$$y_2^* = e^x\left(\frac{1}{4}\ln\left|tg\left(\frac{\pi}{4} - x\right)\right|\cos 2x + \frac{1}{4}\sin 2x\cos 2x -\right.$$

$$\left.-\frac{1}{4}\sin 2x\cos 2x\right) = \frac{1}{4}e^x\ln\left|tg\left(\frac{\pi}{4} - x\right)\right|\cdot\cos 2x.$$

Shunday qilib, berilgan (1) tenglamaning xususiy yechimi quyidagicha bo'ladi.

$$y^* = y_1^* + y_2^* = \frac{3}{4}e^x + \frac{1}{4}e^x \ln\left|tg\left(\frac{\pi}{4} - x\right)\right| \cdot \cos 2x =$$

$$= \frac{1}{4}e^x\left(3 + \ln\left|tg\left(\frac{\pi}{4} - x\right)\right| \cdot \cos 2x\right),$$

Uning umumiy yechimi esa quyidagi funksiya bilan aniqlanadi.

$$y = y + y^* = e^x\left(c_1 \cos 2x + c_2 \sin 2x\right) + \frac{1}{4}e^x\left(3 + \ln\left|tg\left(\frac{\pi}{4} - x\right)\right| \cdot \cos 2x\right) \cdot$$

(2)

Koshi masalasini yechish uchun $y(0) = 3/4$, $y'(0) = 2$ boshlang'ich shartlardan foydalanib, (2) umumiy yechimdagi c_1 va c_2 ixtiyoriy o'zgarmaslarning qiymatlarini hisoblaymiz. y' ni topamiz.

$$y' = e^x\left(c_1 \cos 2x + c_2 \sin 2x\right) + e^x\left(-2c_1 \sin 2x + 2c_2 \cos 2x\right) +$$

$$+\frac{1}{4}e^x\left(3 + \ln\left|tg\frac{\pi}{4} - x\right| \cdot \cos 2x\right) +$$

$$+\frac{1}{4}e^x\left(-\frac{\cos 2x}{tg\left(\frac{\pi}{4} - x\right) \cdot \cos\left(\frac{\pi}{4} - x\right)} - 2\ln\left|tg\left(\frac{\pi}{4} - x\right)\right| \cdot \sin 2x\right).$$

y va y' lar uchun olingan ifodalarga $x = 0$ qiymatni qo'yib, boshlang'ich shartlarni e'tiborg olib, quyidagilaarni hosil qilamız.

$$y(0) = 3/4 = c_1 + 3/4.$$

$$y'(0) = 2 = 2c_2 + 3/4 - 1/2.$$

bu yerdan $c_1 = 0$, $c_2 = 7/4$.

Natijada izlanayotgan xususiy yechim quyidagicha bo'ladi.

$$y=\frac{1}{4}e^{x}\left(3+7\sin 2x-\ln\left|tg\left(\frac{\pi}{4}-x\right)\right|\cdot\cos 2x\right).$$ ◄

11.5. AT

1. Quyidagi ikkinchi tartibli bir jinsli chiziqli differensial tenglamalarning umumiy yechimlarini va fundamentlar yechimlari sistemasini toping.

a) $y''-2y'-4y=0;$

b) $y''+6y'+9y=0;$

v) $y''-6y'+18y=0$.

(*Javob:* a) $y_1=e^{(1+\sqrt{5})x}$, $y_2=e^{(1-\sqrt{5})x}$; $y=c_1e^{(1+\sqrt{5})x}+c_2e^{(1-\sqrt{5})x}$;

b) $y_1=e^{-3x}$, $y_2=xe^{-3x}$; $y=e^{-3x}(c_1+c_2x)$;

v) $y_1=e^{3x}\cos 3x$, $y_2=e^{3x}\sin x$; $y=(c_1\cos 3x+c_2\sin 3x)$.)

2. .Quyidagi yuqori tartibli bir jinsli chiziqli differensial tenglamalarning umum yechimlarini va fundamental yechimlari sistemasini toping.

a) $y'''-5y''+16y'-12y=0;$

b) $y^{IV}-8y''-7y=0;$

v) $y^{V}-6y^{IV}+9y'''=0;$

g) $y^{VI}-3y^{V}+3y^{IV}=0$.

(*Javob:* a) $y_1=e^{x}$, $y_2=e^{2x}\cos 2\sqrt{2}x$, $y_3=e^{2x}\sin 2\sqrt{2}x$; $y=c_1e^{x}+e^{2x}\left(c_1\cos 2\sqrt{2}x+c_2\sin 2\sqrt{2}x\right)$; b) $y_1=e^{x}$, $y_2=e^{-x}$, $y_3=e^{\sqrt{7}x}$, $y_4=e^{-\sqrt{7}x}$; $y=c_1e^{x}+c_2e^{-x}+c_3e^{-\sqrt{7}x}+c_4e^{-\sqrt{7}x}$; v) $y_1=1$, $y_2=x$, $y_3=x^2$, $y_4=e^{3x}$, $y_5=xe^{3x}$, $y=c_1+c_2x+c_3x^2+(c_4+c_5x)\cdot e^{3x}$, g)

$y_1=1,\ y_2=x,\ y_3=x^2,\ y_4=x^3,\ y_5=e^{3x/2}\cos\frac{\sqrt{3}}{2}x,$

$y_6=e^{3x/2}\sin\frac{\sqrt{3}}{2}x;\ y=c_1+c_2x+c_3x^2+c_4x^3+e^{3x/2}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right).)$

Mustaqil ish

Quyidagi bir jinsli chiziqli differensial tenglamalarning umumiy yechimini va fundamental yechimlari sistemasini toping.

1. a) $3y''-2y'-8y=0$; b) $y'''+9y'=0$;

(*Javob:* a) $y=c_1e^{2x}+c_2e^{-4x/3}$,

b) $y=c_1+c_2\cos 3x+c_3\sin 3x$.)

2. a) $y''-6y'+13y=0$; b) $y^{IV}-8y''+16y=0$.

(*Javob:*a) $y=e^{3x}(c_1\cos 2x+c_2\sin 2x)$;

b) $y=(c_1+c_2x)e^{2x}+(c_3+c_4x)e^{-2x}$.)

3. a) $4y'''-8y'+5y=0$; b) $y'''-3y''+3y'-y=0$.

(*Javob:* a) $y=e^x\left(c_1\cos\frac{x}{2}+c_2\sin\frac{x}{2}\right)$;

b) $y=e^x(c_1+c_2x+c_3x)^2$.)

11.6. Auditoriya topshiriqlari

Quyidagi, bir jinsli bo‘lmagan tenglamalarning ko‘rsatilgan boshlang‘ich shartlarni qanoatlantiruvchi, xususiy yechimlarini toping. (Koshi masalasini yeching.)

1. $y''-3y'+2y=e^{3x}(x^2-x),\ y(0)=1,\ y(0)=-2$.

(*Javob:* $y=4(e^x-e^{2x})+\frac{1}{2}(x^2-2x+2)e^{3x}$.)

2. $y'''-y'=-2x,\ y(0)=0,\ y'(0)=y''(0)=2$.

(*Javob:* $y=e^x-e^{-x}+x^2$).

3. $y^{IV}-y=8e^x,\ y'(0)=0,\ y''(0)=1,\ y'''(0)=0$

(*Javob:* $y = 2xe^x - 3e^x + e^{-x} + \cos x + 2\sin x$.)

4. $y'' - 2y' + 2y = 4e^x \cos x,\ y(\pi) = \pi e^\pi,\ y'(\pi) = e^\pi$.

(*Javob:* $y = e^e\left((2x - \pi - 1)\sin x - \pi\cos x\right)$.)

5. $y'' + 4y = 4(\sin 2x + \cos 2x),\ y(\pi) = y'(\pi) = 2\pi$.

(*Javob:* $y = 3\pi\cos 2x + \frac{1}{2}\sin x + (\sin 2x - \cos 2x)$.)

Mustaqil ish

Ko'rsatilgan quyidagi tenglamalarning boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimlarini toping.

1. $y'' - 22y = 2e^x,\ y(1) = -1,\ y'(1) = 0$.

(*Javob:* $y = e^{2x-1} - 2e^x + e + 1$.)

2. $y'' + 4y = x, y(0) = 1,\ y'(0) = \frac{\pi}{2}$.

(*Javob:* $y = \frac{1}{4}x + \cos 2x + \left(\frac{\pi}{4} - \frac{1}{8}\right)\sin 2x$)

3. $y'' + 6y' + 9y = 10\sin x,\ y(0) = -0{,}6,\ y'(0) = 0{,}8$.

(*Javob:* $y = 0{,}8\sin x - 0{,}6\cos x$.)

11.7. Auditoriya topshiriqlari

Quyidagi berilgan bir jinsli bo'lmagan chiziq differensial tenglamalarning har birining xususiy yechimini aniqlang va uning tuzilishini yozing.

1. $y'' - 8y' + 16y = e^{4x}(1 - x)$.
2. $y'' - 3y' = e^{3x} - 28x$.
3. $y'' + 16y = x\sin x$
4. $y''' + y'' = 2x + e^{-x}$

5. $y'' - 4y' = 2\cos^2 4x$

6. $y^{IV} - y = 3xe^x + \sin x$

7. $y'' - 7y' = (x-1)^2$

8. $y^{IV} + y'' = x^2 + 2x$

9. $y'' - 4y' + 13y = e^{2x}\left(x^2 \cos 3x + \sin 3x\right)$.

10. $y^V - y^{IV} = 2xe^x - 4$.

Berilgan chiziqli tenglamalarning umumiy yechimini toping.

11. $y'' + 4y = \cos^2 x$

12. $y'' + 5y' + 6y = e^{-x} + e^{-2x}$

13. $4y'' - y = x^3 - 24x$

14. $y''' + y'' = 6x + e^{-x}$

15. $y'' + 4y = 1/\sin^2 x$

16. $y''' + y' = tgx$

Mustaqil ish

1. $y'' + 4y' + 4y = e^{-2x} \ln x$.

(*Javob:* $y = \left(c_1 + c_2 x + \frac{1}{2}x^2 \ln x - \frac{3}{4}x^2\right)e^{-2x}$.)

2. $y'' + y + ctg^2 x = 0$.

(*Javob:* $y = 2 + c_1 \cos x + c_2 \sin x + \cos \ln\left|tg \frac{x}{2}\right|$.)

3. $y'' - 2y' + y = e^x / \left(x^2 + 1\right)$.

(*Javob:* $y = e^x\left(c_1 + c_2 - \ln\sqrt{x^2+1} + x arctg\, x\right)$.)

11.7. DIFFERENSIAL TENGLAMALAR SISTEMASI

Quyidagi ko'rinishdagi sistema

$$\left.\begin{array}{l} y_1' = f_1(x, y_1, y_2, ..., y_n) \\ y_2' = f_2(x, y_1, y_2, ..., y_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ y_n' = f_n(x, y_1, y_2, ..., y_n) \end{array}\right\} \quad (11.58)$$

(bu yerda $x, y_1, y_2, ..., y_n$ o'zgaruvchili $f_i\left(i = \overline{1,n}\right)$ funksiya $(n+1)$ o'lchamli biror D sohada aniqlangan) $y_1(x)$, $y_2(x), ..., y_n(x)$ noma'lum funksiyali n – *ta birinchi tartibli differensial tenglamalarning normal sistemasi* deb ataladi.

(11.58) sistemaga kiruvchi tenglamalar soni uning *tartibi* deb ataladi.

(11.58) sistemaning (a,b) oraliqdagi yechimi deb, (a,b) oraliqda uzluksiz differensiallanuvchi va o'zining hosilalar bilan (11.58) sistemaning har bir tenglamasini ayniyatga aylantiruvchi $y_1 = y_1(x),\ y_2 = y_2(x), ..., y_n = y_n(x)$ funksiyalar to'plamiga aytiladi.

Birinchi tartibli differensial tenglamalar sistemasi uchun Koshi masalasi quyidagicha ta'riflanadi.

(11.58) sistemaning

$$y_1(x_0) = y_{10},\ \ y_2(x_0) = y_{20}, ..., y_n(x_0) = y_{n0}. \quad (11.59)$$

bu yerda $y_{10}, y_{20}, ..., y_{n0}$ – berilgan sonlar: $x_0 \in (a,b)$ boshlang'ich shartlarni qanoatlantiruvchi $y_1 = y_1(x),\ \ y_2(x), ..., y_n = y_n(x)$ yechimlari topilsin.

Quyidagi o'rinlidir.

Teorema. (Koshi masalasining mavjudligi va yagonaligi haqida).

Agar $f_i\left(i=\overline{1,n}\right)$ *funksiya* $\left(x_0, y_{10}, y_{20}, \dots, y_{n0}\right) \in D$ *nuqtaning atrofida uzluksiz bo'lsa va* $\frac{\partial f_i}{\partial y_i}\left(i=\overline{1,n}\right)$ *uzluksiz xususiy hosilalarga ega bo'lsa, u holda har doim shunday* x_0 *markazli integral topiladiki, (11.58) sistemaning (11.59) boshlang'ich shartlarni qanoatlantiruvchi yagona yechimi mavjud bo'ladi.*

(11.58) sistemaning umumiy yechimi deb, quyidagi boshlang'ich shartlarni qanoatlantiruvchi va n ta $c_1, c_2, \dots, c_n$ ixtiyoriy o'zgarmaslarga bog'liq bo'lgan, n ta $y_i = \varphi_i\left(x, c_1, c_2, \dots, c_n\right)\left(i=\overline{1,n}\right)$ funksiyalar to'plamiga aytiladi.

1) φ_i funksiya $x, c_1, c_2, \dots, c_n$ o'zgaruvchilrning biror o'zgarish sohasida aniqlangan va $\frac{\partial \varphi_i}{\partial x}$ uzluksiz xususiy hosilalarga ega bo'lsa;

2) φ_i to'plam c_i ning ixtiyoriy qiymatlarida (11.58) sistemaning yechimi bo'lsa;

3) Koshi teoremasi o'rinli bo'ladigan c sohadagi har qanday (11.59) boshlang'ich shartlar uchun har doim $c_{10}, c_{20}, \dots, c_{n0}$ boshlang'ich shartlarning shunday qiymatlari topiladiki, $\varphi_{i0} = \varphi_i\left(x_0, c_{10}, c_{20}, \dots, c_{n0}\right)$ tenglik o'rinli bo'ldi.

(11.58) *sistemaning xususiy yechimi* deb, ixtiyoriy o'zgarmaslarning biror xususiy qiymatlarida umumiy yechimdan olingan yechimga aytiladi.

(11.58) sistemani yechish usullaridan biri, uni yuqori tartibli bitta yoki bir necha differensial tenglamalarni yechishga olib kelishdir. (Noma'lumni yo'qotish usuli).

Yuqorida aytilganlarning barchasi, quyidagi ko'rinishdagi, (11.58) sistemaning hususiy holi bo'lgan chiziqli differensial tenglamalar sistemasi uchun o'rinlidir.

$$\left.\begin{aligned} y_1' &= a_{11}(x)y_1 + a_{12}(x)y_2 + \ldots + a_{1n}(x)y_n + f_1(x) \\ y_2' &= a_{21}(x)y_1 + a_{22}(x)y_2 + \ldots + a_{2n}(x)y_n + f_2(x) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n' &= a_{n1}(x)y_1 a_{n2}(x)y_2 + \ldots + a_{nn}(x)y_n + f_n(x) \end{aligned}\right| . \quad (11.60)$$

bu yerda $a_{ij}(x)$, $f_i(x)\left(1, j=\overline{1,n}\right)$ funksiyalar odatda, biror (a,b) oraliqda uzluksiz deb faraz qilinadi. Agar barcha $f_i(x)=0$ bo'lsa, u holda (11.60) sistema *bir jinsli*, aks holda *bir jinsli bo'lmagan* deyiladi. Agar $a_{ij}(x)=const$ bo'lsa, *sistema o'zgarmas koeffitsientli chiziqli sistema* deyiladi. Bunday sistemalarni integrallashga imkon beruvchi usullar mavjuddir.

Shulardan ikkitasini ko'rib chiqamiz.

1. Xarakteristik tenglamasini tuzamiz.

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-\lambda & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn}-\lambda \end{vmatrix} = 0 . \quad (11.61)$$

bu yerda $a_{ij}=const$. Determinantni ochib chiqib, n ta yechimga ega bo'lgan (ularning karraliklarini hisobga olgan holda), λ ga nisbatan darajali, haqiqiy o'zgarmas koeffitsienli algebraik tenglamani hosil qilamiz. Shu bilan birgalikda quyidagi holatlar bo'lishi mumkin.

1. (11.61) xarakteristik tenglamaning ildizlari turlicha va haqiqiy. Ularni $\lambda_1, \lambda_2, \ldots, \lambda_n$ deb belgilaymiz. Ma'lumki, har bir $\lambda_i\left(i=\overline{1,n}\right)$ ildizga quyidagi ko'rinishdagi xususiy yechimlar mos keladi.

$$y_1^{(i)} = \alpha_1^{(i)} e^{\lambda_i x},\ y_2^{(i)} = \alpha_2^{(i)} e^{\lambda_i x}, \ldots, y_n^{(i)} = \alpha_n^{(i)} e^{\lambda_i x} . \quad (11.62)$$

bu yerda $\alpha_1^{(i)}, \alpha_2^{(i)}, ..., \alpha_n^{(i)}$ koeffitsientlar quyidagi chiziqli algebraik tenglamalar sistemasidan topiladi.

$$\left.\begin{array}{l} (a_{11}-\lambda_1)\alpha_1^{(i)} + \quad a_{12}\alpha_2^{(i)} + ... + \ a_{1n}\alpha_n^{(i)} = 0 \\ a_{21}\alpha_1^{(i)} + (a_{22}-\lambda_i)\alpha_2^{(i)} + ... + \ a_{2n}\alpha_n^{(i)} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}\alpha_1^{(i)} + \quad a_2^{(i)} + ... + (a_{nn}-\lambda_i)\alpha_n^{(i)} = 0 \end{array}\right\}. \qquad (11.63)$$

Barcha (11.62) ko'rinishdagi xususiy yechimlar fundamental yechimlar sistemasini tashkil qiladi.

$a_{ij} = const,\ f_i(x) = 0$ bo'lgan holda (11.60) sistemadan olingan bir jinsli o'zgarmas koeffitsientli tenglamaning umumiy yechimi, (11.62) yechimining chiziqli kombinatsiyasini tashkil etuvchi quyidagi funksiyalar to'plamini ifodalaydi.

$$\left.\begin{array}{l} y_1 = \sum_{i=1}^{n} c_i y_1^{(i)} = c_1\alpha_1^{\alpha} e^{\lambda_1 x} + c_2\alpha_1^{(2)} e^{\lambda_2 x} + ... + c_n\alpha_1^{(n)} e^{\lambda_n x} \\ y_2 = \sum_{i=1}^{n} c_i y_2^{(i)} = c_1\alpha_2^{(1)} e^{\lambda_1 x} + c_2\alpha_2^{(2)} e^{\lambda_2 x} + ... + c_n\alpha_2^{(n)} e^{\lambda_n x} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ y_n = \sum_{i=1}^{n} c_i y_n^{(i)} = c_1\alpha_n^{(1)} e^{\lambda_1 x} + c_2\alpha_2^{(2)} e^{\lambda_2 x} + ... + c_n\alpha_n^{(n)} e^{\lambda_n x} \end{array}\right\}. \qquad (11.64)$$

bu yerda c_i – ixtiyoriy o'zgarmaslar.

1-misol. Bir jinsli sistemaning umumiy yechimini toping.

$$\left.\begin{array}{l} y_1' = 3y_1 - y_2 + y_3 \\ y_2' = -y_1 + 5y_2 - y_3 \\ y_3' = y_1 - y_2 + 3y_3 \end{array}\right\}.$$

► Berilgan sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0. \qquad (1)$$

$\lambda_1 = 2$, $\lambda_3 = 3$, $\lambda_3 = 6$ bo'lgan turli haqiqiy ildizlarga ega. Ularning har biri uchun (11.63) ko'rinishdagi sistemani tuzamiz.

$$\left.\begin{array}{r} \alpha_1^{(1)} - \alpha_2^{(1)} + \alpha_3^{(1)} = 0, \\ -\alpha_1^{(1)} + 3\alpha_2^{(1)} - \alpha_3^{(1)} = 0, \\ \alpha_1^{(1)} - \alpha_2^{(1)} + \alpha_3^{(1)} = 0, \end{array}\right| \qquad \left.\begin{array}{r} -\alpha_2^{(2)} + \alpha_3^{(2)} = 0, \\ -\alpha_1^{(2)} + 2\alpha_2^{(2)} - \alpha_3^{(2)} = 0, \\ \alpha_1^{(2)} - \alpha_2^{(2)} = 0, \end{array}\right|. \qquad (2)$$

$$\left.\begin{array}{r} -3\alpha_1^{(3)} - \alpha_2^{(3)} + \alpha_3^{(3)} = 0, \\ -\alpha_1^{(3)} - \alpha_2^{(3)} - \alpha_3^{(3)} = 0, \\ \alpha_1^{(3)} - \alpha_2^{(3)} - 3\alpha_3^{(3)} = 0. \end{array}\right|$$

Bu sistemalarning determinantlari, (1) formulaga asosan, nolga teng, u holda ularning har biri cheksiz ko'p yechimga ega. Bunday holda. Shunday yechimni tanlash mumkinki, ular uchun $\alpha_1^{(1)} = \alpha_1^{(2)} = \alpha_1^{(3)} = 1$ bo'ladi. U holda (2) sistemaning quyidagi yechimlarini hosil qilamiz: agar $\alpha_1 = 2$ bo'lsa, u holda $\alpha_1^{(1)} = 1$, $\alpha_2^{(1)} = 0$, $\alpha_3^{(1)} = 1$; agar $\alpha_2 = 3$ bo'lsa, u holda $\alpha_1^{(2)} = 1$, $\alpha_2^{(2)} = 1$, $\alpha_3^{(2)} = 1$; agar $\alpha = 6$ bo'lsa, u holda $\alpha_1^{(3)} = 1$, $\alpha_2^{(3)} = -2$, $\alpha_3^{(3)} = 1$.

Bundan quyidagi fundamental yechimlar sistemasini hosil qilamiz.

$$y_1^{(1)} = e^{2x}, \quad y_2^{(1)} = 0, \quad y_3^{(1)} = -e^{-2x};$$

$$y_1^{(2)} = e^{3x}, \quad y_2^{(2)} = e^{3x}, \quad y_3^{(2)} = e^{3x};$$

$$y_1^{(3)} = e^{6x}, \quad y_2^{(3)} = -2e^{6x}, \quad y_3^{(3)} = e^{6x}.$$

(11.64) funksiyalar to'plamini e'tiborga olgan holda bu yechimlarning chiziqli kombinatsiyasi dastlabki sistemaning umumiy yechimini beradi.

$$\left.\begin{aligned} y_1 &= c_1e^{2x} + c_2e^{3x} + c_3e^{6x}, \\ y_2 &= \qquad\quad c_2e^{3x} - 2c_3e^{6x}, \\ y_3 &= -c_1e^{2x} + c_2e^{3x} + c_3e^{6x}. \end{aligned}\right\} . \blacktriangleleft$$

2. (11.61) xarakteristik tenglamalari $\lambda_1, \lambda_2, ..., \lambda_n$ ildizlari turlicha, ammo ular orasida kompleks ildizlar mavjud. Ma'lumki, bu holda (11.61) xarakteristik tenglamaning har bir juft qo'shma – kompleks $\lambda_{1,2} = \lambda \pm i\beta$ ildizlariga juft xususiy yechim mos keladi.

$$y_j^{(1)} = \alpha_j^{(1)} e^{(\alpha + i\beta)x}, \tag{11.65}$$

$$y_j^{(2)} = \alpha_j^{(2)} e^{(\alpha - i\beta)x}, \tag{11.66}$$

bu yerda $\lambda = \lambda_1 = \lambda \pm i\beta$ va $\lambda = \lambda_2 = \lambda - i\beta$ lar uchun mos ravishda (11.63) sistemadan $\alpha_j^{(1)}$, $\alpha_j^{(2)}$ koeffitsientlar aniqlanadi. $\alpha_j^{(1)}$, $\alpha_j^{(2)}$ koeffitsientlar, qoida bo'yich, kompleks sonlardir, ularga mos keluvchi $y_j^{(1)}$, $y_j^{(2)}$ funksiyalar ega, kompleks funksiyalardir. $y_j^{(1)}$ va $y_j^{(2)}$ funksiyalarning mavhum va haqiqiy qismlarini ajrutib va haqiqiy koeffitsientli chiziqli tenglamalar uchun yechimlarining mavhum qismi ham, haqiqiy qismi ham yechim ekanligidan foydalanib, bir jinsli sistemaning xususiy haqiqiy juft yechimlarini hosil qilamiz.

2-misol. Sistemaning umumiy yechimini toping.

$$\left.\begin{aligned} y_1' &= 7y_1 + y_2 \\ y_2' &= -2y_1 - 5y_2 \end{aligned}\right\} . \tag{1}$$

► (1) sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} -7-\lambda & 1 \\ -2 & -5-\lambda \end{vmatrix} = \lambda^2 + 12\lambda + 37 = 0 .$$

$\lambda_{1,2} = -6 \pm i$ ildizlarg ega. (11.63) formulaga asosan, quyidagini hosil qilamiz:

$$\left.\begin{array}{l}(-7-\lambda)\alpha_1+\qquad\alpha_2=0\\ -2\alpha_1+(-5-\lambda)\alpha_2=0\end{array}\right\}.$$

$\alpha_1^{(1)}, \alpha_2^{(1)}$ larni hisoblash uchun $\lambda_1=-6+i$ ildizga quyidagi sistema mos keladi.

$$\left.\begin{array}{l}(-7-\lambda_1)\alpha_1^{(1)}+\alpha_2^{(1)}=0,\\ -2\alpha_1^{(1)}+(-5-\lambda_1)\alpha_2^{(1)}=0.\end{array}\right\}\Rightarrow\left|\begin{array}{l}(-1-i)\alpha_1^{(1)}+\alpha_2^{(1)}=0\\ -2\alpha_1^{(1)}+(1-i)\alpha_2^{(1)}=0.\end{array}\right.\Rightarrow\left\{\begin{array}{l}\alpha_1^{(1)}=1\\ \alpha_2^{(1)}=1+i\end{array}\right.$$

(11.65) formulaga asosan, quyidagi xususiy yechimini hosil qilamiz.

$$y_1^{(1)}=\alpha_1^{(1)}e^{(\alpha+i\beta)x}=e^{(\alpha+i\beta)x}=e^{(-6+i)x}=e^{-6x}(\cos x+i\sin x),$$

$$y_2^{(1)}=\alpha_2^{(1)}e^{(\alpha+i\beta)x}=(1-i)e^{(-6+i)x}=e^{-6x}(\cos x-\sin+i(\cos x+\sin x)).\quad(2)$$

(Bu yerda biz Eyler formulasidan foydalandik: $e^{(\alpha+i\beta)x}=e^{\alpha x}(\cos\beta x+i\sin\beta x)$) (2) yechimdan mavhum va haqiqiy qismlarini alohida olib, (1) sistemaning fundamental yechimlar sistemasini hosil qiluvchi, 2 ta haqiqiy ko‘rinishdagi yechimni hosil qilamiz.

$$\begin{array}{ll}\overline{y}_1^{(1)}=e^{-6x}\cos x, & \overline{y}_2=e^{-6x}(\cos x-\sin x),\\ \overline{\overline{y}}_1^{(1)}=e^{-6x}\sin x, & \overline{\overline{y}}_2=e^{-6x}(\cos x+\sin x).\end{array}\qquad(3)$$

U holda (1) sistemaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$\left.\begin{array}{l}y_1=c_1\overline{y}_1^{(1)}+c_2\overline{\overline{y}}_1^{(1)}=e^{-6x}(c_1\cos x+c_2\sin x),\\ y_2=c_1\overline{y}_2^{(1)}+c_2\overline{\overline{y}}_2^{(1)}=e^{-6x}(c_1(\cos x-\sin x)+c_2(\cos x+\sin x)).\end{array}\right|\qquad(4)$$

Endi, $\lambda_2=-6-i$ ikkinchi ildizni foydalanish ortiqchadir, chunki yana (1)-(4) yechimlarni olamiz. Bu mulohaza barcha bir jinsli chiziqli differensial tenglamalar uchun o‘rinlidir. ◀

3. (11.61) xarakteristik tenglamaning $\lambda_1,\lambda_2,...,\lambda_n$ ildizlari orasida karralari mavjud. Bu holda quyidagicha yo‘l tutamiz. Faraz qilaylik, λ – (11.61) xarakteristik tenglamaning k – karrali ildizi bo‘lsin. U holda (11.60) sistemaning

$\left(\alpha_{ij} = const,\ f_i(x) \equiv 0 (i, j = 1, n)\right)$ k karrali ildiziga mos keluvchi yechimini quyidagi ko'rinishda izlaymiz.

$$y_1 = \left(\alpha_{10} + \alpha_{11}x + \alpha_{12}x + ... + \alpha_{1k-1}x^{k-1}\right)e^{\lambda x},$$
$$y_2 = \left(\alpha_{20} + \alpha_{21}x + \alpha_{22}x^2 + ... + \alpha_{2k-1}x^{k-1}\right)e^{\lambda x}, \quad (11.67)$$
$$..$$
$$y_n = \left(\alpha_{n0} + \alpha_{n1}x + \alpha_{n_2}x^2 + ... + \alpha_{nk-1}x^{k-1}\right)e^{\lambda x}.$$

$a_{ij}\left(i = \overline{1, n},\ e = 0, k-1\right)$ sonlarni quyidagicha topamiz. (11.67) dan y_i funksiyani va uning y_i' hosilasini a_{ij} va $f_i(x)$ larga ko'rsatilgan cheklanishlarda dastlabki (11.60) sistemaga qo'yamiz. So'ngra ($e^{\lambda x} \neq 0$ qisqartirganimizdan keyin) olingan tenglikni o'ng va chap tomonlarida x ning bir xil darajalari oldidagi koeffitsientlarni tenglashtiramiz. O'tkazilgan protseduralar natijada barcha λ_{ie} sonlardan ixtiyoriy o'zgarmas sifatida qabul qilinuvchi k soni har doim erkin o'zgaruvchi sifatida qoladi.

(11.61) xarakteristik tenglamaning oddiy (karrali bo'lmagan) yechimlariga mos keluvchi fundamental sistemaning yechimlari, 1 va 2 hollarda ko'rsatilganday aniqlanadi.

3-misol. Sistemaning umumiy yechimini toping.

$$\begin{aligned} y_1' &= \quad\ \ y_2 + y_3, \\ y_2' &= y_1 + y_2 - y_3, \\ y_3' &= \quad\ \ y_2 + y_3. \end{aligned} \quad (1)$$

▶ (1) sistemaning xarakteristik tenglamasi

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & 1-\lambda & -1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = -(\lambda - 1)^2 \lambda = 0 \cdot \quad (2)$$

ikki karrali $\lambda_{1,2}=1$ va bir karrali $\lambda_3=0$ ildizlarga ega. (11.67) formulaga asosan, ikki karrali $\lambda_{1,2}=1$ ildizga quyidagi ko'rinishdagi yechim mos keladi.

$$
\begin{aligned}
&y_1^{(1,2)}=(\alpha_{10}+\alpha_{11}x)e^x,\ y_2^{(1,2)}=(\alpha_{20}+\alpha_{21}x)e^x,\\
&y_3^{(1,2)}=(\alpha_{30}+\alpha_{31})e^x.
\end{aligned}
\tag{3}
$$

$\alpha_{ie}\left(i=\overline{1,3},\ e=\overline{0,1}\right)$ koeffitsientlar $y_1, y_2, y_3, y_1', y_2', y_3'$ lar uchun olingan ifodalarni (1) dastlabki sistemaga qo'yishdan hosil bo'lgan sistemadan aniqlanadi. $e^x\neq 0$ ga qisqartirilgandan so'ng, quyidagiga ega bo'lamiz.

$$
\left.
\begin{aligned}
\alpha_{11}+\alpha_{10}+\alpha_{11}x&=\alpha_{20}+\alpha_{21}x+\alpha_{30}+\alpha_{31}x,\\
\alpha_{21}+\alpha_{20}+\alpha_{21}x&=\alpha_{10}+\alpha_{11}x+\alpha_{20}+\alpha_{21}x-\alpha_{30}-\alpha_{31}x,\\
\alpha_{31}+\alpha_{30}+\alpha_{31}x&=\alpha_{20}+\alpha_{21}x+\alpha_{30}+\alpha_{31}x.
\end{aligned}
\right\}
$$

O'ng va chap tomonlaridagi x ning bir xil darajalari oldidagi koeffitsientlarni tenglashtirib, quyidagi sistemani hosil qilamiz.

$$
\left.
\begin{aligned}
\alpha_{11}+\alpha_{10}&=\alpha_{20}+\alpha_{30},\\
\alpha_{11}&=\alpha_{21}+\alpha_{31},\\
\alpha_{21}+\alpha_{20}&=\alpha_{10}+\alpha_{20}-\alpha_{30},\\
\alpha_{21}&=\alpha_{11}+\alpha_{21}-\alpha_{31},\\
\alpha_{31}&=\alpha_{21}+\alpha_{31},\\
\alpha_{31}+\alpha_{30}&=\alpha_{20}+\alpha_{30},
\end{aligned}
\right|
$$

bundan $\alpha_{20}=\alpha_{31}=\alpha_{11}$, $\alpha_{30}=\alpha_{10}$, $\alpha_{21}=0$ ekanligini topamiz. α_{10} va α_{11} sonlarni ixtiyoriy parametrlar deb hisoblash mumkin. Ularni mos ravishda c_1 va c_2 deb belgilaymiz. U holda (3) yechim quyidagi ko'rinishda yoziladi.

$$
y_1^{(1,2)}=(c_1+c_2x)e^x,\quad y_2^{(1,2)}=c_1e^x,\quad y_3^{(1,2)}=(c_1+c_2x)e^x. \tag{4}
$$

(11.62) formulaga asosan, $\lambda_3=0$ ildizga quyidagi yechim mos keladi.

$$y_1^{(3)} = \alpha_1^{(3)} e^{0x} = \alpha_1^{(3)}, \quad y_2^{(3)} = \alpha_2^{(3)} e^{0x} = \alpha_2^{(3)}, \quad y_3^{(3)} = \alpha_3^{(3)} e^{0x} = \alpha_3^{(3)}, \; (5)$$

bu yerda $\alpha_1^{(3)}, \alpha_2^{(3)}, \alpha_3^{(3)}$ – sonlar quyidagi sistemadan topiladi ((11.63) sistemaga qarang).

$$\left.\begin{array}{c} \alpha_2^{(3)} + \alpha_3^{(3)} = 0, \\ \alpha_1^{(3)} + \alpha_2^{(3)} - \alpha_3^{(3)} = 0, \\ \alpha_2^{(3)} + \alpha_3^{(3)} = 0. \end{array}\right\}$$

Uning yechimi quyidagicha bo‘ladi. $\alpha_1^{(3)} = 2c_3, \; \alpha_2^{(3)} = -c_3, \; \alpha_3^{(3)} = c_3$. Natijada $\lambda_3 = 0$ ildizga mos keluvchi (1) dastlabki sistemaning (5) ko‘rinishdagi yechimi quyidagicha bo‘ladi.

$$y_1^{(3)} = 2c_3, \quad y_2^{(3)} = -c_3, \quad y_3^{(3)} = c_3,$$

bu yerda c_3 – ixtiyoriy o‘zgarmas.

Dastlabki sistemaning umumiy yechimi quyidagicha ko‘rinishda yoziladi.

$$\left.\begin{array}{c} y_1 = y_1^{(1,2)} + y_1^{(3)} = (c_1 + c_2 x) e^x + 2c_3, \\ y_2 = y_2^{(1,2)} + y_2^{(3)} = c_1 e^x - c_3, \\ y_3 = y_3^{(1,2)} + y_3^{(3)} = (c_1 + c_2 x) e^x + c_3. \end{array}\right\} \blacktriangleleft$$

Agar sistema bir jinsli bo‘lmasa, u holda bir jinsli sistemaga mos keluvchi (11.64) ko‘rinishdagi umumiy yechimni bilgan holda, dastlabki bir jinsli bo‘lmagan sistemaning umumiy yechimini (11.64) yechimdagi $c_1, c_2, ..., c_n$ ixtiyoriy o‘zgarmaslarni variatsiyalash usuli bilan topish mumkin. Bu savolni to‘laroq ko‘rib chiqamiz. Bir jinsli bo‘lmagan sistemaning yechimini $c_1, c_2, ..., c_n$ ixtiyoriy o‘zgarmaslarni, ularga mos keluvchi $c_1(x), c_2(x), ..., c_n(x)$ funksiyalarga almashtirib, har doim (11.64) ko‘rinishda yozamiz mumkinligi isbot qilingan. Bu funksiyalar berilgan bir jinsli bo‘lmagan sistemalar yordamida quyidagicha aniqlanadi. Berilgan sistemaga

$y_1, y_2, ..., y_n$, $y'_1, y'_2, ..., y'_n$ larning ifodalari qo'yiladi va $c'_1(x), c'_2(x), ..., c'_n(x)$ larga nisbatan, yechimi har doim mavjud va quyidagi ko'rinishda tasvirlash mumkin bo'lgan:

$$c'_1(x) = \varphi_1(x),\ \ c'_2(x) = \varphi_2(x), ..., c'_n(x) = \varphi_n(x).$$

bu yerda $\varphi_i(x)\left(i = \overline{1,n}\right)$ – ma'lum funksiyalar, n ta algebraik tenglamadan iborat chiziqli tenglamalar sistemasi hosil qilinadi. Bu tengliklarni integrallab quyidagilarni topamiz.

$$c_i(x) = \int \varphi_i(x)dx + c.$$

bu yerda c – ixtiyoriy o'zgarmas.

(11.64) yechimga $c_i = const$ lar o'rniga topilgan $c_i(x)$ larning qiymatlarini qo'yib, bir jinsli bo'lmagan tenglamalar sistemasining umumiy yechimini hosil qilamiz.

4-misol. Koshi masalasini yeching.

$$\left.\begin{aligned} y'_1 &= 4y_1 - 5y_2 + 4x + 1, \\ y_1(0) &= 1,\ y_2(0) = 2 \\ y'_2 &= y_1 - 2y_2 + x. \end{aligned}\right\} \quad (1)$$

► Avval, bir jinsli sistemaga mos keluvchi umumiy yechimni topamiz.

$$\left.\begin{aligned} y'_1 &= 4y_1 - 5y_2 \\ y'_2 &= y_1 - 2y_2 \end{aligned}\right\}. \quad (2)$$

Uning xarakteristik tenglamasining ildizlari: $\lambda_1 = -1$, $\lambda_2 = 3$ umumiy yechimi esa quyidagi ko'rinishda izlaymiz (1 holga qarang):

$$\begin{aligned} y_1 &= c_1e^{-x} + 5c_2e^{3x}, \\ y_2 &= c_1e^{-x} + c_2e^{3x}. \end{aligned} \quad (3)$$

(3) yechimda c_1 va c_2 larni $c_1(x)$ va $c_2(x)$ noma'lum funksiyalar deb hisoblaymiz (o'zgarmasni variatsiyalash usulini

ma'nosi ham shunda), y_1 va y_2 larni (1) dastlabki sistemaning yechimlari bo'lsin deb talab qilamiz va quyidagilarni topamiz.

$$y_1' = c_1'(x)e^{-x} - c_1(x)e^{-x} + 5c_2'(x)e^{3x} + 15c_2(x)e^{3x},$$
$$y_2' = c_1'(x)e^{-x} - c_1(x)e^{-x} + c_2'(x)e^{3x} + 3c_2(x)e^{3x}.$$

(1) sistemaga $y_1, y_2,\ y_1', y_2'$ lar uchun olingan ifodalarni qo'yamizyu. O'xshash hadlarni ihchamlab, quyidagi sistemani hosil qilamiz:

$$\left.\begin{aligned} c_1'(x)e^{-x} + 5c_2'(x)e^{3x} = 4x+1, \\ c_1'(x)e^{-x} + c_2'(x)e^{3x} = x. \end{aligned}\right\}$$

bundan,

$$c_1'(x) = \frac{1}{4}(x-1)e^{x}, \quad c_2'(x) = \frac{1}{4}(3x+1)e^{-3x}.$$

Oxirgi tenglikni integrallab, quyidagiga ega bo'lamiz.

$$c_1(x) = \frac{1}{4}(x-2)e^{x} + c_1, \qquad c_2(x) = -\frac{1}{12}(3x+2)e^{-3x} + c_2.$$

$c_1(x)$ va $c_2(x)$ larni (3) tenglikdagi C_1 va C_2 larning o'rniga qo'yib, dastlabki (1) bir jinsli bo'lmagan sistemaning umumiy yechimini topamiz.

$$y_1 = c_1e^{-x} + 5c_2e^{3x} + \frac{1}{4}(x-2) - \frac{5}{12}(3x+2),$$
$$y_2 = c_1e^{-x} + c_2e^{3x} + \frac{1}{4}(x-2) - \frac{1}{12}(3x+2)$$

Boshlang'ich shartlardan foydalanib, c_1 va c_2 o'zgarmaslarni topish uchun quyidagi sistemadan

$$\left.\begin{aligned} 1 = c_1 + 5c_2 - 1/2 - 5/6 \\ 2 = c_1 + c_2 - 1/2 - 1/6 \end{aligned}\right\}$$

$c_1 = 11/4$, $c_2 = -1/12$ qiymatlarni hosil qilamiz. Shunday qilib, Koshi masalasining yechimi quyidagi ko'rinishda aniqlanadi.

$$y_1 = \frac{11}{4}e^{-x} - \frac{5}{12}e^{3x} + \frac{1}{4}(x-2) - \frac{5}{12}(3x+2).$$

$$y_2 = \frac{11}{4}e^{-x} - \frac{1}{12}e^{3x} + \frac{1}{4}(x-2) - \frac{1}{12}(3x+2). \blacktriangleleft$$

II. (11.60) sistemaning integrallashning ikkinchi usuli (*noma'lumlarni yo'qotish usuli*) quyidagilardan iborat. Ba'zi shartlarni bajarishda, bittadan boshqa, masalan y_1 dan boshqa, barcha noma'lum funksiyalarni yo'qotish mumkin va $y_1(x)$ uchun bitta n – tartibli (agar (11.60) sistemada $a_{ij} = const$ bo'lsa) o'zgarmas koeffitsientli bir jinsli bo'lmagan chiziqli differensial tenglama hosil qilish mumkin. Uni echib, qolgan barcha $y_2(x),...,y_n(x)$ noma'lum funksiyalarni differensiallash amali yordamida topamiz. Bu quyidagicha bajariladi. (11.60) sistemaning ($a_{ij} = const$ deb hisoblaymiz). Birinchi tenglamasining har ikkala tomonini x bo'yicha differensiallaymiz. So'ngra $y_1', y_2',..., y_n'$ larning o'rniga (11.60) sistemadan ularning qiymatlarini qo'yamiz va quyidagini hosil qilamiz.

$$y_1'' = a_{11}y_1' + a_{12}y_2' + ... + a_{1n}y_n' + f_1'(x) = L_2(y_1, y_2,..., y_n) + F_2(x), \quad (11.68)$$

bu yerda $L_2(y_1, y_2,..., y_n)$ $y_1, y_2, y_3,..., y_n$ funksiyalarning o'zgarmas koeffitsientlar bilan ma'lum chiziqli kombinatsiyani, $F_2(x)$ esa $f_1(x)$, $f_2(x),..., f_n(x)$ va $f_1'(x)$ funksiyalarning chiziqli kombinatsiyasini ifodalaydi. (11.68) tenglamaning ikkala tomonini x bo'yicha differensiallab, yana bir jinsli bo'lmagan chiziqli tenglamani hosil qilamiz.

$$y_1''' = L_3(y_1, y_2,..., y_n) + F_3(x).$$

Bu jarayonni davom ettirib, quyidagini topamiz.

$$y_1^{(n)} = L_n(y_1, y_2,..., y_n) + F_n(x).$$

Natijada, n – ta tenglamadan iborat sistemani hosil qilamiz.

$$\left.\begin{aligned} &y_1' = a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n + f_1(x), \\ &y_1'' = L_2(y_1, y_2, \ldots, y_n) + F_2(x), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &y_1^{(n-1)} = L_{n-1}(y_1, y_2, \ldots, y_n) + F_{n-1}(x), \end{aligned}\right| . \tag{11.69}$$

(11.69) sistemaning birinchi $n-1$ tenglamasini, $y_2, y_3, \ldots, y_n$ funksiyalarga nisbatan yechib olamiz (bu qoida bo‘yicha mumkin). Ko‘rinib turibdiki, bu funksiyalar $x, y_1, y_1', y_1'', \ldots, y_1^{(n-1)}$ lar orqali ifodalanadi.

$$\begin{aligned} &y_2 = \varphi_2\left(x, y_1, y_1', y_1'', \ldots, y^{(n-1)}\right), \\ &y_3 = \varphi_3\left(x, y_1, y_1', y_1'', \ldots, y_1^{(n-1)}\right), \\ &y_n = \varphi_n\left(x, y_1, y_1', y_1'', \ldots, y_1^{(n-1)}\right). \end{aligned} \tag{11.70}$$

(11.70) sistemadan $y_2, y_3, \ldots, y_n$ ifodalarni (11.69) tenglamalar sistemasining oxirgi tenglamasiga qo‘yib, n – tartibli o‘zgarmas koeffitsientli, bir jinsli bo‘lmagan chiziqli differensial tenglamaga kelamiz.

$$y_1^{(n)} = F\left(x, y_1, y_1', y_1'', \ldots, y_1^{(n-1)}\right),$$

Buning umumiy yechimi ma’lum metodlar yordamida aniqlanadi (§ 11.5 ga qarang).

$$y_1 = \psi_1(x, c_1, c_2, \ldots, c_n). \tag{11.71}$$

Oxirgi ifodani x bo‘yicha $n-1$ marta differensiallab, $y_1', y_1'', \ldots, y_1^{(n-1)}$ hosilalarni topamiz. Ularni (11.70) sistemga qo‘yamiz va (11.71) funksiya bilan birgalikda dastlabki sistemaning umumiy yechimini topamiz.

$$\begin{aligned} &y_2 = \psi_2(x, c_1, c_2, \ldots, c_n), \\ &y_3 = \psi_3(x, c_1, c_2, \ldots, c_n), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &y_n = \psi_n(x, c_1, c_2, \ldots, c_n) \end{aligned} , \tag{11.72}$$

(11.71)-(11.72) sistemalarni va berilgan boshlang'ich shartlarni e'tiborga olgan holda, Koshi masalasini yechish uchun $C_1, C_2, ..., C_n$ ixtiyoriy o'zgarmaslarni topamiz va ularni (11.71)-(11.72) sistemalarga qo'yamiz.

5-misol. Noma'lumlarni yo'qotish usuli bilan quyidagi sistemaning

$$\left.\begin{aligned} y_1' &= 3y_1 - y_2 + y_3 + e^x, \\ y_2' &= y_1 + y_2 + y_3 + x, \\ y_3' &= 4y_1 - y_2 + 4y_3. \end{aligned}\right\} \quad (1)$$

$$y_1(0) = 0,34, \quad y_2(0) = -0,16, \quad y_3(0) = 0,27 \quad (2)$$

Boshlang'ich shartlarini qanoatlantiruvchi umumiy va xususiy yechimlarni toping.

► (1) sistemaning birinchi tenglamasini x bo'yicha differensiallaymiz va y_1', y_2', y_3' lar o'rniga ularning shu sistemadagi ifodalarini qo'yamiz.

$$y_1'' = 3y_1' - y_2' + y_3' + e^x = 3(3y_1 - y_2 + y_3 + e^x) - (y_1 + y_2 + y_3 - x) +$$
$$+4y_1 - y_2 + 4y_3 + e^x = 12y_1 - 5y_2 + 6y_3 + 4e^x + x.$$

y_1'' ni x bo'yicha differensiallab va yana y_1', y_2', y_3' larni ularning (1) sistemadagi ifodalari bilan almashtirib, quyidagini hosil qilamiz.

$$y_1''' = 12y_1' - 5y_2' + 6y_3' + 4e^x + 1 = 12(3y_1 - y_2 + y_3 + e^x) - 5(y_1 + y_2 + y_3 - x) +$$
$$+6(4y_1 - y_2 + 4y_3) + 4e^x - x = 55y_1 - 23y_2 + 31y_3 + 16e^x + 6x.$$

Bu holda (11.69) sistema quyidagi ko'rinishga ega bo'ladi.

$$\left.\begin{aligned} y_1' &= 3y_1 - y_2 + y_3 + e^x, \\ y_1'' &= 12y_1 - 5y_2 + 6y_3 + 4e^x + x, \\ y_1''' &= 55y_1 - 23y_2 + 31y_3 + 16e^x + 6x. \end{aligned}\right\} \quad (3)$$

Birinchi 2 ta tenglamadan y_2 va y_3 larni topamiz.

$$\begin{aligned} y_2 &= y_1'' - 6y_1' + 6y_1 + 2e^x - x, \\ y_3 &= y_1'' - 5y_1' + 3y_1 + e^x - x. \end{aligned} \quad (4)$$

y_2 va y_3 ifodalarni (3) sistemadagi uchinchi tenglamaga qo'yamiz.

$$y_1''' = 55y_1 - 23\left(y_1' - 6y_1' + 6y_1 + 2e^x - x\right) + 31\left(y_1'' - 5y_1' + 3y_1 + e^x - x\right) +$$

$$+16e^x + 6x = 8y_1'' - 17y_1' + 10y_1 + e^x - 2x .$$

Quyidagi 3-tartibli o'zgarmas koeffitsientini bir jinsli bo'lmagan chiziqli tenglamani hosil qilamiz:

$$y_1''' - 8y_1'' + 17y_1' - 10y_1 = e^x - 2x . \quad (5)$$

Uni ma'lum usulda yechamiz (§ 11.5 ga qarang). Xarakteristik tenglamasini tuzamiz.

$\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$. (6)

Buning yechimlari $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 5$ bo'ladi. (5) tenglamaga mos keluvchi bir jinsli tenglamaning umumiy yechimi y_1 quyidagi ko'rinishda bo'ladi.

$$y_1 = c_1 e^x + c_2 e^{2x} + c_3 e^{5x} .$$

(5) tenglamaning o'ng tomoni (11.50) va (11.51) maxsus ko'rinishdagi 2 ta funksiyaning yig'indisidan iborat.

$$f(x) = f_1(x) + f_2(x), \quad f_1(x) = e^x, \quad f_2(x) = -2x, \quad f_1(x) = e^x .$$

uchun $Z = 1$, ya'ni $\lambda_1 = 1$ ildiz bilan ustma-ust tushadi, shuning uchun $k = 1$. $f_2(x) = -2x$ uchun $Z = 1$ va y (6) xarakteristik tenglama ildizlari orasida yo'q, shuning uchun $k = 0$.

Shunday qilib, (5) tenglamaning xususiy yechimi $y*$ ni quyidagi ko'rinishda izlash kerak.

$$y_1^* = Axe^x + Bx + c .$$

bu yerda A, B, C noaniq son aniqmas koeffitsientlar usuli yordamida topiladi. $y_1^{*'}, y_1^{*''}, y_1^{*'''}$ larni aniqlab, y_1^* bilan birgalikda (5) tenglamaga qo'yamiz va quyidagiga ega bo'lamiz.

$y_1^{*'} = Ae^x + Axe^x + B, \quad y_1^{*''} = 2Ae^x + Axe^x,$

$y_1^{*'''} = 3Ae^x + Axe^x,$

$$3Ae^x + Axe^x - 8\left(2Ae^x + Axe^x\right) + 17\left(Ae^x + Axe^x + B\right) -$$

$$10\left(Axe^x + Bx + c\right) = e^x - 2x,$$

$$4Ae^x + 17B - 10Bx - 10c = e^x - 2x,$$

$$4A = 1, \quad -10B = -2, \quad 17B - 10c = 0,$$

bu yerdan $A = 1/4, \quad B = 1/5, \quad c = 17/50.$

Shunday qilib,

$$y_1^* = \frac{1}{4}xe^x + \frac{1}{5}x + \frac{17}{50}.$$

(5) tenglamaning umumiy yechimi quyidagi formuladan topiladi.

$$y_1 = y_1 + y_1^* = c_1e^x + c_2e^{2x} + c_3e^{5x} + \frac{1}{4}xe^x + \frac{1}{5}x + \frac{17}{50}.$$

y_1', y_1'' hosilalarni topamiz va ularni (4) tenglikka qo'yamiz:

$$y_1' = c_1e^x + 2c_2e^{2x} + 5c_3e^{5x} + \frac{1}{4}xe^x + \frac{1}{5},$$

$$y_1'' = c_1e^x + 4c_2e^{2x} + 25c_3e^{5x} + \frac{1}{2}e^x + \frac{1}{4}xe^x,$$

$$y_2 = c_1e^x + 4c_2e^{2x} + 25c_3e^{5x} + \frac{1}{2}e^x + \frac{1}{4}xe^x - 6\left(c_1e^x + 2c_2e^{2x} + 5c_3e^{5x} + \frac{1}{4}e^x + \frac{1}{4}xe^x + \frac{1}{5}\right) +$$

$$+6\left(c_1e^x + c_2e^{2x} + c_3e^{5x} + \frac{1}{4}xe^x + \frac{1}{5}x + \frac{17}{50}\right) +$$

$$2e^x - x = c_1e^x - 2c_2e^{2x} + c_3e^{5x} - e^x + \frac{1}{4}xe^x + \frac{6}{5}x + \frac{21}{25}.$$

$$y_3 = c_1e^x + 4c_2e^{2x} + 25c_3e^{5x} + \frac{1}{2}e^x + \frac{1}{4}xe^x - 5\left(c_1e^x + 2c_2e^{2x} + 5c_3e^{5x} + \frac{1}{4}e^x + \frac{1}{4}xe^x\right) +$$

$$+3\left(c_1e^x + c_2e^{2x} + c_3e^{5x} + \frac{1}{4}xe^x + \frac{1}{5}x + \frac{17}{50}\right) + e^x - x =$$

$$= c_1e^x - 3c_2e^{2x} + 3c_3e^{5x} + \frac{1}{4}e^x - \frac{1}{4}xe^x - \frac{2}{5}x + \frac{1}{50}.$$

Shunday qilib, (1) sistemaning umumiy yechimi topiladi.

$$\left.\begin{aligned}
&y_1=c_1e^{2x}+c_2e^{2x}+c_3e^{5x}+\frac{1}{4}xe^{x}+\frac{1}{5}x+\frac{17}{50},\\
&y_2=c_1e^{x}-2c_2e^{2x}+c_3e^{5x}-e^{x}+\frac{1}{4}xe^{x}+\frac{6}{5}x+\frac{21}{25},\\
&y_3=-c_1e^{x}-3c_2e^{2x}+3c_3e^{5x}+\frac{1}{4}e^{x}+\frac{1}{4}xe^{x}-\frac{2}{5}x-\frac{21}{50},
\end{aligned}\right|$$

Koshi masalasini yechish uchun boshlang‘ich shartlardan foydalanamiz. c_1,c_2,c_3 ixtiyoriy o‘zgarmaslarni aniqlash uchun quyidagi sistemani hosil qilamiz.

$$\left.\begin{aligned}
\frac{17}{50}&=c_1+c_2+c_3+\frac{17}{50},\\
-\frac{4}{25}&=c_1-2c_2+c_3-1+\frac{21}{25},\\
\frac{27}{100}&=-c_1-3c_2+3c_3+\frac{1}{4}+\frac{1}{50}.
\end{aligned}\right|$$

Bundan $c_1=0,\ c_2=0,\ c_3=0$.

Izlanayotgan xususiy yechim quyidagi ko‘rinishga ega bo‘ladi.

$$\left.\begin{aligned}
y_1&=\frac{1}{4}xe^{x}+\frac{1}{5}x+\frac{17}{50},\\
y_2&=\frac{1}{4}xe^{x}-e^{x}+\frac{6}{5}x+\frac{21}{25},\\
y_3&=-\frac{1}{4}xe^{x}+\frac{1}{4}e^{x}-\frac{2}{5}x+\frac{1}{50}.
\end{aligned}\right|$$ ◄

11.8. AT

1. Noma’lumlarni yo‘qotish usulidan foydlanmasdan, quyidagi bir jinsli tenglamalar sistemasining umumiy yechimini toping.

a) $\begin{cases} y_1'=-7y_1+y_2, \\ y_2'=-2y-5y_2, \end{cases}$ b) $\begin{cases} y_1'=y_1-3y_2, \\ y_2'=3y_1+y_2; \end{cases}$ v) $\begin{cases} y_1'=y_1-y_2+y_3, \\ y_2'=y_1+y_2-y_3, \\ y_3'=2y_1-y_2. \end{cases}$

(*Javob:*

a) $y_1 = e^{-6x}(c_1\cos x + c_2\sin x)$, $y_2 = e^{-6x}((c_1+c_2)\cos x - (c_1 - c_2)\sin x)$;

b) $y_1 = e^x(c_1\sin 3x + c_2\cos 3x)$; $y_2 = e^x(c_1 \sin 3x - c_2\cos 3x)$;

v) $y_1 = c_1e^x + c_2e^{2x} + c_3e^{-x}$, $y_2 = c_1e^x - 3c_3e^{-x}$,

$y_3 = c_1e^x + c_2e^{2x} - 5c_3e^{-x}$.)

2. Quyidagi har bir tenglamalar sistemasining umumiy yechimini noma'lumlarni yo'qotish usuli bilan toping.

a) $\begin{cases} y_1' = -5y_1 + 2y_2 + e^x, \\ y_2' = y_1 + 6y_2 + e^{-2x}. \end{cases}$ b) $\begin{cases} y_1' = 3y_1 - 2y_2 + x, \\ y_2' = 3y_1 - 4y_2; \end{cases}$ v) $\begin{cases} y_1' = 5y_1 + 2y_2 - 3y_3, \\ y_2' = 5y_1 + 5y_2 - 4y_3, \\ y_3' = 6y_1 + 4y_2 - 4y_3. \end{cases}$

(*Javob:* a) $y_1 = c_1e^{-4x} + c_2e^{-7x} + \frac{7}{40}e^x + \frac{1}{5}e^{-2x}$,

$y_2 = \frac{1}{2}c_1e^{-4x} - c_2e^{7x} + \frac{1}{40}e^x - \frac{3}{10}e^{-2x}$;

b) $y_1 = 2c_1e^{2x} + c_2e^{-3x} - \frac{2}{3}x - \frac{5}{18}$, $y_2 = c_1e^{2x} + 3c_2e^{-3x} - \frac{1}{2}x - \frac{1}{12}$;

v) $y_1 = c_1e^x + c_2e^{2x} + c_3e^{3x}$, $y_2 = c_1e^x + 2c_3e^{3x}$; $y_3 = 2c_1e^x + c_2e^{2x} + 2c_3e^{3x}$.)

3. Quyidagi differensial tenglamalar uchun, Koshi masalasini yeching.

a) $\begin{cases} y_1' = y_2, \\ y_2' = y_3, \\ y_3' = y_1, \end{cases}$ $y_1(0) = y_2(0) = y_3(0) = 1$; b)

$\begin{cases} y_1' = y_2 + y_3, \\ y_2' = y_1 + y_3, \\ y_3' = y_1 + y_2. \end{cases}$ $y_1(0) = -1$, $y_2(0) = 1$, $y_3(0) = 0$.

(*Javob:* a) $y_1 = y_2 = y_3 = e^x$; b) $y_1 = -e^{-x}$, $y_2 = e^{-x}$, $y_3 = 0$.)

Mustaqil ish

Differensial tenglamalar sistemasining umumiy yechimini toping.

1. $\begin{cases} y_1' = y_2 + tg^2x - 1, \\ y_2' = -y_1 + tgx. \end{cases}$

(*Javob*:
$y_1 = c_1\cos x + c_2\sin x + tgx,\ y_2 = -c_1\sin x + c_2\cos x + 2$.)

2. $\begin{cases} y_1' = y_1 - y_2, \\ y' = y_1 + y_2 + e^x. \end{cases}$

(*Javob*: $y_1 = ((c_1\cos x + c_2\sin x - 1)e^x,\ y_2 = (c_1\sin x - c_2\cos x)e^x)$.)

3. $\begin{cases} y_1' = y_1 + y_2 - \cos x, \\ y_2' = -2y_1' - y_2 + \sin x + \cos x. \end{cases}$

(*Javob*:
$y_1 = c_1\cos x + c_2\sin x - x\cos x,\ \ y_2 = (c_2 - c_1)\cos x - (c_1 + c_2)\sin x + x(\cos x + \sin x)$.)

11.8. 11 BOBGA INDIVIDUAL UY VAZIFALARI

IUT – 11.1.

Differensial tenglamaning umumiy yechimi (umumiy integral) ni toping.

1.

1.1. $e^{x+3y}dy = xdx$. (*Javob*: $e^{3y} = 3\left(c - xe^{-x} - e^{-x}\right)$.)

1.2. $y'\sin x = y\ln y$. (*Javob*: $\ln y = c * tg(x/2)$.)

1.3. $y' = (2x-1)ctgy$. (*Javob*: $\ln|\cos y| = x - x^2 + c$.)

1.4. $sec^2 xtgydy + \sec^2 ytgxdy = 0$. (*Javob*: $c = tgytgx$.)

1.5. $(1+e^x)ydy - e^y dx = 0$. (*Javob*: $-e^{-y}(y+1) = \ln\frac{e^x}{e^x+1} + c$.)

1.6. $(y^2+3)dx - \frac{e^x}{x}ydy = 0$.(*Javob*: $\ln(y^2+3) = 2\left(c - xe^{-x} - e^{-x}\right)$.)

1.7. $\sin y\cos xdy = \cos y\sin xdx$. (*Javob*: $C = \cos x/\cos y$.)

1.8. $y' = (2y+1)tgx$. (*Javob*: $\sqrt{2y+1} = c/\cos x$.)

1.9. $(sin(x+y)+\sin(x-y))dx+\frac{dy}{cosy}=0$.(*Javob:* $tgy=c+2\cos x$)

1.10. $(1+e^x)yy'=e^x$ (*Javob:* $y^2=2lnC(1+e^x)$)

1.11. $\sin xtgydx-\frac{dy}{\sin x}=0$. (*Javob:* $\ln|\sin y|=c+\frac{1}{2}x-\frac{1}{4}\sin 2x$.)

1.12. $3e^x\sin ydx+(1-e^x)\cos ydy=0$. (*Javob:* $\sin y=c(e^x-1)^3$.)

1.13. $y'=e^{2x}/\ln y$. (*Javob:* $y(\ln y-1)=\frac{1}{2}e^{2x}+c$.)

1.14. $3^{x^2+y}dy+xdx=0$. (*Javob:* $3^{y}=\frac{1}{2}3^{-x^2}+c\ln 3$.)

1.15. $(cos(x-2y)+\cos(x+2y))y'=\sec x$ (*Javob:* $\sin 2y=tgx+c$

1.16. $y'=e^{x^2}x(1+y^2)$. (*Javob:* $arctgy=c+\frac{1}{2}e^{x^2}$.)

1.17. $ctgx\cos^2 ydx+\sin^2 xtgydy=0$. (*Javob:* $tg^2y=ctg^2x+2c$.)

1.18. $\sin x\cdot y'=y\cos x+2\cos x$. (*Javob:* $y=C\sin x-2$.)

1.19. $1+(1+y')e^y=0$. (*Javob:* $c(e^y-1)=e^{-x}$.)

1.20. $y'ctgx+y=2$. (*Javob:* $y=c\cos x+2$.)

1.21. $\frac{e^{-y^2}dy}{x}+\frac{dx}{\cos^2 y}=0$. (*Javob:* $\frac{1}{2}y+\frac{1}{4}\sin 2y=c-\frac{1}{2}e^{x^2}$.)

1.22. $e^x\sin ydx+tgydy=0$. (*Javob:* $\ln\left|tg\left(\frac{\pi}{4}+\frac{y}{2}\right)\right|=c-e^x$.)

1.23. $(1+e^{3y})xdx=e^{3y}dy$. (*Javob:* $\frac{x^2}{2}=\frac{1}{3}\ln(1+e^{3y})+c$.)

1.24. $(\sin(2x+y)-\sin(2x-y))dx=\frac{dy}{\sin y}$. (*Javob:* $ctgy=C-\sin 2x$.)

1.25. $\cos y dx = 2\sqrt{1+x^2}dy + \cos y\sqrt{1+x^2}dy$.(*Javob:* $2\ln\left|tg\left(\frac{\pi}{4}+\frac{y}{2}\right)\right| + y = \ln\left|x+\sqrt{1+x^2}\right| + c$.)

1.26. $y'\sqrt{1-x^2} - \cos^2 y = 0$. (*Javob:* $tgy = \arcsin x + c$.)

1.27. $e^x tgy dx = (1-e^x)\sec^2 y dy$. (*Javob:* $tgy = c/(e^x-1)$.)

1.28. $y' + \sin(x+y) = \sin(x-y)$.

(*Javob:* $\ln\left|tg\frac{y}{2}\right| = c - 2\sin x$.)

1.29. $\cos^3 y \cdot y' - \cos(2x+y) = \cos(2x-y)$.

(*Javob:* $\frac{1}{2}y + \frac{1}{4}\sin 2y = \sin 2x + c$.)

1.30. $3^{y^2-x^2} = yy'/x$. (*Javob:* $3^{-y^2} = 3^{-x^2} - 2c\ln 3$.)

2

2.1. $(xy + x^3y)y' = 1 + y^2$. (*Javob:* $Cx = \sqrt{(1+x^2)(1+y^2)}$.)

2.2. $y'/7^{y-x} = 3$. (*Javob:* $7^{-y} = 3\cdot 7^{-x} + c\ln 7$.)

2.3. $y - xy' = 2(1 + x^2y')$. (*Javob:* $y = Cx/\sqrt{1+2x^2} + 2$.)

2.4. $y - xy' = 1 + x^2y'$. (*Javob:* $y = Cx/(x+1) + 1$.)

2.5. $(x+4)dy - xddx = 0$. (*Javob:* $y = Ce^x/(x+4)^4$.)

2.6. $y' + y + y^2 = 0$. (*Javob:* $y/(y+1) = C - x$.)

2.7. $y^2\ln xdx-(y-1)xdy=0$. (*Javob:* $\frac{1}{y}+\ln y=C+\frac{1}{2}\ln^2 x$.)

2.8. $\left(x+xy^2\right)dy+ydx-y^2dx=0$.(*Javob:* $y+\ln\frac{(y-1)^1}{y}=c+\ln x$.)

2.9. $y'+2y+y^2=0$. (*Javob:* $\sqrt{(y-2)/y}=Ce^x$.)

2.10. $\left(x^2+x\right)ydx+\left(y^2+1\right)dy=0$.(*Javob:* $\frac{y^2}{2}+\ln y=c-\frac{x^2}{3}-\frac{x^2}{2}$.)

2.11. $\left(xy^3+x\right)dx+\left(x^2y^2-y^2\right)dy=0$.(*Javob:* $\sqrt[3]{y^3+1}=c\sqrt{x^2-1}$.)

2.12. $\left(1+y^2\right)dx-\left(y+yx^2\right)dy=0$.(*Javob* $\frac{1}{2}\ln\left(y^2+1\right)=c+arctgx$)

2.13. $y'=2xy+x$. (*Javob:* $\frac{1}{2}\ln|2x+1|=x^2/2+c$.)

2.14. $y-xy'=3\left(1+x^2y\right)$. (*Javob:* $y=c\sqrt[3]{x}/\sqrt[3]{x+3}+3$.)

2.15. $2xyy'=1-x^2$. (*Javob:* $y^2=\ln|x|-\frac{x^2}{2}+c$.)

2.16. $\left(x^2-1\right)y'-xy=0$. (*Javob:* $y=c\sqrt{x^2-1}$.)

2.17. $\left(y^2x+y^2\right)dy+xdx=0$. (*Javob:* $y^3=3\left(C-x+\ln|x+1|\right)$.)

2.18. $(1+x^3)y^3dxy-(y^2-1)x^3dy=0$.(*Javob:* $\ln y+\frac{1}{2y^2}=ctx-\frac{1}{x^2}$.)

2.19. $xy'-y=y^2$. (*Javob:* $y/(y+1)=Cx$.)

2.20. $\sqrt{y^2+1}dx=xydy$. (*Javob:* $\sqrt{y^2+1}=\ln Cx$.)

2.21. $y'-xy^2=2xy$. (*Javob:* $\ln|y|(y+2)|=c+x^2$.)

2.22. $2x^2yy'+y^2=2$. (*Javob:* $\ln|2-y^2|=c+1/x$.)

2.23. $y'=(1+y^2)/(1+x^2)$. (*Javob:* arc .)

2.24. $y'\sqrt{1+y^2}=x^2 \mid y$. (*Javob:* $\sqrt{(1+y^2)^3}=c+x^2$.)

2.25. $(y+1)y'=\frac{y}{\sqrt{1-x^2}}+xy$.(*Javob:* $y+\ln y=\arcsin x+x^2/2+c$.)

2.26. $(1+x^2)y'+y\sqrt{1+x^2}=xy$. (*Javob:* $y=\frac{c\sqrt{1+x^2}}{x+\sqrt{1+x^2}}$.)

2.27. $xyy'=\frac{1+x^2}{1-y^2}$. (*Javob:* $2y^2-y^4=4\ln|x|+2x^2+c$.)

2.28. $(xy-x)^2dy+y(1-x)dx=0$.(*Javob:* $\frac{7}{2}-2y+\ln|y|=\ln|x|+\frac{1}{x}+c$.)

2.29. $(x^2y-y)^2y'=x^2y-y+x^2-1$.(*Javob:* $\frac{y^2}{2}-y+\ln|y+1|=\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+c$.)

2.30. $\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$. (*Javob:* $\sqrt{1-y^2} = \arcsin x + c$.)

3

3.1. $y - xy' = x\sec\frac{y}{x}$. (*Javob:* $\sin\frac{y}{x} = \ln\frac{c}{|x|}$.)

3.2. $(y^2 - 3x^2)dy + 2xydx = 0$. (*Javob:* $(y^2 - x^2)^2\ cx^2y^3$.)

3.3. $(x+2y)dx - xdy = 0$. (*Javob:* $y = cx^2 - x$.)

3.4. $(x-y)dx + (x+y)dy = 0$. (*Javob:* $arctg\frac{y}{x} + \frac{1}{2}\ln\frac{y^2+x^2}{x^2} = \ln\frac{c}{x}$.)

3.5. $(y^2 - 2xy)dx + x^2dy = 0$. (*Javob:* $y/(x-y) = Cx$.)

3.6. $y^2 + x^2y' = xyy'$. (*Javob:* $e^{y/x} = cy$.)

3.7. $xy' - y = xtg(y/x)$. (*Javob:* $\sin(y/x) = Cx$.)

3.8. $xy' = y - xe^{y/x}$. (*Javob:* $e^{-y/x} = \ln Cx$.)

3.9. $xy' - y' = (x+y)\ln((x+y)/x)$. (*Javob:* $\ln|1 + y/x| = Cx$.)

3.10. $xy' = y\cos\ln(y/x)$. (*Javob:* $Ctg\left(\frac{1}{2}\ln\frac{y}{x}\right) = \ln Cx$.)

3.11. $(y + \sqrt{xy})dx = xdy$. (*Javob:* $y = \frac{x}{y}\ln^2 Cx$.)

3.12. $xy' = \sqrt{x^2 - y^2} + y$. (*Javob:* $(y/x) = \ln Cx$.)

3.13. $y = x\left(y' - \sqrt[x]{e^{y}}\right)$. (*Javob:* $-e^{-y/x} = \ln Cx$.)

3.14. $y' = y/x - 1$. (*Javob:* $y = x\ln(c/x)$.)

3.15. $y'x + x + y = 0$. (*Javob:* $y = \frac{c}{x} - \frac{x}{2}$.)

3.16. $ydx + \left(2\sqrt{xy} - x\right)dy = 0$. (*Javob:* $\sqrt{\frac{y}{x}} - \frac{y}{x} = \ln Cx$.)

3.17. $xdy - ydx = \sqrt{x^2 + y^2}\,dx$. (*Javob:* $y + \sqrt{x^2 + y^2} = Cx^2$.)

3.18. $\left(4x^2 + 3xy + y^2\right)dx + \left(4y^2 + 3x + x^2\right)dy = 0$.(*Javob:* $\frac{2}{5}\ln\left(\frac{y+x}{x}\right) +$

$+\frac{9}{5}\ln\left(\frac{y^2 + 4x^2}{x^2}\right) - \frac{3}{10}arctg\frac{y}{2x} = \ln\frac{c}{x}$.)

3.19. $(x - y)ydx - x^2dy = 0$. (*Javob:* $y = x/\ln Cx$.)

3.20. $xy + y^2 = \left(2x^2 + xy\right)y^2$. (*Javob:* $\frac{y}{x} + 2\ln\frac{y}{x} = \ln\frac{c}{x}$.)

3.21. $\left(x^2 - 2xy\right)y' = xy - y^2$. (*Javob:* $\frac{x}{y} + 2\ln\frac{y}{x} = -\ln Cx$.)

3.22. $\left(2\sqrt{xy} - y\right)dx + xdy = 0$. (*Javob:* $y = x\ln^2|Cx|$.)

3.23. $xy' + y\left(\ln\frac{y}{x} - 1\right) = 0$. (*Javob:* $y = xe^{c/x}$.)

3.24. $\left(x^2 + y^2\right)dx + 2xydy = 0$. (*Javob:* $y^2 = c^3/3^x - x^2/3$.)

3.25. $\left(y^2 - 2xy\right)dx - x^2dy = 0$. (*Javob:* $y = x/\ln Cx$.)

3.26. $(x+2y)dx+xdy=0$. (*Javob:* $y=c^3/(3x^2)-x/3$.)

3.27. $(2x-y)dx+(x+y)dy=0$.

Javob: $\frac{1}{2}\ln\left(\frac{y^2+x^2}{x^2}\right)+arctg\frac{y}{x}=\ln Cx$.)

3.28. $2x^3y'=y'(2x^2-y^2)$. (*Javob:* $y^2=x^2/\ln(Cx)^4$.)

3.29. $x^2y'=y(x+y)$. (*Javob:* $y=-x/\ln(Cx)$.)

3.30. $y'=\frac{x}{y}+\frac{y}{x}$. (*Javob:* $y^2=x^2\ln(Cx)^2$.)

4. Differensial tenglamaning xususiy hosilasini (xususiy integralini) toping.

4.1. $(x^2+1)y'+4xy=3$; $y(0)=0$.

(*Javob:* $y=(x^3+3x)/(x^2+1)^2$.)

4.2. $y'+ytgx=\sec x$, $y(0)=0$. (*Javob:* $y=\sin x$.)

4.3. $(1-x)(y'-y)=e^{-x}$, $y(0)=0$.

(*Javob:* $y=e^{-x}\ln\frac{1}{1-x}$.)

4.4. $xy'-2x=2x^4$, $y(1)=0$. (*Javob:* $y=x^4-x^2$.)

4.5. $y'=2x(x^2+y)$, $y(0)=0$. (*Javob:* $y=x^2+1-e^x$.)

4.6. $y'-y=e^x$, $y(0)=1$. (*Javob:* $y=(x+1)e^x$.)

4.7. $xy'+y+xe^{-x^2}=0$, $y(1)=\frac{1}{2e}$. (*Javob:* $y=\frac{e^{-x^2}}{2x}$.)

4.8. $\cos dx = (x + 2\cos y)\sin y dy, \quad y(0) = \pi / 4$

(*Javob:* $x = \left(\sin^2 y - \frac{1}{2}\right)\frac{1}{\cos y}$.)

4.9. $x^2 y' + xy + 1 = 0, \quad y(1) = 0$. (*Javob:* $y = -(\ln x)/x$.)

4.10. $yx' + x = 4y^3 + 3y^2, \quad y(2) = 1$. (*Javob:* $x = y^3 + y^2$.)

4.11. $(2x + y)dy = ydx + 4\ln y dy, \ y(0) = 1$. (*Javob:* $2\ln y + 1 - y$.)

4.12. $y' = y/(3x - y^2), \quad y(0) = 0$. (*Javob:* $x = y^2 - y^3$.)

4.13. $(1 - 2xy)y' = y(y - 1), \quad y(0) = 1$.

(*Javob:* $x(y-1)^2 = (y - \ln y - 1)$.)

4.14. $x(y' - y) = e^x, \quad y(1) = 0$. (*Javob:* $y = e^x \ln x$.)

4.15. $y = x(y' - x\cos x), \ y(\pi/2) = 0$. (*Javob:* $y = (\sin x - 1)x$.)

4.16. $(xy' - 1)\ln x = 2y, \quad y(e) = 0$. (*Javob:* $y = (\ln^3 x - \ln^2 x)/3$.)

4.17. $(2e^y - x)y' = 1, \quad y(0) = 0$. (*Javob:* $x = e^y - e^{-y}$.)

4.18. $xy' + (x+1)y = 3x^2 e^{-x}, \quad y(1) = 0$.

(*Javob:* $y = (x^2 - 1/x)e^{-x}$.)

4.19. $(x + y^2)dy = ydx, \quad y(0) = 1$. (*Javob:* $x = y^2 - y$.)

4.20. $(\sin^2 y + x ctg y)y' = 1, \quad y(0) = \pi / 2$

(*Javob:* $x = -\sin y \cos y$.)

4.21. $(x+1)y' + y = x^3 - x^2, \quad y(0) = 0$

(*Javob:* $y = \dfrac{3x^4 + 4x^3}{12(x+1)}$.)

4.22. $(xy'-2y)+x^2=0, \quad y(1)=0$. (*Javob:* $y=-x^2\ln x$.)

4.23. $xy'+y'=\sin x, \quad y(\pi/2)=2/\pi$.

(*Javob:* $y=(1-\cos x)/x$.)

4.24. $(x^2-1)y'-xy=x^3-x, \quad y(\sqrt{2})=1$.

(*Javob:* $y=x^2-1$.)

4.25. $(1-x^2)y'+xy=1\ y(0)=1$ (*Javob:* $y=x+\sqrt{1-x^2}$)

4.26. $y'ctgx-y=2\cos^2 xCtgx, \quad y(0)=0$.

(*Javob:* $y=\dfrac{6\sin x-2\sin^3 x}{3\cos x}$.)

4.27. $y'+2xy+3, \quad y(1)=-1$. (*Javob:* $y=-1/x$.)

4.28. $y'+2xy=xe^{-x^2}, \quad y(0)=0$. (*Javob:* $y=0,5x^2e^{-x^2}$.)

4.29. $y'-3x^2y-x^2e^{x^3}=0, \quad y(0)=0$.

(*Javob:* $y=\dfrac{1}{3}x^3e^{x^3}$.)

4.30. $xy'+y=\ln x+1, \quad y(1)=0$. (*Javob:* $y=\ln x$.)

5. Differensial tenglamaning umumiy yechimini toping.

5.1. $y'+yx\sqrt{y}$. (*Javob:* $y=\left(xe^{x/2}-2e^{x/2}+c\right)^2e^{-x}$.)

5.2. $ydx+2xdy=2y\sqrt{x}\sec^2 ydy$.

(*Javob:* $x=\left(ytgy+\ln|\cos y|+c\right)^2/y^2$.)

5.3. $y'+2y=y^2e^x$. (*Javob:* $y=/\left(Ce^{2x}+e^x\right)$.)

5.4. $y' = y^4 \cos x + ytgx$. (*Javob:* $y = 1/\left(\cos x\sqrt[3]{c - tgx}\right)$.)

5.5. $xydy = \left(y^2 + x\right)dx$. (*Javob:* $y = x\sqrt{2(c - 1/x)}$.)

5.6. $xy' + 2y + x^5y^3e^x = 0$. (*Javob:* $y = 1/\left(x^2\sqrt{2\left(e^x + c\right)}\right)$.)

5.7. $y'x^3 \sin y = xy' - 2y$. (*Javob:* $x = \sqrt{y/(c - \cos y)}$.)

5.8. $\left(2x^2y\ln y - x\right)y' = y$. (*Javob:* $x = 1/\left(y\left(c - \ln^2 y\right)\right)$.)

5.9. $2y' - \dfrac{x}{y} = \dfrac{xy}{x^2 - 1}$. (*Javob:* $y = \sqrt{c - \sqrt{x^2 - 1}}\sqrt[4]{x^2 - 1}$.)

5.10. $xy' - 2x^2\sqrt{y} = 4y$. (*Javob:* $y = \dfrac{x^4}{4}(c - \ln x)^2$.)

5.11. $xy^2y' = x^2 + y^3$. (*Javob:* $y = x\sqrt[3]{3(c - 1/x)}$.)

5.12. $(x+1)\left(y' + y^2\right) = -y$.

(*Javob:* $y = 1/\left((x+1)\left(c + \ln|x+1|\right)\right)$.)

5.13. $y'x + y = -xy^2$. (*Javob:* $y = 1/\left(x\left(c + \ln x\right)\right)$.)

5.14. $y' - xy = -y^3e^{-x^2}$. (*Javob:* $e^{x^2/2}/\sqrt{2(c + x)}$.)

5.15. $xy' - 2\sqrt{x^3y} = y$. (*Javob:* $y = x\left(x^2/2 + c\right)^2$.)

5.16. $y' + xy = x^3y^3$.

(*Javob:* $y = e^{-x^2/2}/\sqrt{x^2e^{-x^2} + e^{-x^2} + c}$.)

5.17. $y' = \dfrac{x}{y}e^{2x} + y$. (*Javob:* $y = e^x\sqrt{x^2 + c}$.)

5.18. $yx' + x = -yx^2$. (*Javob:* $x = 1/\left(y\left(c + \ln y\right)\right)$.)

5.19. $x(x-1)y' + y^3 = xy$

(*Javob:* $y = (x-1)/\sqrt{2(x-\ln x + c)}$.)

5.20. $2x^3yy' + 3x^2y^2 = 1 = 0$. (*Javob:* $y = \sqrt{c-x}/x^{3/2}$.)

5.21. $\frac{dx}{x} = \left(\frac{1}{y} - 2x\right)dx$. (*Javob:* $x = y/(y^2 + c)$.)

5.22. $y' + x\sqrt[3]{y} = 3y$. (*Javob:* $y = e^{3x}\left(\frac{x}{3}e^{-2x} + \frac{1}{6}e^{-2x} + c\right)$.)

5.23. $xy' + y = y^2\ln x$. (*Javob:* $y = 1/(\ln x + 1 + Cx)$.)

5.24. $xdx = (x^2/y - y^3)dy$. (*Javob:* $x = y\sqrt{c-y^2}$.)

5.25. $y' + 2xy = 2x^3y^3$.

(*Javob:* $y = 2e^{-x^2}/\sqrt{2x^2e^{-2x^2} + e^{-2x^2} + 4c}$.)

5.26. $y' + y = x/y^2$. (*Javob:* $e^{-x}\sqrt[3]{xe^{3x} - \frac{1}{3}e^{3x} + c}$.)

5.27. $y' - ytgx + y^2\cos = 0$. (*Javob:* $y = 1/((x+c)\cos x)$.)

5.28. $y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}$.

(*Javob:* $y = \left(\frac{xtgx + \ln|\cos x| + c}{x}\right)^2$.)

5.29. $y' - y + y^2\cos x = 0$.

(*Javob:* $y = 2e^x/(e^x(\cos x + \sin x) + c)$.)

5.30. $y' = x\sqrt{y} + \frac{xy}{x^2-1}$.

(*Javob:* $y = \left(\frac{1}{3}(x^2-1)^{3/4} + c\right)^2\sqrt{x^2-1}$.)

Namunaviy variantlarning yechilishi

Differensial tenglamaning umumiy yechimini (umumiy integralini) toping.

1. $(xy^2+x)dx+y(y-x^2y)=0.$

▶Berilgan tenglamani quyidagi ko'rinishda yozib olamiz.

$$y(1-x^2)dy=-x(y^2+1)dx.$$

But englama o'zgaruvchilari ajraladigan tenglamadir. O'zgaruvchilarini ajratamiz.

$$\frac{ydy}{y^2+1}=\frac{-xdx}{1-x^2}.$$

Oxirgi tenglikning ikkala tomonini integrallaymiz.

$$\int\frac{ydy}{y^2+1}=-\int\frac{xdx}{1-x^2},\quad \frac{1}{2}\ln(y^2+1)=\frac{1}{2}\ln\left|x^2-1\right|+\frac{1}{2}\ln C,$$

$$y^2+1=C\left|x^2-1\right|,\quad y^2=C\left|x^2-1\right|-1.$$

Shunday qilib, dastlabki tenglamaning umumiy yechimi quyidagicha bo'ladi.

$$y=\pm\sqrt{C\left|x^2-1\right|-1}.\ ◀$$

2. $\sec^2 x tgy\, dx+\sec^2 y\, tgx\, dy=0$

▶Berilgan tenglama o'zgaruvchilari ajraladigan differensial tenglamadir. Ularni ajratib, integrallaymiz va differensial tenglamaning umumiy yechimini hosil qilamiz.

$$\frac{\sec^2 ydy}{tgy}=-\frac{\sec^2 xdx}{tgx},\quad \int\frac{(tgy)}{tgy}=-\int\frac{d(tgx)}{tgx},$$

$$\ln|tgy|=-\ln|tgx|+\ln|C|,\quad tgy=C/tgx,\ tgy\cdot tgx=C.\ ◀$$

3. $y-x\frac{dy}{dx}=x+y\frac{dy}{dx}$

▶Berilgan tenglamadan $\frac{dy}{dx}$ ni topamiz: $\frac{dy}{dx}=\frac{y-x}{x+y}$

Dastlabki tenglama 1-tartibli bir jinsli tenglamadir. Uni o'rniga qo'yish yordamida yechamiz $y=xu(x)$ va quyidagini topamiz.

$$y'=u'x+u,\quad u'x+u=\frac{ux-x}{x+ux},\quad u'x+u=\frac{u-1}{1+u}$$

$$u'x=\frac{u-1}{u+1}-u=\frac{-u^2-1}{u+1},\quad x\frac{du}{dx}=-\frac{u^2+1}{u+1}.$$

O'zgaruvchilari ajraladigan tenglamani hosil qildik. Uni yechamiz.

$$\frac{u+1}{u^2+1}du=-\frac{dx}{x},\int\frac{u+1}{u^2+1}du=-\int\frac{dx}{x},$$

$$\frac{1}{2}\int\frac{2udu}{u^2+1}+\int\frac{du}{u^2+1}=-\ln|x|+\ln|C|.$$

$$\frac{1}{2}\ln(u^2+1)+arctgu=\ln|C/x|,\quad arctgu=\ln\left|\frac{C}{x\sqrt{u^2+1}}\right|,$$

$$arctg\frac{y}{x}=\ln\frac{|C|}{\sqrt{x^2+y^2}},$$

Shunday qilib, dastlabki tenglamaning umumiy yechimini topdik.◀

4. Differensial tenglamaning xususiy yechimini toping.

$$dy-e^{-x}dx+ydx-xdx=xydx;\ y(0)=\ln 5.$$

▶Berilgan tenglamani hosilaga nisabatan yechib, quyidagini hosil qilamiz:

$$\frac{dy}{dx}=\frac{xy+e^{-x}-y}{1-x},\quad \frac{dy}{dx}+\frac{1-x}{1-x}y=\frac{e^{-x}}{1-x}. \tag{1}$$

$\frac{dy}{dx}+y=\frac{e^{-x}}{1-x}$ tenglama 1-tartibli chiziqli tenglamadir. Uni *y=u(x)v(x)* ko'rinishidagi almashtirish yordamida yechamiz va quyidagiga ega bo'lamiz.

$$y'=u'v+uv',\quad u'v+uv'+uv=\frac{e^{-x}}{1-x},\ u'v+u\left(\frac{dv}{dx}+v\right)=\frac{e^{-x}}{1-x}.$$

$\frac{dv}{dx}+v=0$ shartdan *v(x)* funksiyani topamiz.

$$\frac{dv}{dx}=-v,\quad \frac{dv}{dx}=-dx,\quad \int\frac{dv}{v}=-\int dx,\ \ln|v|=-x,\quad v=e^{-x}$$

bu ifodani (1) tenglamadagi *v(x)* funksiyaning o‘rniga qo‘yamiz:

$$\frac{du}{dx}e^{-x}=\frac{e^{-x}}{1-x},\quad \frac{du}{dx}=\frac{1}{1-x},$$

$$du=\frac{dx}{1-x},\ \int du=\int\frac{dx}{1-x},\quad u=-\ln|1-x|+\ln C,\quad u=\ln\frac{C}{|1-x|}.$$

u holda $y=uv=e^{x}\ln\frac{C}{|1-x|}$

Bu dastlabki tenglamaning umumiy yechimidir. Boshlang‘ich shartdan foydalanib, C ni topamiz.

y(0)=lnCc=ln5, C=5

Shunday qilib, dastlabki tenglamaning xususiy yechimi quyidagi ko‘rinishda bo‘ladi. $y=e^{-x}\ln\frac{5}{|1-x|}$ ◄

5. Differensial tenglamaning umumiy yechimini toping.

$$(1+x^2)\frac{dy}{dx}=xy+x^2y^2.$$

►Tenglamaning turini aniqlash uchun uni quyidagicha yozib olamiz:

$$\frac{dy}{dx}-\frac{x}{1+x^2}y=\frac{x^2}{1+x}y^2.$$

Butenglama Bernulli tenglamasidir. Bu tenglamani *y=u(x)v(x)*ko‘rinishidagio‘rnigaqo‘yishyordamidayechamiz. Uholda

$$y'=u'v+v'u.\quad u'v+v'u-\frac{xu\cdot v}{1+x^2}=\frac{x^2}{1+x^2}u^2v^2$$

$$u'v+u\left(\frac{dv}{dx}-\frac{xv}{1+x^2}\right)=\frac{x^2u^2v^2}{1+x^2}\qquad (1)$$

v(x) ni $\frac{dv}{dx}-\frac{xv}{1+x^2}=0$ shartdan topamiz. Bu tenglama o‘zgaruvchilari ajraladigan differensial tenglamadir. Bundan

$$\frac{dv}{dx}-\frac{xv}{1+x^2},\quad \frac{dv}{v}=\frac{xdx}{1+x^2},\ \int\frac{dv}{v}=\int\frac{xdx}{1+x^2},\ln|v|=\frac{1}{2}\ln(1+x^2),v=\sqrt{1+x^2}$$

$v(x)$ ning hosil qilingan ifodasini (1) tenglamaga qo'yamiz:

$$\frac{du}{dx}\sqrt{1+x^2}=\frac{x^2u^2(1+x^2)}{1+x^2},\quad \frac{du}{u^2}=\frac{x^2dx}{\sqrt{1+x^2}},$$

$$\int\frac{du}{u^2}=\int\frac{x^2dx}{\sqrt{1+x^2}},\quad \int\frac{du}{u^2}=-\frac{1}{u}.$$

$$\int\frac{x^2dx}{\sqrt{1+x^2}}=\left|\begin{matrix}u_1(x)=x, & du_1=dx\\ dv=\frac{xdx}{\sqrt{1+x^2}}, & v_1=\sqrt{1+x^2}\end{matrix}\right|=x\sqrt{1+x^2}-\int\sqrt{1+x^2}dx=x\sqrt{1+x^2}-\int\frac{1+x^2}{\sqrt{1+x^2}}dx=$$

$$=x\sqrt{1+x^2}-\int\frac{dx}{\sqrt{1+x^2}}-\int\frac{x^2dx}{\sqrt{1+x^2}}.$$

Oxirgi tenglikdan quyidagini hosil qilamiz.

$$2\int\frac{x^2dx}{\sqrt{1+x^2}}=x\sqrt{1+x^2}-\ln\left|x+\sqrt{1+x^2}\right|-2C,\int\frac{x^2dx}{\sqrt{1+x^2}}=\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right|-C$$

Bundan

$$-\frac{1}{u}=\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right|-C,\ \frac{1}{u}=\frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right|-\frac{1}{2}x\sqrt{1+x^2}+C,$$

$$u=\left(\frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right|-\frac{1}{2}x\sqrt{1+x^2}+C\right)^{-1}.$$

Shunday qilib, dastlabki tenglamaning umumiy yechimi quyidagi formuladan topiladi.

$$y=\frac{\sqrt{1+x^2}}{\frac{1}{2}\ln/x+\sqrt{1+x^2}/-\frac{1}{2}x\sqrt{1+x^2}+C}$$ ◄

11.2. – IUT

1. Differensial tenglamaning xususiy yechimini toping va hosil qilingan $y=\phi(x)$ funksiyaning qiymatini $x=x_0$ da verguldan keyin ikki xona aniqligida hisoblang.

1.1. $y'''=\sin x,\ \ x_0=\pi/2,\ \ y(0)=1,\ \ y'(0)=0,\ \ y''(0)=0.$
(*Javob:* 1,23).

1.2. $y'''=1/x,\ \ x_0=2,\ \ y(1)=1/4,\ \ y'(1)=y''(1)=0.$
(*Javob:*0,38).

1.3. $y'' = 1/\cos^2 x, \quad x_0 = \pi/3, \quad y(0) = 1, \quad y'(0) = 3/5.$
(*Javob:* 2,69)

1.4. $y''' = 6/x^3, \quad x_0 = 2, \quad y(1) = 0, \quad y(1) = 5, \quad y''(1) = 1.$
(*Javob:* 6,07).

1.5. $y'' = 4\cos 2x, \quad x_0 = \pi/4, \quad y(0) = 1, \quad y'(0) = 3.$ (*Javob:* 4,36).

1.6. $y'' = 1/(1 + x^2), \quad x_0 = 1, \quad y(0) = 0, \quad y'(0) = 0.$ (*Javob* 0,44).

1.7. $xy''' = 2, \quad x_0 = 2, \quad y(1) = 1/2, \quad y'(1) = y''(1) = 0.$
(*Javob:* 0,77).

1.8.
$y''' = e^{2x}, \quad x_0 = \frac{1}{2}, \quad y(0) = \frac{9}{8}, \quad y'(0) = \frac{1}{4}, \quad y''(0) = -\frac{1}{2}.$
(*Javob:*1,22).

1.9.
$y''' = \cos^2 x, \quad x_0 = \pi, \quad y(0) = 1, \quad y'(0) = -1/8, \quad y''(0) = 0$
(*Javob:*3,58).

1.10. $y'' = 1 \big/ \sqrt{1 - x^2}, \quad x_0 = 1, \quad y(0) = 2, \quad y'(0) = 3$
(*Javob:*5,57).

1.11. $y'' = \frac{1}{\sin^2 2x}, \quad x_0 = \frac{5}{4}\pi, \quad y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}, \quad y'(\frac{\pi}{4}) = 1.$
(*Javob:*3,93).

1.12. $y'' = x + \sin x, \quad x_0 = 5, \quad y(0) = -3, \quad y'(0) = 0.$
(*Javob:*5,31).

1.13. $y'' = arctgx, \quad x_0 = 1, \quad y(0) = y'(0) = 0.$
(*Javob:*0,15).

1.14. $y'' = tgx \cdot \frac{1}{\cos^2 2x}, \quad x_0 = \pi/4, \quad y = 1/2, \quad y'(0) = 0.$
(*Javob:*-0,39).

1.15.

$y''' = e^{x/2} + 1, \quad x_0 = 2, \quad y(0) = 8, \quad y'(0) = 5, \quad y''(0) = 2.$

(*Javob:*25,08).

1.16.

$y'' = x / e^{2x}, \quad x_0 = -1/2, \quad y_0(0) = 1/4, \quad y'(0) = -1/4.$

(*Javob.*0,34).

1.17.

$y'' = \sin^2 3x, \quad x_0 = \pi/12, \quad y(0) = -\pi^2/16, \quad y'(0) = 0.$

(*Javob:* -0,01).

1.18.

$y''' = x\sin x, \quad x_0 = \pi/2, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0.$

(*Javob:*0,14).

1.19.

$y''' \cdot \sin^4 x = \sin 2x, \quad x_0 = 5\pi/2, \quad y(\pi/2) = \pi/2, \quad y'(\pi/2) = 1 \; y''(\pi/2) = -1.$

(*Javob:*7,85).

1.20. $y'' = \cos x + e^{-x}, \quad x_0 = \pi, \quad y(0) = -e^{-\pi}, \quad y'(0) = 1.$

(*Javob:*1,00).

1.21.

$y'' = \sin^3 x, \quad x_0 = 2{,}5\pi, \quad y(\pi/2) = -7/9, \quad y'(\pi/2) = 0.$

(*Javob:*-0,78).

1.22.

$y''' = \sqrt{x} - \sin 2x, \quad x_0 = 1, \quad y(0) = -1/8, \quad y'(0) = \frac{1}{8}\cos 2, \quad y''(0) = \frac{1}{2}.$

(*Javob:*0,08).

1.23. $y'' = \frac{1}{\cos^2(x/2)}, \quad x_0 = 4\pi, \quad y = 0, \quad y'(0) = 1.$

(*Javob:*12,56).

1.24.

$y'' = 2\sin x\cos^2 x, \quad x_0 = \pi/2, \quad y(0) = -5/9, \quad y'(0) = -2/3.$

(*Javob:*-1,00).

1.25. $y'' = 2\sin^2 x\cos x, \quad x_0 = \pi, \quad y(0) = 1/9, \quad y'(0) = 1.$
(*Javob:*4,14).

1.26.
$y'' = 2\sin x\cos^2 x - \sin^3 x, \quad x_0 = \pi/2, \quad y(0) = 0, \quad y'(0) = 1.$
(*Javob:*1,90).

1.27.

$y'' = 2\cos x\sin^2 x - \cos^3 x, \quad x_0 = \pi/2, \quad y(0) = 2/3, \quad y'(0) = 2.$
(*Javob:*3,47).

1.28. $y'' = x - \ln x, \quad x_0 = 2, \quad y(1) = -5/12, \quad y'(1) = 3/2.$
(*Javob:*1,62).

1.29. $y'' = 1/x^2, \quad x_0 = 2, \quad y(1) = 3, \quad y'(1) = 1.$
(*Javob:*4,31).

1.30.
$y''' = \cos 4x, \quad x_0 = \pi, \quad y(0) = 2, \quad y'(0) = 15/16, \quad y''(0) = 0.$
(*Javob:*5,14).

2. Tartibi pasaytirish mumkin bo'lgan differensial tenglamaning umumiy yechimini toping.

2.1. $(1-x^2)y'' - xy = 2.$

(*Javob:* $y = \arcsin^2 x + C_1 \arcsin_x + C_2.$).

2.2. $2xy'y'' = y'^2 - 1.$

(*Javob:* $9C_2(y - C_2)^2 = 4(C_1x+1)^3, y = \pm x + C.$).

2.3. $x^3y'' + x^2y' = 1.$ (*Javob:* $y = C_1 \ln x + 1/x + C_2.$)

2.4. $y'' + y'tgx = \sin 2x.$

(*Javob:* $y = C_1 \sin x - x - \frac{1}{2}\sin 2x + C_2.$)

2.5. $y''x\ln x = y'.$ (*Javob:* $y = C_1x(\ln x - 1) + C_2$).

2.6. $xy'' - y' = x^2e^x.$ (*Javob:* $y = e^x(x-1) + C_1x^2 + C_2$).

2.7. $y''x\ln x = 2y'$.

(*Javob:* $y = C_1(x\ln^2 x - 2x\ln x + 2x) + C_2$).

2.8. $x^2y'' + xy' = 1$. (*Javob:* $y = (\ln^2 x)/2 + C_1\ln x + C_2$).

2.9. $y'' = -x/y$.

(*Javob:* $y = \frac{C_1^2}{2}\arcsin\frac{x}{C_1} + \frac{x}{2}\sqrt{C_1^2 - x^2} + C_2$).

2.10. $xy'' = y'$. (*Javob:* $y = C_1x^2/2 + C_2$).

2.11. $y'' = y' + x$. (*Javob:* $y = -x^2/2 - x + C_1e^x + C_2$).

2.12. $xy'' = y' + x^2$. (*Javob:* $y = x^3/3 + C_1x^2/2 + C_2$).

2.13. $xy'' = y'\ln(y'/x)$.

(*Javob:* $y = \frac{x}{C_1}e^{C_1x+1} - \frac{1}{C_1^2}e^{C_1x+1} + C_2$).

2.14. $xy'' = y' = \ln x$. (*Javob:* $y = (x + C_1)\ln x - 2x + C_2$).

2.15. $y''\cdot tgx = y' + 1$. (*Javob:* $y = -C_1\ln x - x + C_2$).

2.16. $y'' + 2xy'^2 = 0$. (*Javob:* $y = \frac{1}{2C_1}\ln\frac{x - C_1}{x + C_1} + C_2$).

2.17. $2xy'y'' = y'^2 + 1$. (*Javob:* $y = \frac{2}{3C_1}(C_1x - 1)^{3/2} + C_2$).

2.18. $y'' - \frac{y'}{x-1} = x(x-1)$.

(*Javob:* $y = x^4/8 - x^3/6 + C_1x^2/2 - C_1x + C_2$).

2.19. $y''' + y''tgx = \sec x$.

(*Javob:* $y = -\sin x - C_1\cos x + C_2x + C_3$).

2.20. $y'' - 2y'ctgx = \sin^3 x$.

(*Javob:* $y = -\sin^3 x/3 + C_1x/2 - C_1\sin 2x/4 + C_2$).

2.21. $y'' + 4y' = 2x^3$.

(*Javob:* $y = x^3/6 - x^2/8 + x/16 - C_1e^{-4x}/4 + C_2$).

2.22. $xy'' - y' = 2x^2e^x$.

(*Javob:* $y = 2e^x(x-1) + C_1x^2/2 + C_2$).

2.23. $x(y''+1) + y' = 0$. (*Javob:* $y = -x^2/4 + C_1\ln x + C_2$).

2.24. $y'' + 4y' = \cos 2x$.

(*Javob:* $y = \frac{1}{10}\sin 2x - \frac{1}{20}\cos 2x - \frac{C_1}{4}e^{-4x} + C_2$).

2.25. $y'' + y' = \sin x$.

(*Javob:* $y = -\frac{1}{2}\cos x - \frac{1}{2}\sin x - C_1e^{-x} + C_2$).

2.26. $x^2y'' = y'^2$. (*Javob:* $y = C_1x - C_1^2\ln(x + C_1) + C_2$).

2.27. $2xy''y' = y'^2 - 4$. (*Javob:* $y = \frac{2}{3C_1}(C_1x+4)^{3/2} + C_2$).

2.28. $y'''x\ln x = y''$.

(*Javob:* $y = \frac{C_1x^2}{4}(2\ln x - 3) + C_2x + C_3$).

2.29. $y''ctgx + y' = 2$. (*Javob:* $y = 2x + C_1\sin x + C_2$).

2.30. $(1+x^2)y'' = 2xy$. (*Javob:* $y = C_1x^3/3 + C_1x + C_2$).

3. Tartibi pasaytirilish mumkin bo'lgan differensial tenglamalar uchun Koshi masalasini yeching.

3.1. $y'' = y'e^y$, $y(0) = 0$, $y'(0) = 1$.

(*Javob:* $y = -\ln|1-x|$, $y = 0$).

3.2. $y'^2 + 2yy'' = 0$, $y(0) = 1$, $y'(0) = 1$.

(*Javob:* $y = (1 \pm 3x/2)^{2/3}$, $y = 1$).

3.3. $yy'' + y'^2 = 0$, $y(0) = 1$, $y'(0) = 1$.

(*Javob:* $y = \sqrt{2x+1}$, $y = 1$).

3.4. $y'' + 2yy'^3 = 0, \quad y(0) = 2, \quad y'(0) = 1/3.$

(*Javob:* $x = y^3/3 - y - 2/3, \quad y = 2$).

3.5. $y''tgy = 2y'^2, \quad y(1) = \pi/2, \quad y'(1) = 2.$

(*Javob:* $y = arctg(2 - 2x), \quad y = \pi/2$).

3.6. $2yy'' = y'^2, y(0) = 1.$ (*Javob:* $y = \left(\frac{x}{2} + 1\right)^2, \quad y = 1.$)

3.7. $yy'' - y'^2 = y^4, \quad y(0) = 1, \quad y'(0) = 1$

(*Javob:* $x = \pm\ln\left(1 + \sqrt{2}\right) \pm \ln\frac{y}{1 + \sqrt{y^2 + 1}}.$)

3.8. $y'' = -1/(2y^3), \quad y(0) = 1/2, \quad y'(0) = \sqrt{2}$

(*Javob:* $y = \sqrt{x\sqrt{2} + 1/4}.$)

3.9. $y'' = 1 - y'^2, \quad y(0) = 0, \quad y'(0)$

(*Javob:* $x = \pm\ln\left|e^y + \sqrt{e^y - 1}\right|.$)

3.10. $y''^2 = y', \quad y(0) = 2/3, \quad y'(0) = 1.$

(*Javob:* $y = (x + 2)^3/12, \quad y = 2/3$).

3.11. $2yy'' = y'^2 + 1, \quad y(0) = 2, \quad y'(0) = 1$

(*Javob:* $y = \left(\frac{x + 2}{2}\right)^2 + 1.$)

3.12. $y'' = 2 - y, y(0) = 2, y'(0) = 2$

(*Javob:* $y = 2\sin x + 2.$)

3.13. $y'' = 1/y^3, y(0) = 1, y'(0) = 0.$ (*Javob:* $x = \sqrt{y^2 + 1}.$)

3.14. $yy'' - 2y'^3 = 0, y(0) = 1, y'(0) = 2.$

(*Javob:* $y = \frac{1}{1 - 2x}, y = 1.$)

3.15. $y'' = y' + y'^2, y(0) = 0, y'(0) = 1.$

(*Javob:* $x = \ln\dfrac{2e^y - 1}{e^y}$, $y = 0$.)

3.16. $y'' + \dfrac{2}{1-2}y'^2 = 0$, $y(0) = 0$, $y'(0) = 1.$

(*Javob:* $y = 1 - \dfrac{1}{x+1}$, $y = 0$.)

3.17. $y''(1+y) = 5y'^2, y(0) = 0, y'(0) = 1.$

(*Javob:* $\dfrac{1}{4} - \dfrac{1}{4(1+y)^4}$, $y = 0$.)

3.18. $y''(2y+3) = -2y'^2 = 0, y(0) = 0, y'(0) = 3.$

(*Javob:* $y = \dfrac{3}{2}(e^x - 1)$, $y = 0$.)

3.19. $4y''^2 1 + y'^2, y(0) = 1, y'(0) = 0.$

(*Javob:* $x = 2\ln\dfrac{1}{2}\left|y + 1 + \sqrt{(y+1)^2 - 4}\right|$.)

3.20. $2y'^2 = (y-1) = y'', y(0) = 2, y'(0) = 2.$

(*Javob:* $y = 1 + \dfrac{1}{1-2x}$, $y = 2$.)

3.21. $1 + y''^2 = yy', y(0) = 1, y'(0) = 0.$

(*Javob:* $x = \ln\left|y + \sqrt{(y^2 - 1)}\right|$.)

3.22. $y'' + yy'^3 = 0$, $y(0) = 1$, $y'(0) = 2.$

(*Javob:* $y = \sqrt[3]{6x+1}$, $y = 1$.)

3.23. $yy'' - y'^2 = 0$, $y(0) = 1$, $y'(0) = 2.$

(*Javob:* $y = e^{2x}$, $y = 1$.)

3.24. $yy'' - y'^2 = y^2 \ln y, y(0)=1, y'(0)=1.$

(*Javob:* $x = \ln\left|\ln y + \sqrt{\ln^2 y + 1}\right|.$)

3.25. $y(1-\ln y)y'' + (1+\ln y)y'^2 = 0, y(0)=1, y'(0)=1.$

(*Javob:* $x = \dfrac{1}{1-\ln y} - 1, \quad y = 1.$)

3.26. $y''(1+y) = y'^2 + y', y(0)=2, y'(0)=2.$

(*Javob:* $y = 2e^x, \quad y = 2.$)

3.27. $y'' = y' / \sqrt{y}, y(0)=1, y'(0)=2.$

(*Javob:* $y = (x+1)^2, \quad y = 1.$)

3.28. $y'' = 1 \big/ (1+y'^2), \quad y(0)=0, \quad y'(0)=0.$

(*Javob:* $x = 2arctg\sqrt{e^y - 1}.$)

3.29. $yy'' - 2yy'\ln y = y'^2, \quad y(0)=1, \quad y'(0)=1.$

(*Javob:* $y = e^{tgx}, y = 1.$)

3.30. $y'' = 1/\sqrt{y}, \quad y(0) = y'(0) = 0.$ (*Javob:* $x = \dfrac{2}{3} y^{3/4}.$)

4. Quyidagi tenglamalarni integrallang.

4.1. $\dfrac{1}{x} dy - \dfrac{y}{x^2} dx = 0.$ (*Javob:* $y/x = C.$)

4.2. $\dfrac{xdy - ydx}{x^2 + y^2} = 0.$ (*Javob:* $arctg(x/y) = C.$)

4.3. $(2x - y + 1)dx + (2y - x - 1)dy.$

(*Javob:* $x^2 + y^2 - xy + x - y = C.$)

4.4. $xdx + ydy + \dfrac{ydx - xdy}{x^2 + y^2} = 0.$

(*Javob:* $\frac{x^2+y^2}{2}+arctg\frac{x}{y}+C.$)

4.5. $\left(\frac{x}{\sqrt{x^2-y^2}}-1\right)dx-\frac{ydy}{\sqrt{x^2-y^2}}=0.$

(*Javob:* $\sqrt{x^2-y^2}-x=C.$)

4.6. $\frac{2x(1-e^y)}{(1+x^2)^2}dx+\frac{e^y}{1+x^2}dy=0.$ (*Javob:* $\frac{e^y-1}{1+x^2}=C.$)

4.7. $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0.$ (*Javob:* $\frac{x^2}{y^3}-\frac{1}{y}=C.$)

4.8. $(1-e^{x/y})dx+e^{x/y}(1-x/y)dy=0.$

(*Javob:* $x+ye^{x/y}=C.$)

4.9. $x(2x^2+y^2)+y(x^2+2y^2)y'=0.$

(*Javob:* $x^4+x^2y^2+y^4=C.$)

4.10. $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0.$

(*Javob:* $x^3+3x^2y^2+y^4=C.$)

4.11.

$$\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{1}{y^2}\right)dy=0$$

(*Javob:* $\sqrt{x^2+y^2}+\ln|xy|+\frac{x}{y}=C.$)

4.12. $\left(3x^2tgy-\frac{2y^3}{x^3}\right)dx+\left(x^3\sec^2y+4y^3+\frac{3y^2}{x^2}\right)dy=0.$

(*Javob:* $x^3tgy+y^4+\frac{y^3}{x^2}=C.$)

4.13. $\left(2x+\frac{x^2+y^2}{x^2+y}\right)dx=\frac{x^2+y^2}{xy^2}dy.$

(*Javob:* $x^2+\frac{x}{y}-\frac{y}{x}=C.$)

4.14. $\left(\frac{\sin 2x}{y}+x\right)dx+\left(y-\frac{\sin^2 x}{y^2}\right)dy=0.$

(*Javob:* $\frac{x^2+y^2}{2}+\frac{\sin^2 x}{y}=C.$)

4.15. $(3x^2-2x^2-y)dx+(2y-x+3y^2)dy=0.$

(*Javob:* $x^3+y^3-x^2-xy+y^2=C.$)

4.16. $\frac{xdx+ydy}{\sqrt{x^2+y^2}}+\frac{xdx-ydy}{x^2}=0$

(*Javob:* $\frac{y}{x}+\sqrt{x^2+y^2}=C.$)

4.17. $(3x^2y+y^3)dx+(x^3+3xy^2)dy=0.$

(*Javob:* $xy(x^2+y^2)=C.$)

4.18. $y(x^2+y^2+a^2)dy+x(x^2-y^2-a^2)dx=0.$

(*Javob:* $(x^2+y^2)^2+2a^2(y^2-x^2)=C.$)

4.19. $\left(\sin y+y\sin x+\frac{1}{x}\right)dx+\left(x\cos y-\cos x+\frac{1}{y}\right)dy=0.$

(*Javob:* $tgxy-\cos x-\cos y=C.$)

4.20. $\frac{y+\sin x\cos^2 yx}{\cos^2 yx}dx+\left(\frac{x}{\cos^2 yx}-\sin y\right)dy=0.$

(*Javob:* $tgxy-\cos x-\cos y=C.$)

4.21. $(3x^2-y\cos xy+y)dx+(x-x\cos xy)dy=0.$

(*Javob:* $x^3 - \sin xy + xy = C.$)

4.22. $$\left(12x^3 - e^{x/y}\frac{1}{y}\right)dx + \left(16y + \frac{x}{y^2}e^{x/y}\right)dy = 0.$$

(*Javob:* $3x^4 + 8y^2 - e^{x/y} = C.$)

4.23.

$$\left(\frac{y}{2\sqrt{xy}} + 2xy\sin x^2y + 4\right)dx + \left(\frac{x}{2\sqrt{xy}} + x^2\sin x^2y\right)dy = 0$$

(*Javob:* $\sqrt{xy} - \cos x^2y + 4x = C.$)

4.24. $y3^{xy}\ln 3dx + (x3^{xy}\ln 3 - 3)dy = 0.$

(*Javob:* $3^{xy} - 3y = C.$)

4.25.

$$\left(\frac{1}{x-y} + 3x^2y^7\right)dx + \left(7x^3y^6 - \frac{1}{x-y} - \sin y\right)dy = 0.$$

(*Javob:* $\ln|x-y| + x^3y^7 = C.$)

4.26. $$\left(\frac{2y}{x^3} + y\cos xy\right)dx + \left(\frac{1}{x^2}x + \cos xy\right)dy = 0.$$

(*Javob:* $\sin xy - \dfrac{y}{x^2} = C.$)

4.27. $$\left(\frac{y}{\sqrt{-x^2y^2}} - 2x\right)dx + \frac{xdy}{\sqrt{1-x^2y^2}} = 0$$

(*Javob:* $\arcsin xy - x^2 = C.$)

4.28. $(5x^4y^4 + 28x^6)dx + (4x^5y^3 - 3y^2)dy = 0.$

(*Javob:* $x^5y^4 - y^3 + 4x^7 = C.$)

4.29. $(2xe^{x^2+y^2} + 2)dx + (2ye^{x^2+y^2} - 3)dy = 0.$

(*Javob:* $e^{x^2+y^2} + 2x - 3y = C.$)

4.30. $(3y^3\cos 3x+7)dx+(3y^2\sin 3x-2y)dy=0.$

(Javob: $y^3\sin 3x-y^2+7x+C.$)

5. Agar y ning ixtiyoriy nuqtasidagi burchak koeffitsiyenti, shu nuqtaning *k* marta kattalashtirilgan ordinatasiga teng ekanligi ma'lum bo'lsa, $A(x_0, y_0)$ nuqtadan o'tuvchi egri chiziq tenglamasini yozing.

5.1. *A(0,2), k=3. (Javob:* $y=-2e^{3x}$*).*

5.2. *A(0,5), k=7. (Javob:*$y=5e^{7x}$*).*

5.3. *A(-1,3), k=2. (Javob:* $y=3e^{2x+2}$*).*

5.4. *A(-2,4), k=6. (Javob:* $y=4e^{6x+12}$*).*

5.5. *A(-2,1), k=5. (Javob:* $y=-e^{5x+10}$*).*

5.6. *A(3,-2), k=4. (Javob:* $y=-2e^{4x-12}$*).*

Ixtiyoriy nuqtasidagi urinmaning burchak koeffitsiyenti, shu nuqtani koordinata boshi bilan tutashtiruvchi to'g'ri chiziqning burchak koeffitsiyentidan *n* marta katta ekanligi ma'lum bo'lsa, $A(x_0, y_0)$ nuqtadan o'tuvchi egri chiziq tenglamasini yozing.

5.7. *A(2,5), n=8.(Javob:*$y=5x^8/256$

5.8. *A(3,1), n=3/2.(Javob:*$y=-x\sqrt{x}/(3\sqrt{3}))$

5.9. *A(-6,4), n=9.(Javob:*$y=-x^9/11664$.

5.10. *A(-8,-2), n=3.(Javob:*$y=-x^3/256$.

Egri chiziqning ixtiyoriy nuqtasiga o'tkazilgan normalning ordinata o'qidan ajratgan kesmasiningning uzunligi shu nuqtadan koordinata boshigacha bo'lgan masofaga tengligima'lum bo'lsa, $A(x_0,y_0)$ nuqtadan o'tuvchi egri chiziq tenglamasini yozing.

5.11. *A(0,4), (Javob:* $y=-x^2/16+4$*).*

5.12. *A(0,-8), (Javob:* $y=x^2/32-8$.

5.13. *A(0,1), (Javob:* $y=-x^2/4+1$.

5.14. *A(0,-3), (Javob:* $y=x^2/12-3$.

$A(x_0, y_0)$ nuqtadan o'tuvchi va quyidagi xossaga ega bo'lgan to'g'ri chiziq tenglamasini tuzing: koordinata boshidan egri chiziqning urinmasiga o'tkazilgan perpendikulyarning uzunligi, urinish nuqtasining absissasiga teng.

5.15. *A(2,3), (Javob:* $(x-13/4)^2+u^2=169/16$*).*

5.16. *A(-4,1), (Javob:* $(x+17/8)^2+u^2=289/64$*).*

5.17. *A(1,2), (Javob:* $(x-2,5)^2+u^2=6,25$*).*

5.18. $A(-2,-2)$, (*Javob:* $(x+2)^2+u^2=4$).

5.19. $A(4,-3)$, (*Javob:* $(x-25/8)^2+u^2=625/64$).

5.20. $A(5,0)$, (*Javob:* $(x-2,5)^2+u^2=6,25$.

$A(x_0, y_0)$ nuqtadan o‘tuvchi va quyidagi xossaga ega bo‘lgan egri chiziq tenglamasini tuzing.

Egri chiziqning ixtiyoriy nuqtasining urinmasining *Ou* o‘qdan ajratgan kesmasi, urinish nuqtasi absissasining kvadratiga teng.

5.21. $A(4,1)$, (*Javob:* $y=17x/4-x^2$.

5.22. $A(-2,5)$, (*Javob:* $y=-9x/2-x^2$.

5.23. $A(3,-2)$, (*Javob:* $y=7x/3-x^2$).

5.24. $A(-2,-4)$, (*Javob:* $y=4x-x^2$).

5.25. $A(3,0)$, (*Javob:* $y=3x-x^2$).

5.26. $A(2,8)$, (*Javob:* $y=6x-x^2$).

Egri chiziqqa urinmaning ordinata o‘qidan ajratgan kesmasi, urinish nuqtasining koordinatalari yig‘indisining yarmiga teng ekanligi ma’lum bo‘lsa, $A(x_0, y_0)$ nuqtadan o‘tuvchi egri chiziq tenglamasini yozing.

5.27. $A(9,4)$, (*Javob:* $y=\frac{2}{3}\sqrt{x}-x)$

5.28. $A(4,10)$, (*Javob:* $y=7\sqrt{x}-x)$

5.29. $A(18,-2)$, (*Javob:* $y=4\sqrt{x}-x)$

5.30. $A(1,-7)$, (*Javob:* $y=-6\sqrt{x}-x)$

Namunaviy variantlar yechimi

1. Differensial tenglamaning xususiy yechimini toping va hosil qilingan funksiyaning $x=-3$ dagi qiymatini verguldan keyin ikki xonagacha aniqlikda hisoblang:

$$y''(x+2)^5=1,\ y(-1)=1/12,\ y'(-1)=-1/4.$$

►Berilgan tenglamaning umumiy yechimini topamiz (§11.5ga qarang, 1 xildagi tenglama):

$$y''=\frac{1}{(x+2)^2};\quad y'=\int\frac{dx}{(x+2)^5}=-\frac{1}{4(x+2)^4}+C_1$$

$$y=\int\left(-\frac{1}{4(x+2)^4}+C_1\right)dx=\frac{1}{12(x+2)^3}+C^1x+C^2$$

Boshlang‘ich shartlardan foydalanib, C_1 va C_2 larningqiymatlarinianiqlaymiz:

$$y(-1)=1/12-C_1+C_2=1/12,\ C_1-C_2=0,$$

$$y'(-1)=-1/4+C_1=-1/4,\ C_1=0,\ C_2=0.$$

Boshlang‘ich shartlarni qanoatlantiruvchi, dastlabki tenglamalarning xususiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$y=1/(12(x+2)^3)$$

$y(x)$ funksiyaning $x=-3$ dagi qiymatini hisoblaymiz.

$$y(-3)=\frac{1}{12(-3+2)}=-\frac{1}{12}=-0{,}08 \blacktriangleleft$$

2. Tartibi pasaytirilish mumkin bo‘lgan differensial tenglamaning umumiy yechimini toping.

$$y'(e^x+1)+y''=0.$$

►Berilgan tenglama II xildagi tenglamadir. (§11.5. 2-misolga qarang).

Shuning uchun $y'=z(x)$ almashtirish bajaramiz. U holda $y''=\frac{dz}{dx}$

$$\frac{dz}{dx}(e^x+1)+z=0, \qquad \frac{dz}{dx}(e^x+1)=-z,$$

$$\frac{dz}{z}=\frac{dx}{-e^x+1}, \qquad \int\frac{dz}{z}=-\int\frac{dx}{e^x+1}.$$

va $e^x+1=t$ o‘zgaruvchilarni almashtirish yo‘li bilan quyidagini topamiz.

$$\ln|z|=\ln(e^x+1)-\ln e^x+\ln C$$

Oxirgi ifodani potensirlab, dastlabki tenglamaning umumiy yechimini topamiz:

$$z=C_1\frac{e^x+1}{e^x}dx=C_1(x-e^x)+C_2 \quad y=C_1\int\frac{e^x+1}{e^x}dx=C_1(x-e^{-x})+C_2 \blacktriangleleft$$

3. $y(1)=1, y'(1)=0,$ boshlang‘ich shartlarni qanoatlantiruvchi, tartibi pasayuvchi, $y^3\ y''=1$ differensial tenglamaning yechimini toping.

►Berilgan tenglama III tipga tegishlidir. (§11.5. 2-misolga qarang). Shuning uchun, $y' = p(y)$ almashtirish yordamida tenglamaning tartibini pasaytiramiz. U holda,

$$y'' = p\frac{dp}{dy}.$$

Bundan,

$$y^3 p\frac{dp}{dy} = -1, \qquad pdp = -\frac{dy}{y^3}$$

$$\int pdp = \int\frac{dy}{y^3}, \qquad \frac{p^2}{2} = \frac{1}{2y^2}C^1$$

$$p^2\frac{1}{y^2} + 2C^1, \qquad p = \pm\sqrt{\frac{1}{y^2} + 2C^1},$$

$$\frac{dy}{dx} = \pm\frac{\sqrt{1+2C_1y^2}}{y}, \qquad dx = \pm\frac{ydy}{\sqrt{1+2C_1y^2}}$$

$$x = \pm\int\frac{ydy}{\sqrt{1+2C_1y^2}} + C^2 = \frac{\pm 1}{4C^1}\int(1+2C^1y^2)^{-1/2}d(1+C_1y^2),$$

$$x = \pm\frac{1}{2C_1}\sqrt{1+C_1y^2} + C_2.$$

ya'ni, dastlabki tenglamaning umumiy yechimini hosil qildik. Endi boshlang'ich shartlardan foydalanib, C_1 va C_2 larning qiymatlarini aniqlaymiz. $x=1$, $y=1$ va $y'=0$ da quyidagiga ega bo'lamiz.

$$1 = \pm\frac{1}{2C_1}\sqrt{1+2C_1} + C_2, 0 = \pm\sqrt{1+2C_1}$$

bundan, $1+2C_1=0$, $C_1=-1/2$, $C_2=1$.

Natijada, dastlabki yechim quyidagi ko'rinishga ega bo'ladi:

$$x = \mp\sqrt{1-y^2} + 1$$

Geometrik nuqtai nazardan bu yechim $(x-1)^2+y^2=1$ aylananing o'ng yoki chap tomonining yarmini tasvirlaydi.◄

4. Tenglamani integrallang.

$$\left(\frac{1}{x}-y^3+4\right)dx+\left(-\frac{1}{y}-3xy^2\right)dy=0.$$

Quyidagi belgilashlarni kiritamiz:

$P(x,y)=1/x-y^3+4$, $Q(x,e)=-1/y-3xy^2$ (11.26) tenglamaga qarang.

U holda, $\frac{dP}{dy}=-3y^2$, $\frac{dQ}{dx}=-3y^2$

$\frac{dP}{dy}=\frac{dQ}{dx}$ bo'lganligidan, berilgan tenglama to'la differensial tenglamadir. Uning umumiy integrali (11.24) formuladan topiladi:

$$\int_{x_0}^{x}\left(\frac{1}{x}-y^3+4\right)dx+\int_{y0}^{y}\left(-\frac{1}{y}-3x_0\, y^2\right)dy=C_0$$

Quyidagiga ega bo'lamiz:

$$\int_{x_0}^{x}\frac{dx}{x}-\int_{x_0}^{x}y^3dx+4\int_{x_0}^{x}dx-\int_{y_0}^{y}\frac{dy}{y}-3x_0\int_{y_0}^{y}y^2dy=C_0,$$

$$\ln|x|\Big|_{x_2}^{x}-y^3x\Big|_{x}^{x^0}+4x\Big|_{x_0}^{x}-\ln|y|\Big|_{y_0}^{y}-3x_0\frac{y^2}{3}\Big|_{y_0}^{y}=C_0,$$

$$\ln|x|-\ln|x_0|-xy^3+x_0y^3+4x-4x_0-\ln|y|+\ln|y_0|-x_0y^3+x_0\,y_0^3=C_0,$$

$$\ln\left|\frac{x}{y}\right|-xy^3+4x=C,$$

bu yerda,

$$C=C_0+\ln\left|\frac{x_0}{y_0}\right|+4x_0-x_0y_0^3.$$

◄

5. Agar koordinata o'qlari bilan hamda egri chiziqning ixtiyoriy nuqtasiga o'tkazilgan urinma va urinish nuqtasining koordinatasi bilan chegaralangan trapetsiyaning yuzi o'zgarmas son bo'lib, 3 ga teng ekanligi ma'lum bo'lsa, (11.3-rasmga qarang) A(2,2) nuqtadan o'tuvchi egri chiziqning tenglamasini yozing.

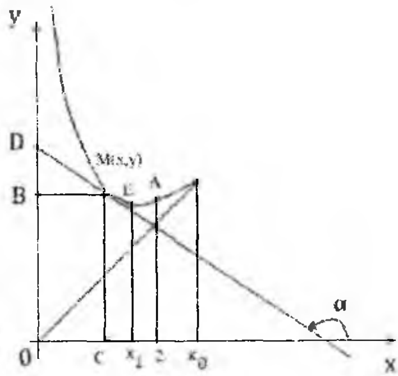
►Quyidagiga ega bo'lamiz:

$$S_{DMCO} = \frac{|MC| + |DO|}{2}|OC|$$

$|MC|=y$, $|DO|=$

$$\pm|DB| + BO = \pm|DB| + |MC| = \pm|DB| + y.$$

$OC|=x, \pm DB|=-|BM|tg\alpha=-|BM|y'=xy'$.



11.3-rasm

bu yerda, agar $y^{|}= tg\alpha< 0$ bo'lsa, ($x<x_1$, 11.3-rasmga qarang) |DB| oldiga "+" belgisi qo'yiladi, agar $y^{|}= tg\alpha> 0$, bo'lsa, ($x>x_1$) oldiga "-" belgisi qo'yiladi.

Shuning uchun ikkala holda ham $|DO|=-xy^{|}+u$. Shularni e'tiborga olib, quyidagilarni topamiz:

$$S_{DMCO}=\frac{y-xy^{1}+y}{2}\cdot x=3, -\frac{1}{2}x^{2}y^{1}+xy=3,$$

$$-\quad x^2y^{|}+2xu=6,\ y^{|}-\frac{2}{x}y=-\frac{6}{x^2},\ x\neq 0.$$

Birinchi tartibli chiziqli tenglamani hosil qildik. Buni yechamiz:

$$u=u\vartheta,\ y'=u'\vartheta+u\vartheta',\ \ u'\vartheta+u\vartheta'-\frac{2u\vartheta}{x}=-\frac{6}{x^2},$$

$$u'\vartheta+u(\frac{dv}{dx}-\frac{2\vartheta}{x})=-\frac{6}{x^2} \qquad (1)$$

$$\frac{d\vartheta}{\vartheta}-\frac{2\vartheta}{x}=0, \qquad \frac{d\vartheta}{\vartheta}=\frac{2dx}{x},$$

$$\int\frac{d\vartheta}{\vartheta}=2\int\frac{dx}{x}, \ln|\vartheta|=2\ln|x|, \vartheta=x^2$$

Topilgan $\vartheta=x^2$ ifodani (1) tenglamaga qo'yamiz: $u'=-\frac{6}{x^4}$,

$$M=-6\int\frac{dx}{x^4}=\frac{2}{x^3}+C$$

U holda $y=u\cdot\vartheta=(\frac{2}{x^2}+C)x^2=\frac{2}{x}+Cx^2$

Egri chiziq A(2,2) nuqtadan o'tganligi uchun, $2=2/2+4C$, $C=1/4$. Izlanayotgan egri chiziq tenglamasi quyidagicha bo'ladi.

$$u=\frac{2}{x}+\frac{x^2}{4}, 0<x\le x_0=\sqrt[3]{16}$$

Ushbu chiziq 11.3-rasmda tasvirlangan bo'lib, $x_1=\sqrt[3]{4}$ da minimumga ega bo'ladi.◄

11.3. Individual uy vazifalari

1. Differensial tenglamaning umumiy yechimini toping.

1.1. *a) y''+4u=0; b) y''-10y'+25u=0; v) y''+3y' +2y=0.*

1.2. *a) y''-y'-2y=0; b) y''+9y=0; v) y''+4y' +4y=0.*

1.3. *a) y''-4y'=0; b) y''-4y'+13y=0; v) y''-3y' +3y=0.*

1.4. *a) y''-5y'+6y=0; b) y''+3y'=0; v) y''+2y' +5y=0.*

1.5. *a) y''-2y'+10y=0; b) y''+y'-2y=0; v) y''-2y=0.*

1.6. *a) y''-4y=0; b) y'' y''+2y'+17y=0; v) y''-y' -12y=0.*

1.7. *a) y'' +y'-6y=0; b) y''+9y'=0; v) y''-4y'+20y=0.*

1.8. *a) y''-49y=0; b) y''-4y'+5y=0; v) y''+2y'-3y=0.*

1.9. *a) y''+7y'=0; b) y''-5y'+4y=0; v) y''+16y=0.*

1.10. *a) y''-6y'+8y=0; b) y''+4y'+5y=0; v) y''+5y'=0.*

1.11. *a) y''-8y'+3y=0; b) y''-3y'=0; v) y''-2y'+10y=0.*

1.12. *a) y''+4y'+20y=0; b) y''-3y'-10y=0; v) y''-16y=0.*

1.13. *a) 9 y''+6y'+y=0; b) y''-4y'-21y=0; v) y''+y=0.*

1.14. *a) 2 y'' +3y'+y=0; b) y''+4y'+8y=0; v) y'' -6y'+9y=0.*

1.15. *a)* $y''-10y'+21y=0$; *b)* $y''-2y'+2y=0$; *v)* $y''+4y'=0$.

1.16. *a)* $y''+6y'=0$; *b)* $y''+10y'+29y=0$; *v)* $y''-8y'+7y=0$.

1.17. *a)* $y''+25y=0$; *b)* $y''+6y'+9y=0$; *v)* $y''+2y'+2y=0$.

1.18. *a)* $y''-3y'=0$; *b)* $y''-7y'-8y=0$; *v)* $y''+4y'+13y=0$.

1.19. *a)* $y''-3y'-4y=0$; *b)* $y''+6y'+13y=0$; *v)* $y''+2y'=0$.

1.20. *a)* $y''+25y'=0$; *b)* $y''-10y'+16y=0$; *v)* $y''-8y'+16y=0$.

1.21. *a)* $y''-3y'-18y=0$; *b)* $y''-6y'=0$; *v)* $y''+2y'+5y=0$.

1.22. *a)* $y''-6y'+13y=0$; *b)* $y''-2y'-15y=0$; *v)* $y''-8y'=0$.

1.23. *a)* $y''+2y'+y=0$; *b)* $y''+6y'+25y=0$; *v)* $y''-4y'=0$.

1.24. *a)* $y''+10y'=0$; *b)* $y''-6y'+8y=0$; *v)* $4y''+4y'+y=0$.

1.25. *a)* $y''+5y=0$; *b)* $9y''-6y'+y=0$; *v)* $y''+6y'+8y=0$.

1.26. *a)* $y''+6y'+10y=0$; *b)* $y''-4y'+4y=0$; *v)* $y''-5y'+4y=0$.

1.27. *a)* $y''-y=0$; *b)* $4y''+8y'-5y=0$; *v)* $y''-6y'+10y=0$.

1.28. *a)* $u''+8y'+25y=0$; *b)* $y''+9y'=0$; *v)* $9y''+3y'-2y=0$.

1.29. *a)* $6y''+7y'-3y=0$; *b)* $y''+16y=0$; *v)* $4y''-4y'+y=0$.

1.30. *a)* $9y''-6y'+y=0$; *b)* $y''+12y'+37y=0$; *v)* $y''-2y'=0$.

2.2.1. $y''+y'=2x-1$. (*Javob:* $y=C_1+C_2e^{-x}+x^2-3x$)

2.2. $y''-2y'+5y=10e^{-x}\cos x$. (*Javob:* $y=e^{x}(C_1\cos 2x+C_2\sin 2x)+e^{-x}\cos 2x$).

2.3. $y''-2y'-8y=12\sin 2x-36\cos 2x$. (*Javob:* $y=C_1e^{-2x}+C_2e^{4x}+3\cos 2x$).

2.4. $y''-12y'+36y=14e^{6x}$. (*Javob:* $y=C_1e^{6x}+C_2e^{6x}+7x^2e^{6x}$).

2.5. $y''-y'+2y=(34-12x)e^{-x}$. (*Javob:* $y=C_1e^{x}+C_2e^{2x}+(4-2x)e^{-x}$).

2.6. $y''-6y'+10y=51e^{-x}$. (*Javob:* $y=e^{3x}(C_1\cos x+C_2\sin x)+3e^{-x}$).

2.7. $y''+y=2\cos x-(4x+4)\sin x$. (*Javob:* $y=C_1\cos x+C_2\sin x+(x^2+2x)\cos x$).

2.8. $y''+6y'+10y=74e^{3x}$. (*Javob:* $y=e^{3x}+(C_1\cos x+C_2\sin x)+2e^{3x}$).

2.9. $y''-3y'+2y=3\cos x+19\sin x$. (*Javob:* $y=C_1e^{x}+C_2e^{2x}+6\cos x+\sin x$).

2.10. $y''+6y'+9y=(48x+8)e^{x}$. (*Javob:* $y=C_1e^{-3x}+C_{2x}e^{-3x}+(3x-1)e^{x}$).

2.11. $y''+5y'=72e^{2x}$. (*Javob:* $y=C_1+C_{2x}e^{-5x}+3e^{2x}$).

2.12. $y''-5u'-6y=3\cos x+19\sin x$. (*Javob:* $y=C_1e^{-x}+C_2e^{6x}+\cos x-2\sin x$).

2.13. y''-$8u'+12y=36x-96x^2+16x-2$. *(Javob: $y=C_1e^{2x}+C_1e^{6x}+3x-x^2$).*

2.14. y''-$8u'+25y=18e^{5x}$. *(Javob: $y=e^{-4x}(C_1\cos 3x+C_2\sin 3x)+\frac{1}{5}e^{5x}$)*

2.15. y''-$9u'+20y=126e^{-2x}$. *(Javob: $y=C_1e^{4x}+S_2e^{5x}+3e^{-2x}$)*

2.16. $y''+36y=36+66x-36x^3$. *(Javob: $y=C_1\cos 6x+C_2\sin 6x-x^3+2x+1$).*

2.17. $y''+y=-4\cos x+2\sin x-36x^3$.*(Javob: $y=C_1\cos x+C_2\sin x+x(\cos x-2\sin x)$.).*

2.18. $y''+2y'-24y=6\cos 3x-33\sin 3x$.*(Javob: $y=C_1e^{-6x}+C_2e^{4x}+\sin 3x$).*

2.19. $y''+6y'+13y=-75\sin 2x$.*(Javob: $y=e^{-3x}(C_1\cos 2x+C_2\sin 2x)+4\cos 2x$- $-3\sin 2x$).*

2.20. $y''+5y'=39\cos 3x-105\sin 3x$. *(Javob: $y=(C_1+C_2e^{-5x}+4\cos 3x+5\sin 3x$).*

2.21. $y''-4y'+29y=104\sin 5x$.

(Javob: $y=e^{2x}(C_1\cos 5x+C_2\sin 5x)+5\cos 5x+\sin 5x$).

2.22. $y''-4y'+5y=(24\sin x+8\cos x)e^{-2x}$.

(Javob: $y=e^{2x}(C_1\cos x+C_2\sin x)+C_2)+e^{-2x}(\cos x+\sin x)$.).

2.23. $y''+16y=8\cos 4x$. *(Javob: $u=C_1\cos 4x+C_2\sin 4x+x\sin 4x$).*

2.24.$y''+9y=9x^4+12x^2-27$. *(Javob: $C_1\cos 3x+C_2\sin 3x+x^4-3$).*

2.25. $y''-12y'+40y=2e^{6x}$. *(Javob: $y=e^{6x}(C_1\cos 2x+C_2\sin 2x)+\frac{1}{2}e^{6x}$).*

2.26. $y''+4y'=e^x(24\cos 2x+2\sin 2x)$.*(Javob: $C_1+C_2e^4+2e^x\sin 2x$).*

2.27. $y''+2y'+y=6e^{x}$).*(Javob: $y=C_1+e^{-x}+C_2xe^4+2e^{-x}+3x^2e^{-x}$).*

2.28. $y''+2y'+37y=37x^2$-$33x+74$. *(Javob: $y=e^{x}x(C_1\cos 6x+C_2\sin 6x)+x^2-x+2$).*

2.29. $6y''-y'-y=3e^{2x}$. *(Javob: $y=C_1e^{x/2}+C_2e^{-x/3}+e^{2x}$).*

2.30. $2y''+7y'+3y=222\sin 3x$. *(Javob: $y=(C_1e^{-3x}+C_2e^{-x/2}+7\cos 3x+5\sin 3x$).*

3.3.1. $y''- 8y' + 17y = 10e^{2x}$;(Javob: $y=e^{4x}(C_1cos+C_2sin\ x)+2l^{2x}$.)

3.2. $y''+y'-6y= (6x+1)e^{3x}$;(Javob: $y=C_1 e^{-3x}+C_2 e^{2x}$! $(x-1)e^{3x}$.)

3.3. $y''- 7y' + 12y = 3e^{4x}$;(Javob: $y= C_1 e^{3x} +C_2 e^{4x}+3xe^{4x}$.)

3.4. $y''- 2y'=6 + 12y = 24x^2$;(Javob: $y= C_1 +C_2 e^{2x}+4x^3+3x^2+3x$.

3.5. $y''- 6y' + 34y = 18\ cos\ 5x$! $60\ sin\ 5x$;
(Javob: $y=e^{3x}(C_1cos\ 5x+C_2sin\ 5x)+2\ cos\ 5x$.)

3.6. $y''- 2y'= (4x+4)e^{2x}$;(Javob: $y= C_1 +C_2 e^{2x}+ (x^2+x)e^{2x}$.)

3.7. $y''+ 2y'+y= 4x^3 +24x^2+22x-4$;(Javob: $y= C_1e^{-x}+C_2xe^{-x}+4x^x-2x$.)

3.8. $y''- 4y'= 8–16x$;(Javob: $y= C_1 +C_2 e^{4x}+ 2x^2–x$.)

3.9. $y''-2y'+y= 4e^x$;(Javob: $y= C_1 e^x+C_2 e^x+2x^2e^x$.)

3.10. $y''- 8y' + 20y = 16\ sin\ 2x-cos\ 2x$;
(Javob: $y=e^{4x}(C_1cos\ 2x+C_2sin\ 2x)+sin\ 2x$.)

3.11. $y''- 6y' + 13y = 34e^{-3x}sin\ 2x$;
(Javob: $y=e^{3x}(C_1cos\ 2x+C_2sin\ 2x)+2–^{3x}cos\ 2x$.)

3.12. $y''+ 2y'-3y = (12x^2+6x-4)e^x$.;(Javob: $y=C_1e^{-3x}+C_2 e^x+(x^3-x)e^x$.)

3.13. $y''+ 4y' + 4y = 6e^{-2x}$;(Javobi:$y=C_1e^{-2x}+C_2xe^{-2x}+3x^22^{-ex}$.)

3.14. $y''+ 3y' =10–6x$;(Javob: $y=C_1+C_2 e^{-3x} -x^2\ 4x$.)

3.15. $y''+ 10y'+25y =40+52x-240x^{2–}200x^3$;(Javob: $y=C_1e^{-5x}+C_2 e^{-5x} -8x^3+4x$.)

3.16. $y''+ 4y'+20y =4cos\ 4x-52\ sin\ 4x$;
(Javob: $y=e^{-2x}(C_1cos\ 4x+C_2 sin\ 4x)+3\ cos\ 4x-sin\ 4x$.)

3.17. $y''+ 4y'+5y =5x^{2–}32x+5$;(Javob: $y=e^{-2x}(C_1cosx+C_2 sinx)+x^{2–}8x+7$.)

3.18. $y''+ 2y'+y = (12x-10)e^{-x}$;(Javob: $y=C_1 e^{-x}+C_2 xe^{-x}+(2x^{3–}5x^2)e^{-x}$.)

3.19. $y''-4y = (-2x-10)e^{2x}$;(Javob: $y=C_1cos\ 2x+C_2\ sin\ 2x+(3x^2+x)e^{2x}$.)

3.20. $y''+ 6y'+9y = 72e^{3x}$;(Javob: $y=C_1 e^{-3x}+C_2 xe^{-3x}+2e^{3x}$.)

3.21. $y''+ 16y = 80e^{2x}$;(Javob: $y=C_1cos\ 4x +C_2\ sin\ 4x+4e^{2x}$.)

3.22. $y''+ 4y' = 15e^x$;(Javob: $y=C_1+C_2 e^{-4x}+3e^x$.)

3.23. $y''+ y'-2y = 9cos\ x-7\ sin\ x$; (Javob: $y=C_1e^{-2x} +C_2 e^x+3sin\ x-2cos\ x$.)

3.24. $y''+2y'+y=(18x+8)e^{x}$;(*Javob:* $y=C_1e^{x}+C_2xe^{x}+(3x^3+4x^2)e^{x}$.)

3.25. $y''-14y'+49y=144\sin 7x$;(*Javob:* $y=C_1e^{7x}+C_2xe^{7x}+2\cos 7x$.)

3.26. $y''+9y=10e^{3x}$;(*Javob:* $y=C_1\cos 3x+C_2\sin 3x+e^{3x}$.)

3.27. $4y''-4y'+y=-25\cos x$;(*Javob:* $y=C_1e^{x/2}+C_2e^{x/2}x+3\cos x+4\sin x$.)

3.28. $3y''-5y'-2y=6\cos 2x+38\sin x\, 2x$;

(*Javob:* $y=C_1e^{-x/3}+C_{2x}e^{x/2}+\cos 2x-2\sin 2x$.)

3.29. $y''+4y'+29y=26e^{-x}$;(*Javob:* $y=e^{-2x}(C_1\cos 5x+C_2\sin 5x)+e^{-x}$.)

3.30. $4y''+3y'-y=11\cos x-7\sin x$;

(*Javob:* $y=e^{-2x}(C_1e^{x/4}+C_2e^{x}+2\sin x-\cos x$.)

4. Boshlang'ich shartlarni qanoatlantiruvchi differensial tenglamaning xususiy yechimini toping.

4.1. $y''-2y'+y=-12\cos 2x-9\sin 2x$, $y(0)=2$, $y'(0)=0$.

(*Javob:* $y=-2e^{x}-4xe^{x}+3\sin 2x$.)

4.2. $y''-6y'+9y=9x^2-39x+65$, $y(0)=-1$, $y'(0)=1$.

(*Javob:* $y=-6e^{3x}+22xe^{3x}+x^2-3x+5$.)

4.3. $y''+2y'+2y=2x^2+8x+6$, $y(0)=1$, $y'(0)=4$.

(*Javob:* $y=e^{-x}(\cos x+3\sin x)+x^2+2x$.)

4.4. $y''-6y'+25y=9\sin 4x-24\cos 4x$, $y(0)=2$, $y'(0)=-2$.

(*Javob:* $y=e^{3x}(2\cos 4x-3\sin 4x)+\sin 4x$.)

4.5. $y''-14y'+53y=53x^3-42x^2+59x-14$, $y(0)=0$, $y'(0)=7$.

(*Javob:* $y=3e^{7x}\sin 2x+x^3+x$.)

4.6. $y''+6y=e^{x}(\cos 4x-8\sin 4x)$, $y(0)=0$, $y'(0)=5$.

(*Javob:* $y=\sin 4x-\cos 4x+e^{x}\cos 4x$.)

4.7. $y''-4y'+20y=16xe^{2x}$, $y(0)=1$. $y'(0)=2$.

(*Javob:* $y=e^{2x}(\cos 4x-1/4\sin 4x)+xe^{2x}$.)

4.8. $y''-12y'+36y=32\cos 2x+24\sin 2x$. $y(0)=2$, $y'(0)=4$.

(*Javob:* $y=e^{6x}-2xe^{6x}+\cos 2x$.)

4.9. $y''+y=x^3-4x^2+7x-10$, $y(0)=2$, $y'(0)=3$.

(*Javob:* $y=4\cos x+2\sin x+x^3-4x^2+x-2$.)

4.10. $y''-y=(14-16x)e^{-x}$, $y(0)=0$, $y'(0)=-1$.

(*Javob:* $y=e^{x}-e^{-x}+(4x^2-3x)e^{-x}$.)

4.11. $y''+8y'+16y=16x^2-16x+66$, $y(0)=3$, $y'(0)=0$.

(Javob: $y=-2e^{-4x}-6xe^{-4x}+x^2-2x+5$.)

4.12. $y''+10y'+34y=-9e^{-5x}, y(0)=0, y'(0)=6$.

(Javob: $y=e^{-5x}(\cos 3x+2\sin 3x)-e^{-5x}$.)

4.13. $y''-6y'+25y=(32x-12)\sin x-36x\cos 3x, y(0)=4, y'(0)=0$.

(Javob: $y=e^{3x}(4\cos 4x-3\sin 4x)+2x\sin 3x$.)

4.14. $y''+25y=e^x(\cos 5x-10\sin 5x), y(0)=3, y'(0)=-4$.

(Javob: $y=2\cos 5x-\sin 5x+e^x\cos 5x$.)

4.15. $y''+2y'+5y=-8e^{-x}\sin 2x, \quad y(0)=2, y'(0)=6$.

(Javob: $y=e^{-x}(2\cos 2x+3\sin 2x)+2xe^{-x}\cos 2x$.)

4.16. $y''-10y'+25y=e^{5x}, \quad y(0)=1, y'(0)=0$.

(Javob: $y=3e^{5x}-2xe^{5x}+x^2e^{5x}$.)

4.17. $y''+y'-12y=(16x+22)e^{4x}, y(0)=3, y'(0)=5$.

(Javob: $y=e^{3x}+e^{-4x}+(2x+1)e^{4x}$.)

4.18. $y''-2y'+5y=5x^2+6x-12, y(0)=0, y'(0)=2$.

(Javob: $y=e^x(2\cos 2x-\sin 2x)+x^2+2x-2$.)

4.19. $y''+8y'+16y=16x^3+24x^2-10x+8, y(0)=1, y'(0)=3$.

(Javob: $y=4xe^{-4x}+x^2-x+1$.)

4.20. $y''-2y'+3y=36e^x\cos 6x, y(0)=0, y'(0)=6$.

(Javob: $y=e^x\sin 6x+3xe^x\sin 6x$.)

4.21. $y''-8y'=16+48x^2-128x^x, y(0)=-1, y'(0)=14$.

(Javob: $y=2e^{8x}-3+4x^4-2x$.)

4.22. $y''+12y'+36=72x^3-18, y(0)=-1, y'(0)=0$.

(Javob: $y=\cos 6x+8\sin 6x+2x^3-2x$.)

4.23. $y''+3y'=(40x+58)e^{2x}, \quad y(0)=0, y'(0)=2$.

(Javob: $y=4e^{-3x}-7+(4x+3)e^{2x}$.)

4.24. $y''-9y'+18y=26\cos x-8\sin x, \quad y(0)=0, y'(0)=2$.

(Javob: $y=2e^{6x}-3e^{3x}-\sin x+\cos x$.)

4.25. $y''+8y'=18x+60x^2-32x^3, \quad y(0)=5, y'(0)=2$.

(Javob: $y=3+2e^{-8x}-x^4+3x^3$.)

4.26. $y''-3y'+2y=-\sin x-7\cos x, y(0)=2, y'(0)=7$.

(Javob: $y=e^x+2e^{2x}-\cos x+2\sin x$.)

4.27. $y''+2y'=6x^2+2x+1, \quad y(0)=2, y'(0)=2$.

(Javob: $y=3-e^{-2x}+x^3-x^2$.)

4.28. $y''+16y=32e^{4x}, \quad y(0)=2, y'(0)=2$.

(Javob: $y=\cos 4x+\sin 4x+e^{4x}$.)

4.29. $y''+5y+6y=52\sin 2x, \quad y(0)=-2, y'(0)=-2$.

(Javob: $y=2e^{-2x}+e^{-3x}-5\cos 2x+\sin 2x$.)

4.30. $y''-4y=8e^{2x}$, $y(0)=1$, $y'(0)=-8$.

(Javob: $y=3e^{-2x}+2e^{2x}+2xe^{2x}$.)

5. f (x) funksiyaning ko'rinishi bo'yicha chiziqli bir jinsli bo'lmagan differensial tenglamaning y* xususiy yechimning tuzilishini aniqlang va yozing.

5.1. $2y''-7y'+3y=f(x)$; a) $f(x)=(2x+1)e^{3x}$; b) $f(x)=\cos 3x$.

5.2. $3y''-7y'+2y=f(x)$; a) $f(x)=3xe^{2x}$; b) $f(x)=\sin 2x-3\cos 2x$.

5.3. $2y''+y'-y=f(x)$; a) $f(x)=(x^2-5)e^{-x}$; b) $f(x)=x\sin x$.

5.4. $2y''-9y'+4y=f(x)$; a) $f(x)=-2e^{4x}$; b) $f(x)=e^x\cos 4x$.

5.5. $2y''+49y=f(x)$; a) $f(x)=x^3+4x$; b) $f(x)=3\sin 7x$.

5.6. $3y''+10y'+3y=f(x)$; a) $f(x)=e^{-3x}$; b) $f(x)=2\cos 3x-\sin 3x$.

5.7. $y''-3y'+2y=f(x)$; a) $f(x)=x+2e^x$; b) $f(x)=3\cos 4x$.

5.8. $y''-4y'+4y=f(x)$; a) $f(x)=\sin 2x+2e^x$; b) $f(x)=x^2-4$.

5.9. $y''-y'+y=f(x)$; a) $f(x)=e^x\cos x$; b) $f(x)=7x+2$.

5.10. $y''-3y=f(x)$; a) $f(x)=2x^2-5x$; b) $f(x)=e^{-x}\sin 2x$.

5.11. $y''+3y'-4y=f(x)$; a) $f(x)=3xe^{-4x}$; b) $f(x)=x\sin x$.

5.12. $y''+36y=f(x)$; a) $f(x)=4xe^{-x}$; b) $f(x)=2\sin 6x$.

5.13. $y''-6y'+9y=f(x)$; a) $f(x)=(x-2)e^{3x}$; b) $f(x)=4\cos x$.

5.14. $4y''-5y'+y=f(x)$; a) $f(x)=(4x+2)e^x$; b) $f(x)=e^x\sin 3x$.

5.15. $4y''+7y'-2y=f(x)$; a) $f(x)=3e^{-2x}$; b) $f(x)=(x-1)\cos 2x$.

5.16. $y''-y'-6y=f(x)$; a) $f(x)=2xe^{3x}$; b) $f(x)=9\cos x-\sin x$.

5.17. $y''-16y=f(x)$; a) $f(x)=-3e^{4x}$; b) $f(x)=\cos x-4\sin x$.

5.18. $y''-4y=f(x)$; a) $f(x)=(x-2)e^{4x}$; b) $f(x)=3\cos 4x$.

5.19. $y''-2y'+2y=f(x)$; a) $f(x)=(2x-3)e^{4x}$; b) $f(x)=e^x\sin x$.

5.20. $5y''-6y'+y=f(x)$; a) $f(x)=x^2e^x$; b) $f(x)=\cos x-\sin x$.

5.21. $5y''+9y'-2y=f(x)$; a) $f(x)=x^3-2^x$; b) $f(x)=2\sin 2x-3\cos 2x$.

5.22. $y''-2y'-15y=f(x)$; a) $f(x)=4xe^{3x}$; b) $f(x)=x\sin 5x$.

5.23. $y''-3y=f(x)$; a) $f(x)=2x^3-4x$; b) $f(x)=2e^{3x}\cos x$.

5.24. $y''-7y'+12y=f(x)$; a) $f(x)=xe^{3x}+2e^x$; b) $f(x)=3x\sin 2x$.

5.25. $y''+9y=f(x)$; a) $f(x)=x^2+4x-3$; b) $f(x)=xe^{2x}\sin x$.

5.26. $y''-4y'+5y=f(x)$; a) $f(x)=-2xe^x$; b) $f(x)=x\cos 2x-\sin 2x$.

5.27. $y''+3y'+2y=f(x)$; a) $f(x)=(3x-7)e^{-x}$; b) $f(x)=\cos x-3\sin x$.

5.28. $y''-8y'+16y=f(x)$; a) $f(x)=2xe^{4x}$; b) $f(x)=\cos 4x+2\sin 4x$.

5.29. $y''+y'-2y=f(x)$; a) $f(x)=(2x-1)e^{x}$; b) $f(x)=3x\cos 2x$.
5.30. $y''+3y'-4y=f(x)$; a) $f(x)=6xe^{x}$; b) $f(x)=x^2\sin 2x$.

Namunaviy variant yechimi.

Differensial tenglamalarning umumiy yechimlarini toping.

1. a) $4y''-11y'+6y=0$; b) $4y''-4y'+y=0$; v) $y''-2y'+37y=0$;

Berilgan har bir tenglama uchun xarakteristik tenglama tuzamiz va uni yechamiz. Xarakteristik tenglamaning olingan ildizlarining ko'rinishiga qarab, differensial tenglamaning umumiy yechimini yozamiz (11.48 formulaga va § 11.6 dagi 5-misolga qarang).

a) $4\lambda^2-11\lambda+6=0$, $\lambda_1=3/4$, $\lambda_2=2$ ildizlar har xil va haqiqiydir, shuning uchun tenglamaning umumiy yechimi quyidagicha bo'ladi $y=C_1e^{3x/4}+C_2e^{2x}$;

b) $4\lambda^2-4\lambda+1=0$, $\lambda_1=\lambda_2=1/2$ - ildizlar bir-biriga teng va haqiqiydir, bundan kelib chiqadiki, tenglamaning umumiy yechimi quyidagicha bo'ladi.

$y=C_1e^{x/2}+C_2xe^{x/2}$

v) $\lambda^2-2\lambda+37=0$, $\lambda_{1,2}=\lambda_2=1\pm 6i$ - ildizlar qo'shma kompleksdir, shuning uchun tenglamaning umumiy yechimi quyidagicha bo'ladi.

$y=e^x(C_1\cos 6x+C_2\sin 6x)$.

2. $y''-3y'-4y=6xe^{x}$;

► $\lambda^2-3\lambda-4=0$ xarakteristik tenglamasi $\lambda_1=4, \lambda=-1$ ildizlarga ega. Bundan kelib chiqadiki, bir jinsli tenglamaning umumiy yechimi quyidagi formula bilan aniqlanadi.

$$y=C_1e^{4x}+C_2e^{-x};$$

Tenglamaning o'ng tomonida joylashgan $f(x)=6xe^{-x}$ funksiya bo'yicha xususiy yechim tuzilishini yozib olamiz ((11.50) formulaga qarang).

$$y^{*}=(Ax+B)e^{-x}\cdot x$$

Bu yerda, $z=a+ib=-1$ xarakteristik tenglamaning ildizi bo'lganligidan $(Ax+Bx)e^{-x}$ ifodani x ga ko'paytirdik. A va V

koeffitsiyentlarni noma'lum koeffitsiyentlar usuli bilan aniqlaymiz. Buning uchun quyidagilarni tuzamiz.

$$y^{*'} = (2Ax + B)e^{-x} - (Ax^2 + Bx)e^{-x}$$

$$y^{*''} = 2Ae^{-x} + (Ax^2 + Bx)e^{-x} - 2(2Ax + B)e^{-x};$$

$y^{*'}$, $y^{*''}$ lar uchun topilgan ifodalarni berilgan tenglamaga qo'yib, uning ikkala tomonini e^{-x} ga bo'lib, x^2, x va x^0 lar oldidagi koeffitsiyentlarini tenglashtiramiz va A va V larni topish uchun sistema hosil qilamiz. Shunday qilib, yuqorida bayon qilinganlarga mos ravishda quyidagilarga ega bo'lamiz:

$$2A+Ax^2+Bx-4Ax-2B-6Ax-3B+3Ax^2+3Bx-4Ax^2-4Bx=6x,$$

$$\begin{array}{c|l} x^2 & A + 3A - 4A = 0 \\ x & B - 4A - 6A - 3B - 4B = 6 \\ x^0 & 2A - 2B - 3B = 0 \end{array}$$

Bu yerda $A = -\frac{3}{5}$, $B = -\frac{6}{25}$.

U holda $y^* = -\left(\frac{3}{5}x^2 + \frac{6}{25}x\right)e^{-x}$

Berilgan, bir jinsli bo'lmagan tenglamaning umumiy yechimi quyidagi formuladan aniqlanadi.

$y = \tilde{y} + y^* = C_1e^{4x} + C_2e^{-x} - \left(\frac{3}{5}x^2 + \frac{6}{25}x\right)e^{-x}$.◄

3. $y'' + y' = 5x + \cos 2x$.

►Tenglamaning $\lambda^2 + \lambda = 0$ xarakteristik tenglamasining ildizlarini topamiz. $\lambda_1 = 0$, $\lambda_2 = -1$. Bundan kelib chiqadiki, bir jinsli tenglamaga mos keluvchi umumiy yechim quyidagi ko'rinishga ega bo'ladi.

$$\tilde{y} = C_1 + C_2e^{-x}.$$

Tenglamaning o'ng tomonidan turgan $f(x) = 5x + \cos 2x$ funksiya, $f_1(x) = 5x$ va $f_2(x) = \cos 2x$ funksiyalarning yig'indisidan iborat. Ularga quyidagi 2ta xususiy yechim mos keladi:

$y_1^* = Ax^2 + B$, $y_2^* = A_1\cos 2x + B_1\sin 2x$,

ya'ni, $y^* = y_1^* + y_2^*$ Quyidagilarni topamiz.

$y^{*'} = 2Ax + B - 2A_1\sin 2x + 2B_1\cos 2x$, $y^{*''} = 2A - 4A_1\cos 2x - 4B_1 \sin 2x$.

$y^{*'}$ va $y^{*''}$ lar uchun olingan ifodalarni dastlabki tenglamaga qo'yamiz va A, B, A_1,B_1 koeffitsiyentlarni hisoblaymiz.

$2A-4A_1\cos2x-4B_1\sin2x+2Ax+B-$
$2A_1\sin2x+2B_1\cos2x=5x+\cos2x,$

$$\left.\begin{array}{l|l} x & 2A=5 \\ x^0 & 2A+B=0, \end{array}\right\}$$

$$\left.\begin{array}{l|l} \cos2x & -4A_1+2B_1=1 \\ \sin2x & -2A_1-4B_1=0 \end{array}\right\}, \left.\begin{array}{l} 10B_1=1, \\ A_1=-2B_1, \end{array}\right\}$$

bundan $A=5/2$, $B=-5$, $A_1=-1/5$, $B_1=1/10$.

Shunday qilib, dastlabki tenglamaning xususiy yechimi quyidagi ko'rinishga ega bo'ladi:

$$y^*=\frac{5}{2}x^2-5x-\frac{1}{5}\cos2x+\frac{1}{10}\sin2x$$

uning umumiy yechimi esa quyidagicha bo'ladi

$$y=\bar{y}+y^*=C_1+C_2e^{x}+\frac{5}{2}x^2-5x-\frac{1}{5}\cos2x+\frac{1}{10}\sin2x. \blacktriangleleft$$

4. Berilgan boshlang'ich shartlarni qanoatlantiruvchi, differensial tenglamaning xususiy yechimini toping.

$$y''+16y=(34x+13)e^x,\ y(0)=-1,\ y'(0)=5.$$

$\lambda^2+16=0$ xarakteristik tenglama $\lambda_{1,2}=\pm 4i$ mavhum yechimlarga ega. Bir jinsli tenglamaga mos keluvchi umumiy yechimi quyidagi formula bilan aniqlanadi.

$$\bar{y}=C_1\cos4x+C_2\sin4x,$$

Uning xususiy yechimi esa quyidagi ko'rinishda bo'ladi.

$$y^*=(Ax+B)e^{x}.$$

Quyidagilarni tuzamiz:

$$y^{*\prime}=Ae^{x}-(Ax+B)e^{x},\ y^{*\prime\prime}=-2Ae^{x}+(Ax+B)e^{x}.$$

$y^{*\prime}$ va $y^{*\prime\prime}$ larning ifodalarini dastlabki tenglamaga qo'yamiz.

$-2A+Ax+B+16Ax+16B=34x+13$, hosil qilingan ayniyatdan $A=2$, $B=1$ larni tuzamiz. U holda

$$y^*=(2x+1)e^{x}$$

bo'ladi va dastlabki tenglamaning umumiy yechimi quyidagi ko'rinishda bo'ladi

$$y=C_1\cos4x+C_2\sin4x+(2x+1)e^{x}.$$

$y(0)=-1$, $y'(0)=5$ boshlang'ich shartlardan foydalanib, C_1 va C_2 larning qiymatlarini hisoblash uchun quyidagi sistemani tuzamiz.

$$y(0)=-1=C_1+1$$

$$y'(0)=5=4C_2+2-1$$

bundan $C_1=-2$, $C_2=1$. C_1 va C_2 larning qiymatlarini umumiy yechimga qo'yib, dastlabki tenglamaning xususiy yechimini topamiz.

$$y=\sin 4x-2\cos 4x+(2x+1)e^x. ◄$$

5. $f(x)$ funksiya ko'rinishi bo'yicha $y''-9y=f(x)$ chiziqli bir jinsli bo'lmagan differensial tenglamaning y^* xususiy yechimini aniqlang va ko'rinishini yozing

a) $f(x)=(5-x)e^{3x}$; b) $f(x)=x\sin 2x$.

► $\lambda^2-9=0$, tenglamaning yechimlari $\lambda_1=-3$, $\lambda_2=3$ lardir.

a) $f(x)=(5-x)e^{3x}$ bo'lganligidan, uning xususiy yechimi quyidagi ko'rinishida bo'ladi.

$$y^*=(Ax+B)e^{3x}x=(Ax^2+Bx)e^{3x}.$$

Bu yerda, $z=a+ib=3$ va $k=1$ bo'lganligidan x ko'paytuvchi hosil bo'ladi.

b) $f(x)=x\sin 2x$ bo'lganligi uchun xususiy yechim quyidagicha bo'ladi:

$$y^*=(A_1x+B_1)\cos 2x+(A_2x+B_2)\sin 2x. ◄$$

11.4 –IUT

1. Chiziqli bir jinsli differensial tenglamaning xususiy yechimini toping.

1.1. $y'''-7y''+6y'=0$, $y(0)=0$, $y'(0)=0$, $y''(0)=30$.(*Javob:* $y=5-6e^x+e^{6x}$.)

1.2. $y^V-9y'''=0$, $y(0)=1$, $y'(0)=-1$, $y''(0)=0$, $y'''(0)=0$, $y^{IV}(0)=0$. (*Javob:* $y=1-x$.)

1.3. $y'''-y''=0$, $y(0)=0$, $y'(0)=0$, $y''(0)=-1$.(*Javob:* $y=1+x-e^x$.)

1.4. $y'''-4y'=0$, $y(0)=0$, $y'(0)=2$, $y''(0)=4$.(*Javob:* $y=e^{2x}-1$.)

1.5. $y'''+y'=0$, $y(0)=0$, $y'(0)=1$, $y''(0)=1$.(*Javob:* $y=1-\cos x-\sin x$.)

1.6. $y'''-y'=0$, $y(0)=0$, $y'(0)=2$, $y''(0)=4$.(*Javob:* $y=-4+e^{-x}+3e^x$.)

1.7. $y^{IV}+2y'''-2y'-y=0$, $y(0)=0$, $y'(0)=2$, $y''(0)=0$, $y'''(0)=8$. (*Javob:* $y=2e^{x}-4xe^{-x}-4x^2e^{x}-2e^x$.)

1.8. $y'''+y''-5y'+3y=0$, $y(0)=0$, $y'(0)=1$, $y''(0)=-14$. (*Javob:* $y=e^x-3xe^x-e^{-3x}$.)

1.9. $y'''+y''=0,\ y(0)=0,\ y'(0)=1,\ y''(0)=-1.$
(Javob: $y=1-e^{-x}$.*)*
1.10. $y'''-5y''+8y'-4y=0,\ y(0)=1,\ y'(0)=-1,\ y''(0)=0.$

(Javob: $y=\frac{1}{2}e^{x}+\frac{1}{2}e^{2x}-\frac{5}{8}xe^{2x}$.

1.11. $y'''+3y''+2y'=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=2.$
(Javob: $y=1-2e^{-x}+e^{-2x}$.*)*
1.12. $y'''+3y''+3y'+y=0,\ y(0)=-1,\ y'(0)=0,\ y''(0)=1.$
(Javob: $y=-e^{-x}(1+x)$.*)*
1.13. $y'''2y''+9y'-18y=0,\ y(0)=-2{,}5,\ y'(0)=0,\ y''(0)=0.$

(Javob: $y=-\frac{45^{e}}{26}\ 2x-\frac{10}{13}\cos 2x+\frac{15}{13}\sin 2x$).

1.14. $y'''+9y'=0,\ y(0)=0,\ y'(0)=9,\ y''(0)=-18.$
(Javob: $y=-2+2\cos 3x+3\sin 3x$.*)*
1.15. $y'''-13y''+12y'=0,\ y(0)=0,\ y'(0)=1,\ y''(0)=133$
(Javob: $y=10-11e^{x}+e^{12x}$.*)*
1.16. $y^{IV}-5y''+4y=0,\ y(0)=-2,\ y'(0)=1,\ y''(0)=2,\ y'''(0)=0.$

(Javob: $y=-e^{x}\frac{7}{3}e^{-x}+\frac{7}{12}e^{2x}+\frac{3}{4}e^{-2x}$.*)*

1.17. $y^{IV}-10y''+9y=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=8,\ y'''(0)=24.$
(Javob: $y=-2e^{x}+e^{-x}+e^{3x}$.*)*
1.18. $y'''-y''+y'-y=0.\ y(0)=0,\ y'(0)=1,\ y''(0)=0.$
(Javob: $y=\sin x$.*)*
1.19. $y'''-3y''+3y'-y=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=4.$
(Javob: $y=2x^{2}e^{x}$.*)*
1.20. $y'''-y''+4y'-4y=0,\ y(0)=-1,\ y'(0)=0,\ y''(0)=-6.$
(Javob: $y=-2e^{x}+\cos 2x+\sin 2x$.*)*
1.21. $y^{IV}-2y+y''=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=1,\ y'''(0)=2.$
(Javob: $y=1-e^{x}+xe^{x}$.*)*
1.22. $y^{IV}-y=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=0,\ y'''(0)=-4.$
(Javob: $y=e^{-x}+2\sin x$.*)*
1.23. $y^{IV}-16y=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=0,\ y'''(0)=-8.$

(Javob: $y=\frac{1}{4}e^{2x}-\frac{1}{4}e^{2x}+\frac{1}{2}\sin 2x$.)

1.24. $y'''+y''-4y'-4=0,\ y(0)=0,\ y'(0)=0,\ y''(0)=12.$

(*Javob*: $y=e^{2x}+3e^{-2x}-4e^{-x}$.)

1.25. $y'''+y''+9y'+18y=0,\ y(0)=1,\ y'(0)=-3,\ y''(0)=-9.$

(*Javob*: $y=\cos 3x-\sin 3x$.)

1.26. $y^{V}-y^{IV}+9\ y''=0,\ \ y(0)=\ y'(0)=\ y''(0)=\ y'''(0)=0,$ $y^{IV}(0)=27.$

(*Javob*: $y=1+2x+\dfrac{3}{2}x^2-e^{-3x}+xe^{3x}$.)

1.27. $y'''+2y''+y'+18y=0,\ y(0)=0,\ y'(0)=2,\ y''(0)=-3.$

(*Javob*: $y=1-e^{-x}+xe^{-x}$.)

1.28. $y'''-y''-y'+y=0,\ y(0)=-1,\ y'(0)=0,\ y''(0)=1.$

(*Javob*: $y=-4e^{x}+7xe^{x}+3e^{-x}$.)

1.29. $y^{IV}+5y''+4y=0,\ y(0)=1,\ y'(0)=4,\ y''(0)=-1,\ y'''(0)=-16.$

(*Javob*: $y=2\sin 2x+\cos x$.)

1.30. $y^{IV}+10y''+9y=0,\ y(0)=1,\ y'(0)=3,\ y''(0)=-9,\ y'''(0)=-27.$

(*Javob*: $y=\cos 3x+\sin 3x$.)

2. Differensial tenglamalar sistemasini ikki usulda yeching:

a) yuqori differensial tenglamaga keltirish yo'li bilan;

b) xarakteristik tenglama yordamida.

2.1. $\begin{cases} x'=2x+y, \\ y'=3x+4y. \end{cases}$ .... $\left(Javob: \begin{cases} x=C_1e^{5t}+C_2e^{t} \\ y=3C_1e^{5t}-C_2e^{t} \end{cases}\right)$

2.2. $\begin{cases} x'=x-y, \\ y'=-4x+y. \end{cases}$ .... $\left(Javob: \begin{cases} x=C_1e^{3t}+2C_2e^{t} \\ y=-2C_1e^{3t}+2C_2e^{-t} \end{cases}\right)$

2.3. $\begin{cases} x'=-x+8y, \\ y'=x+y. \end{cases}$ .... $\left(Javob: \begin{cases} x=C_1e^{3t}+2C_2e^{-3t} \\ y=\dfrac{1}{2}C_1e^{3t}-\dfrac{1}{4}C_2e^{-3t} \end{cases}\right)$

2.4. $\begin{cases} x'=-2x-3y, \\ y'=-x. \end{cases}$ .... $\left(Javob: \begin{cases} x=C_1e^{-3t}+C_2e^{t} \\ y=\dfrac{1}{3}C_1e^{-3t}-C_2e^{t} \end{cases}\right)$

2.5. $\begin{cases} x' = x - y, \\ y' = -4x + 4y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 + C_2 e^{5t} \\ y = C_1 - 4C_2 e^{5t} \end{cases}\right)$

2.6. $\begin{cases} x' = -2x + y, \\ y' = -3x + 2y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{t} + C_2 e^{-t} \\ y = 3C_1 e^{t} - C_2 e^{-t} \end{cases}\right)$

2.7. $\begin{cases} x' = 6x - y, \\ y' = 3x + 2y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{3t} + C_2 e^{5t} \\ y = 3C_1 e^{-t} - C_2 e^{5t} \end{cases}\right)$

2.8. $\begin{cases} x' = 2x + y, \\ y' = -6x - 3y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 + C_2 e^{-t} \\ y = -2C_1 - 3C_2 e^{-t} \end{cases}\right)$

2.9. $\begin{cases} x' = y, \\ y' = x. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{t} + C_2 e^{-t} \\ y = C_1 e^{t} - 3C_2 e^{-t} \end{cases}\right)$

2.10.

$\begin{cases} x' = -x - 2y, \\ y' = 3x + 4y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{t} + C_2 e^{2t} \\ y = -C_1 e^{t} - \frac{2}{3} C_2 e^{2t} \end{cases}\right)$

2.11. $\begin{cases} x' = -2x, \\ y' = y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 + C_2 e^{-2t} \\ y = -C_1 e^{t} + C_2. \end{cases}\right)$

2.12.

$\begin{cases} x' = 4x + 2y, \\ y' = 4x + 6y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{2t} + C_2 e^{8t} \\ y = -C_1 e^{2t} + 2C_2 e^{8t} \end{cases}\right)$

2.13.

$\begin{cases} x' = 8x - 3y, \\ y' = 2x + y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{2t} + C_2 e^{7t} \\ y = 2C_1 e^{2t} + \frac{1}{3} C_2 e^{7t} \end{cases}\right)$

2.14. $\begin{cases} x' = 3x + y, \\ y' = x + 3y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 e^{2t} + C_2 e^{4t} \\ y = -C_1 e^{2t} + C_2 e^{4t} \end{cases}\right)$

2.15.

$$\begin{cases} x' = 2x + 3y, \\ y' = 5x + 4y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{-t} + C_2 e^{7t} \\ y = -C_1 e^{-t} + \frac{5}{3} C_2 e^{7t} \end{cases}\right)$$

2.16.

$$\begin{cases} x' = x + 2y, \\ y' = 3x + 6y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{t} + C_2 e^{7t} \\ y = -\frac{1}{2} C_1 + 3C_2 e^{7t} \end{cases}\right)$$

2.17.

$$\begin{cases} x' = 5x + 4y, \\ y' = 4x + 5y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{t} + C_2 e^{9t} \\ y = -C_1 e^{t} + C_2 e^{9t} \end{cases}\right)$$

2.18.

$$\begin{cases} x' = x + 2y, \\ y' = 4x + 3y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{t} + C_2 e^{3t} \\ y = -C_1 e^{t} + 2C_2 e^{3t} \end{cases}\right)$$

2.19.

$$\begin{cases} x' = x + 4y, \\ y' = x + y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{-t} + C_2 e^{3t} \\ y = -\frac{1}{2} C_1 e^{-t} + \frac{1}{2} C_2 e^{3t} \end{cases}\right)$$

2.20.

$$\begin{cases} x' = 3x - 2y, \\ y' = 2x + 8y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{4t} + C_2 e^{7t} \\ y = -\frac{1}{2} C_1 e^{4t} - 2C_2 e^{7t} \end{cases}\right)$$

2.21.

$$\begin{cases} x' = x + 4y, \\ y' = 2x + 3y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{-t} + C_2 e^{5t} \\ y = -\frac{1}{2} C_1 e^{-t} + C_2 e^{5t} \end{cases}\right)$$

2.22.

$$\begin{cases} x' = 7x + 3y, \\ y' = x + 5y. \end{cases} \text{............} \quad \left(Javob: \begin{cases} x = C_1 e^{4t} + C_2 e^{8t} \\ y = -C_1 e^{4t} + \frac{1}{3} C_2 e^{8t} \end{cases}\right)$$

2.23. $\begin{cases} x' = 4x - y, \\ y' = -x + 4y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{3t} + C_2e^{5t} \\ y = C_1e^{3t} - C_2e^{5t} \end{cases}\right)$

2.24.

$\begin{cases} x' = 2x + 8y, \\ y' = x + 4y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1 + C_2e^{6t} \\ y = -\frac{1}{4}C_1 + \frac{1}{2}C_2e^{6t} \end{cases}\right)$

2.25.

$\begin{cases} x' = 5 + 8y, \\ y' = 3x + 3y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{-t} + C_2e^{9t} \\ y = -\frac{3}{4}C_1e^{-t} + \frac{1}{2}C_2e^{9t} \end{cases}\right)$

2.26.

$\begin{cases} x' = 3x + y, \\ y' = 8x + y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{-t} + C_2e^{5t} \\ y = -4C_1e^{-t} + 2C_2e^{5t} \end{cases}\right)$

2.27.

$\begin{cases} x' = x - 5y, \\ y' = -x - 3y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{-4t} + C_2e^{2t} \\ y = C_1e^{-4t} + \frac{1}{5}C_2e^{2t} \end{cases}\right)$

2.28.

$\begin{cases} x' = -5x + 2y, \\ y' = x - 6y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{-4t} + C_2e^{-7t} \\ y = \frac{1}{2}C_1e^{-4t} - C_2e^{-7t} \end{cases}\right)$

2.29.

$\begin{cases} x' = 6x + 3y, \\ y' = -8x - 5y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{-2t} + C_2e^{3t} \\ y = -\frac{8}{3}C_1e^{-2t} - C_2e^{3t} \end{cases}\right)$

2.30.

$\begin{cases} x' = 4x - 8y, \\ y' = -8x + 4y. \end{cases}$ .... $\left(Javob: \begin{cases} x = C_1e^{-4t} + C_2e^{12t} \\ y = C_1e^{-4t} - C_2e^{12t} \end{cases}\right)$

3. Differensial tenglamani ixtiyoriy o'zgarmasni variatsiyalash usuli bilan yeching.

3.1. $y''-y=\dfrac{e^x}{e^x+1}$

$(Javob: y=(-\frac{e^x}{2}+\frac{1}{2}\ln(e^x+1)+e_1)-e^{-x}+(\frac{1}{2}\ln\frac{e^x}{e^x+1}+e_2)e^x.)$

3.2. $y''+4y=\dfrac{1}{\cos 2x}$.

$(Javob: y=(-\frac{1}{4}\ln|\cos 2x|+C_2)\cos 2x+(\frac{1}{2}x+C_2)\sin 2x.)$

3.3. $y''-4y+5y'=\dfrac{e^{2x}}{\cos x}$.

$(Javob: y=(\ln|\cos x|+C_1)e^{2x}\cos x+(x+C_2)e^{2x}\sin 2x.)$

3.4. $y''+y'=\dfrac{\sin^x}{\cos^2 x}$.

$(Javob: y=\frac{1}{\cos}C_1+(\ln|\cos x|+C_2)\cos x+(x-tgx+C_3)\sin x.)$

3.5. $y''+9y=\dfrac{1}{\cos 3x}$.

$(Javob: y=(-\frac{1}{3}x+C_1)\cos 3x+(\frac{1}{9}\ln|\sin 3x|+C_2)\sin 3x.)$

3.6. $y''+2y'+y=xe^x+\dfrac{1}{xe^x}$.

$(Javob: y=C_1e^{-x}+C_2xe^{-x}+\frac{1}{4}e^x-\frac{1}{4}e^x-xe^{-x}+xe^{-x}\ln x.)$

3.7. $y''+2y'+2y=\dfrac{e^{-x}}{\cos x}$.

$(Javob: y=(\ln|\cos x|+C_1)e^{-x}\cos x+(x+C_2)e^{-x}\sin x.)$

3.8. $y''-2y'+2y=\frac{e^x}{\sin^2 x}$.

$(Javob: y=(\ln(ctg\frac{x}{2})+C_1)e^x\cos x+(\frac{1}{\sin x}+C_2)e^x\sin x.)$

3.9. $y''+2y'+2y=e^{-x}ctgx$.

$(Javob: y=C_1e^{-x}\cos x+C_2e^{-x}\sin x+e^{-x}\sin x\cdot\ln|tg(x/2)|.)$

3.10. $y''-2y'+2y=e^x/\sin x$.

$(Javob: y=(-x+C_1)e^x\cos x+(\ln|\sin x|+C_2)e^x\sin x.)$

3.11. $y''-2y'+y=e^x/x^2$.

$(Javob: y=(-\ln x+C_1)e^x+(-1/x+C_2)xe^x$.

3.12. $y''+y=tg\,x\cdot$

$(Javob: y=C_1\cos x+C_2\sin x-\cos x\cdot\ln|tg(x/2+\pi/4)|)$.

3.13. $y''+4y=ctg\,2x\cdot$

$Javob: y=C_1\cos 2x+C_2\sin 2x+\frac{1}{4}\sin 2x\cdot\ln|tgx|.)$.

3.14. $y''+y=ctg\,x$

$Javob: y=C_1\cos x+C_2\sin x+\sin x\cdot\ln|tgx(x/2)|.)$.

3.15. $y''-2y'+y=e^x/x$.

$Javob: y=(-x+C_1)e^x+(\ln x+C_2)xe^x)$.

3.16. $y''+2y'+y=e^{-x}/x$

$Javob: y=(-x+C_1)e^{-x}+(\ln x+C_2)xe^{-x})$.

3.17. $y''+y'=1/\cos x$.

$Javob: y=(\ln|\cos x|+C_1)\cos x+(x+C_2)\sin x)$.

3.18. $y''+y=1/\sin x$.

$Javob: y=(-x+C_1)\cos x+(\ln|\sin x|+C_2)\sin x)$.

3.19. $y''+4y=1/\sin 2x$.

$Javob: y=(-\frac{x}{2}+C_1)\cos 2x+(\frac{1}{4}\ln|\sin 2x|+C_2)\sin 2x).$

3.20. $y''+4y=tg\,2x.$

$Javob: y=C_1\cos 2x+C_2\sin 2x-\frac{1}{4}\ln\left|tg(x+\frac{\pi}{4})\right|\cos 2x).$

3.21. $y''+4y'+4y=e^{-2x}/x^3.$

$Javob: y=C_1+C_2x+1/(2x))x^{-2x}).$

3.22. $y''-4y'+4y=e^{2x}/x^3.$

$Javob: y=C_1e^{2x}+C_2xe^{2x}+e^{2x}/2x).$

3.23. $y''+2y'+y=3e^{-x}\sqrt{x+1}$

$Javob: y=(-\frac{6}{5}\sqrt{(x+1)^5}+2\sqrt{(x+1)^3}+C_1)e^{-x}+(2\sqrt{(x+1)^3}+C_2)xe^{-x}).$

3.24. $y''+y=-ctg^2x.$

$Javob: y=C_1\cos x+C_2\sin x+\cos x\cdot\ln|tg(x/2)|+2.)$

3.25. $y''-y'=e^{2x}\cos(e^x).$

$Javob: y=C_1+C_2e^x-\cos(e^x).)$

3.26. $y''-y'=e^{2x}\cdot\sin(e^x).$

$Javob: y=C_1+C_2e^x-\sin(e^x).)$

3.27. $y''+y=tg^2x.$

$Javob: y=C_1\cos x+C_2\sin x+\sin x\cdot\ln\left|tg(\frac{x}{2}+\frac{\pi}{4})\right|-2.)$

3.28. $y''+y=2/\sin^2x.$

$Javob: y=C_1\cos x+C_2\sin x+2\cos x\cdot\ln|ctg(x/2)|-2).$

3.29. $y''+2y'+5y=\frac{e^x}{\sin 2x}.$

$Javob: y=(-\frac{x}{2}+C_1)e^{-x}\cos 2x+(\frac{1}{4}\ln|\sin 2x|+C_2)e^{-x}\sin 2x.)$

3.30. $y''+9y=\dfrac{1}{\cos 3x}$.

$Javob: y=(\frac{1}{9}\ln|\cos 3x|+C_1)\cos 3x+(\frac{x}{3}+C_2)\sin 3x.)$

4. Quyidagi masalalarni yeching:

4.1. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing: Egri chiziqqa urinma, urinish nuqtasidan abssissa o'qiga tushirilgan perpendikulyar va abssissa o'qi bilan chegaralangan uchburchakning yuzi o'zgarmas kattalik bo'lib b^2 ga teng.

(*Javob*:$y=2b^2/(C\pm x)$).

4.2. Egri chiziqqa ixtiyoriy urinmaning abssissa o'qi bilan kesishish nuqtasi, urinish nuqtasi va koordinata boshidan bir xil uzoqlikda ekanligi ma'lum bo'lsa, egri chiziq tenglamasini yozing.

(*Javob*:$y=C(x^2+y^2)$).

4.3. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing: koordinata o'qlari, egri chiziqqa urinma va urinish nuqtasidan abssissa o'qiga tushirilgan perpendikulyar bilan chegaralangan trapetsiyaning yuzi o'zgarmas kattalik bo'lib, $3a^2$ ga teng.

(*Javob*:$y=Cx^2+2a^2/x$)

4.4. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing: urinma, abssissa o'qi va koordinata boshidan urinish nuqtasigacha bo'lgan kesma bilan chegaralangan uchburchakning yuzi a^2 ga teng bo'lgan o'zgarmas kattalikdir. (*Javob*:$x=a^2/y+Cy$).

4.5. Ixtiyoriy urinmadan koordinata boshigacha bo'lgan masofa, urinish nuqtasining abssissasiga tengligi ma'lum bo'lsa, egri chiziq tenglamasini yozing. (*Javob*:$Cx=x^2+y^2$.)

4.6. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing:

ixtiyoriy urinmaning abssissa o'qi bilan kesishish nuqtasi, urinish nuqtasining abssissasidan ikki marta kichik bo'lgan abssissasiga ega.

(*Javob*:$y=Cx^2$).

4.7. Urinma, urinish nuqtasidan abssissa o'qiga tushirilgan perpendikulyar va abssissa o'qi bilan chegaralangan uchburchakning katetlari yig'indisi o'zgarmas kattalik bo'lib, agar teng bo'lgan xossaga ega egri chiziq tenglamasini yozing: (*Javob:* $\pm x = C + a\ln y - y(0 < y < a)$.)..

4.8. Ixtiyoriy urinmasining abssissa o'qi bilan kesishish nuqtasi, urinish nuqtasi abssissasining 2/3 qismiga teng abssissaga ega bo'lgan egri chiziq tenglamasini yozing. (*Javob:* $y=Cx^3$)

4.9. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing:

Egri chiziqning ixtiryoriy nuqtasidan o'tkazilgan urinma va normalning abssissa o'qidan ajratgan kesmaning uzunligi 2 1 ga teng.

(*Javob:* $x=C+l\cdot ln(l\pm\sqrt{e^2-y^2})\pm\sqrt{(e^2-y^2)}$.)

4.10. *A(2,4)* nuqtadan o'tuvchi va quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing: Egri chiziqning ixtiyoriy nuqtasiga o'tkazilgan urinmaning abssissa o'qidan ajratgan kesmasining uzunligi, urinish nuqtasi abssissaning kubiga teng. (*Javob:* $y=2\sqrt{3}\ x/\sqrt{x^2-1}$)

4.11. *A(1,5)* nuqtadan o'tuvchi va quyidagi xossaga ega bo'lgan egri chiziq tenglamasini tuzing: ixtiyoriy urinmaning ordinata o'qidan ajratgan kesmasining uzunligi, urinish nuqtasi abssissasining uchlangani ga teng.

(*Javob:* $y=3x\ln x+5x$.)

4.12. *A(1,2)* nuqtadan o'tuvchi va quyidagi xossaga ega bo'lgan egri chiziq tenglamasini tuzing: ixtiyoriy nuqtasining ordinatasining shu nuqta abssissasiga nisbati, izlanayotgan egri chiziqqa shu nuqtada o'tkazilgan urinmaning burchak koeffitsiyentiga proporsional. Proporsionallik koeffitsiyenti 3 ga teng.(*Javob:* $y^2=8x$.)

4.13. Ixtiyoriy nuqtasidagi urinmaning burchak koeffitsiyenti, urinish nuqtasi ordinatasining kvadratiga proporsional ekanligi ma'lum bo'lsa, *A(2.-1)* nuqtadan o'tuvchi egri chiziq tenglamasini tuzing. Proporsionallik koeffitsiyenti 6 ga teng. (*Javob:* $y=e^{6x-12}$.)

4.14. Ixtiyoriy nuqtasidagi urinmaning burchak koeffitsiyentining urinish nuqtasi koordinatalarining yig'indisiga ko'paytmasi, shu nuqta ordinatasining ikkilanganiga teng ekanligi ma'lum bo'lsa, *A (1,2)* nuqtadan o'tuvchi egri chiziq tenglamasini yozing. (*Javob:* $y=2(y-x)^2$.)

4.15. Ixtiyoriy nuqtasiga urinmaning burchak koeffitsiyenti, shu nuqta ordinatasining uchlanganiga teng ekanligi ma'lum bo'lsa, *A(0,-2)* nuqtadan o'tuvchi egri chiziq tenglamasini yozing. (*Javob:* $y=-2e^{3x}$.)

4.16. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini yozing:

Urinmaga koordinata boshidan tushirilgan perpendikulyarning uzunligi urinish nuqtasi absissasiga teng. (*Javob:* $y^2=Cx-x^2$.)

4.17. Biror nuqtasiga urinmaning burchak koeffitsiyenti, shu nuqtani koordinata boshi bilan tutashtiruvchi to'g'ri chiziqning burchak koeffitsiyentidan n marta katta bo'lgan xossaga ega egri chiziq tenglamasini yozing. (*Javob:* $y=Cx^n$)

4.18. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini tuzing:

Egri chiziqqa urinmaning koordinata o'qlari bilan chegaralangan kesmasi, urinish nuqtasida teng ikkiga bo'linadi. (*Javob:* $xy=C$)

4.19. Egri chiziqning biror nuqtasiga o'tkazilgan normalning ordinata o'qidan ajratgan kesmasining uzunligi, shu nuqtadan koordinata boshigacha bo'lgan masofaga teng degan xossaga ega egri chiziq tenglamasini tuzing.

(*Javob:* $y=\frac{1}{2}(Cx^2-\frac{1}{C})$.)

4.20. Egri chiziqning biror nuqtasining abssissasining shu nuqtaga o'tkazilgan normalning OU o'qidan ajratgan kesmasi uzunligiga ko'paytmasi shu nuqtadan koordinata boshigacha bo'lgan masofa kvadratining ikkilanganiga teng bo'ladigan egri chiziq tenglamasi tuzilsin.

($Javob: x^2+y^2=Cx^4$.)

4.21. Ou o'qi, urinish nuqtasining radius vektori va urinmasidan tashkil topgan teng yonli uchburchak uchun egri chiziq tenglamasini tuzing.

($Javob: x^2+y^2=Cy,\ y^2=C^2-2Cx,\ x\cdot y=C$)

4.22. $A(2,0)$ nuqtadan o'tuvchi va quyidagi xossaga ega bo'lgan egri chiziq tenglamasini tuzing: Urinish nuqtasi va Ou o'qi orasidagi urinmaning kesmasi, o'zgarmas kattalik bo'lib, 2 ga teng. ($Javob: \pm y=\sqrt{4-x^2}+\ln(\frac{2-\sqrt{4-x^2}}{2+\sqrt{x-x^2}})$

4.23. Barcha urinmalari koordinata boshidan o'tuvchi egri chiziq tenglamasini yozing. ($Javob: y=Cx$.)

4.24. Har bir urinmasi, urinish nuqtasi abssissasining ikkilanganiga teng abssissali nuqtada $u=1$ to'g'ri chiziqni kesib o'tuvchi egri chiziq tenglamasini yozing. ($Javob: y=C/x+1$.)

4.25. Quyidagi xossaga ega bo'lgan egri chiziq tenglamasini tuzing: Agar ixtiyoriy nuqtasidan koordinata o'qlari bilan kesishguncha, ularga parallel to'g'ri chiziqlar o'tkazilsa, u holda hosil bo'lgan to'g'ri to'rtburchak yuzi egri chiziq bilan ikki qismga ajraladi va ulardan birining yuzasi ikkinchisining yuzasidan ikki marta katta bo'ladi.($Javob: y=Cx^2$.)

4.26. Agar egri chiziqqa urinmaning Ou o'qidan ajralgan kesmasi uzunligi bo'yicha urinish nuqtasi koordinatalari yig'indisining $\frac{1}{n}$ ga teng. bo'lsa, egri chiziq tenglamasini toping. ($Javob: y=C\cdot x^{(n-1)/n}-x$.)

4.27. $M(x,u)$ nuqtadagi normalining Ox o'qidan ajratgan kesmasining uzunligi u^2/x ga teng bo'lgan egri chiziq tenglamasini yozing.

($Javob: y=x\sqrt{2\ln(C/x)}$)

4.28. Urinmasining Ou o'qidan ajratgan kesmasining uzunligi, urinish nuqtasi abssissasining kvadratiga teng bo'lgan egri chiziq tenglamasini yozing.

($Javob: y=Cx-x^2$.)

4.29. *M(x,u)* nuqtadagi normalining Ou o'qidan ajratgan kesmasining uzunligi x^2/u ga teng bo'lgan egri chiziq tenglamasini yozing.

(Javob: $C=x^2/(2y^2)+\ln y$.*)*

4.30. Egri chiziq, ordinatasi 2 ga teng nuqtada Ou o'qiga 45° ostida og'gan. Uning ixtiyoriy urinmasi abssissa o'qidan, uzunligi bo'yicha urinish nuqtasi ordinatasining kvadratiga teng kesma ajratadi. Berilgan egri chiziq tenglamasini yozing.

(Javob: $x=(5-y)y$.*)*

Namunaviy variantni yechish.

1. Chiziqli bir jinsli differensial tenglamaning xususiy yechimini toping.

$y^{IV}-y=0,\ y(0)=5,\ y'(0)=3,\ y''(0)=y'''(0)=0.$

Xarakteristik tenglamasini tuzamiz va uni yechamiz:

$\lambda^4-1=0,\ (\lambda^2-1)(\lambda^2+1)=0,\ \lambda_1=-1,\ \lambda_2=1,\ \lambda_{3,4}=\pm i.$

Berilgan tenglamaning umumiy yechimi quyidagi ko'rinishda bo'ladi.

$y=C_1e^{-x}+C_2e^{x}+C_2\cos x+C_4\sin x.$

Quyidagilarni topamiz:

$y'=-C_1e^{-x}+C_2e^{x}-C_3\sin x+C_4\cos x,$

$y''=C_1e^{-x}+C_2e^{x}-C_3\sin x-C_4\sin x,$

$y'''=-C_1e^{-x}+C_2e^{x}-C_3\sin x-C_4\cos x.$

Boshlang'ich shartlardan foydalanib, C_1, C_2, C_3, C_4 larning qiymatlarini topish uchun sistema tuzamiz va uni yechamiz:

$$\left.\begin{aligned} C_1+C_2+C_3&=5\\ -C_1+C_2+C_4&=3\\ C_1+C_2-C_3&=0\\ -C_1+C_2-C_4&=5 \end{aligned}\right\} \qquad \left.\begin{aligned} 2C_1+2C_2&=5\\ -2C_1+2C_2&=3 \end{aligned}\right\}$$

bu yerdan $C_1=1/2,\ C_2=2,\ C_3=5/2,\ C_4=3/2.$

Berilgan tenglamaning xususiy yechimi quyidagi ko'rinishda bo'ladi.

$$y=\frac{1}{2}e^{-x}+2e^{x}+\frac{5}{2}\cos x+\frac{3}{2}\sin x.$$

2. Quyidagi tenglamalar sistemasini 2 xil usulda yeching.

a) yuqori tartibli differensial tenglamaga keltirish yo'li bilan;

b) xarakteristik tenglama yordamida.

$x'=-7x+y,\ x=x(t),\ x'=dx|dt,$

$y'=-2x-5y,\ y=y(t),\ y'=dy|dt.$

Berilgan sistemaning birinchi tenglamasini differensiallab, quyidagini hosil qilamiz. $x''=-7x'+y'$

So'ngra oxirgi tenglamada y' ni berilgan sistemadagi ikkinchi tenglamasidagi ifodasi bilan almashtiramiz: $x''=-7x'-2x-5y$. Oxirgi tenglamada y ni sistemasining birinchi tenglamasidan topilgan $y=x'+7x$ ifoda bilan almashtiramiz. Natijada, ikkinchi tartibli differensial tenglamani hosil qilamiz.

$$x''=-7x-2x-5(x'+7x),\ x''+12x'+37x=0.$$

Oxirgi tenglamani ma'lum usulda yechamiz (§ 11.7ga qarang)

$$\lambda^2+12\lambda+37=0,\quad \lambda_{1,2}=-6\pm\sqrt{36-37}=-6\pm i,$$

$x=e^{-6t}(C_1\cos t+C_2\sin t).$

Bundan quyidagini topamiz.

$x'=-6e^{-6t}(C_1\cos t+C_2\sin t)+e^{6t}(-C_1\sin t+C_2\cos t).$

x va x' lar uchun olingan ifodalarni $y=x'+7x$ ga qo'yib, quyidagini hosil qilamiz.

$y'=-6e^{-6t}(C_1\cos t+C_2\sin t)+e^{-6t}(-C_1\sin t+C_2\cos t)+7e^{-6t}(C_1\cos t+C_2\sin t).$

Shunday qilib, izlanayotgan yechimlar quyidagi funksiyalar bo'ladi.

$x=e^{-6t}(C_1\cos t+C_2\sin t),$

$y=e^{-6t}(C_1(\cos t-\sin t)+C_2(\cos t+\sin t)).$

b) xarakteristik tenglamasini tuzamiz va uni yechamiz:

$$\begin{vmatrix}-7-\lambda & 1\\ -2 & -5-\lambda\end{vmatrix}=0,\qquad (7+\lambda)(5+\lambda)+2=0$$

$$\lambda^2+12\lambda+37=0,\quad \lambda^{1,2}=-6\pm i,\quad \lambda_{1,2}=-6+i,$$

$\lambda_1=-6+i$ uchun quyidagi sistemani hosil qilamiz. (§ 11.7 dagi 2-misol bilan solishtiring)

$$\left.\begin{aligned} (-7+6-i)\alpha+\beta=0 \\ -2\alpha+(-5+6-i)\beta=0 \\ -(1+i)\alpha+\beta=0 \\ -2\alpha+(1-i)\beta=0 \end{aligned}\right\}$$

$\alpha=1$, $\beta=1+I$ deb olib, dastlabki tenglamaning birinchi xususiy yechimini topamiz.

$x_1=e^{(-6+i)t}$, $y_1=(1+i)e^{(-6+i)t}$

$\lambda_1=-6-i$ uchun

$$\left.\begin{aligned} (-7+6+i)\alpha+\beta=0 \\ -2\alpha+(-5+6+i)\beta=0 \\ (-1+i)\alpha+\beta=0 \\ -2\alpha+(1+i)\beta=0 \end{aligned}\right\}$$

$\alpha=1\ va\ \beta=1-i$ deb faraz qilib, dastlabki tenglamaning ikkinchi xususiy yechimini hosil qilamiz.

$$x_2=e^{(-6-i)t},\ y_2=(1-i)e^{(-6-i)t}$$

Quyidagi formulalar bo'yicha yangi fundamentall yechimlar sistemasiga o'tamiz.

$\bar{x}_1=(x_1+x_2)/2,\ \bar{x}_2=(x_1-x_2)/(2i),$

$\bar{y}_1=(y_1+y_2)/2,\ \bar{y}_2=(y_1-y_2)/(2i),$

Eyler formulasidan foydalanib, $e^{(\alpha\pm\beta i)^t}=e^{\alpha t}\ (cos\ \beta\ t \pm isin\beta t),$ quyidagilarni topamiz

$\bar{x}_1=e^{-6t}cost,\ \bar{x}_2=e^{-6t}sint,$

$\bar{y}_1=e^{-6t}(cost-sint),\ \bar{y}_2=e^{-6t}(cost+sint),$

Dastlabki sistemaning umumiy yechimi quyidagi ko'rinishga ega bo'ladi.

$x=C_1\bar{x}_1+C_2\bar{x}_2,\ y=C_1\bar{y}_1+C_2\bar{y}_2,$

ya'ni,

$x=e^{-6t}(C_1cos\ t+C_2sin\ t),$

$y=e^{-6t}\ C_1(cos\ t-sin\ t)+C_2(cos\ t+sin\ t)).$

3. Differensial tenglamani ixtiyoriy o'zgarmasni variatsiyalash usuli bilan yeching

$$y''\text{-}y=\frac{2e^x}{e^x-1}.$$

▶ Berilgan tenglamaga mos keluvchi bir jinsli tenglamani yechamiz.

$y''\text{-}y=0,\ \lambda^2-1=0,\ \lambda_1=-1,\ \lambda_2=1$

Bir jinsli tenglamaning umumiy yechimi quyidagicha bo‘ladi.

$y=C_1e^{-x}+C_2e^x$

C_1 va C_2 larni x ning funksiyasi deb hisoblaymiz, ya’ni,

$y=C_1(x)e^{-x}+C_2(x)e^x$

quyidagi sistemadan $C_1(x)$ va $C_2(x)$ larni aniqlaymiz ((11.39) sistemaga qarang).

$$\left.\begin{aligned} C_1'(x)y_1+C_2'(x)y_2&=0,\\ C_1'(x)y_1'+C_2'(x)y_2'&=f(x).\end{aligned}\right\}$$

berilgan tenglama uchun bu sistema quyidagi ko‘rinishga ega.

$$\left.\begin{aligned} C_1'(x)e^{-x}+C_2'(x)e^x&=0,\\ -C_1'(x)e^{-x}+C_2'(x)e^x&=2e^x/(e^x-1).\end{aligned}\right\}$$

Bu sistemadan avval $C_2'(x)$, $C_1'(x)$, larni, keyin esa $C_2(x)$ va $C_1(x)$ larni topamiz.

$$2C_2'(x)e^x=\frac{2e^x}{e^x-1},\ C_2'(x)=\frac{1}{e^x-1},$$

$$C_2(x)=\int\frac{dx}{e^x-1}=\left|\begin{matrix} t=e^x,\ x=\ln t,\\ dx=dt/t\end{matrix}\right|=\int\frac{dt}{t(t-1)}=\int\frac{dt}{t-1}-\int\frac{dt}{t}=$$

$$\ln \quad |t-1|-\ln|t|+C_2=\ln\left|\frac{t-1}{t}\right|+C_2=\ln\left|\frac{e^x-1}{e^x}\right|+C_2,$$

$$C_1'(x)=-C_2'(x)e^{2x}=e^{-2x}/(e^x-1),$$

$$C_1(x)=-\int\frac{e^{2x}}{e^x-1}dx=\left|\frac{t=e^x, dt=e^x dx,}{x=\ln t}\right|=$$

$$=-\int\frac{tdt}{t-1}=-\int\frac{t-1+1}{t-1}dt=-t-\ln|t-1|+C_1=-e^x-\ln|e^x-1|+C_1.$$

Shunday qilib, (11.38) formulaga asosan, dastlabki tenglamaning umumiy yechimi quyidagicha bo'ladi.

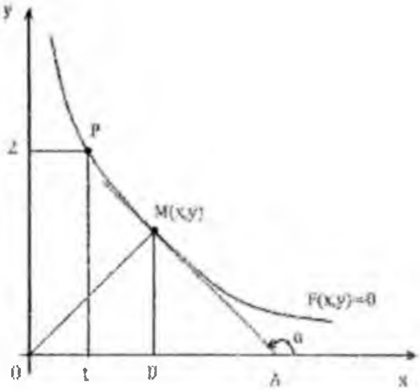
$$y=(-e^{x}-\ln|e^x-1|+C_1)e^{x}+(\ln\left|\frac{e^x-1}{e^x}\right|+C_2,)e^x=$$

$$=C_1e^{x}+C_2e^x+e^x\ln\left|\frac{e^x-1}{e^x}\right|-e^{x}\ln|e^x-1|-1. \blacktriangleleft$$

4. *R(1,2)* nuqtadan o'tuvchi va quyidagi xossaga ega bo'lgan egri chiziq tenglamasini tuzing.

Egri chiziqning ixtiyoriy nuqtasining radius-vektori, shu nuqtaga urinma va abssissa o'qidan tashkil topgan uchburchakning yuzi 2 ga teng.

11.4-rasmdan ko'rinib turibdiki, $|OA|=|OB|+|AB|=x+|AB|$. BMA uchburchakdan quyidagini hosil qilamiz.



11.4. –rasm

$$\frac{|BA|}{y} = ctg(\pi - \alpha) = -ctg\ \alpha,\ |BA| = -yctg\ \alpha,$$

$$|BA| = -\frac{y}{tg\ \alpha} = -\frac{y}{\frac{dy}{dx}} = -y\frac{dx}{dy},\ |OA| = |OB| + |BA| = x - y\frac{dx}{dy},$$

$C_{OMA}=0,5|OA|\ |MB| = 2.$

Oxirgi tenglikka $|OA|$va $|MV|$ lar uchun hosil qilingan ifodalarni qo'yib, quyidagi differensial tenglamani hosil qilamiz.

$$\frac{1}{2}(x - y\frac{dx}{dy})y = 2,\ xy - y^2\frac{dx}{dy} = 4,$$

$$y^2\frac{dx}{dy} = xy - 4,\ \frac{dx}{dy} - \frac{x}{y} = -\frac{4}{y^2}.$$

ya'ni, $x=x(y)$ funksiyaga nisbatan chiziqli, 1-tartibli bo'lgan tenglamani hosil qildik. Bu tenglamani $x=y\ \vartheta$ almashtirish yordamida yechamiz va quyidagiga ega bo'lamiz.

$$y'\vartheta + u\vartheta' - \frac{u\vartheta}{y} = -\frac{4}{y^2},\ u'\vartheta + u(\frac{du}{dy} - \frac{\vartheta}{y}) = -\frac{4}{y^2},$$

$$\frac{d\vartheta}{dy}-\frac{\vartheta}{y}=0, \quad \frac{d\vartheta}{\vartheta}=\frac{dy}{y}, \quad \int\frac{d\vartheta}{\vartheta}=\int\frac{dy}{y},$$

$$\ln|\vartheta|=\ln|y|, \quad \vartheta=y, \quad \frac{du}{dy}y=-\frac{4}{y^{2}},$$

$$du=-\frac{4dy}{y^{3}}, \quad u=\frac{2}{y^{2}}+C, \quad x=(\frac{2}{y^{2}}+C)y=Cy+\frac{2}{y}.$$

Izlanayotgan egri chiziq $R(1,2)$ nuqtadan o'tadi. Shuning uchun $1=2C+1$, $C=0$. Natijada, uning tenglamasi $x=2/y$ yoki $xy=2$ bo'ladi, ya'ni berilgan egri chiziq giperboladir.

11.9 11-bobga qo'shimcha masalalar

1. Lokomativning tezlanishi tortishish kuchi F ga to'g'ri proporsional va poezd massasi m ga teskari proporsional. Lokomativning boshlang'ich tezligi ϑ_0, tortishish kuchi $F=b-k\cdot\vartheta$, bu yerda ϑ-tezlik b, k-o'zgarmas sonlar. Agar boshlang'ich vaqtda $t=0$ da $F=F_0=b-k\vartheta_0$ bo'lsa, lokomativning t vaqt ichidagi tortishish kuchini aniqlang.

(*Javob*: $F=F_0e^{-kt/m}$,)

2. Uzunligi l va ko'ndalang bo'lgan kesim yuzi S bo'lgan po'latsim qiymati R gacha o'suvchi o'zgarmas kuch bilan cho'zilmoqda. Agar simning cho'zilishi quyidagi formula bilan aniqlansa: $\Delta l=k\cdot\frac{p}{F}l_0$, bu yerda k-cho'zilish koeffitsiyenti; l_0-simning boshlang'ich uzunligi bo'lsa, cho'zilish kuchining bajargan ishini aniqlang.

(*Javob*: $A=\frac{kl_0}{2F}P^2$)

3. Motorli qayiq ko'lda $\vartheta_0=20$ km/s tezlik bilan harakatlanmoqda. Motori o'chirilgandan so'ng 40 sekund o'tgach

qayiqning tezligi ϑ_0=8m/s gacha kamayadi. Motor o‘chirilgandan so‘ng 2 minutdan keyingi qayiqning tezligini aniqlang? (suvning qarshilik kuchi qayiq harakatining tezligiga proporsional)

(*Javob: 1,28 km/soat*)

4. Suv bilan to‘ldirilgan balandligi N va asosining yuzasi C_1 ga teng silindrik idishning asosida yuzasi C_2 ga teng teshik bor. Suvning teshikdan to‘la oqib tushib ketish vaqtini aniqlang. (Oqib tushish tezligi quyidagi formula bilan aniqlanadi: $\vartheta = \sqrt{2gh}$ bu yerda h-o‘sha vaqtdagi suv qatlami balandligi, g-erkin tushish tezlanishi)

(*Javob:* $T=\frac{C_1}{C_2}\sqrt{\frac{2H}{g}}$.)

5. Zanjirli ko‘prik arqonining uchlaridan biri $R=5m$ balandlikda, uning o‘rtasi esa, ko‘prikdan o‘tish qismidan $N=4m$ balandlikda joylashgan. Ko‘prikning uzunligi $2l=20m$. Arqonning egilish egri chizig‘ini toping.

(*Javob:* $y-4=x^2/100$.)

6. Tog‘ jinsining bo‘lagida *100mg* uran va *14 mg* uranli qo‘rg‘oshin bor. Agar uranning yarim tarqalish davri $4{\cdot}5{\cdot}10^9$ yildan iborat va *238* g uranning to‘liq tarqalishida *206* g uranli qo‘rg‘oshin hosil bo‘lsa, tog‘ jinsining yoshini aniqlang. (tog‘ jinsining paydo bo‘lishi tarkibida qo‘rg‘oshin bo‘lmagan va tezda tarqaladigan oraliq birikmalarda uran va qo‘rg‘oshin tarqalishi e’tiborga olinmagan deb hisoblansin)

(*Javob:* $975{\cdot}10^6$ *yil.*)

7. Raketaning massasi to'liq yonilg'I zahirasi bilan *M* ga, yonilg'isiz esa m ga teng, yonilg'I mahsulotining tugash tezligi – s, raketaning boshlang'ich tezligi 0 (nolga) teng. Raketaning og'irlik kuchini va havoning qarshiligini e'tiborga olmagan holda, uning yonilg'i yonib bo'lgandan keyingi tezligini aniqlang.

(*Javob:* $C \cdot \ln(M/m)$.)

8. Jism yer sathidan *18m* balandlikdan *30m/s* tezlik bilan yuqoriga vertikal holatda tashlangan. Balandlikni vaqtning funksiyasi deb qarab, jismning t vaqtdagi balandligini toping. Jism ko'tarilishining eng katta balandligini aniqlang.

(*Javob:* $S=h=-\frac{1}{2}gt^2+30t+18, h_{e.katta}=63,9m$.)

9. Ma'lumki, havoda jismning sovush tezligi jism va havo temperaturalarining ayirmasiga proporsional. 20 minut davomida jismning temperaturasi 100^0C dan 60^0C gacha kamayadi. Havoning temperaturasi 20^0C ga teng. Jism temperaturasining 250^0C gacha kamayish vaqtini aniqlang.

(*Javob: 1 soat 20 min.*)

ILOVA

1. Nazorat ishi. “Aniqmas integrallar” (2 soat)

Aniqmas integarallarni toping.

1.1. $\int 2^x \cdot 3^{2x} dx.$

1.2. $\int \frac{\sin x dx}{\sqrt[3]{7+2\cos x}}.$

1.3. $\int \sqrt{\sin x} \cos^5 x dx.$

1.4. $\int \frac{x+(\arccos 3x)^2}{\sqrt{1-9x^2}} dx.$

1.5. $\int \frac{\sqrt{1+\ln x}}{x} dx.$

1.6. $\int \frac{3^{arctgx}}{1+x^2} dx.$

1.7. $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}.$

1.8. $\int \frac{\cos x dx}{\sqrt[3]{\sin^2 x}}.$

1.9. $\int \frac{\sin x dx}{\sqrt[3]{3+2\cos x}}.$

1.10. $\int \frac{x-1}{x^3-x} dx.$

1.11. $\int x^3 arctg\, x dx.$

1.12. $\int \frac{3x-1}{4x^2-4x+17} dx.$

1.13. $\int \frac{dx}{2\sin x - 3\cos x}.$

1.14. $\int \frac{x-8}{\sqrt{3+2x-x^2}} dx.$

1.15. $\int x^2 \cdot 2^x dx.$

1.16. $\int \frac{dx}{x\sqrt{1-\ln^2 x}}.$

1.17. $\int \frac{dx}{x^2\sqrt{x^2+1}}.$

1.18. $\int \frac{2^{\ln x}}{x\sqrt{1+4^{\ln x}}}.$

1.19. $\int \sin 5x \cdot \cos x dx.$

1.20. $\int \frac{x+2}{x^3-2x^2+2x} dx.$

1.21. $\int \frac{\sin 5x}{1+\cos^2 5x} dx.$

1.22. $\int \frac{3-2ctg^2 x}{\cos^2 x} dx.$

1.23. $\int \frac{3x^2+x^2+5x+1}{x^3+x} dx.$

1.24. $\int \frac{dx}{\sqrt{x(x-7)}}.$

1.25. $\int \frac{dx}{(1+x^2)(arctg\, x-3)}.$

1.26. $\int \frac{dx}{\cos^2 x(1+tgx)^3} dx.$

1.27. $\int \frac{e^{2x}dx}{e^{4x}-5}.$

1.28. $\int (x^2+3)e^{-2x} dx.$

1.29. $\int (x+2)\ln x dx.$

1.30. $\int \frac{81^x-3^x}{9^x} dx$

2.

2.1. $\int \arcsin x dx$

2.2. $\int x\ln(x^2+1)dx$

2.3. $\int \frac{8x-11}{\sqrt{5+2x-x^2}}dx$

2.4. $\int \frac{5x-11}{3x^2+2+1}dx$

2.5. $\int \frac{dx}{\sin^2 x\cos^2 x}$

2.6. $\int x^2 e^{-x/2}dx.$

2.7. $\int x^2\cos 3x dx$

2.8. $\int \frac{\arcsin x}{\sqrt{1+x}}dx$

2.9. $\int \frac{inx}{x^2}dx.$

2.10. $\int \frac{x+2}{x^2+2x+5}dx.$

2.11. $\int \frac{\sqrt{(4-x^2)^3}}{x^4}dx.$

2.12. $\int \frac{3x-4}{\sqrt{6x-x^2-8}}dx.$

2.13. $\int \frac{\sin 2x dx}{3\sin^2 x+4}.$

2.14. $\frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$

2.15. $\int \frac{x^3+2}{x^4+3x^2}dx.$

2.16. $\int \frac{3x-1}{\sqrt{x^2+2x+2}}dx.$

2.17. $\int \sqrt{x}\ln x dx.$

2.18. $\int \frac{\sqrt{x^3}-\sqrt[3]{x}}{6\sqrt[4]{x}}dx$

2.19. $\int (1-x)\sin x dx.$

2.20. $\int \frac{e^{2x}}{e^x-1}dx$

2.21. $\int arctg\sqrt{x}dx.$

2.22. $\int \frac{x+2}{\sqrt{4x^2-4x+3}}dx.$

2.23. $\int \frac{5x-3}{\sqrt{2x^2+8x+1}}dx.$

2.24. $\int (x^2+3)\cos x dx$

2.25. $\int \frac{3x+2}{x^2+4x+12}dx.$

2.26. $\int \frac{x+4}{\sqrt{7+6x-x^2}}dx.$

2.27. $\int \frac{dx}{x^4-x^2}.$

2.28. $\int \frac{dx}{\sqrt{3-x-x^2}}dx.$

2.29. $\int \frac{x^2+x+5}{x(x+3)(x-2)}dx.$

2.30. $\int \frac{2x+1}{\sqrt{1+6x-3x^2}}dx.$

3.

3.1. $\int \frac{x+1}{5x^2+3x+1}dx.$

3.2. $\int \frac{3x-13}{x^2-4x+8}dx.$

3.3. $\int \frac{x^2dx}{1-x^4}dx.$

3.4. $\int \frac{xdx}{\cos^2 x}.$

3.5. $\int x^3e^{x^2}dx.$

3.6. $\int \frac{x+5}{\sqrt{3x^2+6x+1}}dx.$

3.7. $\int \frac{x+1}{4x^2-12x+3}dx.$

3.8. $\int \frac{x+2}{\sqrt{3+2x-x^2}}dx.$

3.9. $\int \frac{dx}{4x^2-x}.$

3.10. $\int \frac{x^2dx}{(x+2)^2(x+4)^2}.$

3.11. $\int \frac{1-2x}{\sqrt{1-4x^2}}dx.$

3.12. $\int \frac{xdx}{\sqrt{1+x^4}}.$

3.13. $\int \frac{3x-7}{x^3+x^2+4x+4}dx$

3.14. $\int \frac{2x^2-x-1}{x^3-x^2-6x}dx.$

3.15. $\int e^{2x}\sin(e^{-2x})dx.$

3.16. $\int \frac{\sin 4x}{1+\cos 4x}dx.$

3.17. $\int \frac{x+5}{2x^2+2x+3}dx.$

3.18. $\int \frac{dx}{\sqrt{4x-3x^2-1}}dx.$

3.19. $\int \frac{2x-10}{\sqrt{1+x-x^2}}dx.$

3.20. $\int \frac{\sqrt[3]{arctg^2 x}}{\sqrt{1+x^2}}dx$

3.21. $\int \frac{x-4}{\sqrt{x^2-2x+3}}dx.$

3.22. $\int \frac{xdx}{\sin^2 x}.$

3.23. $\int \frac{\sqrt[6]{x}}{1+\sqrt[3]{x}}dx.$

3.24. $\int \frac{\sqrt{2-x^2}+\sqrt{2+x^2}}{\sqrt{4-x^4}}dx$

3.25. $\int \frac{\ln x}{x^3}dx.$

3.26. $\int e^{2x}\sin 2xdx.$

3.27. $\int \frac{5x+3}{\sqrt{4x+5-x^2}}dx.$

3.28. $\int \frac{dx}{x\ln^5 x}.$

3.29. $\int \frac{2x+3}{x^2-5x+7}dx.$

3.30. $\int \frac{xdx}{\sqrt{2x+1}+1}dx.$

4.

4.1. $\int \frac{x+3}{(x+2)(x^2+x+1)}dx.$

4.2. $\int \frac{(\sqrt{x}-1)(\sqrt[6]{x}+1)}{\sqrt[3]{x^2}}dx.$

4.3. $\int \frac{x^2dx}{9-x^4}.$

4.4. $\int \frac{dx}{x^3+4x-x^2-4}.$

4.5. $\int \frac{3x-1}{x^2-6x+10}dx.$

4.6. $\int \frac{dx}{\sqrt{1+e^x}}.$

4.7. $\int \frac{2x^2+1}{x^3+x^2+2x+2}dx.$

4.8. $\int \frac{2x^2+1}{x^3+2x^2+2x}dx.$

4.9. $\int \frac{dx}{3\sin x+4\cos x}.$

4.10. $\int x^2\cos 6x dx.$

4.11. $\int \frac{2x\ \ -1}{5x^2-x+2}dx.$

4.12. $\int \frac{x\ \ -1}{x^3+8}dx.$

4.13. $\int \frac{3x\ \ -7}{x^3+x^2+4x+4}dx.$

4.14. $\int \frac{x-arctg 2x}{1+4x^2}dx.$

4.15. $\int \frac{\sin^2 x}{\cos^2 x}dx.$

4.16. $\int \frac{2xdx}{(x+1)(x^2+x+2)}.$

4.17. $\int \frac{dx}{5-4\sin x}.$

4.18. $\int x^2\cdot 5^{x/2}dx.$

4.19. $\int \frac{dx}{\sqrt[3]{3x+1}-1}.$

4.20. $\int \sqrt[3]{x^2}\ln x dx.$

4.21. $\int \frac{dx}{x^4-16}.$

4.22. $\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}}.$

4.23. $\int \frac{dx}{4\sin x+3\cos x+5}.$

4.24. $\int \frac{xdx}{2x^2+2x+5}.$

4.25. $\int \frac{2x\ \ -x}{(7-x)^3}dx.$

4.26. $\int \frac{x^2-2x+1}{x^3+2x^2+x}dx.$

4.27. $\int \frac{dx}{x\sqrt{2x-9}}.$

4.28. $\int \frac{dx}{x^4-6x^3+9x^2}.$

4.29. $\int \frac{dx}{\sqrt{x+1}+1}.$

4.30. $\int \sin(\ln x)dx.$

5.

5.1. $\int \frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}}.$

5.2. $\int \frac{dx}{\sin^3 x}.$

5.3. $\int \frac{\cos x}{1+\sin x}dx.$

5.4. $\int \cos 3x \cos x dx.$

5.5. $\int \frac{x+2}{x^3-2x^2+2x}dx.$

5.6. $\int \frac{xdx}{x^3-1}.$

5.7. $\int \frac{\sqrt{1+x^2}}{x}dx.$

5.8. $\int \frac{x^2dx}{\sqrt{(2-x^2)^3}}.$

5.9. $\int \frac{xdx}{\sqrt[3]{1+3}}.$

5.10. $\int x^5 \sqrt[3]{(1+x^3)^2}dx$

5.11. $\int \ln^2 xdx.$

5.12. $\int \cos 2x \cos^2 xdx.$

5.13. $\int \frac{e^x+1}{e^x-1}dx.$

5.14. $\int x^2e^{2x}dx.$

5.15. $\int x^2 \sin xdx.$

5.16. $\int \frac{dx}{\sin^2 x \cos^2 x}.$

5.17. $\int \frac{dx}{\sqrt{(1+x^2)^3}}.$

5.18. $\int \sin x \sin 3xdx.$

5.19. $\int (1-\sin 2)^2 dx.$

5.20. $\int \frac{dx}{\sqrt{x-x^2+1}}.$

5.21. $\int \frac{x^2dx}{\sqrt{4-x^2}}.$

5.22. $\int ctg^4 xdx.$

5.23. $\int \frac{xdx}{2x^4+5}.$

5.24. $\int \frac{x^4}{x^4-16}dx.$

5.25. $\int \frac{\sqrt{2x-3}}{x}dx.$

5.26. $\int \frac{dx}{5-3\cos x}.$

5.27. $\int \ln(x^2+1)dx.$

5.28. $\int \frac{x-1}{\sqrt{2x-1}}dx.$

5.29. $\int \frac{dx}{\sqrt{(1-x^2)\arcsin x}}$

5.30. $\int \frac{x^2-3}{x^4-5x^2+4}dx.$

6.

6.1. $\int \frac{\sin x dx}{\sqrt[3]{7+2\cos x}}$.

6.2. $\int \frac{x^4+2x-2}{x^4-1} dx$.

6.3. $\int (x^2+1)\cdot 3^x dx$.

6.4. $\int \sqrt{4-x^2} dx$

6.5. $\int \frac{\sqrt[4]{x-1}}{\sqrt[6]{x^5}+\sqrt[4]{x^3}} dx$.

6.6. $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx$.

6.7. $\int \cos 2x \sin^2 x dx$.

6.8. $\int \sin^2 x \cos^2 x dx$.

6.9. $\int \frac{2x-1}{\sqrt{x^2-4x+1}} dx$.

6.10. $\int \frac{dx}{\sin^2 3x \cos^2 3x}$.

6.11. $\int \sin\ 2x \cos 5x dx$.

6.12. $\int \frac{x+1}{\sqrt[3]{3x+1}} dx$.

6.13. $\int x\cdot 5^x dx$.

6.14. $\int \frac{x+1}{x^2+x+1} dx$.

6.15. $\int \frac{\sqrt{1+x^2}}{x} dx$.

6.16. $\int \frac{dx}{\sqrt[3]{1+x}-\sqrt{1+x}}$.

6.17. $\int \frac{5x^3-8}{x^3-4x} dx$.

6.18. $\int \frac{dx}{x^4+2x^3+2x^2}$.

6.19. $\int \frac{2x^2-5x+1}{x^3-2x^2+x} dx$.

6.20. $\int \sin 5x \cos 3x dx$.

6.21. $\int \frac{\sin^3 x+1}{\cos^2 x} dx$

6.22. $\int \frac{x^2+1}{(x-1)^3(x+3)} dx$

6.23. $\int (x+1)e^x dx$.

6.24. $\int \sin^2 x \cos^4 x dx$.

6.25. $\int (1+\sin^4 x) dx$.

6.26. $\int \frac{\sqrt[3]{x}}{x(\sqrt{x}+\sqrt[3]{3x})} dx$.

6.27. $\int \frac{dx}{3+5\sin x+3\cos x}$.

6.28. $\int \frac{\sin^3 x}{1+\cos x} dx$.

6.29. $\int \frac{dx}{tg^3 3x}$.

6.30. $\int \frac{dx}{2\sin x-\cos x}$.

2. Nazorat ishi.
"Differensial tenglamalar" (2 soat).

Berilgan differensial tenglamalarni yeching.

1.

1.1. $y'-y/x-1/(\sin(y/x))=0.$

1.2. $xdy-ydx=\sqrt{x^2+y^2}dx.$

1.3. $x^2u'=xy+y^2.$

1.4. $xdy=(x^4-2y)dx.$

1.5. $y'+3y/x-2/x^3=0.$

1.6. $x^2dy+y^2dx=3(x^2-y^2)dx.$

1.7. $y'=4+y/x+(y/x)^2.$

1.8. $(x^2+y^2)dx-xydy=0.$

1.9. $xy'-y=x^2\cos x.$

1.10. $y'-\frac{3}{x}y=x.$

1.11. $y'+2xy=2xy^3.$

1.12. $x^3y'+x^2y+x+1=0.$

1.13. $y'+2y/x=\frac{e^{-x^2}}{x}.$

1.14. $y'+2xy=xe^{-x^2}.$

1.15. $xy+y^2=(2x^2+xy)y'.$

1.16. $xy'+y=\sin x$

1.17. $xy'+y=\sin x.$

1.18. $xy'-y=x\,tg\,(y/x).$

1.19. $y'-y/x=e^{y/x}$

1.20. $y'+y\,tg\,x=1/\cos x.$

1.21. $y'\cos x-y\sin x=\sin x.$

1.22. $xy'=y+x\,e^{y/x}.$

1.23. $y'+xy=-x^3.$

1.24. $x\ln(x/y)dy-ydx=0.$

1.25. $(xy\,e^{x/y}+y^2)dx=x^2\,e^{x/y}\,dy.$

1.26. $x^2y'=2xy+3.$

1.27. $dy=(y+x^2)dx.$

1.28. $(x^2-1)y'-xy=x^3-x.$

1.29. $y'-2xy=xe^{-x^2}$

1.30. $xy'=3y-x^4y^2.$

1.31. $y'-y=e^x.$

2.

2.1. $y'\cos^2 x+y=tg\,x.$

2.2. $y'+y\cos x=\cos x.$

2.3. $\ln\cos y dx+x\,tg\,y dy=0.$

2.4. $y'=tg\,x\cdot tg\,y.$

2.5. $y'\cos x\ln y=y.$

2.6. $e^{1+x^2}tg\,y dx=\frac{e^{2x}}{x-1}dy.$

2.7. $y'=2^{x-y}.$

2.8. $(1+e^{2x})y^2dy=e^x dx.$

2.16. $y^2dx=(xy-x^2)dy.$

2.9. $3e^x tg\,y dx=(1+e^x)\sec^2 y dy.$

2.10. $yy'/x+e^y=0.$

2.11. $y'+y=e^x\sin x.$

2.12. $(x+y)dx+xdy=0.$

2.13. $1+(1+y')e^y=0.$

2.14. $x\cos\frac{y}{x}(ydx+xdy)=x^2\sin\frac{y}{x}dx.$

2.15. $y'+y/(x+1)+x^2=0.$

2.17. $y' + \frac{1-2x}{x^2} y = 1.$

2.18. $y' + \frac{4xy}{x^2+1} = \frac{1}{x^2+1}$

2.19. $xy' = y - xy.$

2.20. $(x^2 - 2y^2)dx + 2xydy = 0.$

2.21. $x + y = xy'.$

2.22. $y' + \frac{3}{x} y = \frac{2}{x^3}.$

2.23. $y'x + y = -xy^2.$

2.24. $xy' ln \frac{y}{x} = x + y \ln \frac{y}{x}.$

2.25. $y^2 + x^2 y' = xyy'.$

2.26. $y = y' ln\, y.$

2.27. $(x^2 - x^2 y)y' + y^2 + xy^2 = 0.$

2.28. $3 e^x tg\, ydx = (1 - e^x)\sec^2 ydy.$

2.29. $(1+y^2)dx - \sqrt{x}dy = 0$

2.30. $x + xy + y'(y + xy) = 0.$

3.

3.1. $y'' cos^2 x = 1.$

3.2. $y''\, tg\, y = 2(y')^2.$

3.3. $y'' x\, ln\, x = y'.$

3.4. $(1+x^2)y'' = 3.$

3.5. $y'' + 2y(y')^3 = 0.$

3.6. $y'' + y'\, tg\, x = sin\, 2x.$

3.7. $y'' = 4\, cos\, 2x.$

3.8. $yy'' + y'^2 = 0.$

3.9. $x^3 y'' + x^2 y' = 1.$

3.10. $x^3 y''' = 6.$

3.11. $y''' sin^4 x = sin\, 2x.$

3.12. $yy'' + 1 = y'^2.$

3.13. $x^2 y''' = y''^2.$

3.14. $y'^2 + 2yy'' = 0.$

3.15. $y'' = 2yy'.$

3.16. $2xy' y'' = y'^2 - 1.$

3.17. $2yy'' = 1 + y'^2.$

3.18. $y''^2 = y'^2 + 1.$

3.19. $xy'' - y' = x^2 e^x.$

3.20. $x^2 y'' + y'^2 = 0.$

3.21. $x(y''+1) + y' = 0.$

3.22. $xy'' = y' + x^2.$

3.23. $y'' + \frac{1}{x} y' = 0.$

3.24. $x^2 y'' = 4.$

3.25. $y'' = \sqrt{1 - y'^2}.$

3.26. $y^3 y'' - 3 = 0.$

3.27. $xy'' + 2y' = 0.$

3.28. $1 + y'^2 + yy'' = 0.$

3.29. $yy'' = y'^2.$

3.30. $y'' = 2 - y.$

4

4.1. $y''-5y'+6y=x, \quad y(0)=0, y'(0)=1.$

4.2. $4y''-8y'+5y=5\cos x, \quad y(0)=0, y'(0)=-1/13.$

4.3. $y''+6y'+13y=26x-1, y(0)=0, y'(0)=1.$

4.4. $2y''-y'=1+x, y(0)=0, y'(0)=1.$

4.5. $y''-4y=2-x, \quad y(0)=11/2, y'(0)=1/4.$

4.6. $y''-y=\cos 2x, y(0)=-1/5, y'(0)=1.$

4.7. $y''-2y'+5y=5x^2-4x+2, y(0)=0, y'(0)=2.$

4.8. $y''+3y'-10y=xe^{-2x}, y(0)=0, y'(0)=0.$

4.9. $y''-2y'=e^x(x^2+x-3), y(0)=2, y'(0)=2.$

4.10. $y''-4y'+4y=\sin x,, y(0)=0, y'(0)=0.$

4.11. $y''-3y'+2y=-e^{-2x}, y(0)=1, y'(0)=0.$

4.12. $y''+y=-\cos 3x, y(\pi/2)=4, y'(\pi/2)=1.$

4.13. $y''-y=e^{2x}, y(0)=1, y'(0)=2.$

4.14. $y''-4y=3e^{-x}, y(0)=0, y'(0)=0.$

4.15. $y''+4y=\sin x, y(0)=0, y'(0)=0.$

4.16. $y''-2y'+2y=2x, y(0)=0, y'(0)=0.$

4.17. $2y''+y'-y=2e^x, y(0)=0, y'(0)=1.$

4.18. $y''-4y'+3y=2e^5, y(0)=3, y'(0)=9.$

4.19. $y''+4y=5e^x, y(0)=0, y'(0)=1.$

4.20. $y''+6y'+8y=3x^2+2x+1, y(0)=17/64, y'(0)=0.$

4.21. $y''+y=xe^x, y(0)=0{,}5, y'(0)=1.$

4.22. $y''-y=2(1-x), y(0)=0, y'(0)=1.$

4.23. $y''-y=9xe^{2x}, y(0)=0, y'(0)=-5.$

4.24. $y''-6y'+9y=e^{3x}, y(0)=1, y'(0)=0.$

4.25. $y''+4y=xe^{-2x}, y(0)=0, y'(0)=0.$

4.26. $y''-4y+5y=xe^{2x}, y(0)=-1, y'(0)=0.$

4.27. $y''-3y'-4y=17\sin x, y(0)=-4, y'(0)=0.$

4.28. $y''-3y'+2y=e^{3x}(3-4x), y(0)=0, y'(0)=0.$

4.29. $y''+2y'+y=9e^{2x}+x, y(0)=1, y'(0)=2.$

4.30. $y''+y=\sin 2x, y(0)=0, y'(0)=0.$

5

5.1. $y''+4y'+4y=e^{-2x}/x^3$

5.2. $y''+3y'+2y=1/e^x+1$.

5.3. $y''+4y=\dfrac{1}{\cos 2x}$

5.4. $y''+y=\dfrac{1}{\sqrt{\cos 2x}}$.

5.5. $y''+5y'+6y=\dfrac{1}{1+e^{2x}}$.

5.6. $y''+4y=ctg\,2x$.

5.7. $y''-y=sh\,x$.

5.8. $y''-3y'+2y=2^x$

5.9. $y''-4y'+5y=\dfrac{e^{2x}}{\cos x}$.

5.10. $y''+4y=\cos^2 x$.

5.11. $y''-6y'+9y=\dfrac{9x^2+6x+2}{x^3(3x-2)}e^{3x}$

5.12. $y''+2y'+y=3e^{-x}\sqrt{x+1}$.

5.13. $y''+y'=tg\,x$.

5.14. $y''+4y=\dfrac{1}{\cos 2x}$.

5.15. $y''-y=\dfrac{e^{2x}}{e^x-1}$.

5.16. $y''-6y'+9y=36\sqrt{xe^{3x}}$

5.17. $y''+y=\dfrac{1}{\cos 2x}$

5.18. $y''+4y=2\,tg\,x$.

5.19. $y''-y'=\dfrac{1}{1+e^x}$.

5.20. $y''+y=\dfrac{1}{\sin x}$.

5.21. $y''+2y'+y=\dfrac{e^{-x}}{x}$.

5.22. $y''-2y'+y=\dfrac{e^x}{\sqrt{4-x^2}}$

5.23. $y''+y=\dfrac{2+\cos^3 x}{\cos^2 x}$.

5.24. $y''+y=tg^2 x$.

5.25. $y''-3y'+2y=1+\dfrac{1}{1+e^x}$.

5.26. $y''+4y=\dfrac{4}{\sin^2 x}$.

5.27. $y''-2y'+y=\dfrac{e^x}{x}$

5.28. $y''+y=\dfrac{1}{\cos^3 x}$.

5.29. $y''+y=ctg\,x$.

5.30. $y''+4y'+4y=e^{-2x}\ln x$.

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